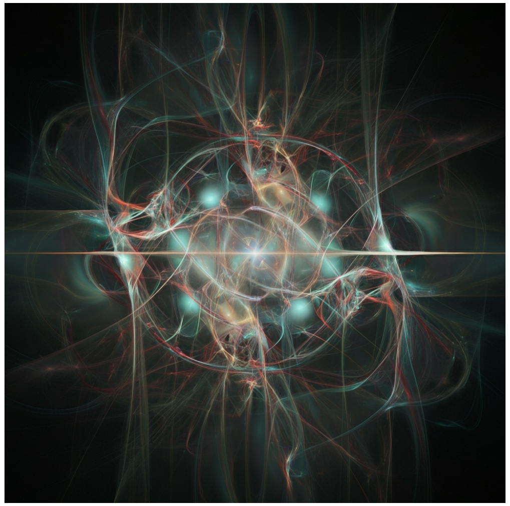
Quantum Information Science at the intersections of Nuclear and AMO Physics, U Mass Boston, January 12-15, 2025

Entanglement and thermalization in high-energy collisions



Google AI

Dmitri Kharzeev

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Office of Science

Center for Nuclear Theory
Stony Brook University

C²QA Co-design Center for Quantum Advantage

Brookhaven National Laboratory

Outline

- Why real-time?
- Why quantum computing?
- Integrable and ergodic quantum systems
- The puzzle of thermalization in high energy collision
- Quantum simulations: apparent thermalization through a maximal entanglement
- Experimental evidence for maximal entanglement

- Why real-time?

An interest in real-time processes is natural for living beings:

"Life is a process. We are a process.
The Universe is a process."

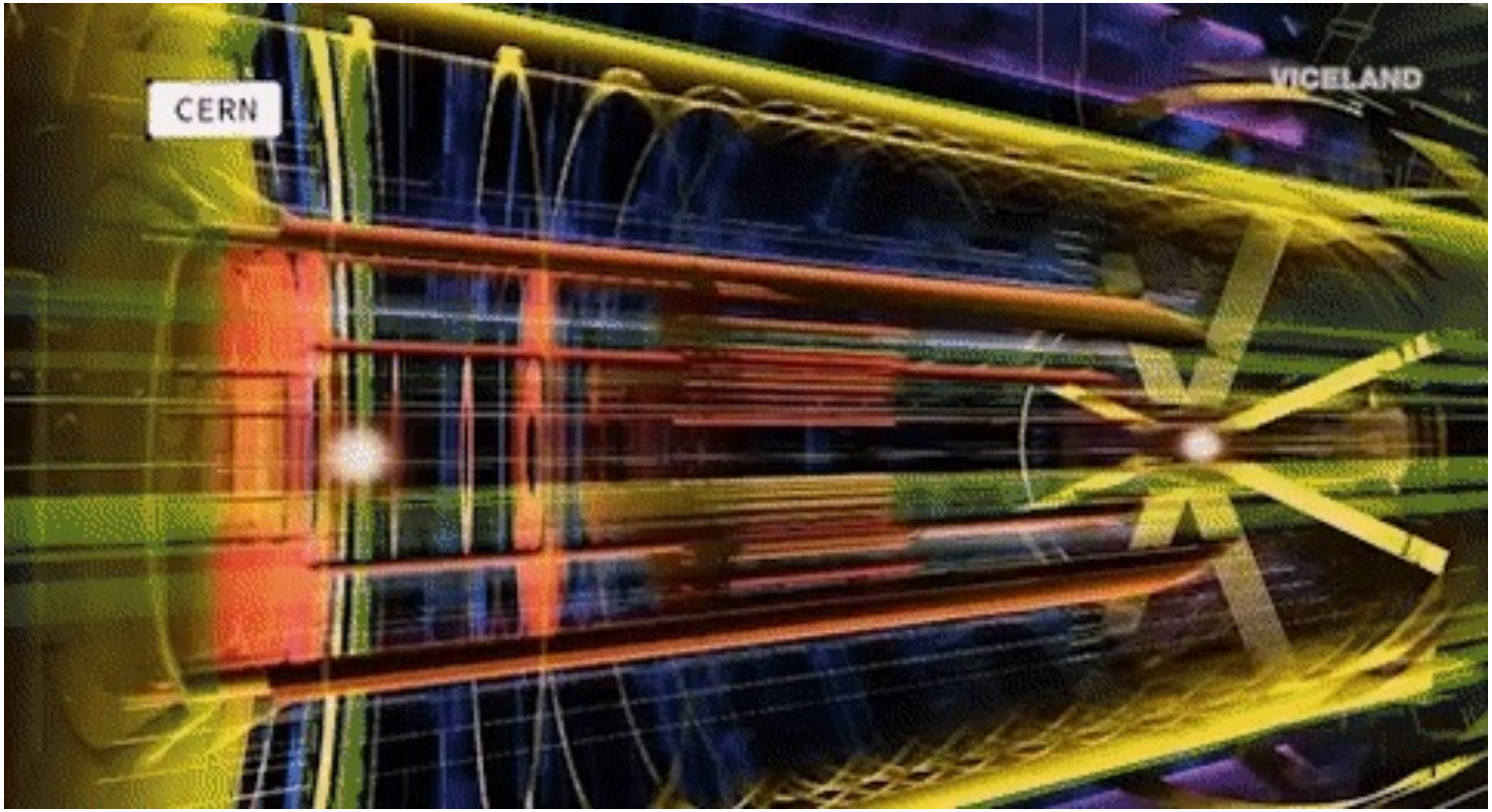
Anne Wilson Schaef

In physics, we are interested in real-time processes: scattering, reactions, decays, …

Beyond physics: self-driving cars, financial markets, healthcare monitoring, decision-making, …

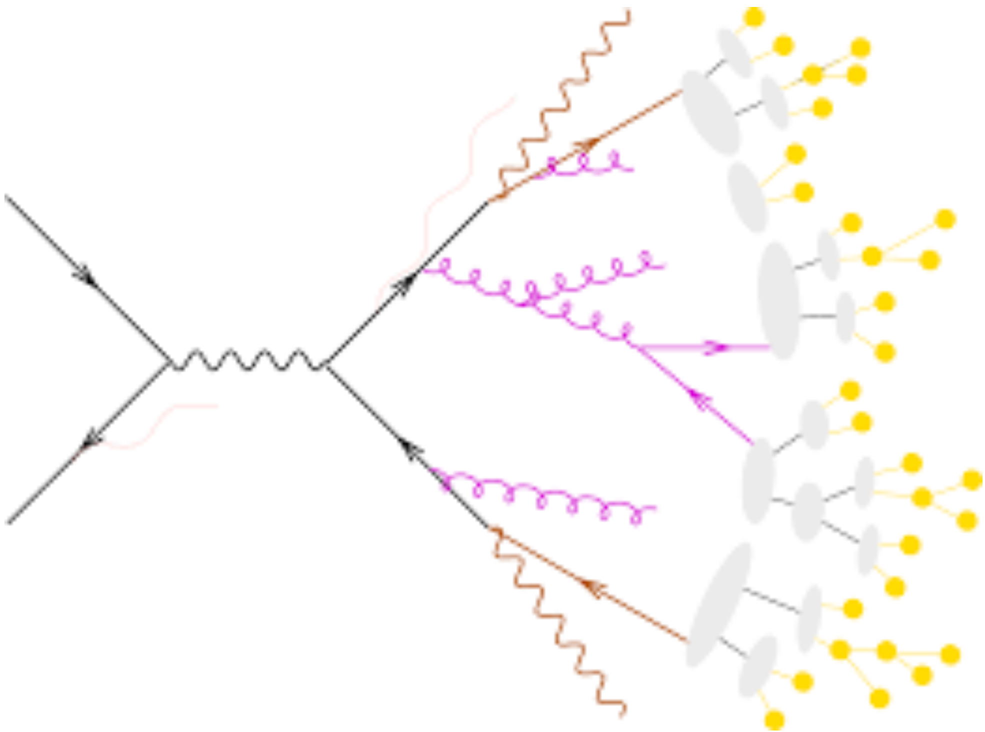
Why real-time?

QCD processes studied at colliders (RHIC, LHC, EIC) evolve in real time



Why quantum computing?

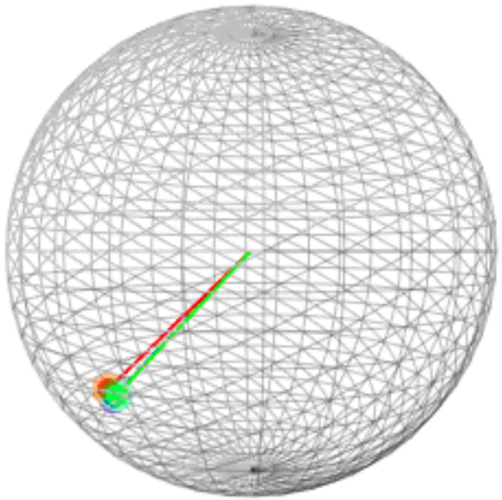
Quantum Field Theory describes quantum systems with a very large (or infinite) number of degrees of freedom



Very large dimension N of state vectors, Hamiltonian is N x N dimensional matrix – huge amount of memory needed!

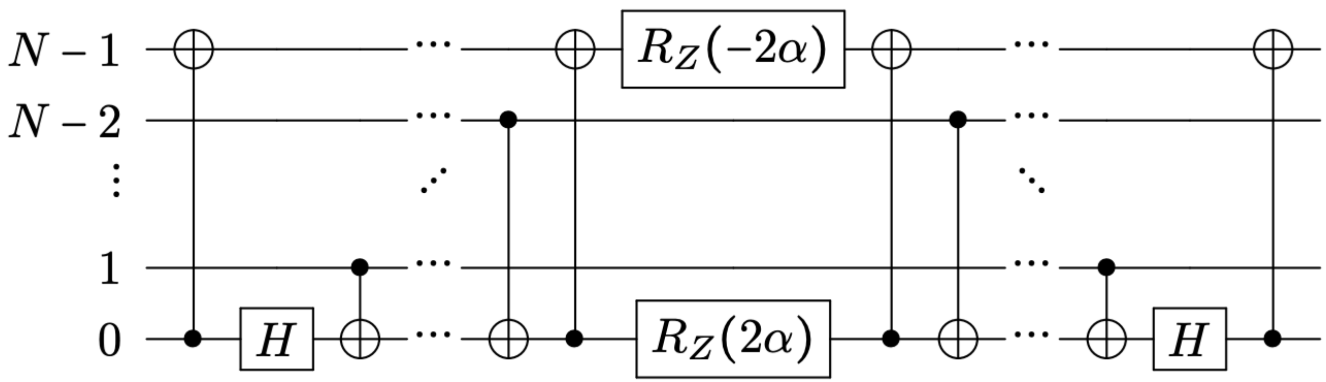
Why quantum computing?

Quantum computing allows manipulation of entire vectors in Hilbert space, offering exponential speed-up:



N bits – 2^N possible states; probability vector

Quantum state vector in 2^N dimensional Hilbert space



Circuit from:
DK, Y. Kikuchi, Phys.Rev.Res.2(2020)

Two types of real-time evolution: quantum chaotic and integrable

Let us begin with integrable systems – what are they?

Louis Armstrong

Integrable systems

Nigel Hitchin

Integrable Systems

Twistors, Loop Groups, and Riemann Surfaces

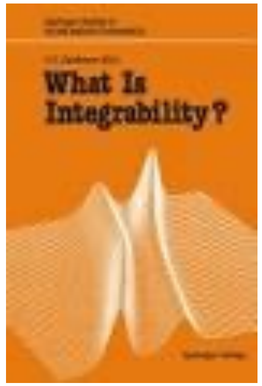
Based on lectures given at a conference on integrable systems organized by N.M.J. Woodhouse and held at the Mathematical Institute, University of Oxford, in September 1997.

N. J. Hitchin
Savilian Professor of Geometry
University of Oxford

G. B. Segal
vndean Professor of Astronomy and Geometry
University of Cambridge

R. S. Ward
Professor of Mathematics
University of Durham

Integrable systems, what are they? It's not easy to answer precisely. The question can occupy a whole book (Zakharov 1991), or be dismissed as Louis Armstrong is reputed to have done once when asked what jazz was—'If you gotta ask, you'll never know!'



If we steer a course between these two extremes, we can say that integrability of a system of differential equations should manifest itself through some generally recognizable features:

- the existence of many conserved quantities;
- the presence of algebraic geometry;
- the ability to give explicit solutions.

How to distinguish integrable and ergodic systems?

Subject of active ongoing research. One direction is based on geometry: so-called “quantum geometric tensor”

Riemannian Structure on Manifolds of Quantum States

J. P. Provost and G. Vallee

Physique Théorique, Université de Nice*,**

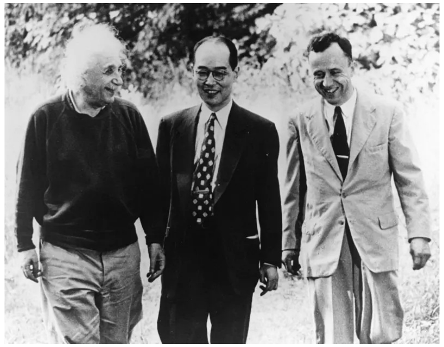
Commun. Math. Phys. 76, 289–301 (1980)

Abstract. A metric tensor is defined from the underlying Hilbert space structure for any submanifold of quantum states. The case where the manifold is generated by the action of a Lie group on a fixed state vector (generalized coherent states manifold hereafter noted G.C.S.M.) is studied in details; the geometrical properties of some wellknown G.C.S.M. are reviewed and an explicit expression for the scalar Riemannian curvature is given in the general case. The physical meaning of such Riemannian structures (which have been recently introduced to describe collective manifolds in nuclear physics) is discussed. It is shown on examples that the distance between nearby states is related to quantum fluctuations; in the particular case of the harmonic oscillator group the condition of zero curvature appears to be identical to that of non dispersion of wave packets.

How to distinguish integrable and ergodic systems?

The roots of Riemannian approach to the Hilbert space geometry:

PHYSICAL REVIEW VOLUME 89, NUMBER 5 MARCH 1, 1953



Nuclear Constitution and the Interpretation of Fission Phenomena

DAVID LAWRENCE HILL*
Vanderbilt University, Nashville, Tennessee, and Los Alamos Scientific Laboratory, Los Alamos, New Mexico

AND

JOHN ARCHIBALD WHEELER†
Princeton University, Princeton, New Jersey, and Los Alamos Scientific Laboratory, Los Alamos, New Mexico

(Received October 14, 1952)

Collective subspaces for large amplitude motion and the generator coordinate method

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Institut für Kernphysik, Kernforschungsanlage Jülich, D-5170 Jülich, West Germany and Institut für Kernphysik, Universität Mainz, D-6500 Mainz, West Germany

K. Goeke
Institut für Kernphysik, Kernforschungsanlage Jülich, D-5170 Jülich, West Germany and Physik-Department, Universität Bonn, D-5300 Bonn, West Germany

(Received 9 April 1979)

Main idea: scalar product between two states in Hilbert space defines a "distance" between them ⇨ Riemannian metric tensor ("quantum information metric")

How to distinguish integrable and ergodic systems?

Parameter-dependent Hamiltonian:

$$H(\vec{\lambda})\left|n(\vec{\lambda})\right\rangle = E_n(\vec{\lambda})\left|n(\vec{\lambda})\right\rangle$$

Real part of the QGT:
quantum metric tensor

Distance:

$$ds^2 \equiv 1 - \left|\left\langle n(\vec{\lambda})\middle|n(\vec{\lambda}+d\vec{\lambda})\right\rangle\right|^2$$

Imaginary part of the QGT:
Berry curvature

Quantum geometric tensor (QGT):

$$ds^2 \equiv g_{\alpha\beta}^{(n)} d\lambda_\alpha d\lambda_\beta + \mathcal{O}(|d\vec{\lambda}|^3) =$$

$$= \langle \partial_\alpha n | \partial_\beta n \rangle - \langle \partial_\alpha n | n \rangle \langle n | \partial_\beta n \rangle + \mathcal{O}(|d\vec{\lambda}|^3)$$

How to distinguish integrable and ergodic systems?

A very recent proposal:

Hilbert space geometry and quantum chaos

Rustem Sharipov,[1,*] Anastasiia Tiutiakina,[2,*] Alexander Gorsky,[3] Vladimir Gritsev,[4] and Anatoli Polkovnikov[5]

arXiv: 2411.11968 (Nov 2024)

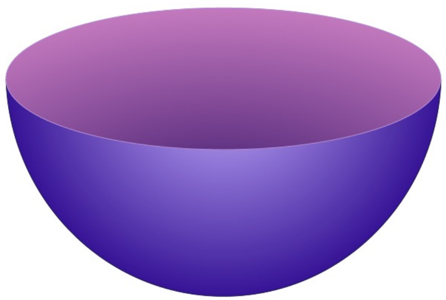


FIG. 1: Isometric manifold $\mathcal{M}_{ch}$

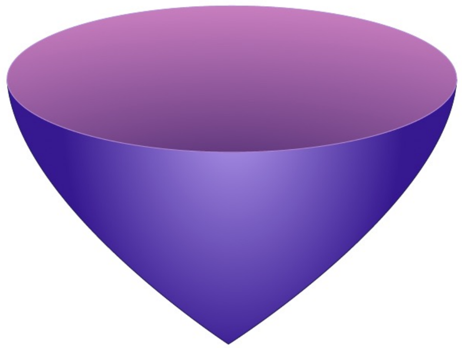


FIG. 2: Isometric manifold $\mathcal{M}_{int}$

Ergodic:
Locally equivalent to a surface of a sphere

Integrable:
Conical singularity

(previously observed near quantum critical points)

A lot of ongoing work on quantum simulations of real-time dynamics – unfortunately, I cannot review them here.

Talks by: D. Lee, F. Ringer, S. Grieninger, K. Ikeda, …

Recent reviews:

PRX QUANTUM **4,** 027001 (2023)

Roadmap

Quantum Simulation for High-Energy Physics

Christian W. Bauer,[1,*] Zohreh Davoudi,[2,†] A. Baha Balantekin,[3] Tanmoy Bhattacharya,[4] Marcela Carena,[5,6,7,8] Wibe A. de Jong,[1] Patrick Draper,[9] Aida El-Khadra,[9] Nate Gemelke,[10] Masanori Hanada,[11] Dmitri Kharzeev,[12,13] Henry Lamm,[5] Ying-Ying Li,[14,15] Junyu Liu,[16,17] Mikhail Lukin,[18] Yannick Meurice,[19] Christopher Monroe,[20,21,22,23] Benjamin Nachman,[1] Guido Pagano,[24] John Preskill,[25] Enrico Rinaldi,[26,27,28] Alessandro Roggero,[29,30] David I. Santiago,[31,32] Martin J. Savage,[33] Irfan Siddiqi,[31,32,34] George Siopsis,[35] David Van Zanten,[5] Nathan Wiebe,[36,37] Yukari Yamauchi,[2] Kübra Yeter-Aydeniz,[38] and Silvia Zorzetti[6]

Quantum Information Science and Technology for Nuclear Physics

Input into U.S. Long-Range Planning, 2023

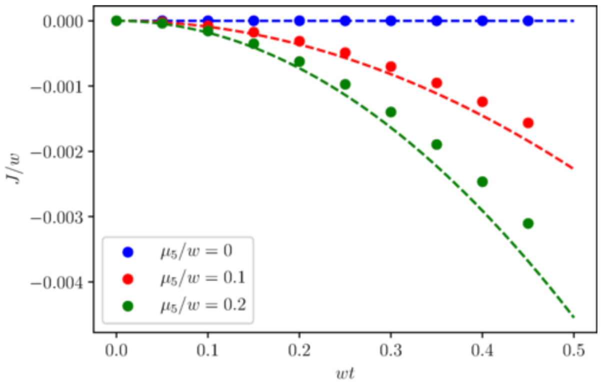
arXiv:2303.00113

Some examples of real-time quantum simulations of integrable models relevant for nuclear physics

Real-time chiral dynamics from a digital quantum simulation

Dmitri E. Kharzeev[1,2,3,*] and Yuta Kikuchi[3,†]

Phys. Rev. Research **2**, 023342 – **Published 16 June, 2020**

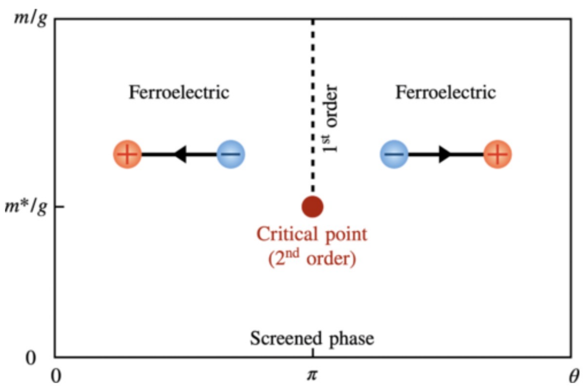


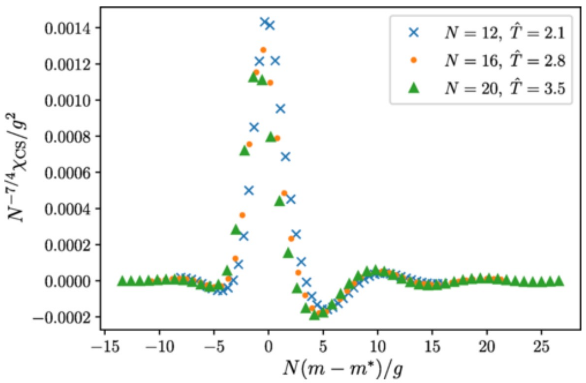
Real-time dynamics of Chiral Magnetic Effect

Real-time dynamics of Chern-Simons fluctuations near a critical point

Kazuki Ikeda[1,*], Dmitri E. Kharzeev[2,3,4,†], and Yuta Kikuchi[3,‡]

Phys. Rev. D **103**, L071502 – **Published 21 April, 2021**





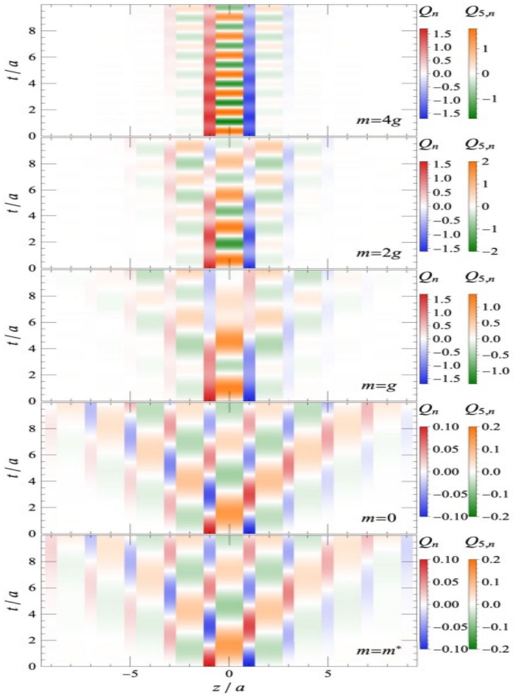
Analogous to topological fluctuations near the critical point of the QCD phase diagram

Some examples of real-time quantum simulations of integrable models relevant for nuclear physics

Nonlinear chiral magnetic waves

Kazuki Ikeda[1,2,*], Dmitri E. Kharzeev[2,3,†], and Shuzhe Shi[4,2,‡]

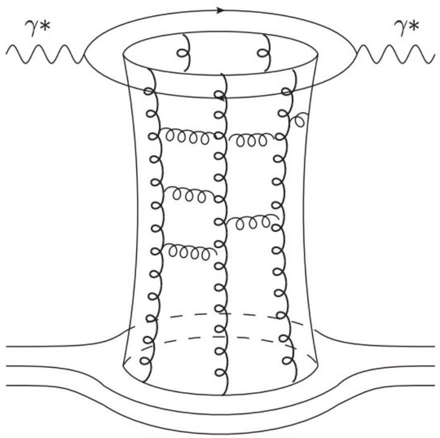
Phys. Rev. D **108**, 074001 – **Published 2 October, 2023**



New nonlinear “thumper” solutions found for the chiral magnetic waves for large fermion masses in Schwinger model(nonlinear scalar theory)

Entanglement entropy production in deep inelastic scattering

Kun Zhang[1], Kun Hao[2,3,*], Dmitri Kharzeev[4,5,†], and Vladimir Korepin[3,6,‡]



Phys. Rev. D **105**, 014002 – **Published 4 January, 2022**

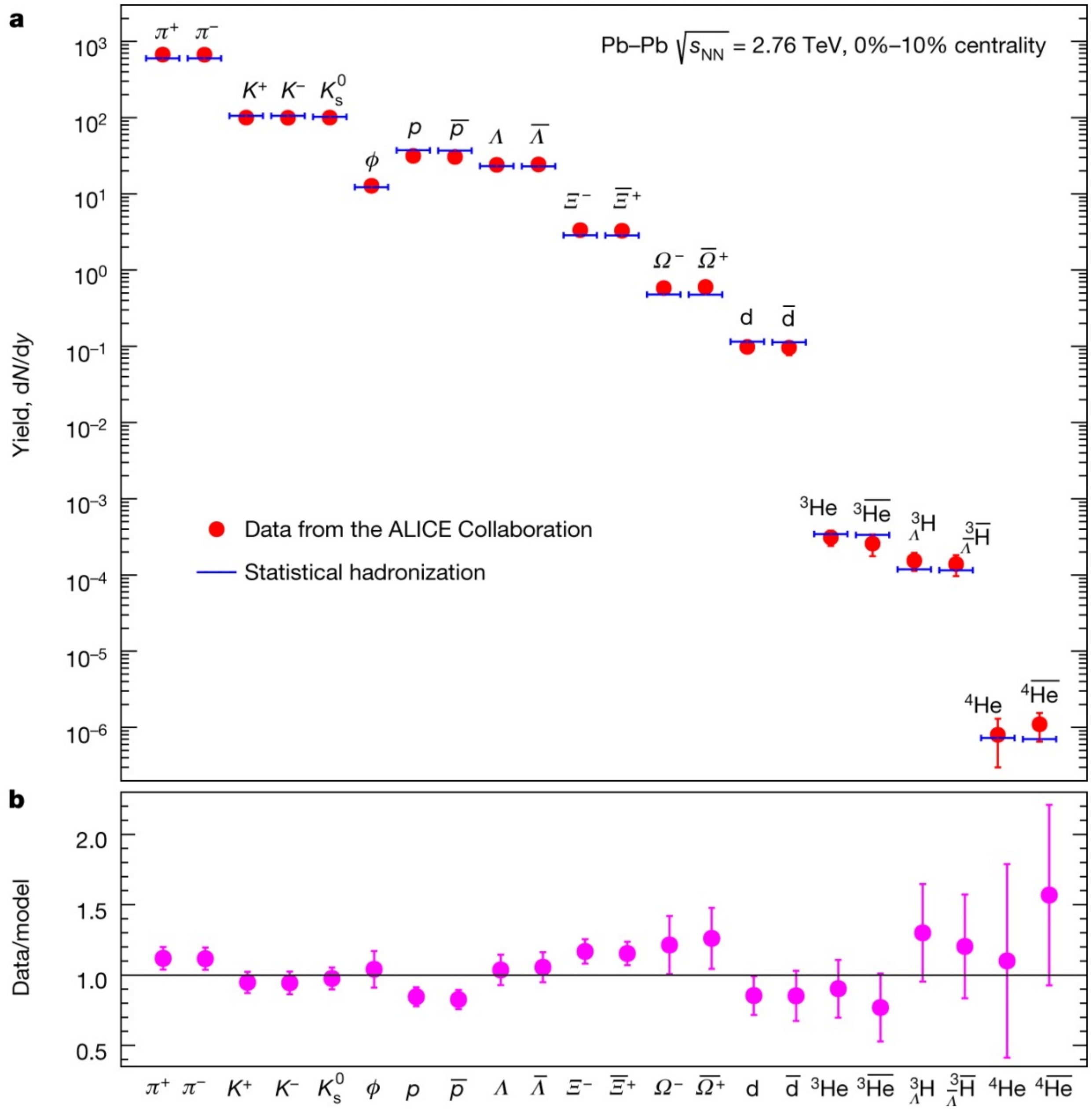
Real-time dynamics of Lipatov’s XXX spin chain (integrable effective theory of high energy QCD)

Real-time dynamics of a non-integrable theory, QCD.

The puzzle of “early thermalization”:

There is an ample evidence from experiments at RHIC, LHC and elsewhere that high energy heavy ion (and even pp and e^+e^- collisions) lead to some kind of fast thermalization:

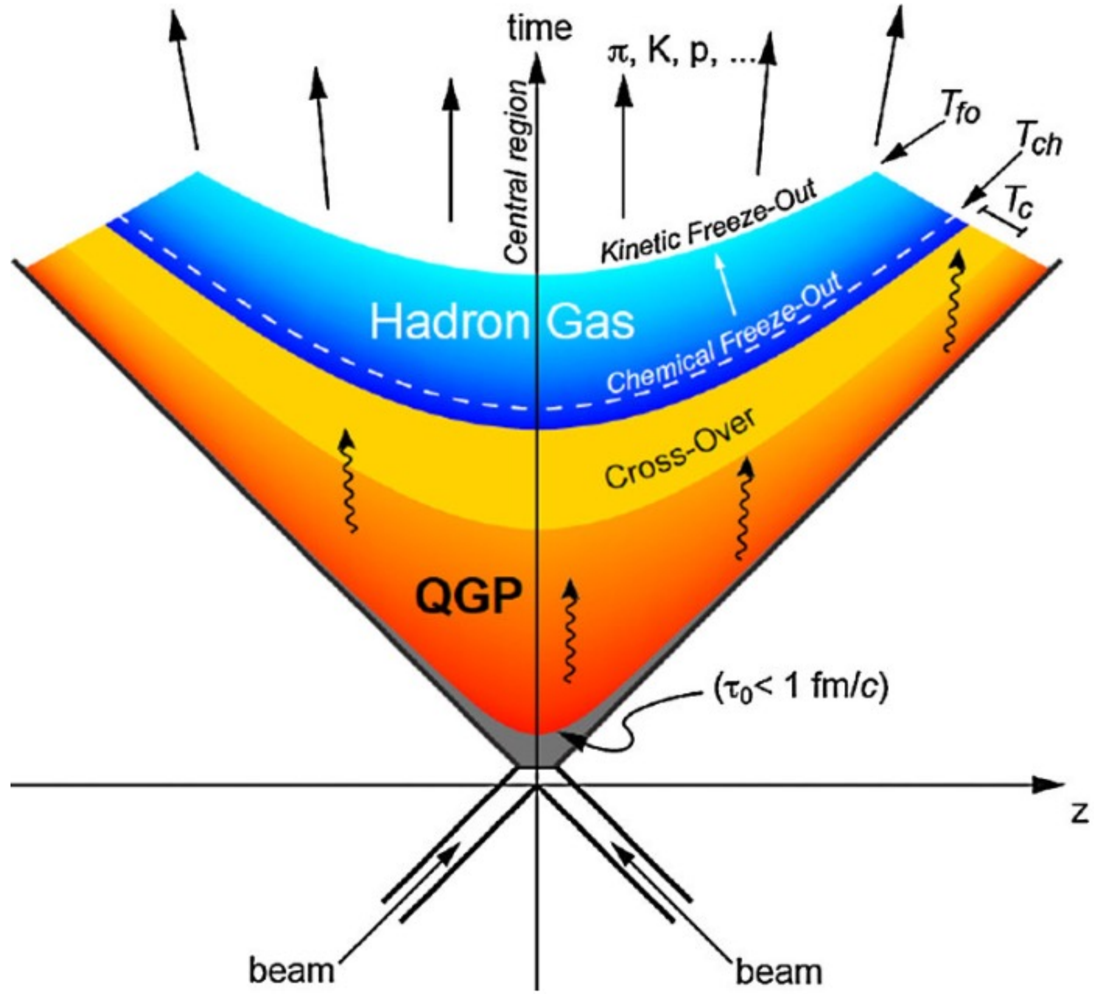
- Hadron abundances look thermal
- Hydrodynamics describes remarkably well the momentum spectra and azimuthal correlations of produced hadrons, assuming that the initial conditions are provided at a very early time $\tau \sim 0.5$ fm



A.Andronic, P.Braun-Munzinger, K.Redlich, J. Stachel, Nature 561 (2018) 321

What is the mechanism of this thermalization?

How can it happen so fast in a rapidly expanding system?



Can a mechanism of thermalization be related to the **entanglement** among the produced quarks and gluons?

To answer this question, we need a **real-time quantum simulation** in a theory that is simple enough to solve numerically and still shares some common properties with QCD.

Schwinger model is similar to QCD in a number of ways: confinement, chiral condensate, anomaly, …

Perform a real-time quantum simulation of e^+e^- annihilation in massive Schwinger model, with the goal of understanding the possible **link between entanglement and thermalization**

Center for Nuclear Theory
Stony Brook University

The team:

C²QA Co-design Center for Quantum Advantage


David Frenklakh (SBU->BNL)


Adrien Florio (SBU->BNL)


Kazuki Ikeda (SBU->UMass)


Shuzhe Shi (SBU->Tsinghua)


Eliana Marroquin (SBU)


Vladimir Korepin (SBU)


Kwangmin Yu (BNL)


Sebastian Grieninger (SBU)


Andrea Palermo (SBU)

Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

Adrien Florio[1,*], David Frenklakh[2,†], Kazuki Ikeda[2,3,‡], Dmitri Kharzeev[1,2,3,§], Vladimir Korepin[4,‖], Shuzhe Shi[5,2,¶], and Kwangmin Yu[6,**]

Phys. Rev. Lett. **131**, 021902 – **Published 13 July, 2023**

Quantum real-time evolution of entanglement and hadronization in jet production: Lessons from the massive Schwinger model

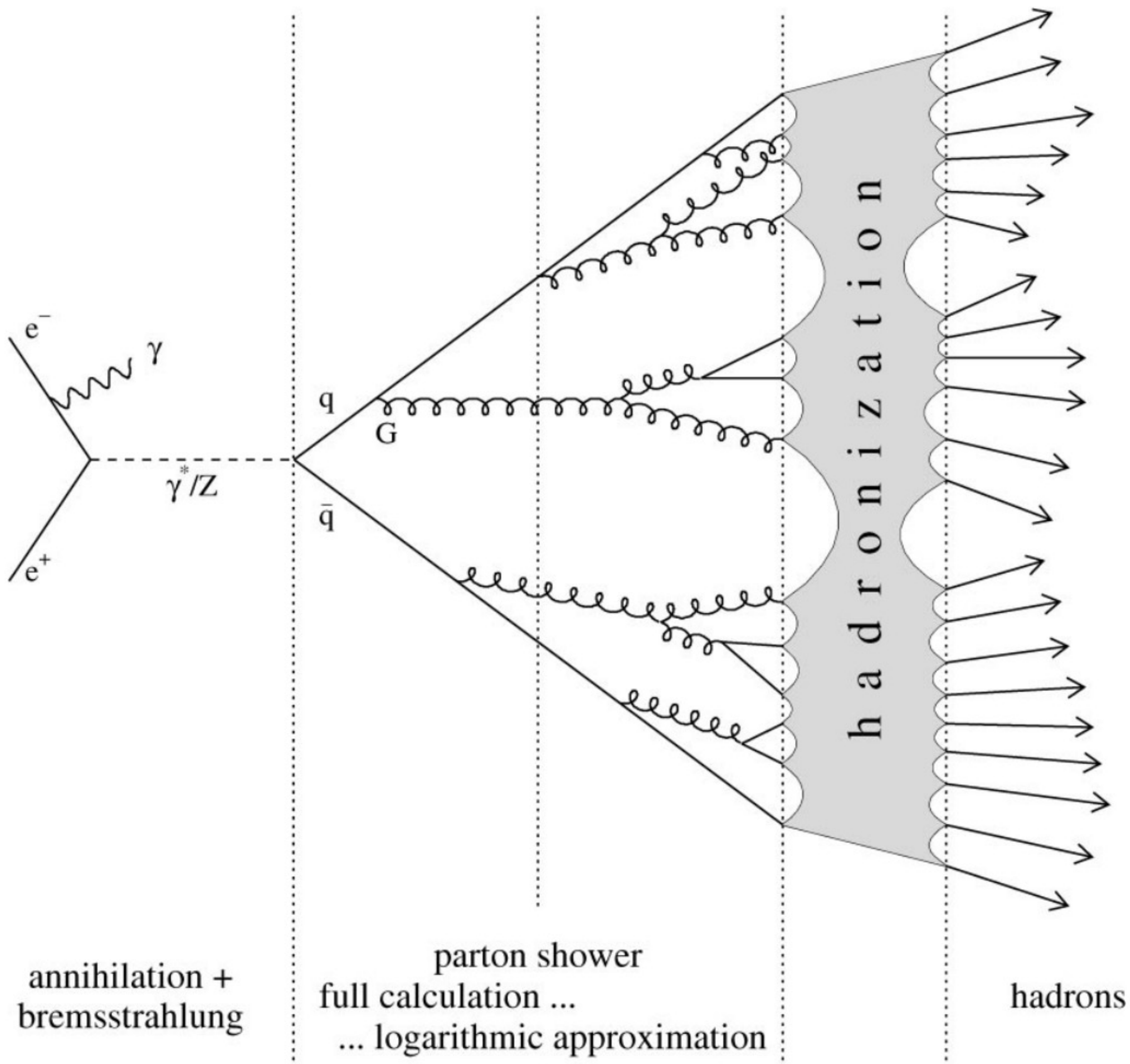
Adrien Florio[1,2,*], David Frenklakh[3,†], Kazuki Ikeda[2,3,‡], Dmitri Kharzeev[1,2,3,§], Vladimir Korepin[2,4,‖], Shuzhe Shi[5,3,¶], and Kwangmin Yu[6,**]

Phys. Rev. D **110**, 094029 – **Published 15 November, 2024**

+ to appear

The setup

O. Biebel / Physics Reports 340 (2001) 165–289



PHYSICAL REVIEW D VOLUME 10, NUMBER 2 15 JULY 1974

Vacuum polarization and the absence of free quarks

A. Casher,* J. Kogut,† and Leonard Susskind‡
Tel Aviv University, Ramat-Aviv, Tel Aviv, Israel
(Received 29 June 1973; revised manuscript received 4 October 1973)

This paper is addressed to the question of why isolated quark partons are not seen. It is argued that in vector gauge theories it is possible to have the short-distance and light-cone behavior of quark fields without real quark production in deep-inelastic reactions. The physical mechanism involved is the flow of vacuum-polarization currents which neutralize any outgoing quarks. Our ideas are inspired by arguments due to Schwinger and an intuitive picture of Bjorken. Two-dimensional (1 space, 1 time) vector gauge field theories provide exactly soluble examples of this phenomenon. The resulting picture of deep-inelastic final states predicts jets of hadrons and logarithmically rising multiplicities as conjectured by Bjorken and Feynman.

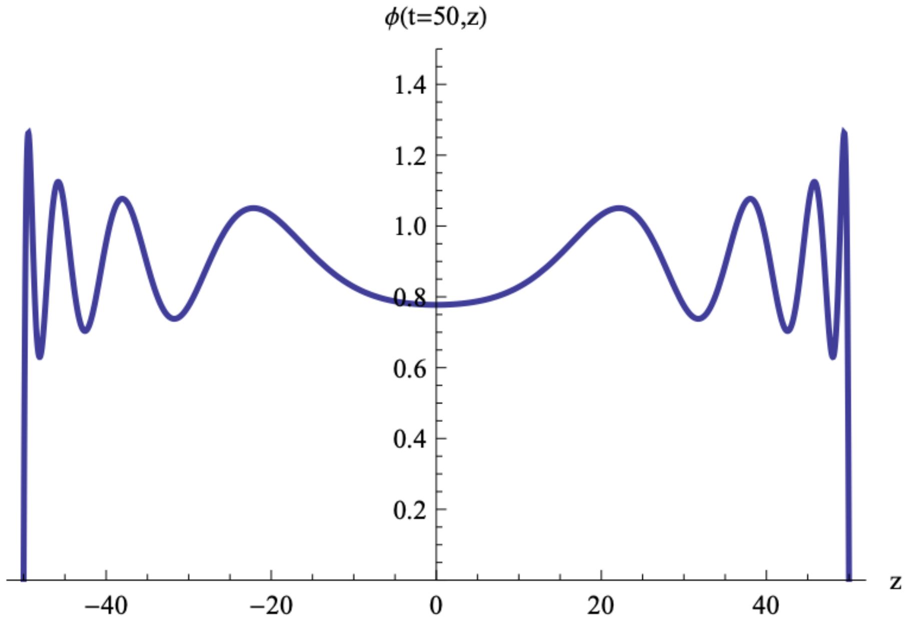
Massless Schwinger model coupled to external sources:

$$j_0^{\text{ext}} = g\delta(z-t), \quad j_1^{\text{ext}} = g\delta(z-t) \qquad \text{for } z>0,$$

$$j_0^{\text{ext}} = -g\delta(z+t), \quad j_1^{\text{ext}} = g\delta(z+t) \qquad \text{for } z<0,$$

In the massless case, can be solved exactly:

$$\phi(x) = \theta(t^2 - z^2)[1 - J_0(m\sqrt{t^2 - z^2})]$$



DK, F. Loshaj
Phys Rev D87 (2013) 7, 077501

String breaking due to production of quark-antiquark pairs;
the produced mesons form a rapidity plateau

To address thermalization, one needs to consider interacting mesons – this leads to the massive Schwinger model.

Non-integrable, no analytical solutions can be found – use digital quantum simulations!

$$H^L(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N-1}[L_{\mathrm{dyn},n} + L_{\mathrm{ext},n}(t)]^2.$$

Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

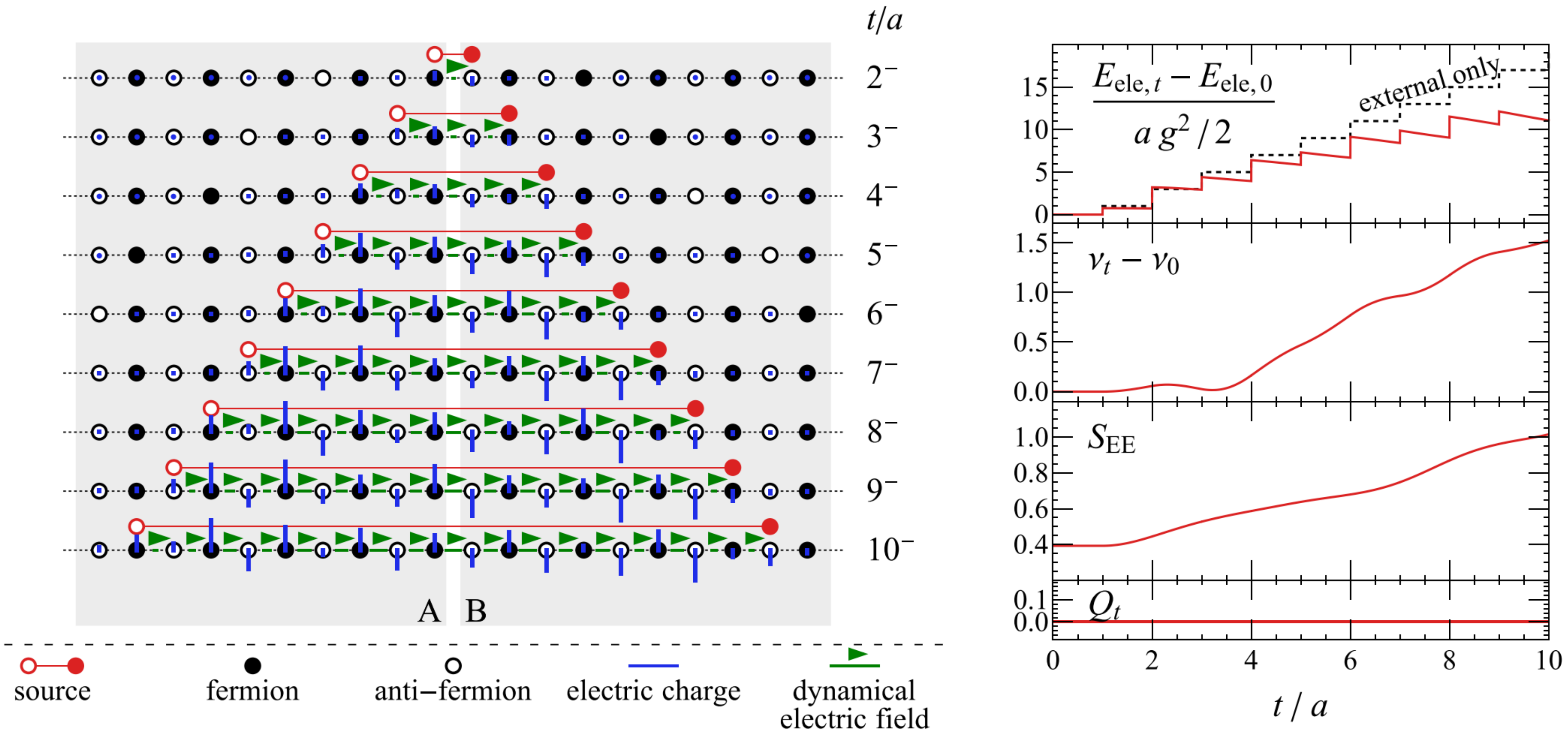
Adrien Florio, David Frenklakh, Kazuki Ikeda, Dmitri Kharzeev, Vladimir Korepin, Shuzhe Shi, and Kwangmin Yu
Phys. Rev. Lett. **131**, 021902 – Published 13 July 2023

Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

Adrien Florio, David Frenklakh, Kazuki Ikeda, Dmitri Kharzeev, Vladimir Korepin, Shuzhe Shi, and Kwangmin Yu

Phys. Rev. Lett. **131**, 021902 – Published 13 July 2023

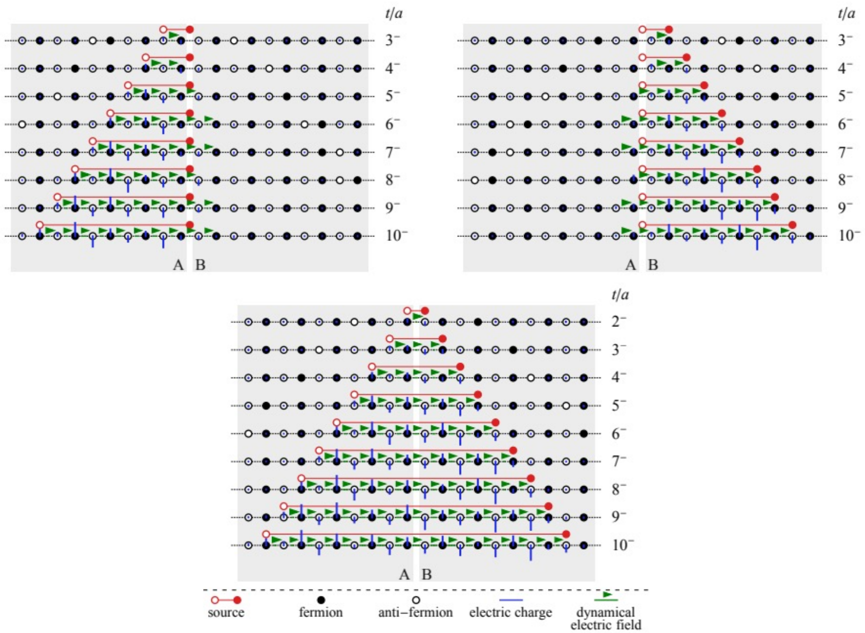
PHYSICAL REVIEW LETTERS **131,** 021902 (2023)

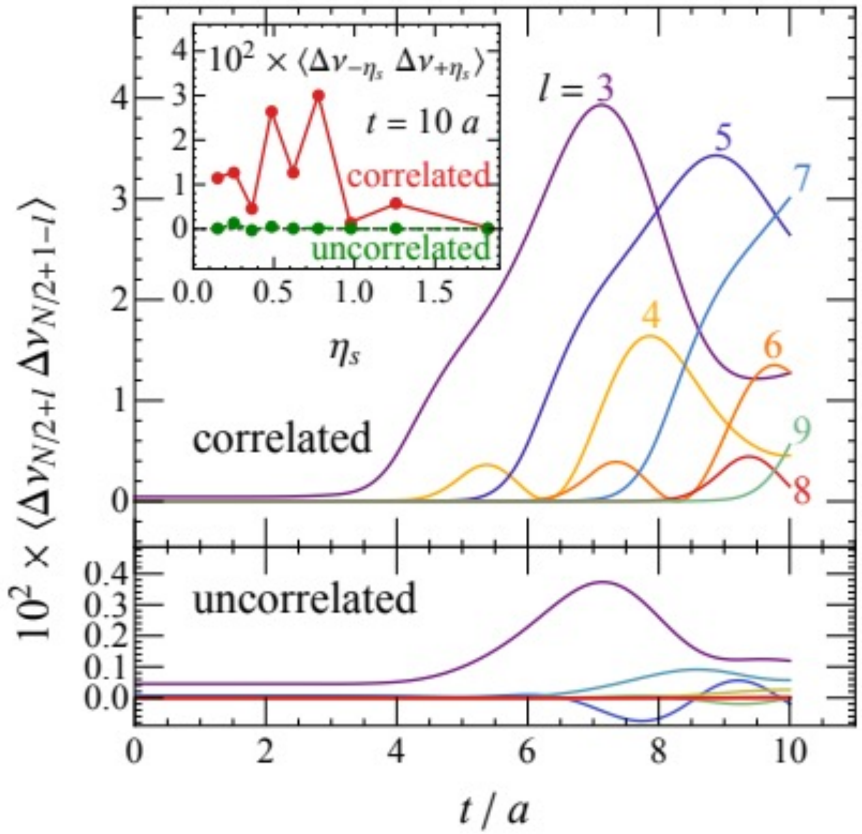


Screening of electric field, modification of the vacuum, growth of entanglement entropy!

Entanglement among the produced jets is a phenomenon that is not present in perturbative QCD (independent fragmentation functions).

This entanglement is “screened” by the produced pairs, and is limited to relatively small rapidity interval (a step towards thermalization?)



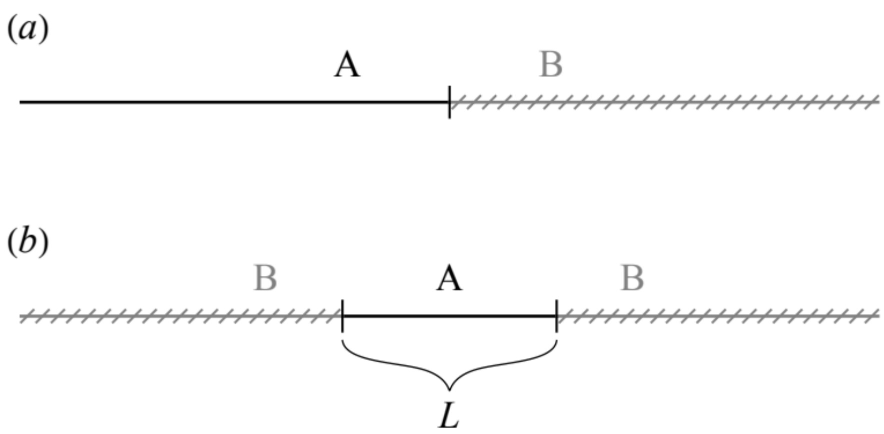


What can we do to understand a possible approach to thermalization in our system?

Quantum simulation of entanglement and hadronization in jet production: lessons from the massive Schwinger model

Adrien Florio,[1,2,*] David Frenklakh,[3,†] Kazuki Ikeda,[2,3,‡] Dmitri Kharzeev,[1,2,3,§]
Vladimir Korepin,[2,4,¶] Shuzhe Shi,[3,5,**] and Kwangmin Yu[6,††]

Let us start by examining the entanglement spectrum:



The entanglement spectrum

$$\rho(t) = \sum_{i=1}^{2^{N/2}} \lambda_i(t) |\psi_i(t)\rangle\langle\psi_i(t)|,$$

At late times, a huge number of entanglement eigenstates start to contribute, with comparable eigenvalues – approach to the maximal entanglement and thermalization?

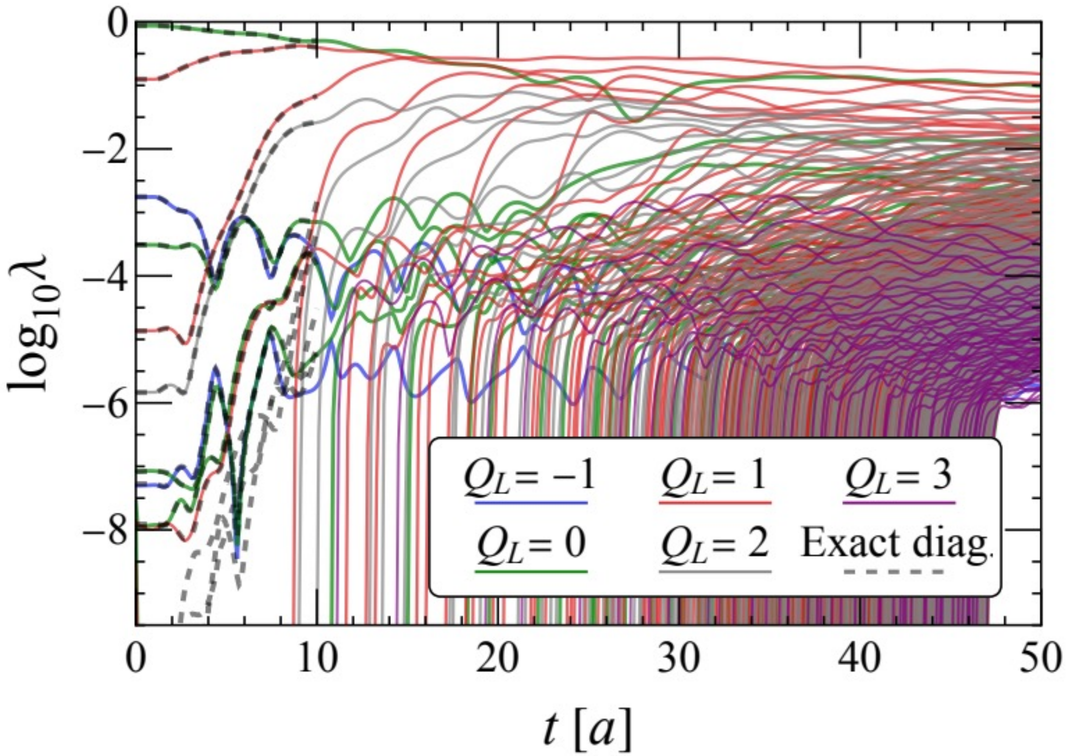


FIG. 2. Symmetry-resolved entanglement spectrum evolution for the lattice size $N = 100$, $m = 1/(4a), g = 1/(2a)$. For comparison the spectrum obtained with exact diagonalization for $N = 20$ at the same mass and coupling is shown as dashed curves.

Tests of maximal entanglement

Renyi entropy

$$S_\alpha(t) \equiv \frac{\ln \mathrm{Tr}_L(\rho_L(t)^\alpha)}{1-\alpha} = \frac{\ln \sum_{i=1}^{2^{N/2}} \lambda_i^\alpha}{1-\alpha}.$$

"Entangleness"

$$\mathcal{E} \equiv \frac{1-\mathrm{tr}\rho_L^2}{1-2^{-N/2}} = \frac{1-\sum_{i=1}^{2^{N/2}} \lambda^2}{1-2^{-N/2}}.$$

$$\mathcal{E}[\mathrm{MES}] = 1.$$

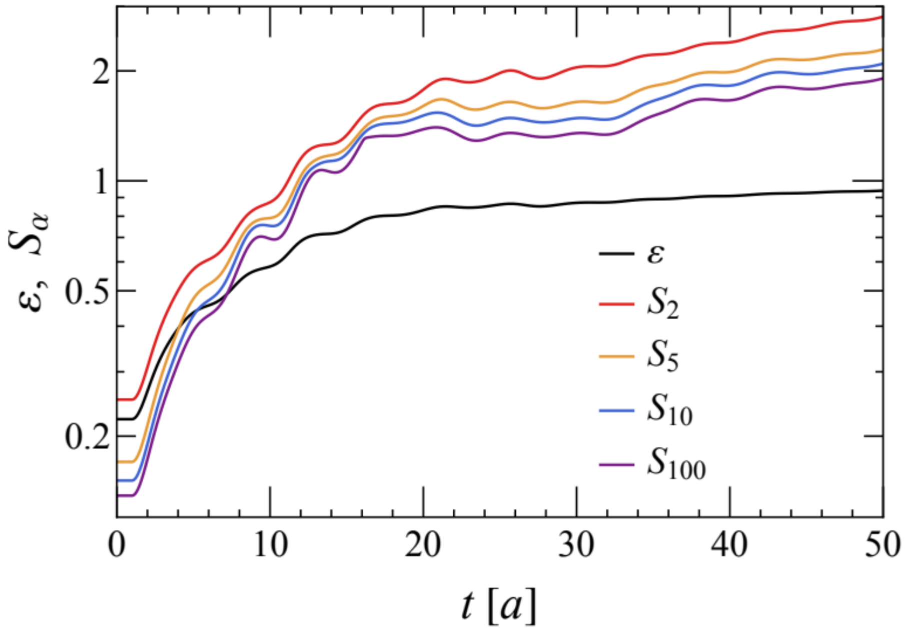


FIG. 3. Entangleness (black) and Rényi entropy with $\alpha = 2$ (red), 5 (gold), 10 (blue), and 100 (purple).

Approach to maximal entanglement!

(in a subspace of the full Hilbert space)

The physical meaning of Schmidt states

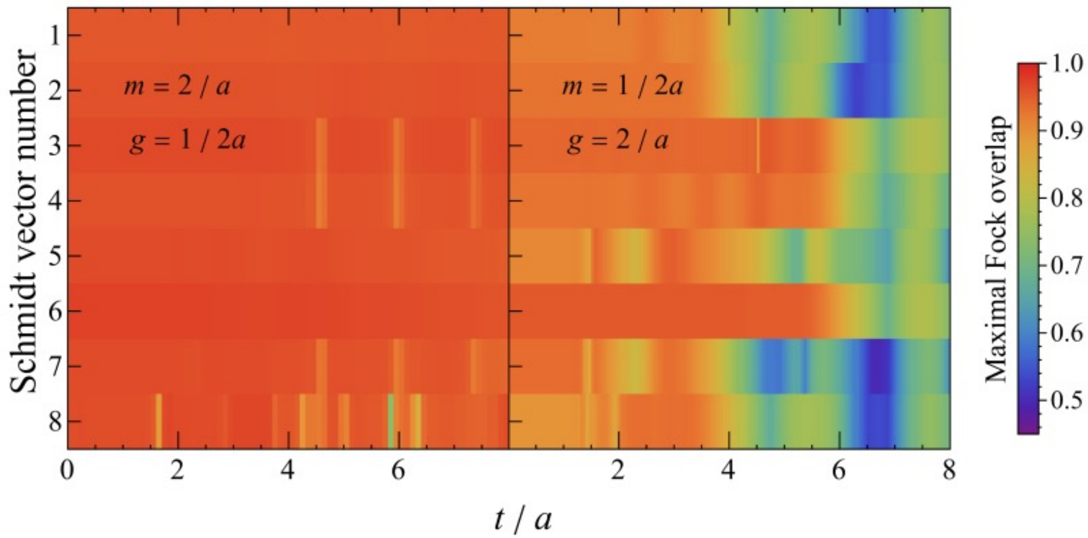


FIG. 5. Maximal overlap of each Schmidt vector with any Fock state. Comparison between $m = 2/a, g = 1/(2a)$ on the left panel and $m = 1/(2a), g = 2/a$ on the right panel is shown. In both cases, $N = 16$. To study continuous evolution, we choose to consider the 8 leading Schmidt vectors in the vacuum state at $t = 0$ and follow their evolution. Because of the level crossing in Schmidt spectrum, at later times these vectors are not necessarily the 8 leading Schmidt vectors.

Transition from "quark-antiquark" states at early times to "mesons" at late times –

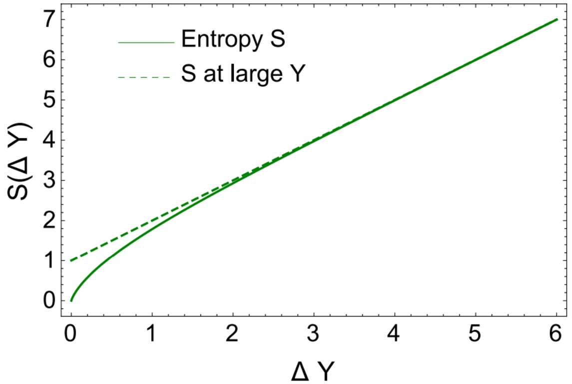
Hadronization seen in real time!

Maximal entanglement at high energies (small x)

Deep inelastic scattering as a probe of entanglement

Dmitri E. Kharzeev[1,2,*] and Eugene M. Levin[3,4,†]

Phys. Rev. D **95**, 114008 – **Published 13 June, 2017**



Quantum information approach to high energy interactions

Dmitri E. Kharzeev

Published: **20 December 2021** https://doi.org/10.1098/rsta.2021.0063

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A

At high energies, the phase of the wave function cannot be measured, and has to be traced over ("Haar scrambling"). This leads to the (probabilistic) parton model description:

$$\hat{\rho}_{\rm parton} = {\rm Tr}_{\varphi}\hat{\rho} = \int_0^{2\pi} \frac{{\rm d}\varphi}{2\pi} {\rm e}^{{\rm i}(n'-n)\varphi} \sum_{n,n'} \alpha_n\, \alpha^*_{n'} |n\rangle\langle n'| = \sum_n |\alpha_n|^2 |n\rangle\langle n|.$$

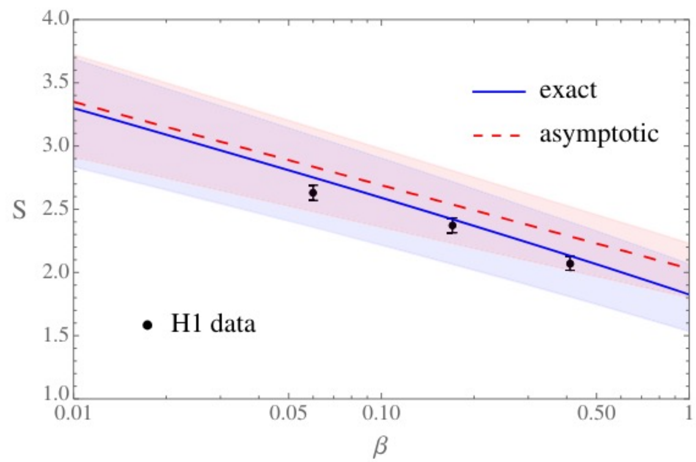
Signature: $S_{hadrons} = \ln[xG(x)]$

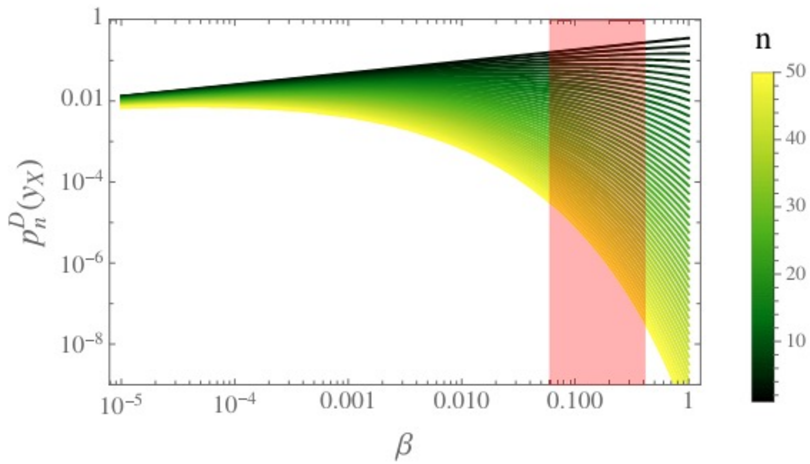
Maximal entanglement: experimental tests at HERA and EIC

Probing the Onset of Maximal Entanglement inside the Proton in Diffractive Deep Inelastic Scattering

Martin Hentschinski, Dmitri E. Kharzeev, Krzysztof Kutak, and Zhoudunming Tu

Phys. Rev. Lett. **131**, 241901 – Published 13 December 2023

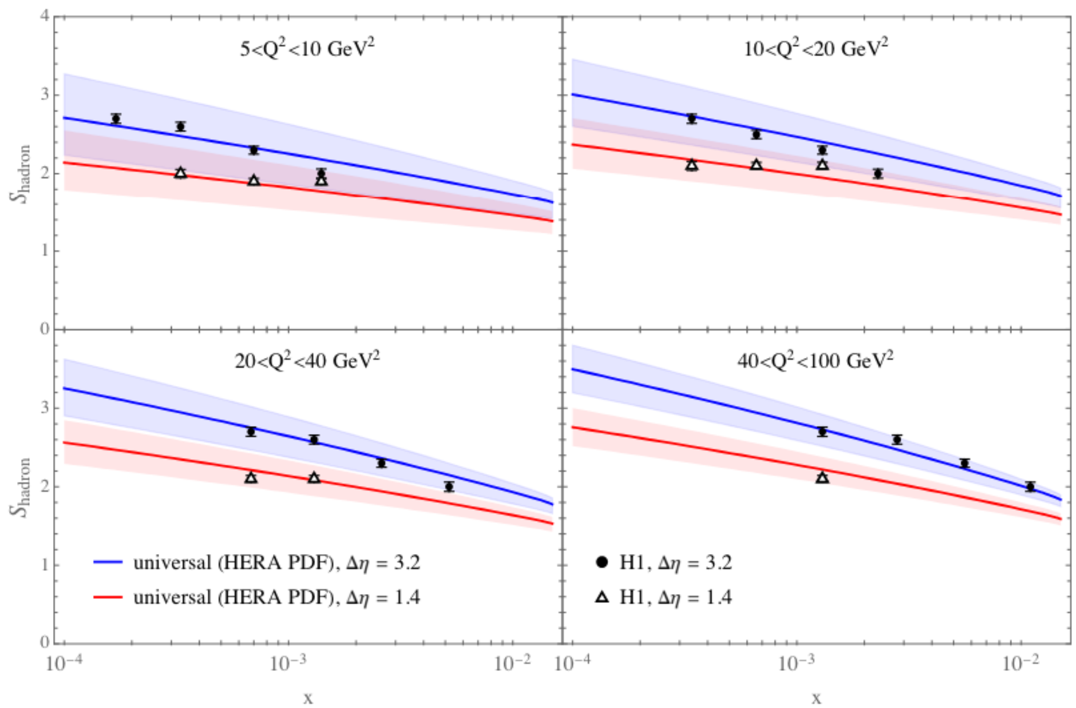




QCD evolution of entanglement entropy

Martin Hentschinski,[1,*] Dmitri E. Kharzeev,[2,3,†] Krzysztof Kutak,[4,‡] and Zhoudunming Tu[3,§]

arXiv:2408.01259, Rep.Prog.Phys.(2025)



Entanglement as a probe of hadronization

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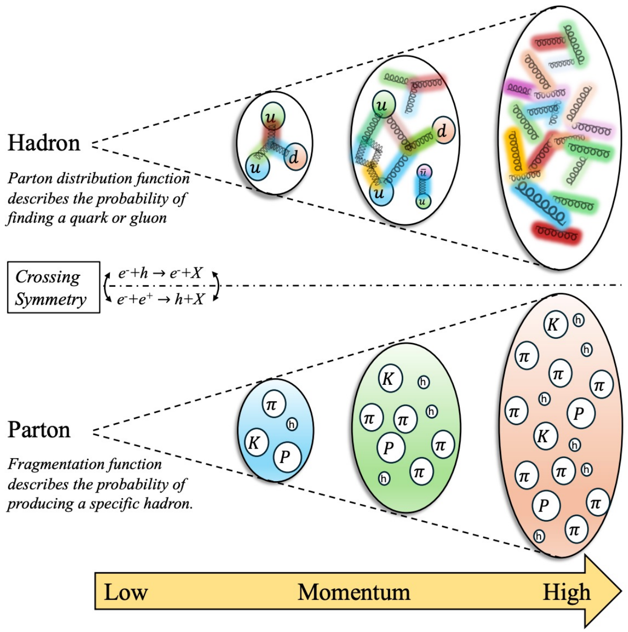
arXiv:2410.22331



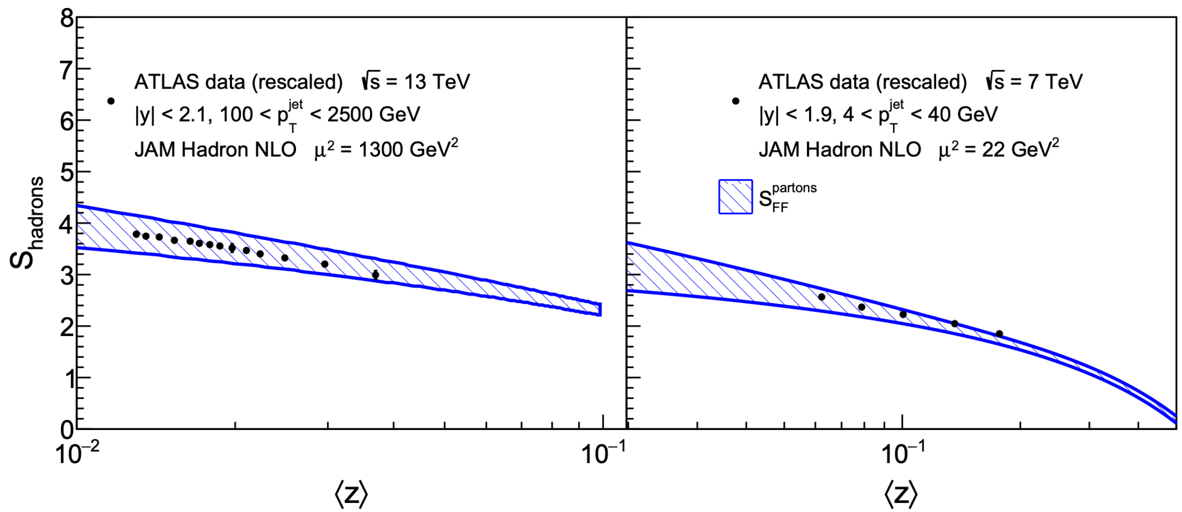


FIG. 3. The entropy S_{hadrons} as a function of $\langle z\rangle$ for $S_{\text{FF}}^{\text{partons}}$ — incorporating gluons, u-(anti)quarks, and d-(anti)quarks — is shown using JAM fragmentation functions at NLO for $\mu^2 = 1300$ GeV^2, compared with ATLAS data at $\sqrt{s} = 13$ TeV [45] (left). Additionally, the results at $\mu^2 = 22$ GeV^2 are compared with ATLAS data at $\sqrt{s} = 7$ TeV [43] (right). The uncertainties are calculated at the 1σ level. The total entropy $S_{\text{FF}}^{\text{partons}}$ is derived from the sum of the individual entropies of each parton, with each contribution normalized by the average fraction of jets produced by that parton from PYTHIA simulation.

Evidence for maximal entanglement from jet fragmentation

Summary:

- High energy collisions seem to lead to the maximally entangled states – experimental signatures at RHIC, LHC, **EIC**

- Saturation of local observables in time, consistency with thermal expectation values, transition of entanglement entropy from area law to volume law detected – thermalization**?**

Many open questions remain –
fundamental quantum science
addressed through **quantum simulations**