A lattice field theory of quantum circuits

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in collaboration with:

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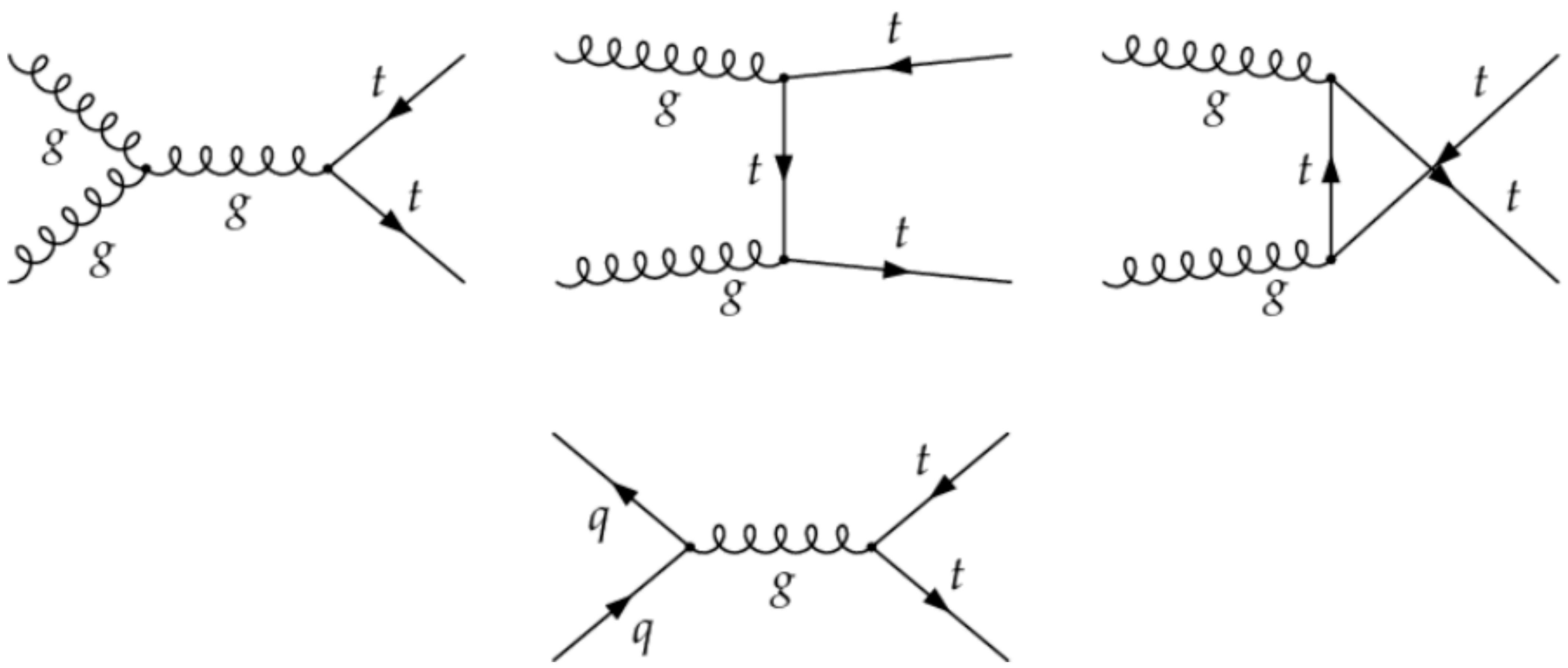
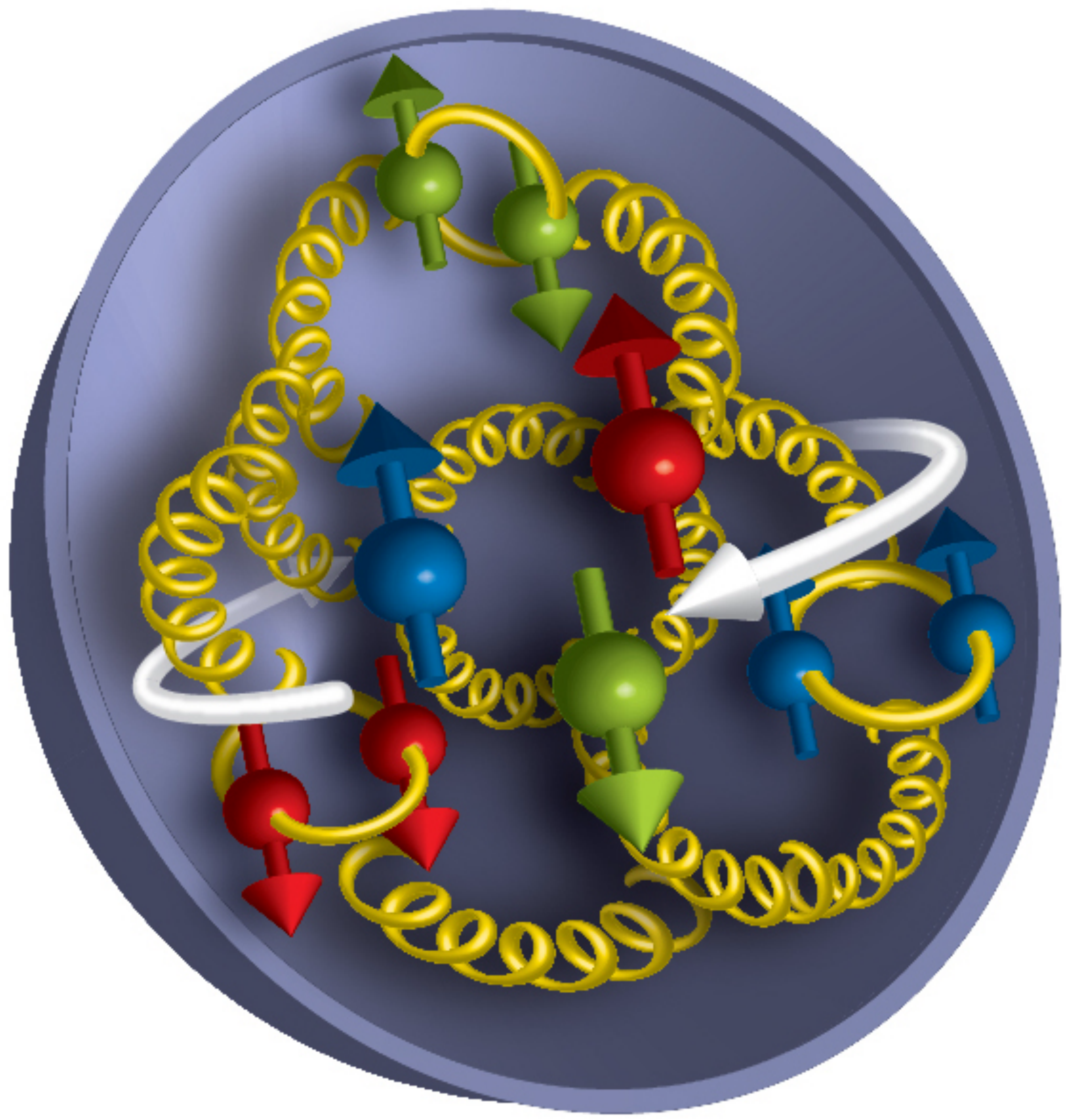
NIST NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY U.S. DEPARTMENT OF COMMERCE
C^{2}QA Co-design Center for Quantum Advantage

"Quantum Information Science on the Intersections of Nuclear and AMO Physics"
January 15th 2025

An analogy:

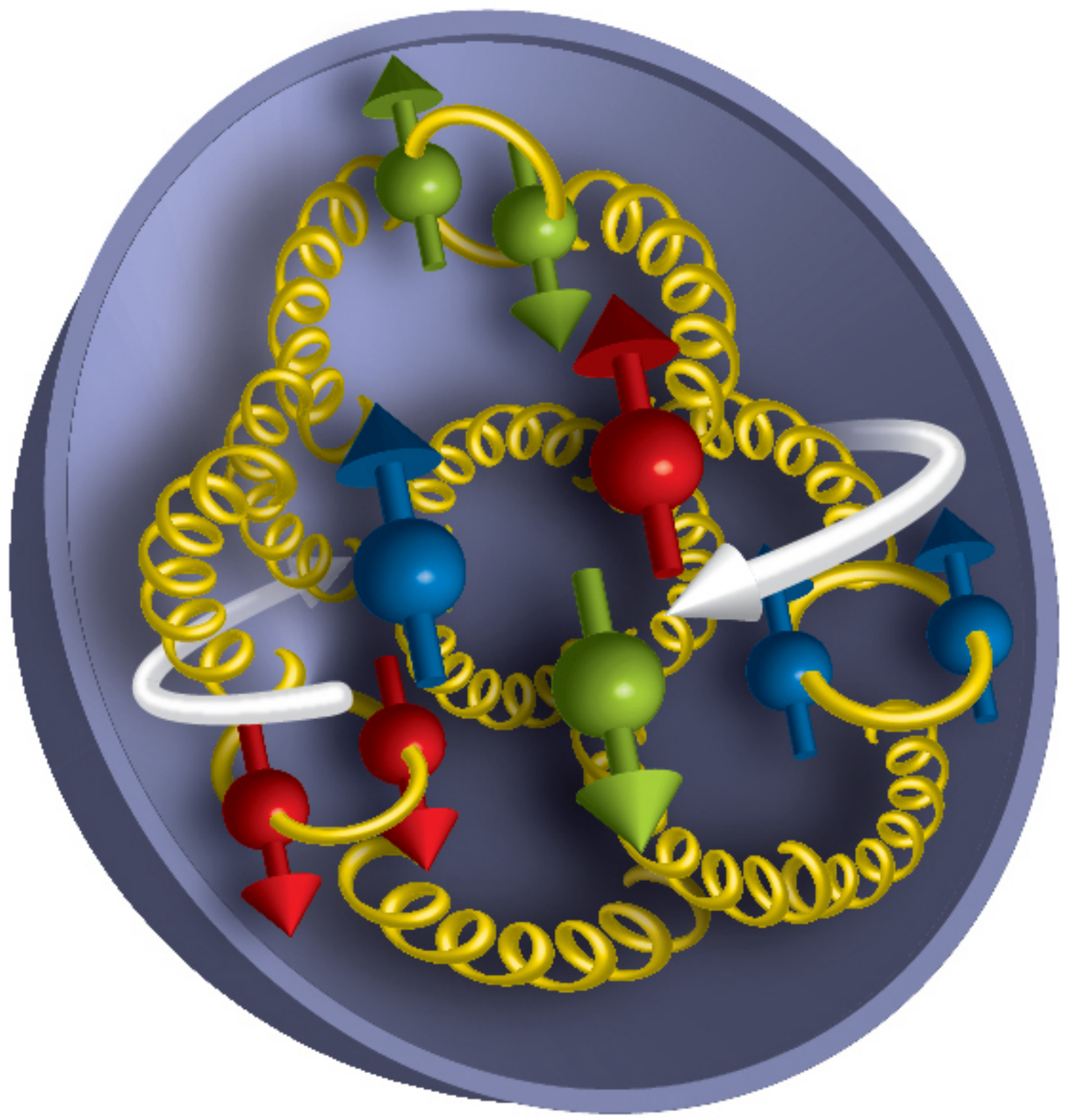
$$\mathcal{L} = \bar{q}(i\not{\partial} - g\not{A} - m)q - \frac{1}{2}\mathrm{tr}\, G_{\mu\nu}G^{\mu\nu}$$

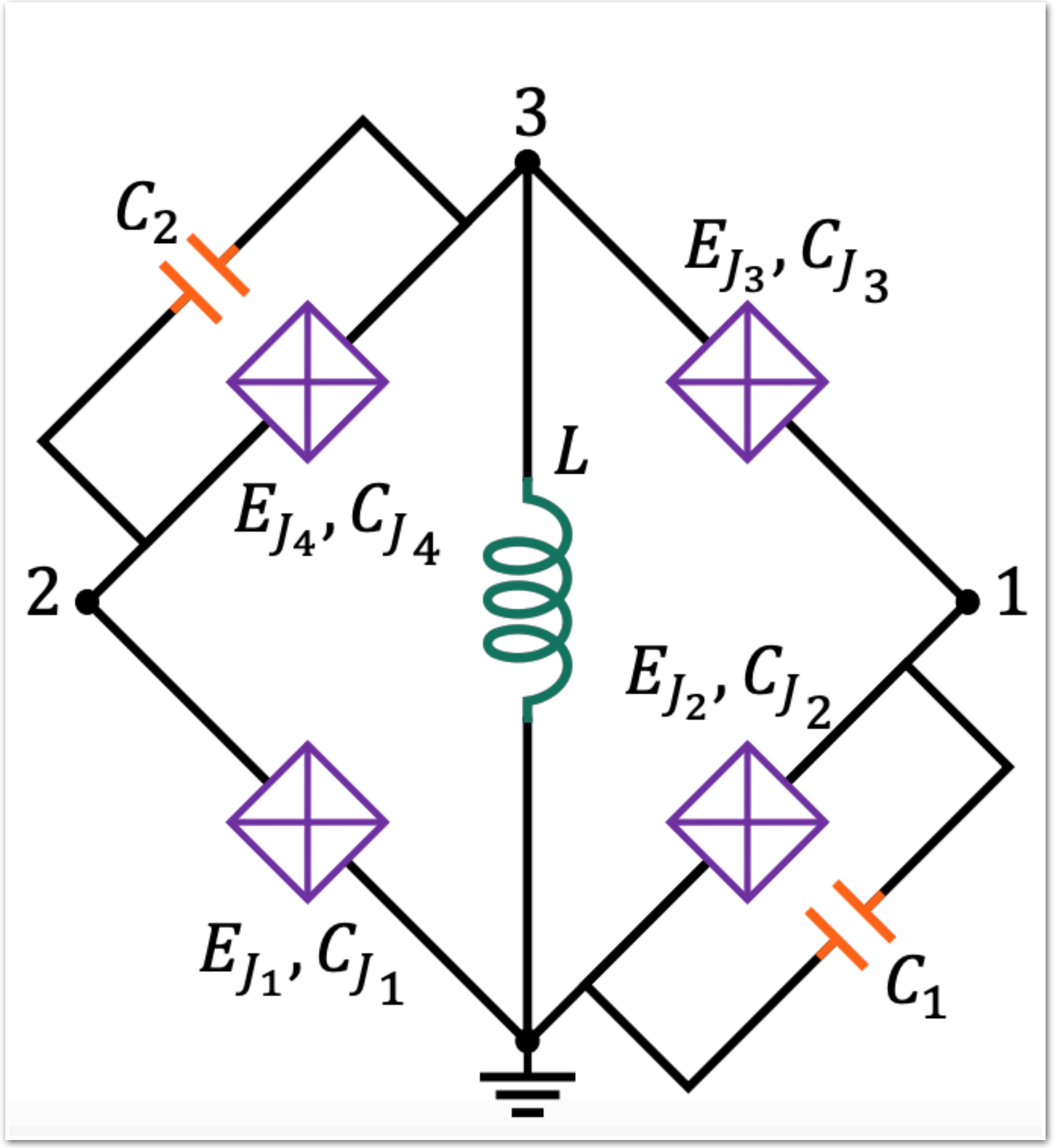




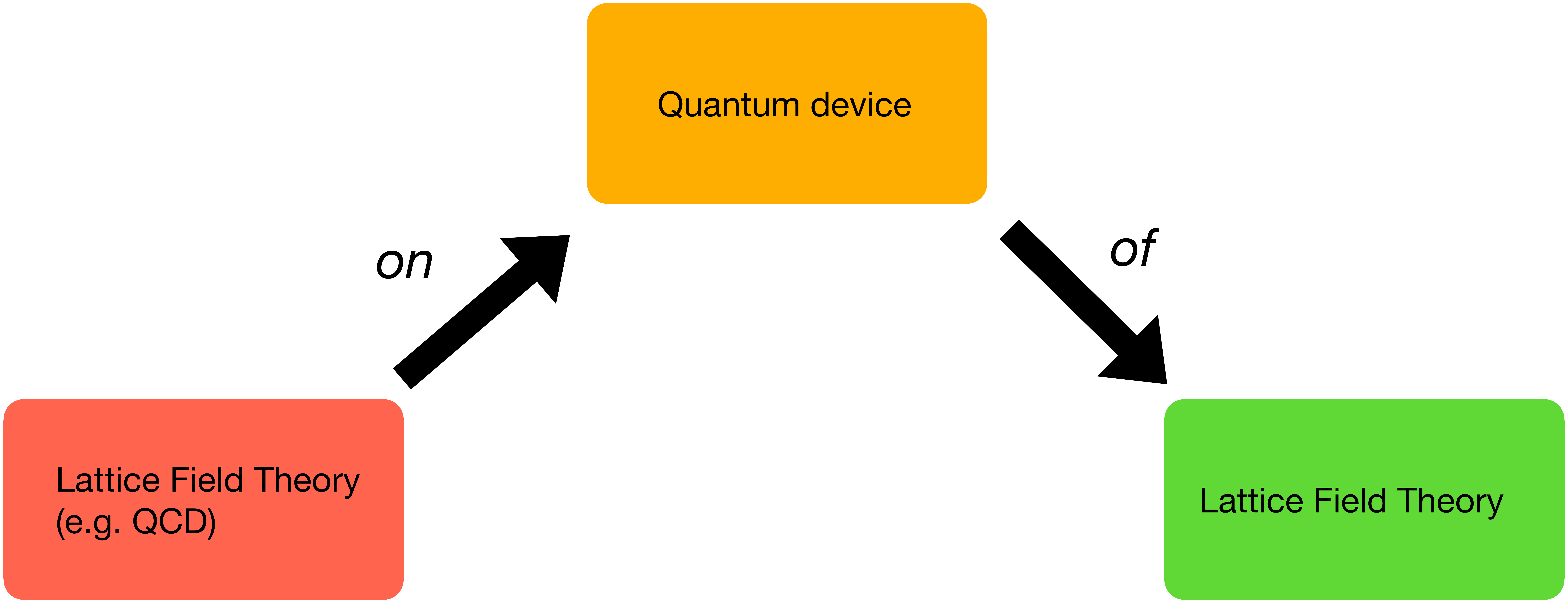
BNL Newsroom

An analogy:

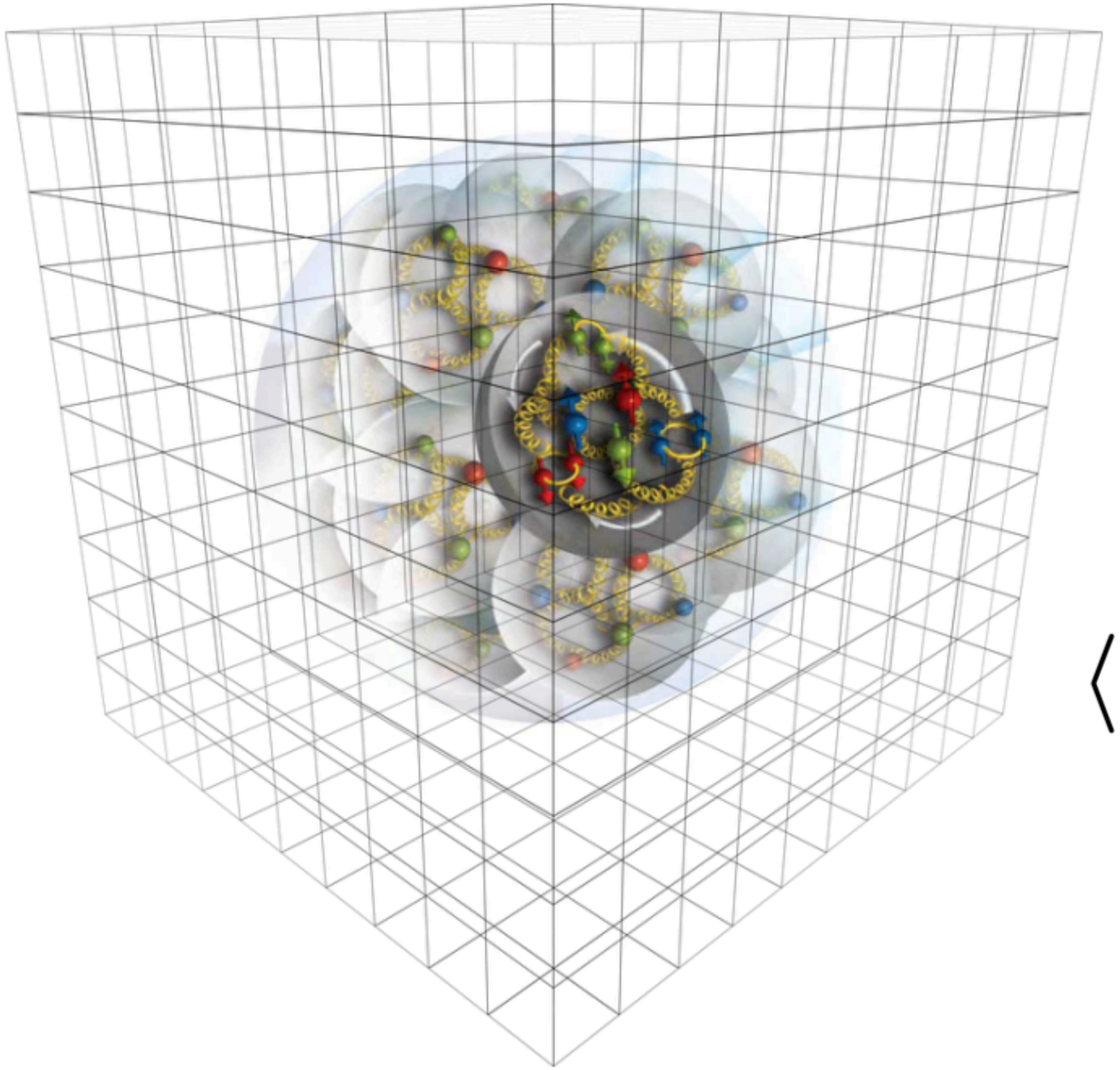




Rajabzadeh et. al. Quantum 7, 1118 (2023).


Quantum device
on
of
Lattice Field Theory
(e.g. QCD)
Lattice Field Theory

Lattice field theory:


Credit: Phiala Shanahan

1. Restrict spacetime to a lattice with spacing a .
2. Write everything as path integrals.

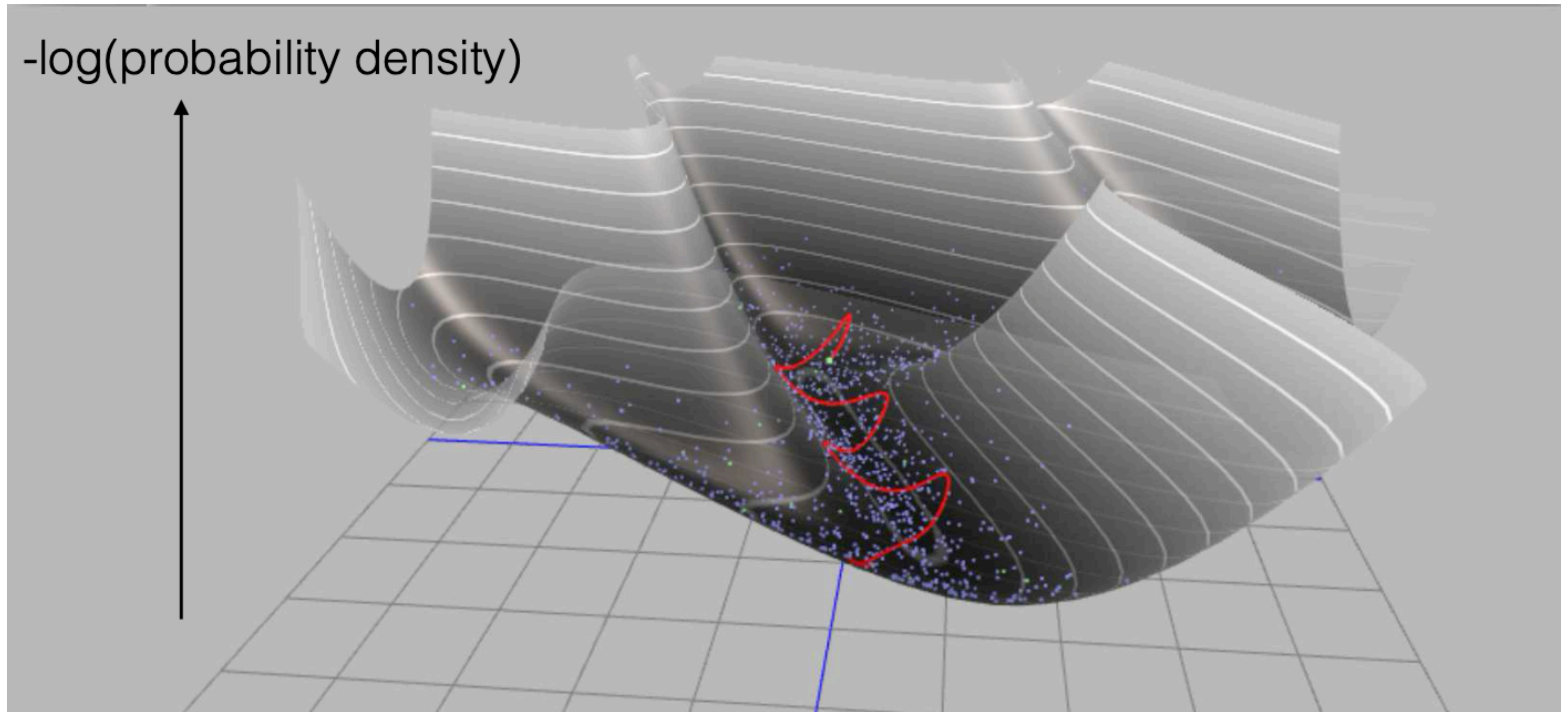
$$\langle \mathcal{O} \rangle = \frac{\int DADq\, e^{-S(A,q)}\, \mathcal{O}(A,q)}{\int DADq\, e^{-S(A,q)}}$$

$$= \langle \mathcal{O} \rangle_{\text{exact}} + O(a^n)$$

Monte Carlo:

$$\langle \mathcal{O} \rangle = \frac{\int DADq\, e^{-S(A,q)}\, \mathcal{O}(A,q)}{\int DADq\, e^{-S(A,q)}}$$

$$= \int DADq\; p(A,q)\, \mathcal{O}(A,q)$$



Credit: Phiala Shanahan

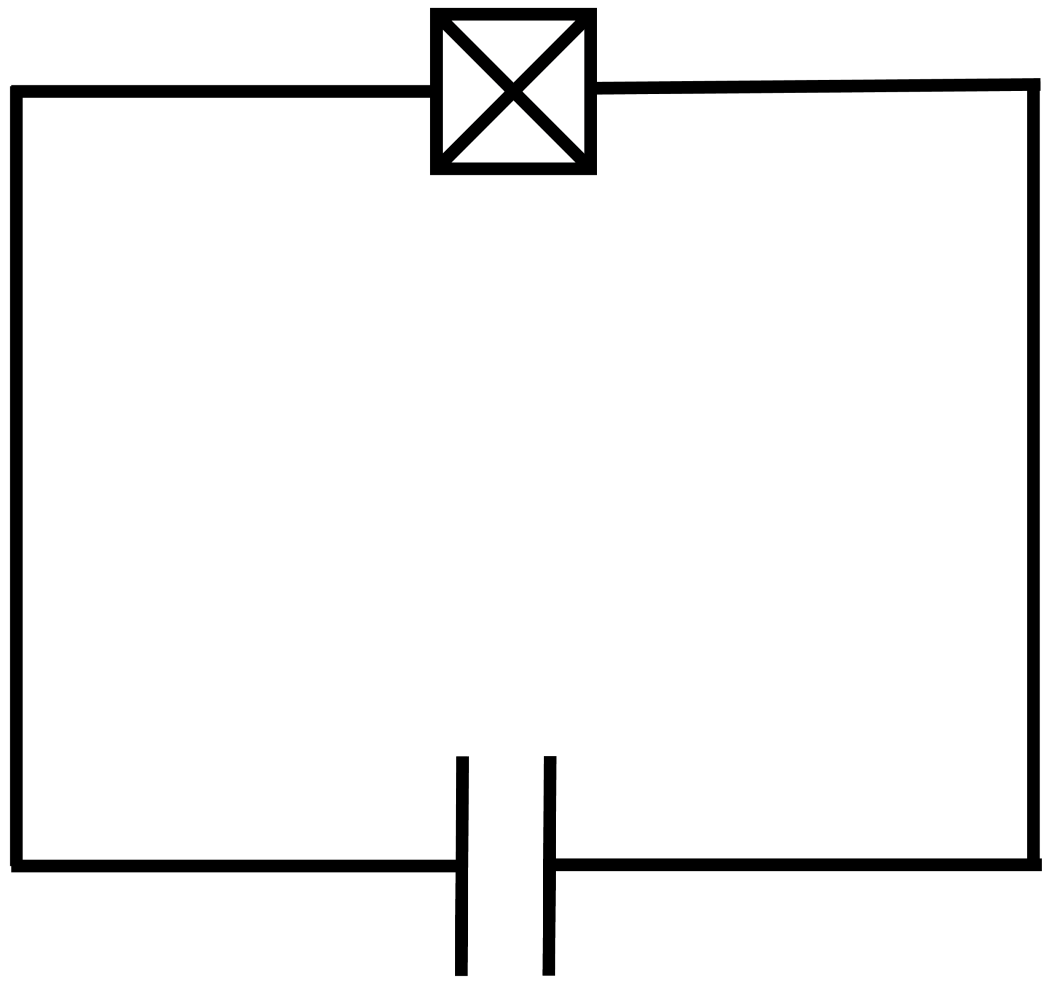
where:

$$p(A,q) = \frac{e^{-S(A,q)}}{\int DADq\, e^{-S(A,q)}}$$

Stochastic estimate:

$$\frac{1}{N}\sum_{i=1}^{N} \mathcal{O}(A_i, q_i) = \langle \mathcal{O} \rangle + O(N^{-1/2})$$

Superconducting quantum circuits:



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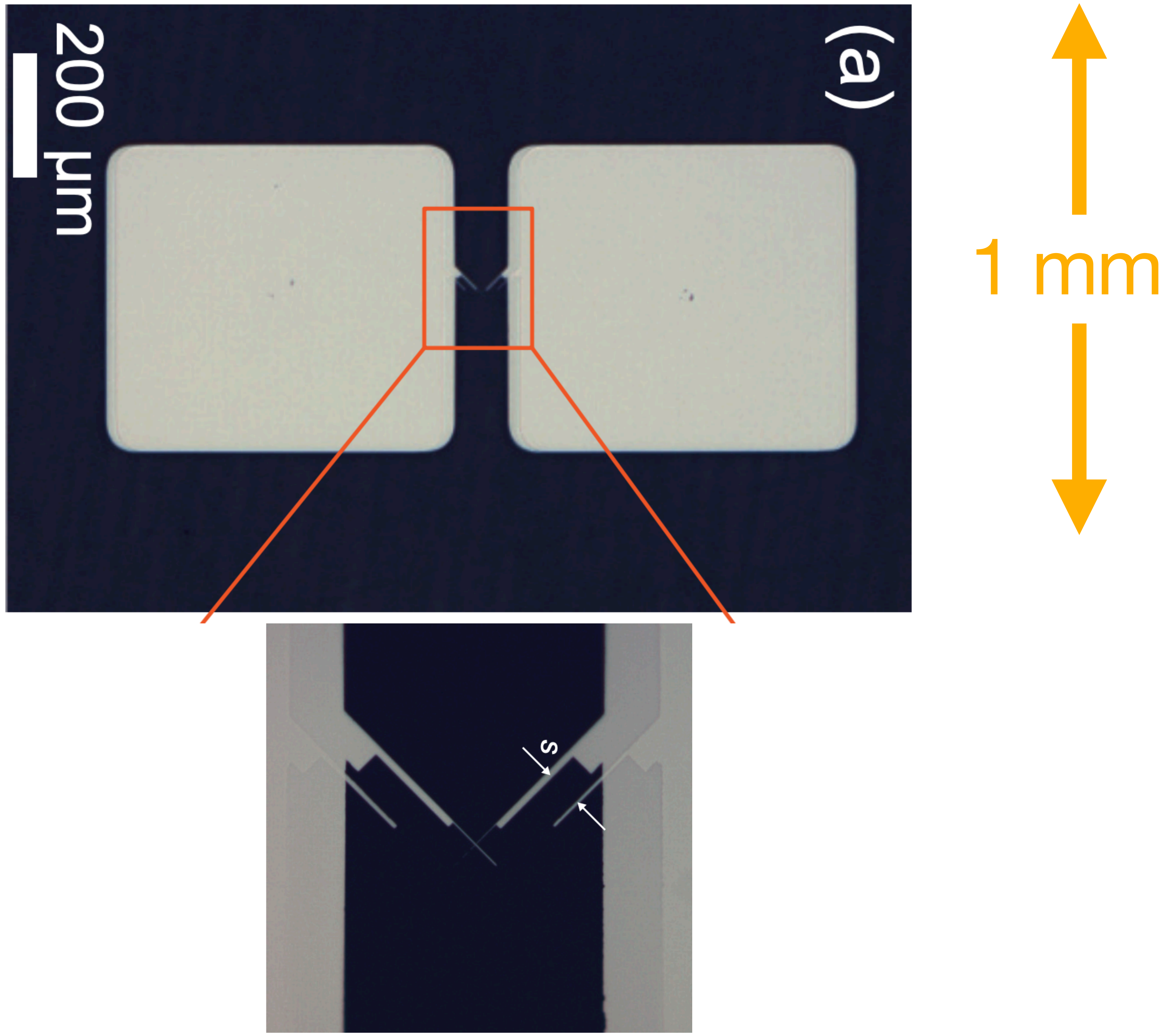
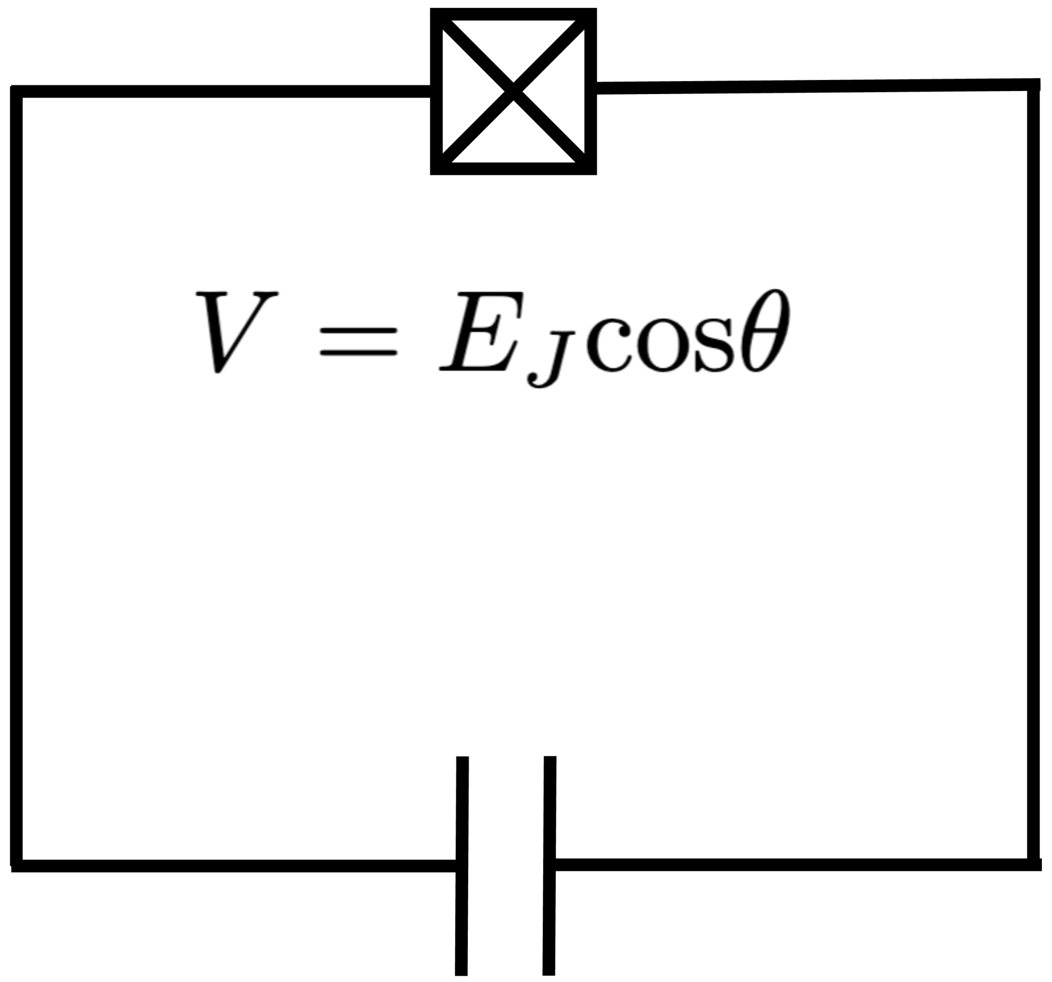
I. Tsioutsios, K. Serniak et. al. *AIP Advances* 10, 065120 (2020)



Superconducting quantum circuits:

$$\theta \in U(1)$$



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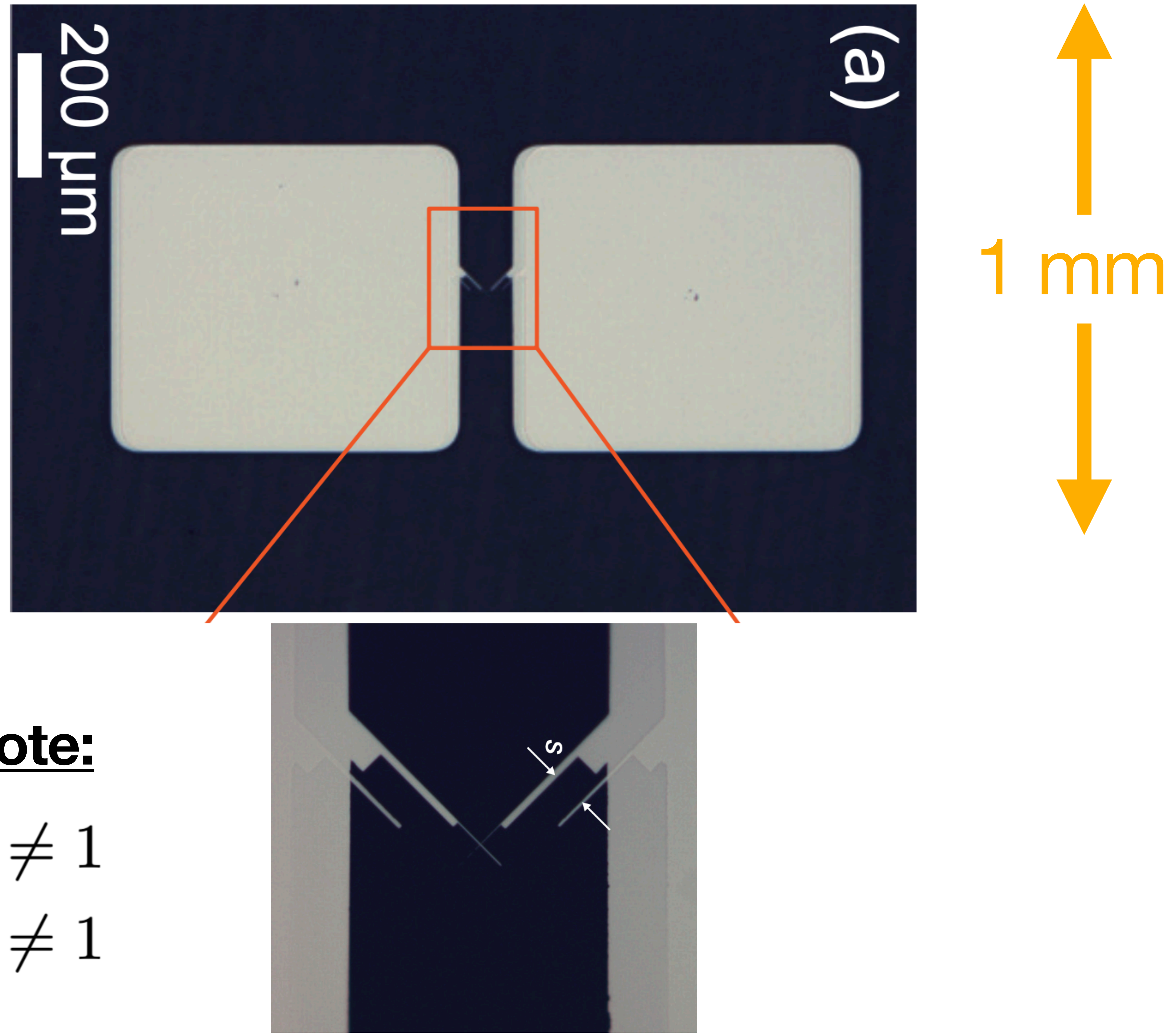
[I. Tsioutsios, K. Serniak et. al. *AIP Advances* 10, 065120 (2020)]



$$H = 4E_C n^2 - E_J \cos(\theta)$$

where $[\theta, n] = i$

Note:

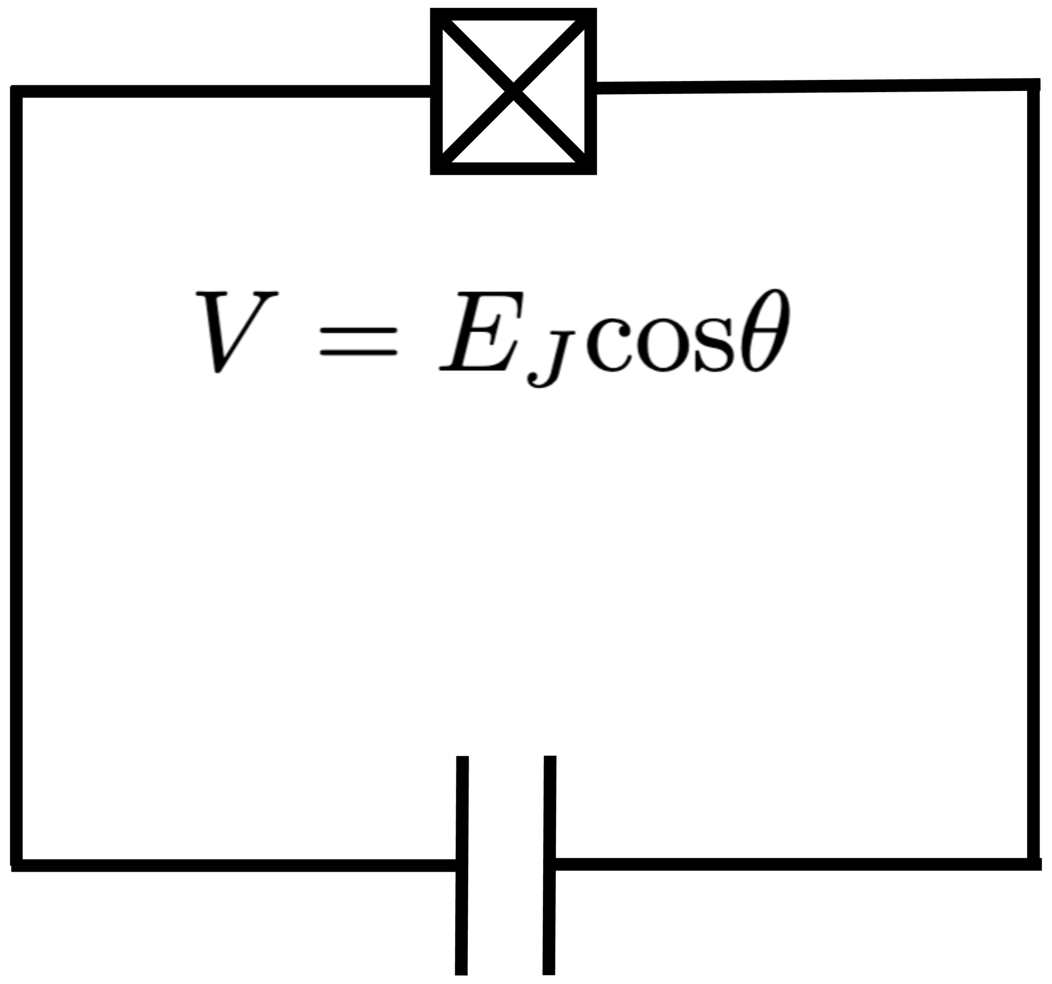
$\hbar \neq 1$

$k_B \neq 1$

Superconducting quantum circuits:

$$\theta \in U(1)$$

$$V = E_J \cos\theta$$



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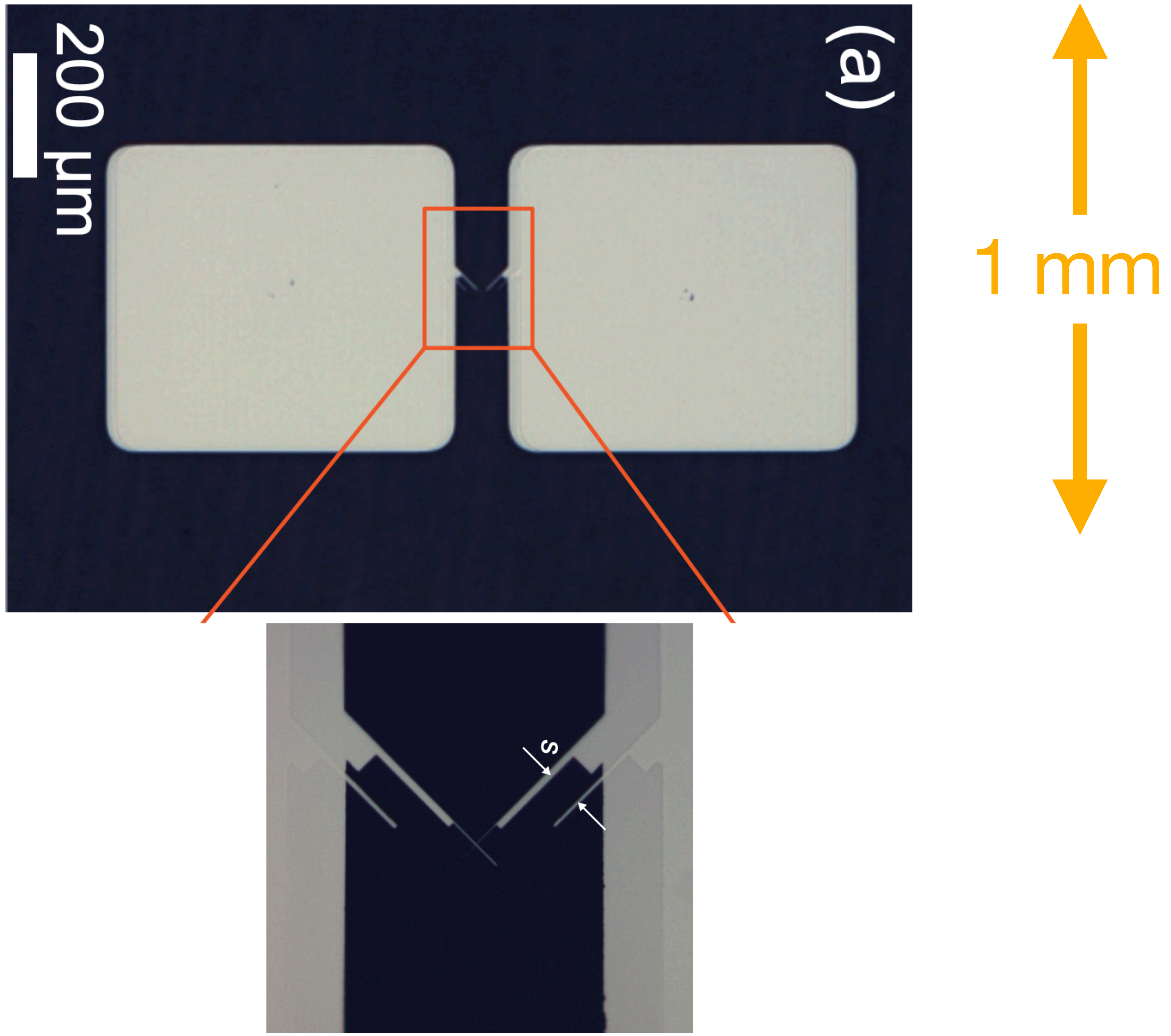
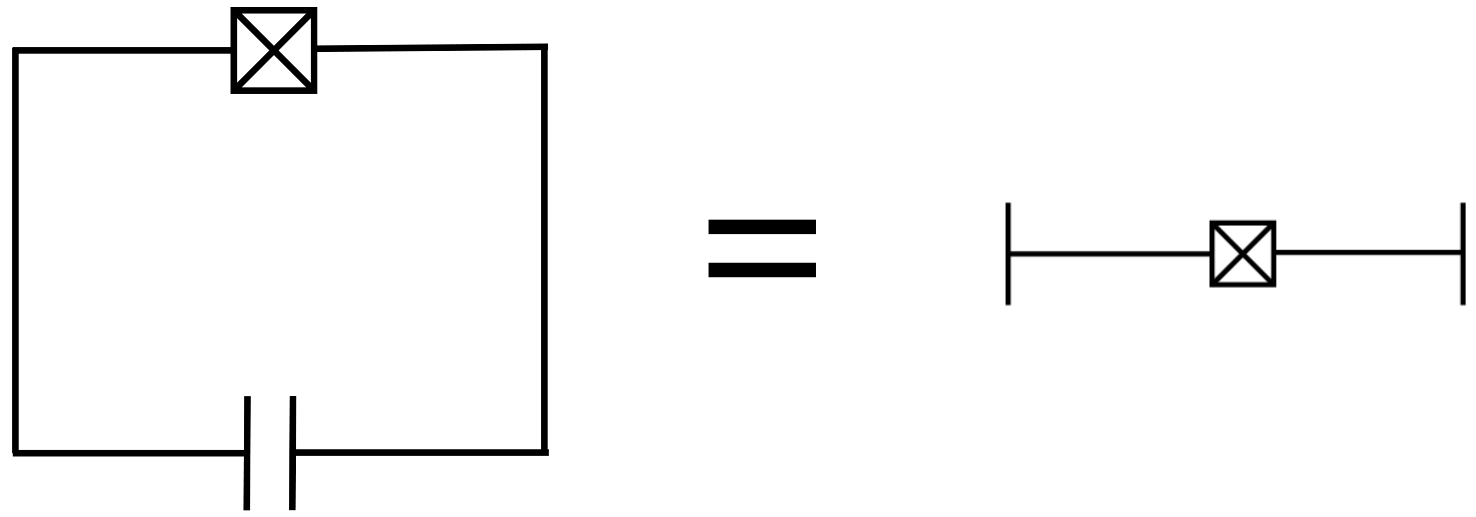
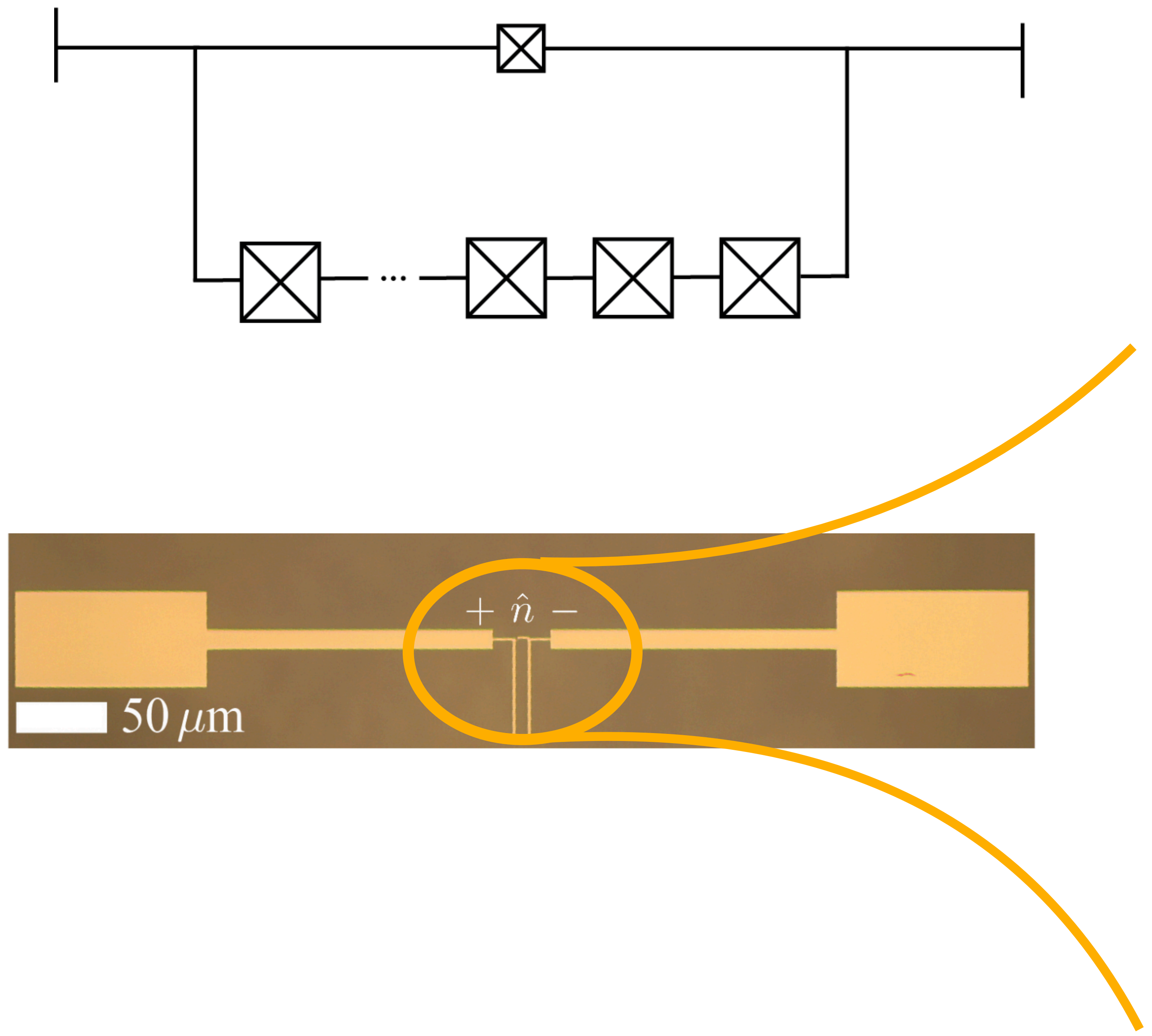
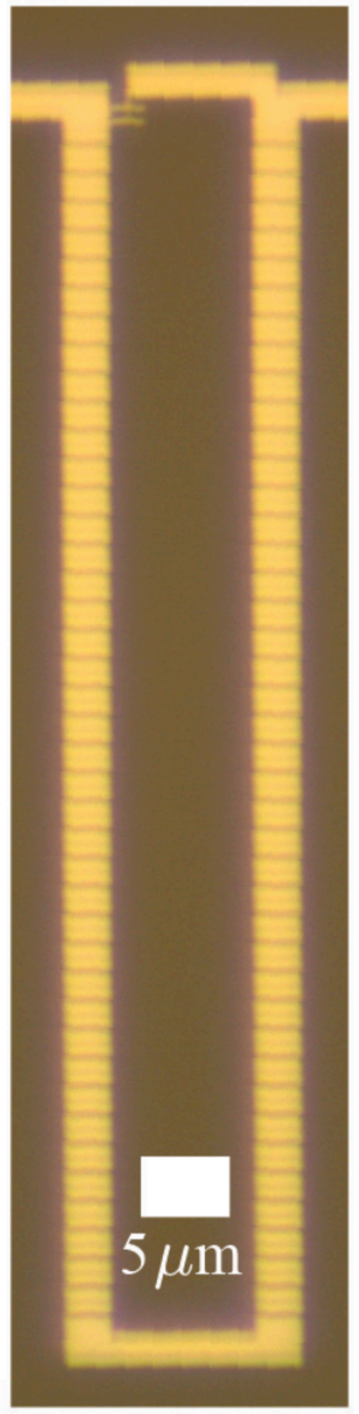




Superconducting quantum circuits:

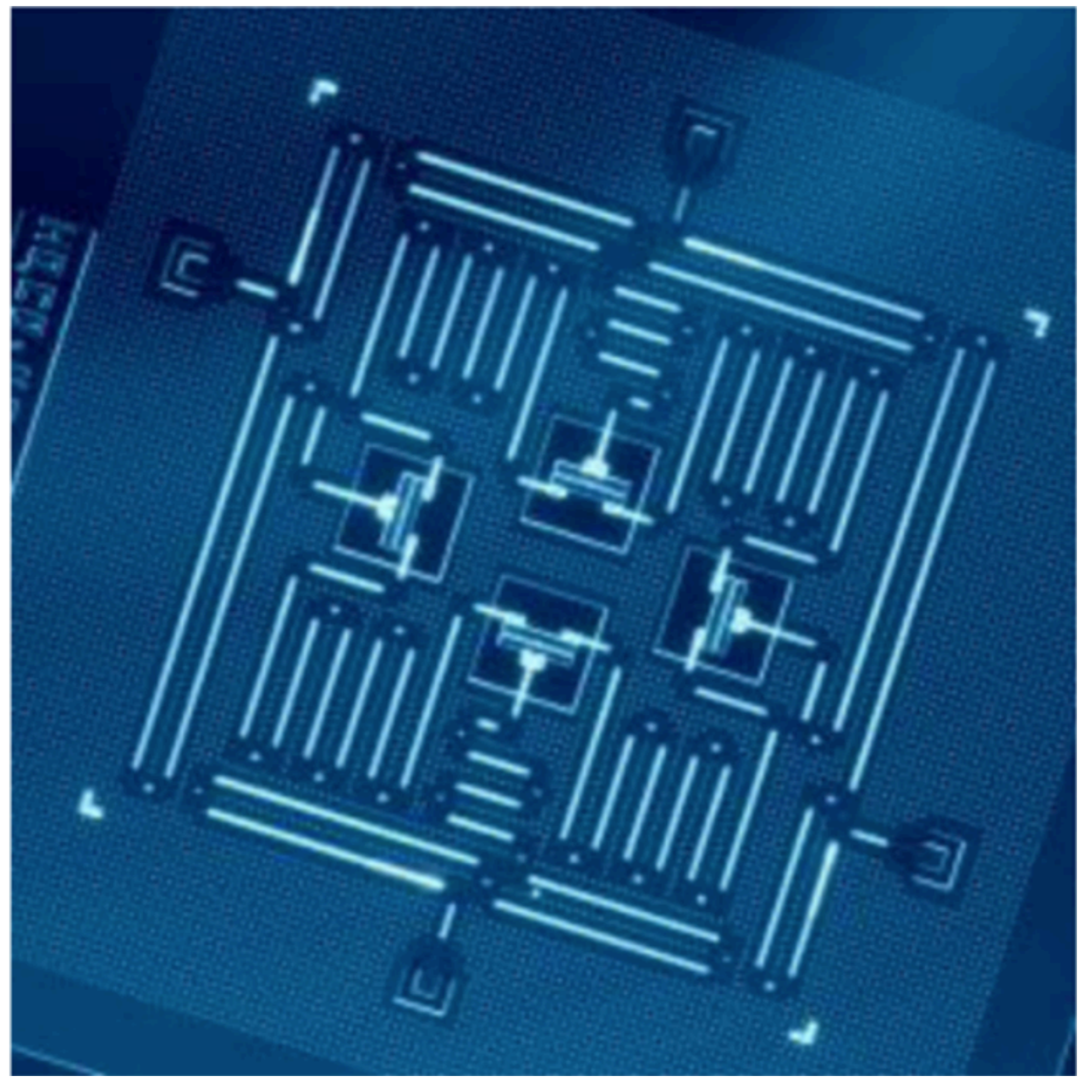
Qubits





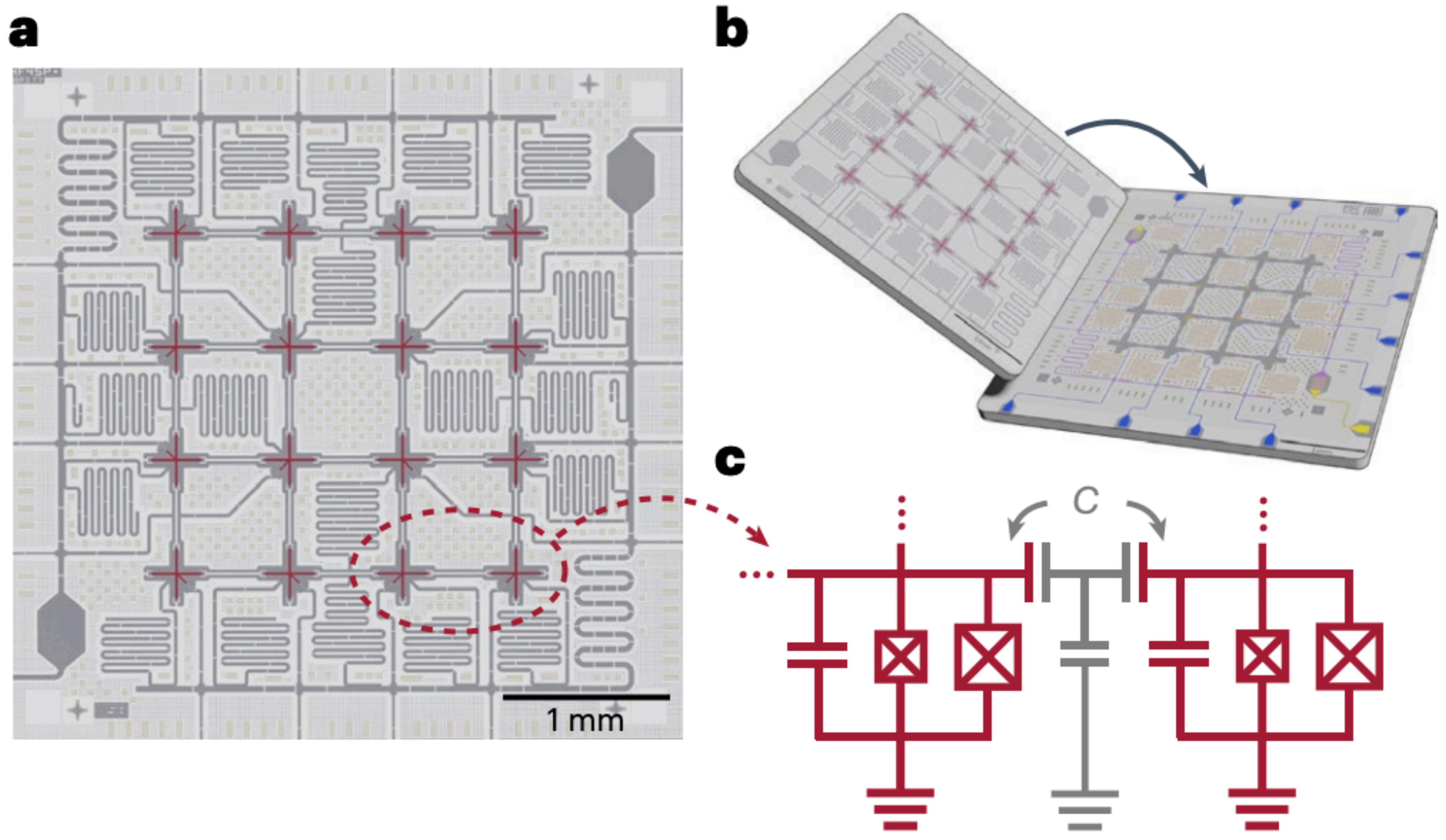
Somoroff et. al. PRL 130, 267001 (2023)

Quantum computers



Gambetta et. al.
npj Quantum Information (2017)

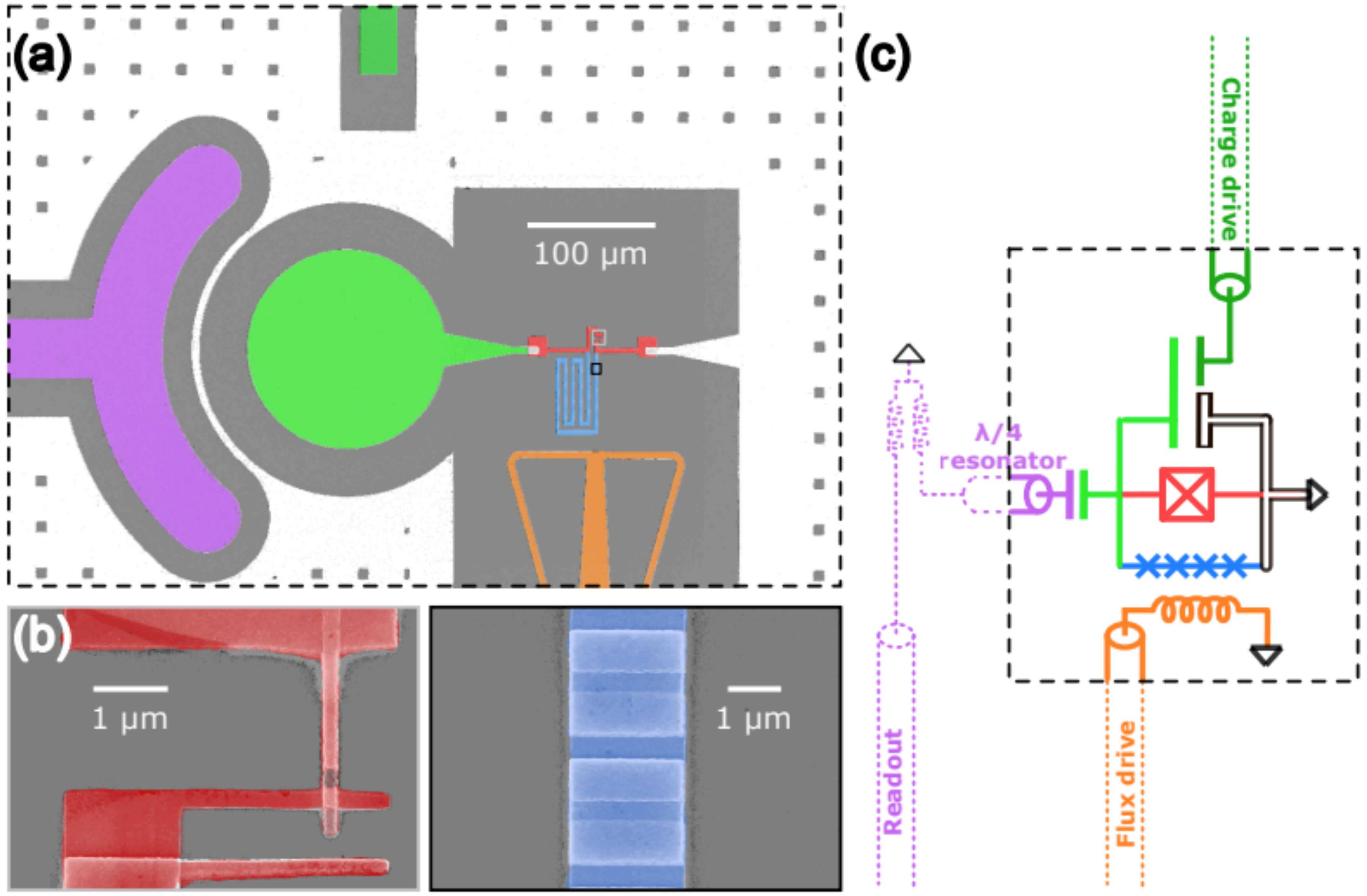
Quantum Simulators



Rosen et. al. Nature Physics 20 (2024)

Superconducting quantum circuits:

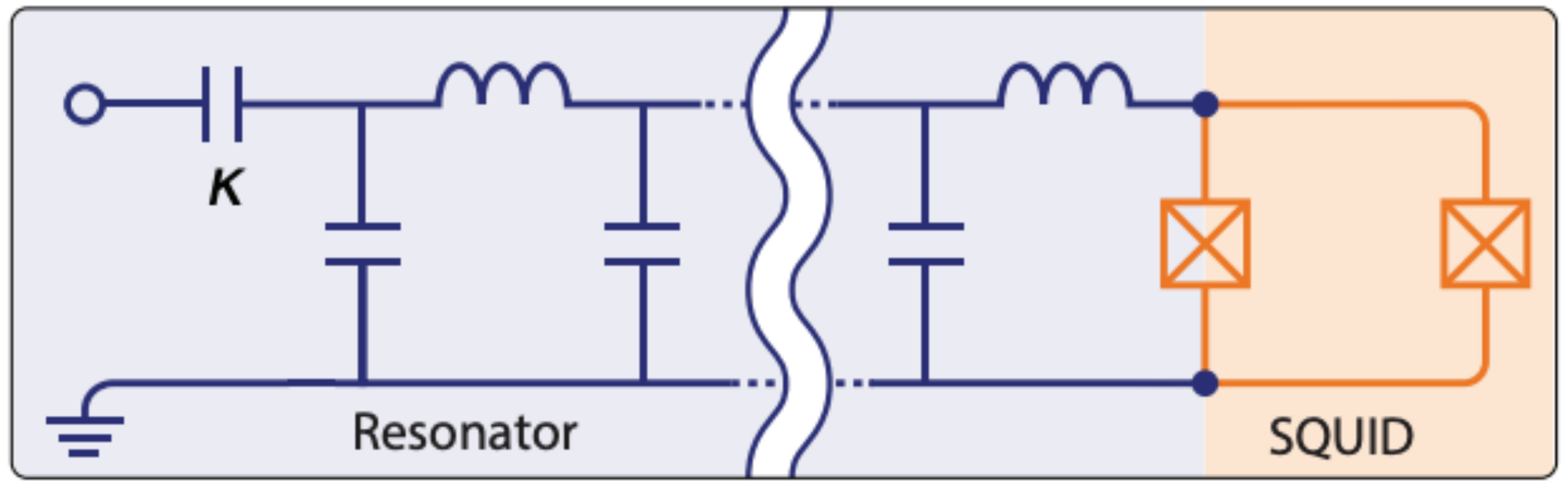
Quantum sensors



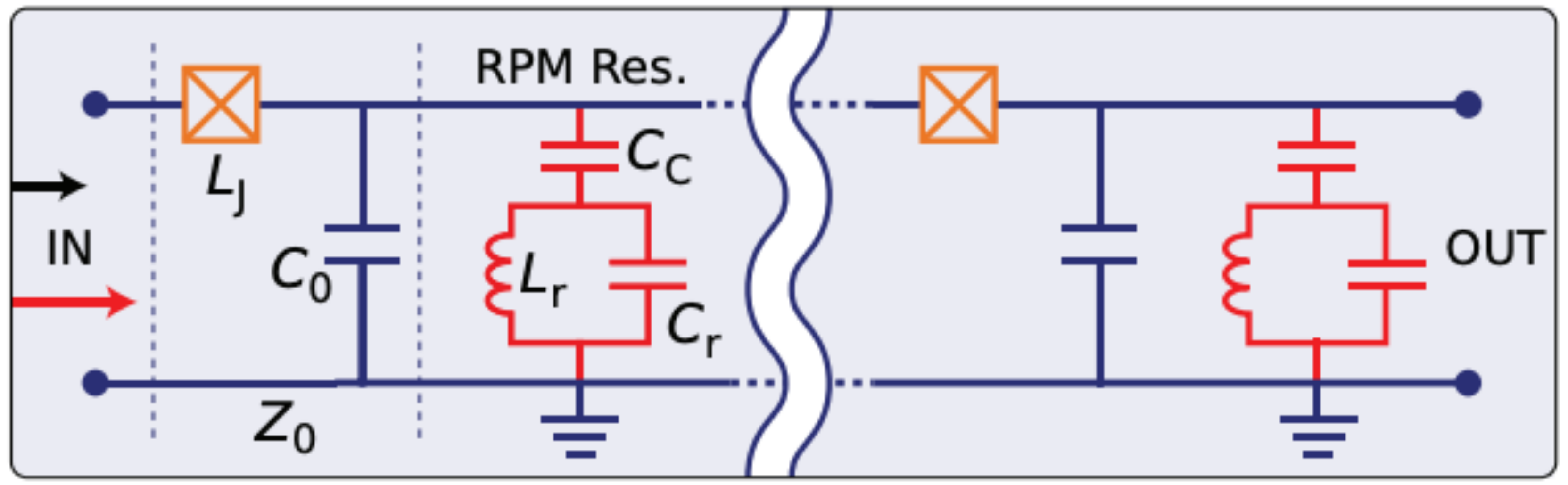
Najera-Santos et. al. Phys. Rev. X **14**, 011007 (2024)

Quantum amplifiers

(a) Josephson parametric amplifier (JPA)



(a) Josephson traveling wave parametric amplifier (JTWPA)



Krantz et. al. Applied Physics Reviews 6, 021318 (2019)

Circuit quantization:

(Devoret, Les Houches 1995)

position coordinate: θ_x

momentum coordinate: n_x

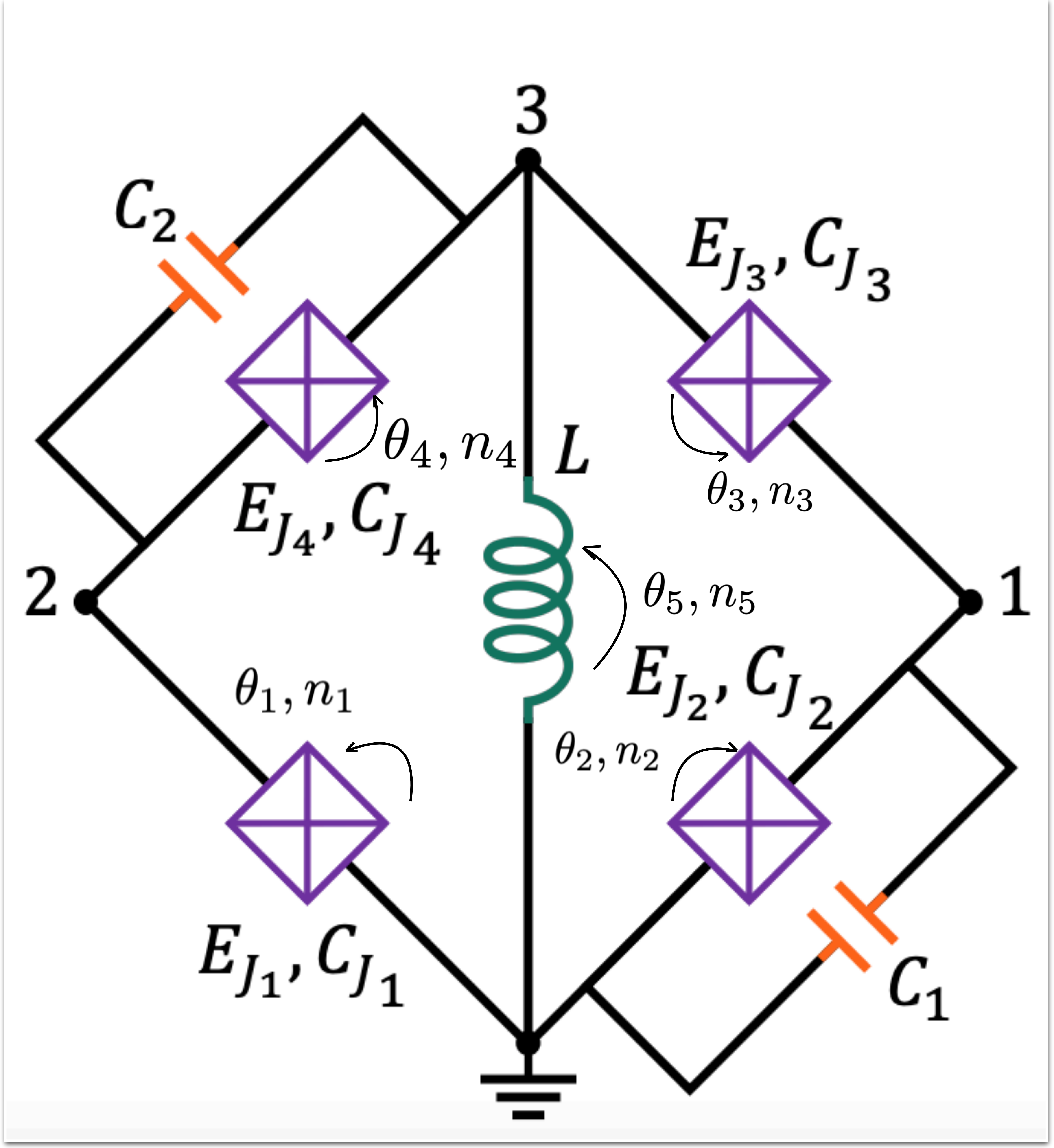
quantum condition: $[\theta_x, n_y] = i\delta_{xy}$

capacitor: $\Delta E \sim \frac{e^2}{2C} n^2$

JJ: $\Delta E \sim -E_J \cos\theta$

inductor: $\Delta E \sim \frac{1}{2} E_L \theta^2$

Hamiltonian: $H = (2e)^2 n^T C^{-1} n + U(\theta)$

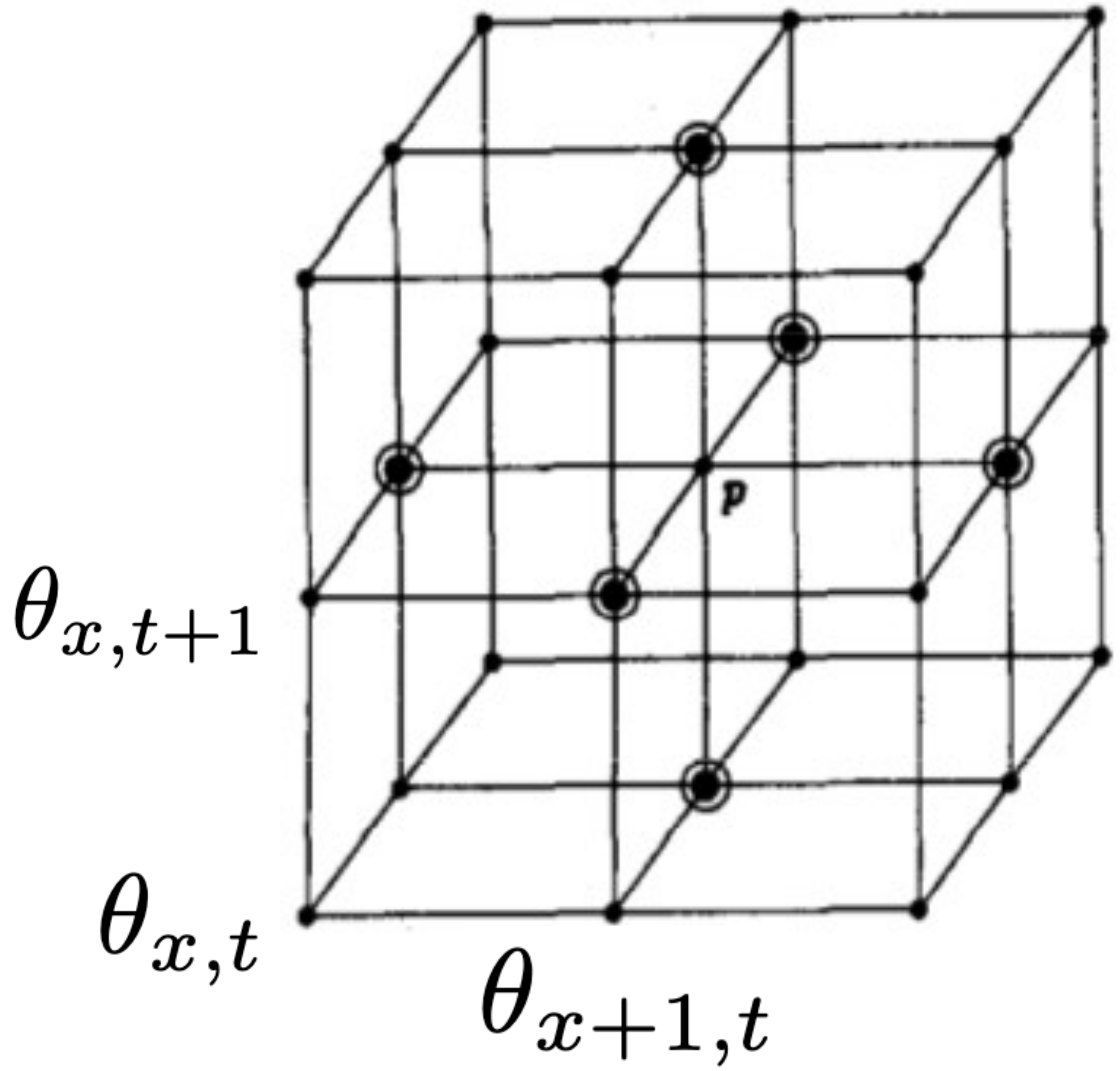


Quantum circuits:

$$\langle \mathcal{O} \rangle = \frac{\mathrm{tr}\left(e^{-\beta H}\mathcal{O}\right)}{\mathrm{tr}\left(e^{-\beta H}\right)} = \frac{\int D\theta \; e^{-S(\theta)}\mathcal{O}(\theta)}{\int D\theta \; e^{-S(\theta)}}$$

$$= \langle \mathcal{O} \rangle_{\text{exact}} + O(\Delta t) \quad , \quad \beta^{-1} = k_B T$$

$$\Delta t = \beta / N_t$$



where $$S = \Delta t \Big[\frac{1}{2}\left(\frac{1}{2e}\right)^2 \sum_{x,y,t} \frac{(\theta_{t+1,x} - \theta_{t,x})}{\Delta t} C_{xy} \frac{(\theta_{t+1,y} - \theta_{t,y})}{\Delta t} + \sum_t U(\vec{\theta}_t) \Big]$$

Correlator method:

Correlators: $\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \frac{\int D\theta\ e^{-S}\mathcal{O}(t)\mathcal{O}(0)}{\int D\theta\ e^{-S}}$

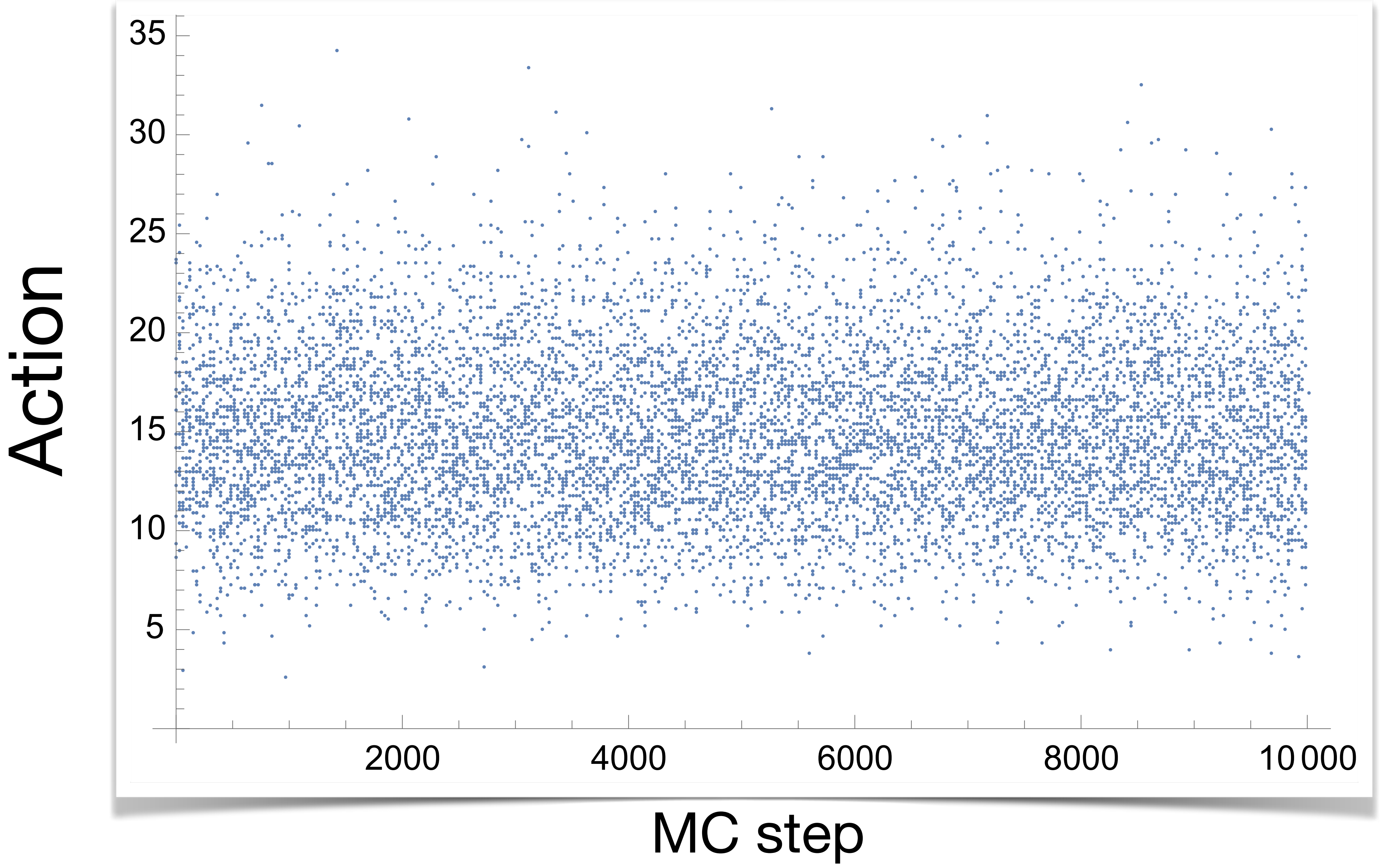
1. Make it cold: $\lim_{\beta\to\infty}\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \to \langle 0|\mathcal{O}(t)\mathcal{O}(0)|0\rangle$

2. Fit: $\langle 0|\mathcal{O}(t)\mathcal{O}(0)|0\rangle = \sum_m e^{-t(E_m-E_0)}\left|\langle m|\mathcal{O}|0\rangle\right|^2 \longrightarrow e^{-\Delta E^* t}|\langle m^*|\mathcal{O}|0\rangle|^2$

Example:

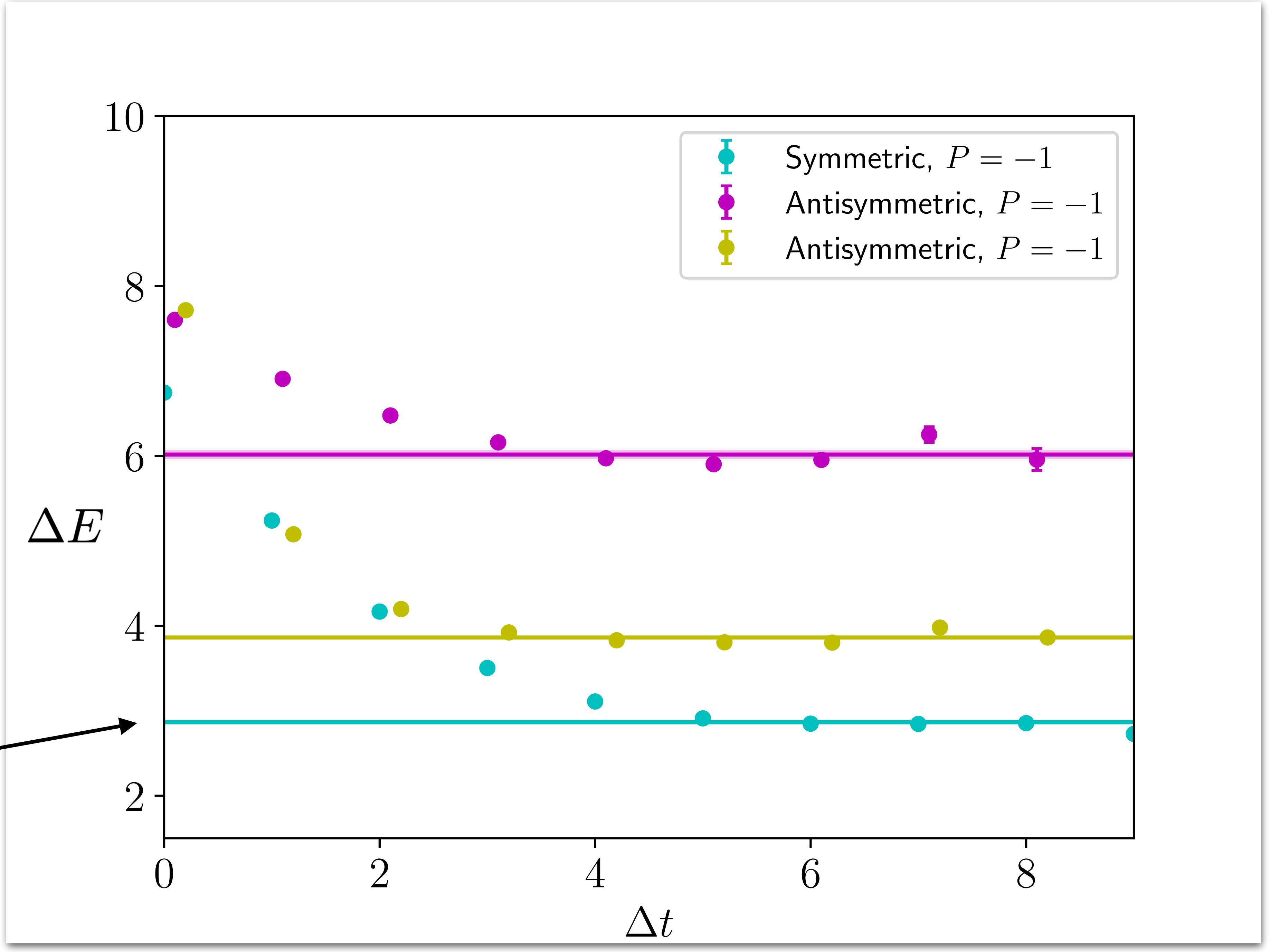
$$H = \sum_{x=1}^{2} \left(4E_C \hat{n}_x^2 - E_J \cos\theta_x\right) - E_J^b \cos\left(\theta_1 + \theta_2 - \varphi_{\text{ext}}\right)$$

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \longrightarrow e^{-\Delta E^* t} |\langle m^* | \mathcal{O} | 0\rangle|^2$$



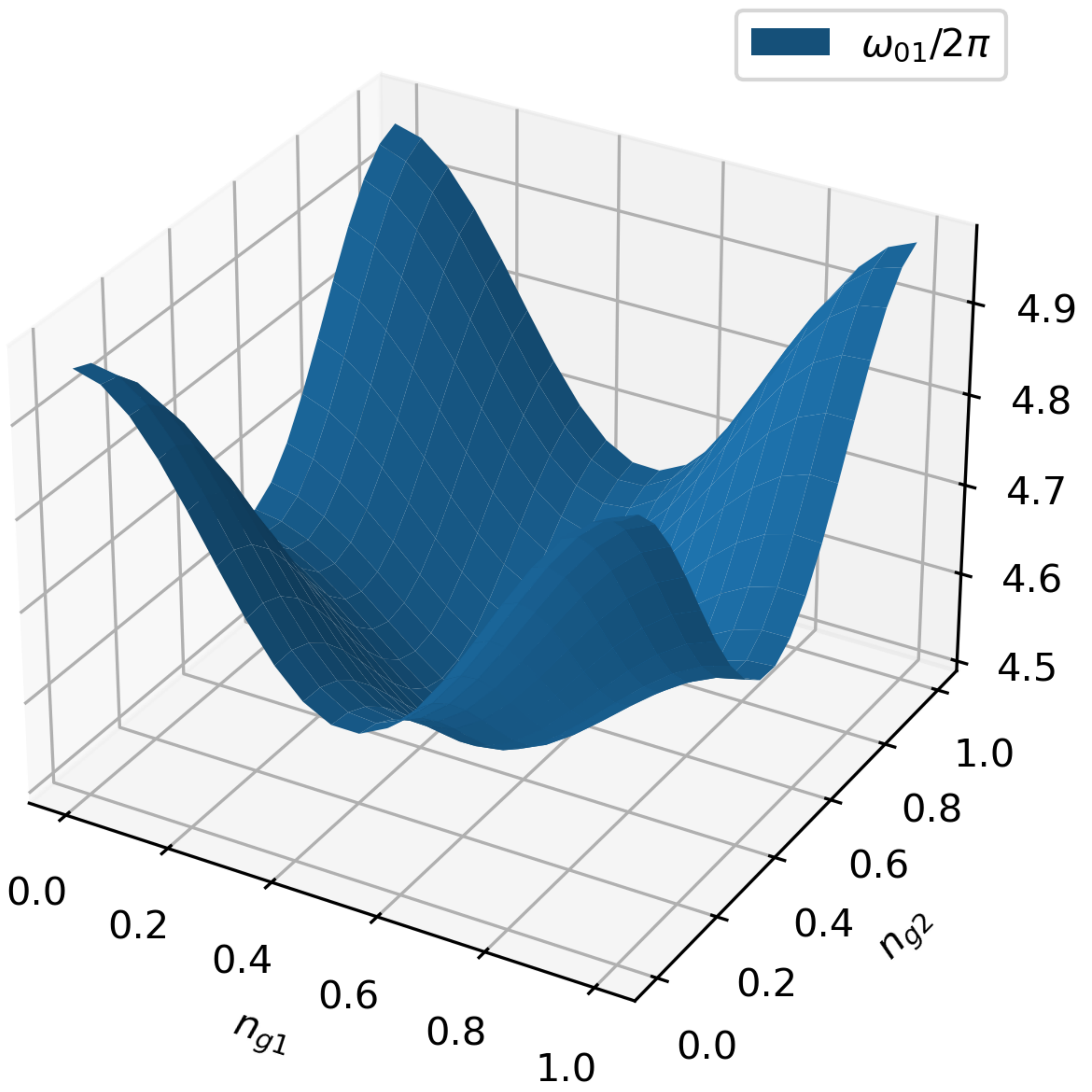
	exact	lattice
$E_1 - E_0$	3.001	3.0(1)
$E_2 - E_0$	3.946	3.9(1)
$E_3 - E_0$	5.715	6.0(2)
$\langle 1 \| \mathcal{O} \| 0\rangle$	0.445	0.46(3)

$$\mathcal{O} = \sum_{x=1,2} \sin\theta_x$$

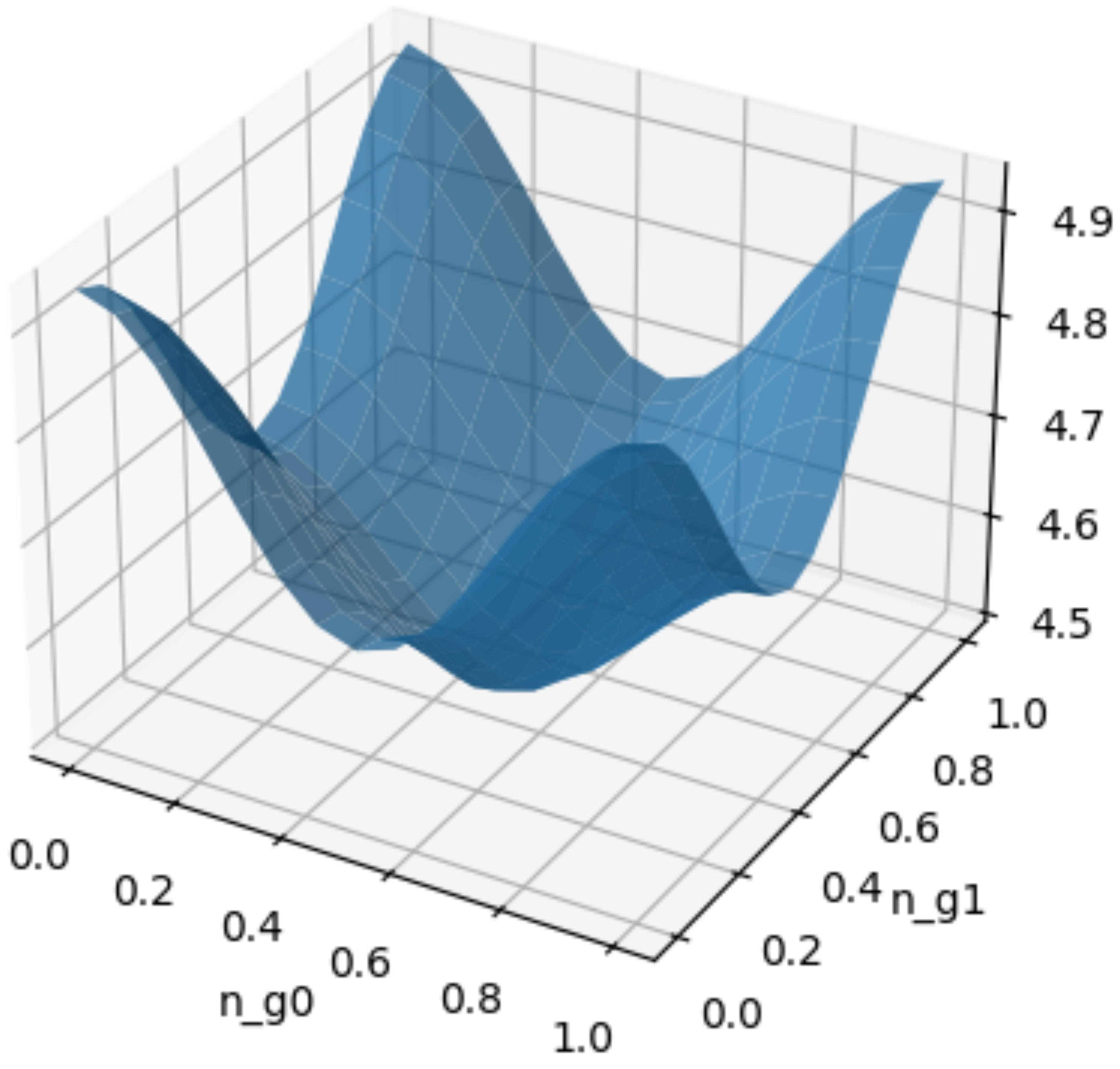


Example:

$$H = \sum_x 4E_C(n_x - n_{gx})^2 - E_J\cos(\theta_x) - E_J^b\cos(\theta_1 + \theta_2 - \varphi_{\text{ext}})$$

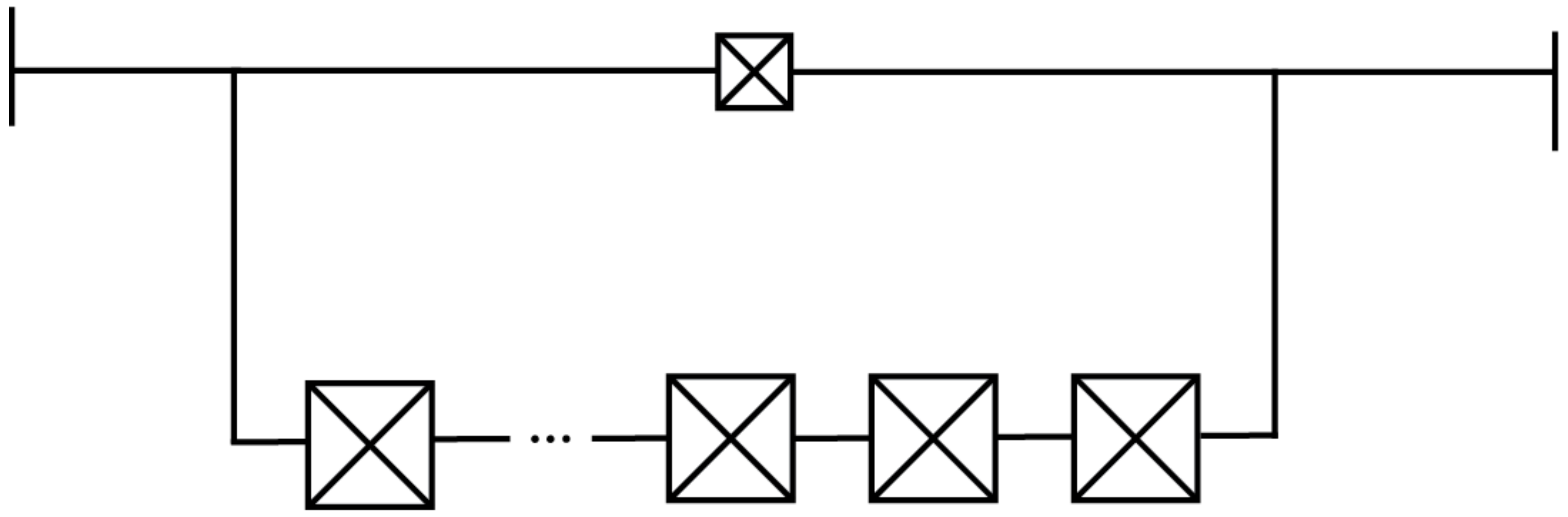


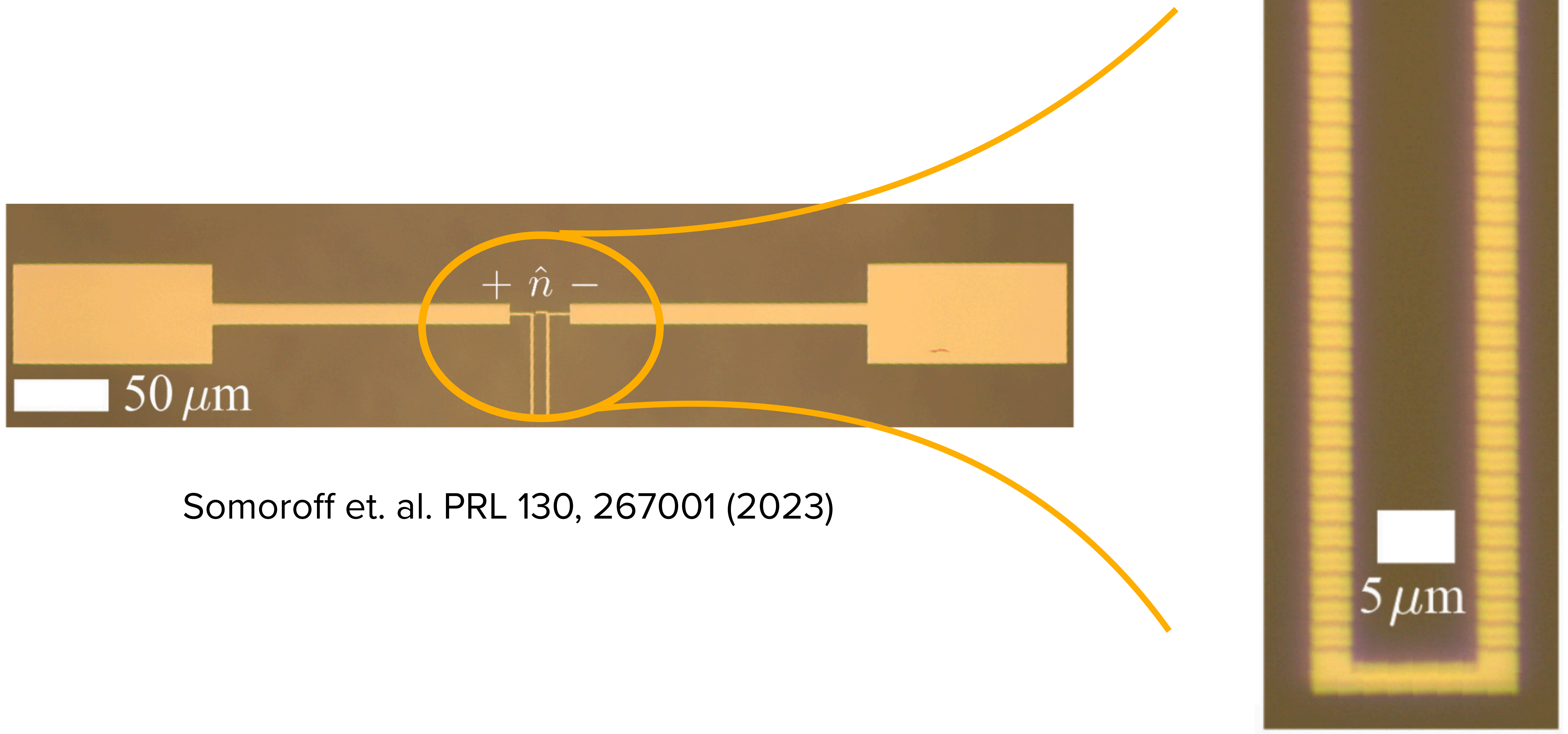
PDE Solver



Lattice

Fluxonium:



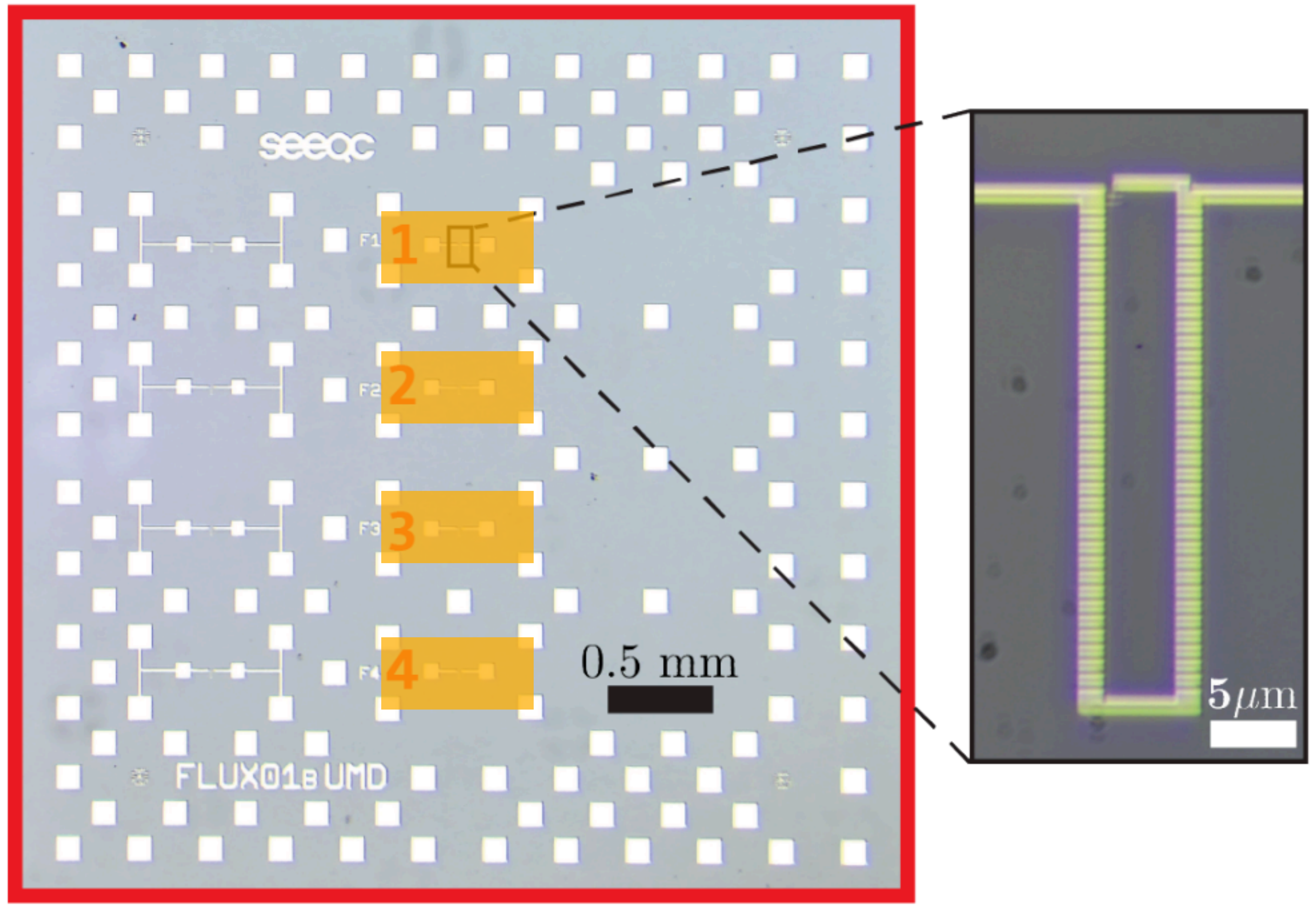


Somoroff et. al. PRL 130, 267001 (2023)

	T2	anharm
fluxonium	~1.0 ms	>100%
transmon	~0.1 ms	5%

T2:
Somoroff et al PRL 130, 267001 (2023), [arxiv:2106.11352]

anharmonicity:
Oliver et. al. PRX 031035, Schoelkopf PRA 76 (2007)



Somoroff et. al. [arxiv:2303.01481] (2023)

Recent progress in transoms here: [*npj Quantum Information*](#) **volume** **10**, Article number: 78 (2024)

Fluxonium:

$$H = (2e)^2 \sum_{xy} (n_x - n_{gx}) C^{-1}_{xy} (n_y - n_{gy}) - E_J^a \sum_x \cos\theta_x - E_J^b \cos\left(\sum_x \theta_x - \varphi_{\text{ext}}\right)$$

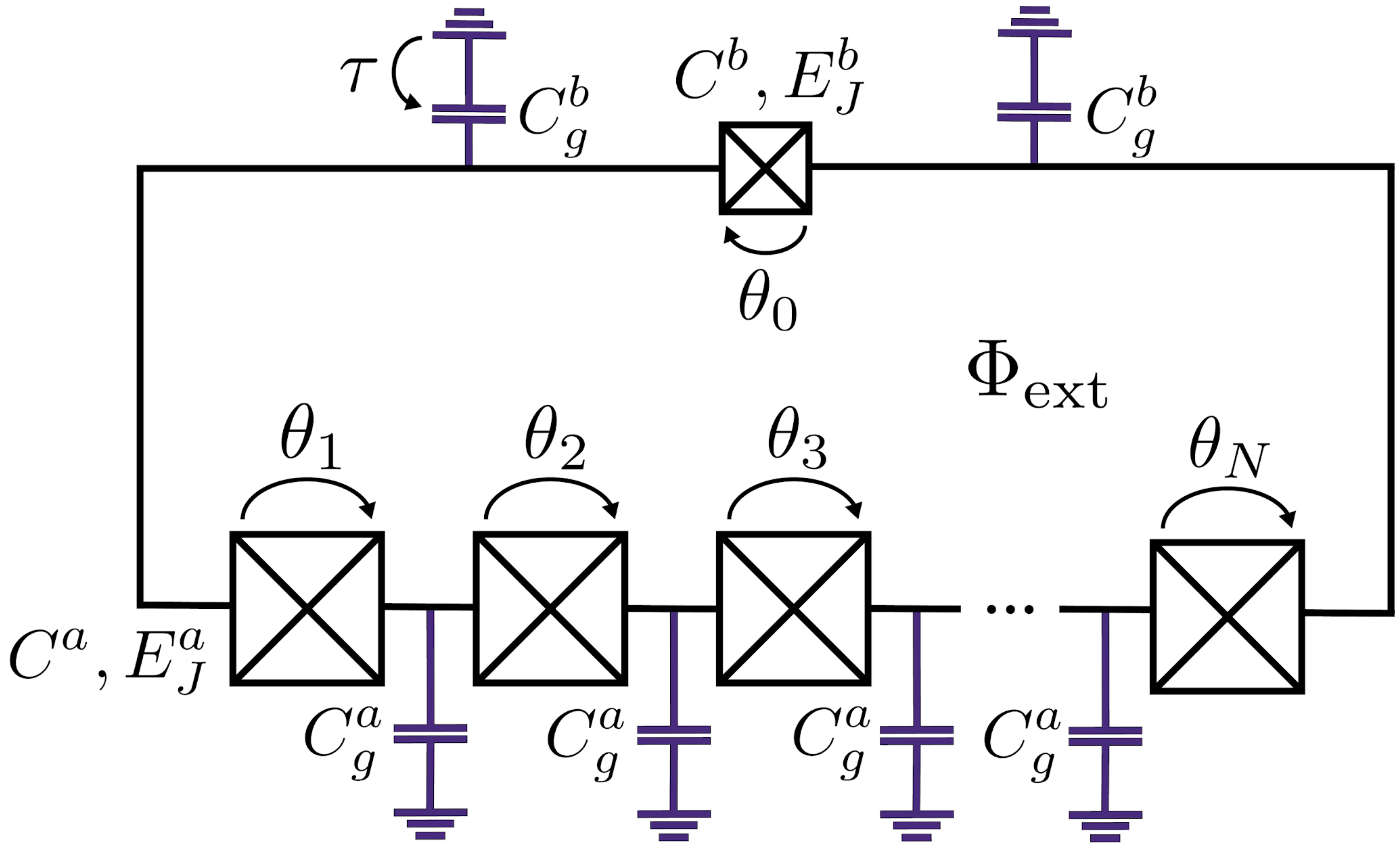
Tunable parameters:

$$N, C^a, C^b, C_g^a, C_g^b, E_J^a, E_J^b$$

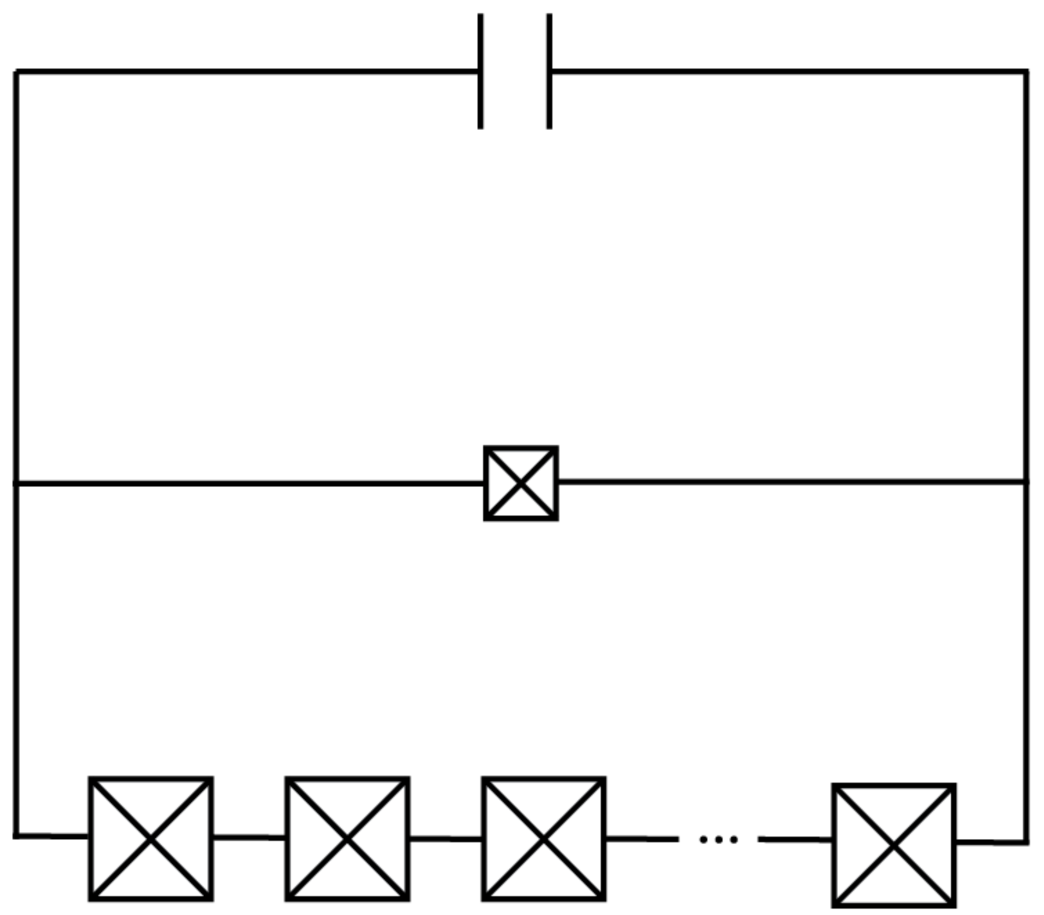
Fabrication:

$$C = S_c A$$

$$E_J = \frac{\Phi_0}{2\pi} j_c A$$



Application:



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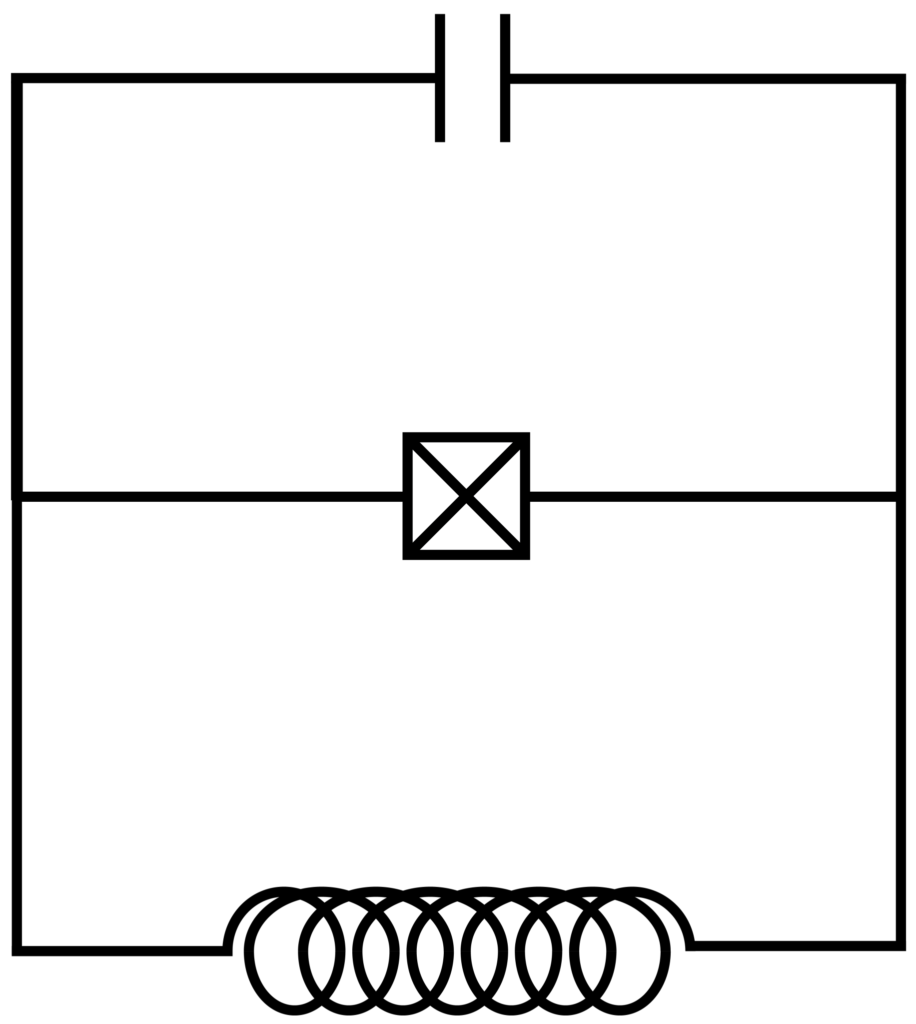


+ ‘small’

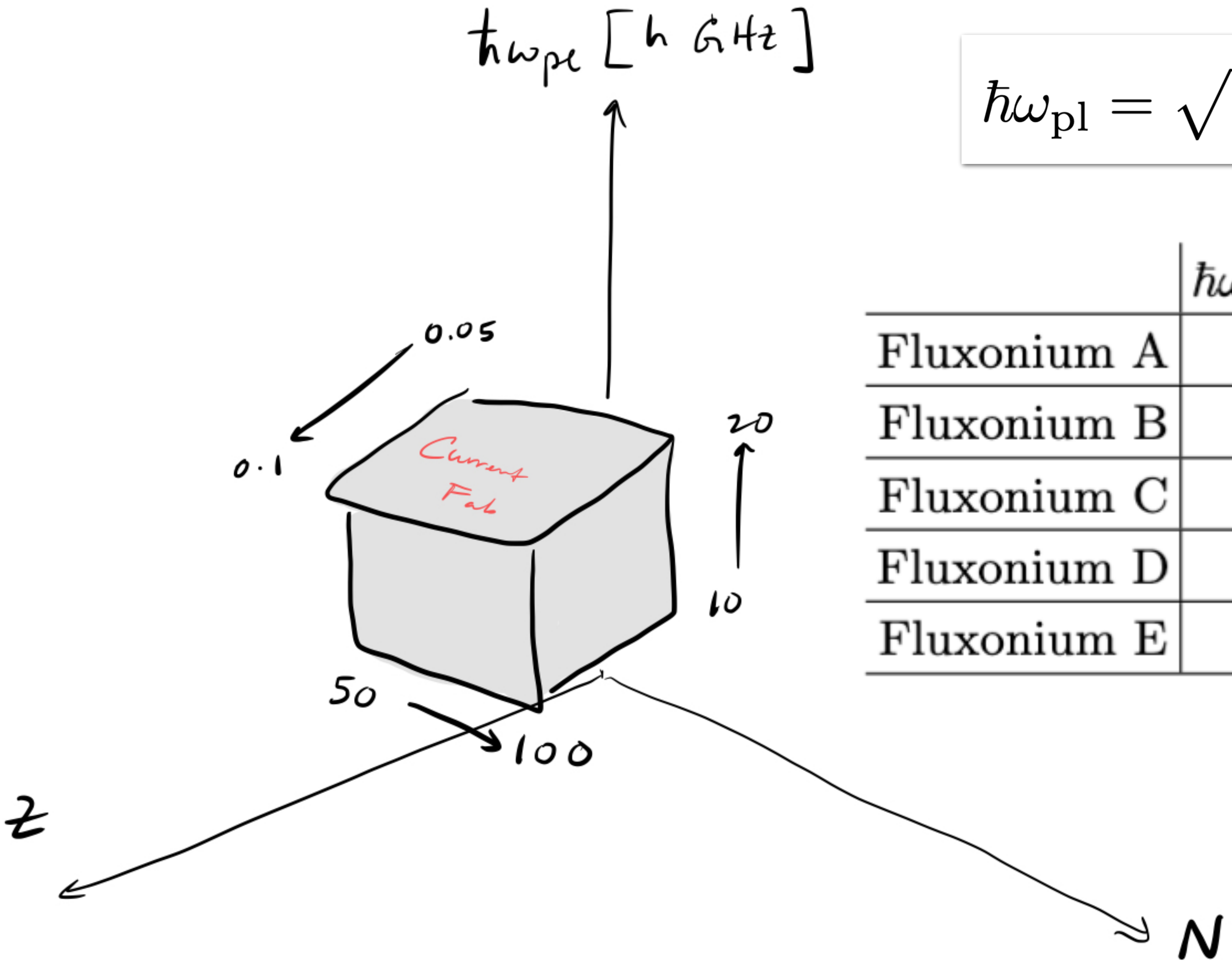
$$H(\theta_1, ..., \theta_N) = \left\{4E_C n^2 + \frac{1}{2}E_L\varphi^2 - E_J\cos(\varphi - \varphi_{\text{ext}})\right\} + \Delta H$$

Typical fabrication:

$$\hbar\omega_{\rm pl} = \sqrt{8E_C^a E_J^a}$$

$$z = \pi^{-1}\sqrt{2E_C^a/E_J^a}$$

N



	$\hbar\omega_{\rm pl}$ [h GHz]	z	N	source
Fluxonium A	8.18	0.06	40	Manucharyan et. al. Science 326, 113-116 (2009)
Fluxonium B	13.4	0.09	43	Manucharyan et. al. Phys. Rev. B **85**, 024521 (2012)
Fluxonium C	17.4	0.07	43	Manucharyan et. al. Phys. Rev. B **85**, 024521 (2012)
Fluxonium D	N/A	N/A	102	Ding et. al. Phys. Rev. X 13, 031035 (2023)
Fluxonium E	N/A	N/A	102	Ding et. al. Phys. Rev. X 13, 031035 (2023)

1. What are “safe” directions in parameter space?

2. What is out there?

Coherence:

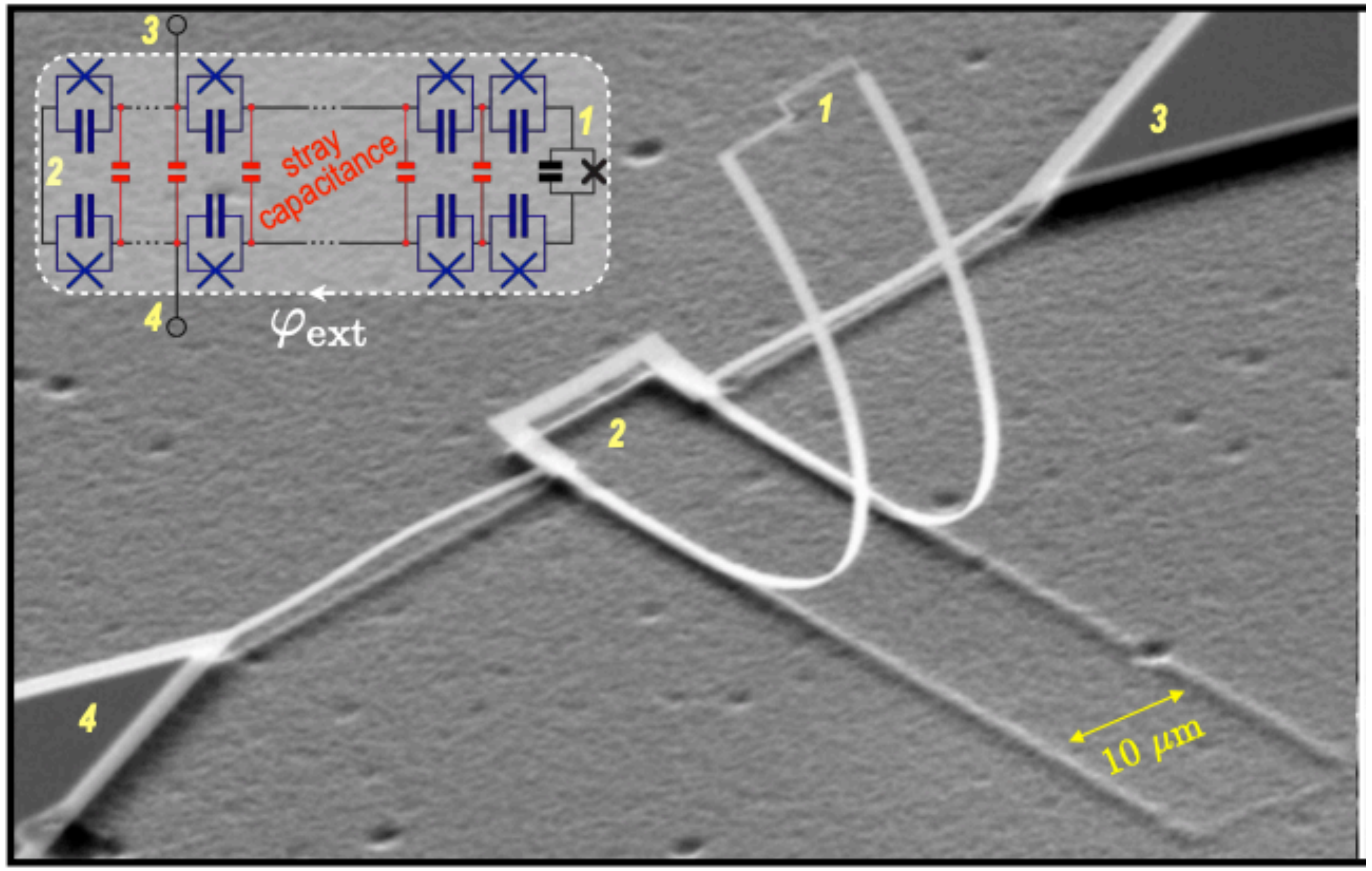
Charge noise in array can limit coherence

$$\Gamma_1 = \Gamma_1^{\text{charge}} + \Gamma_1^{\text{flux}} + \ldots$$

$$\Gamma_\varphi = \Gamma_\varphi^{\text{charge,1st}} + \Gamma_\varphi^{\text{flux,1st}} + \ldots$$

$$\propto \frac{d\omega_{01}}{dn_g} = 1/T_\varphi$$

(a)



Pechenezhskiy et. al. Nature 585, 368 (2020)

$$H = (2e)^2 \sum_{xy} (n_x - n_{gx}) C^{-1}_{xy} (n_y - n_{gy}) - E_J^a \sum_x \cos\theta_x - E_J^b \cos\left(\sum_x \theta_x - \varphi_{\text{ext}}\right)$$

Coherence:

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$$\Gamma_\varphi = \Gamma_\varphi^{\text{charge,1st}} + \Gamma_\varphi^{\text{flux,1st}} + \dots$$

↑

$$\propto \frac{d\omega_{01}}{dn_g} = 1/T_\varphi$$

<u>Outstanding questions:</u>

1. Dependence on $z = \pi^{-1}\sqrt{2E_C^a/E_J^a}$?

2. Dependence on Cg ?

Coherence:

$$\Gamma_1 = \Gamma_1^{\text{charge}} + \Gamma_1^{\text{flux}} + \dots$$

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↑

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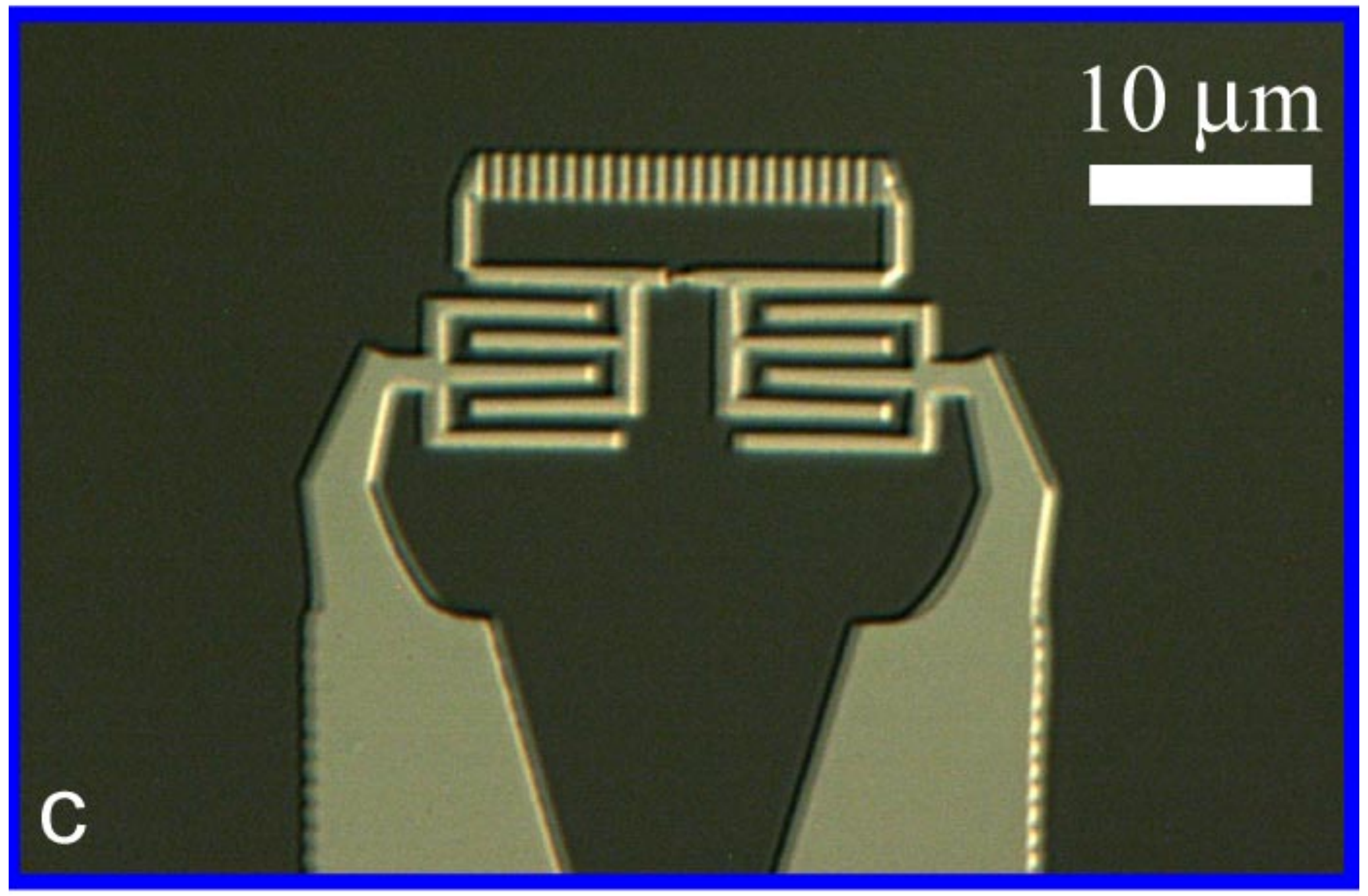
Outstanding questions:

1. Dependence on $z = \pi^{-1}\sqrt{2E_C^a/E_J^a}$?

2. Dependence on Cg ?

Dependence on z:

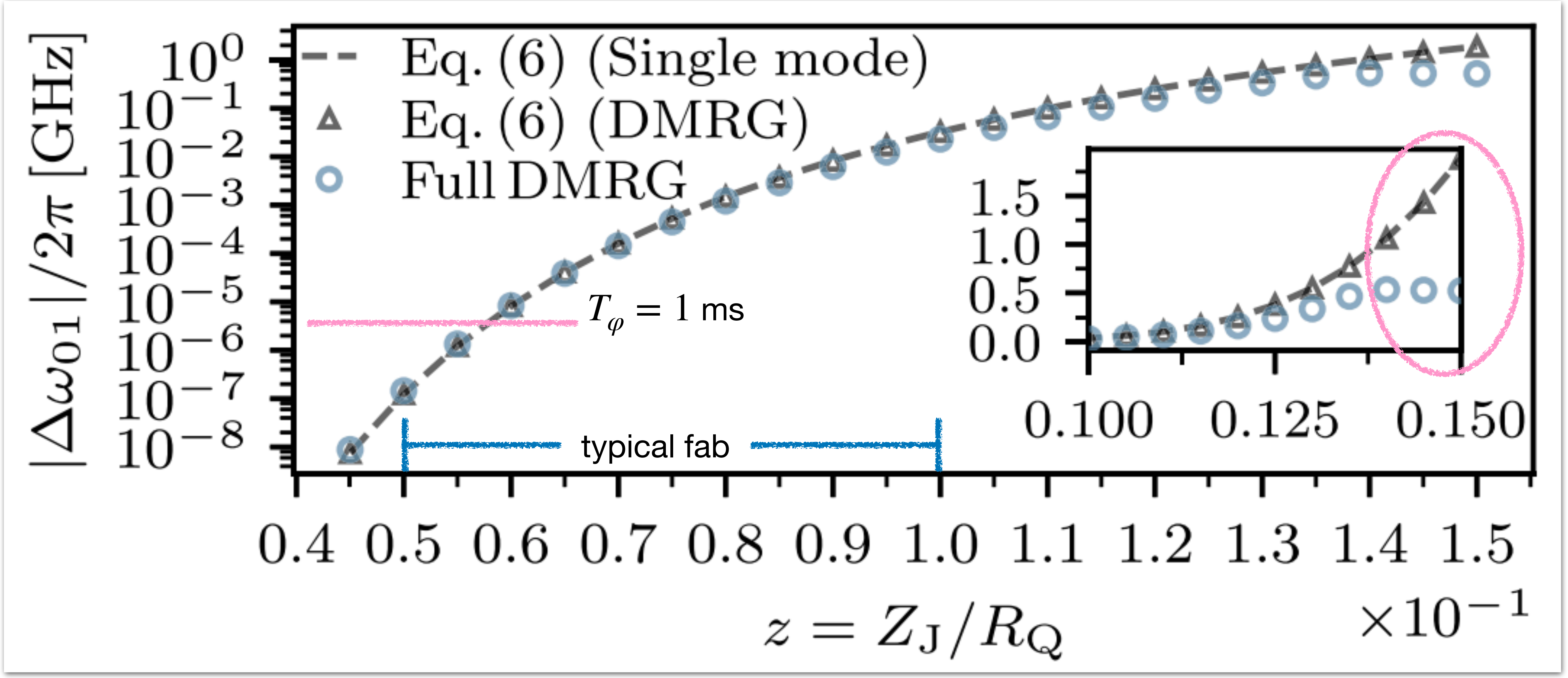
N = 43 device



Manucharyan et. al. Phys. Rev. B 85, 024521 (2012)

$$\hbar\omega_{\rm pl} = 13.4 \text{ h GHz}$$
$$z = 0.09$$

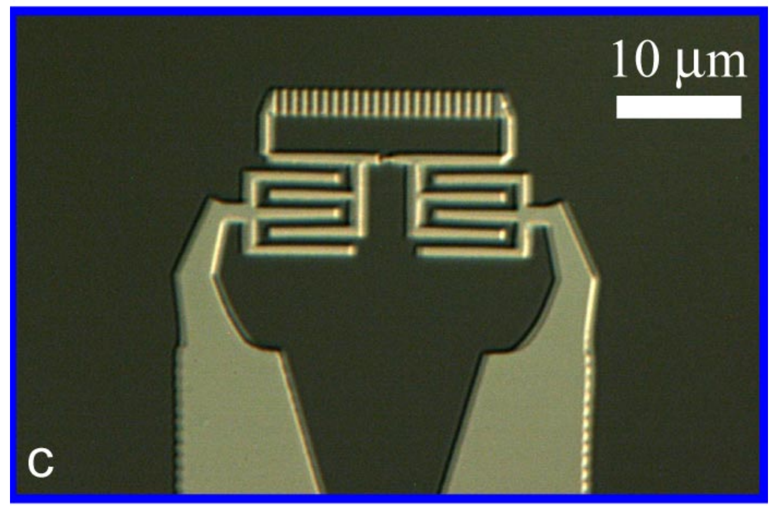
Tensor network simulations @ N=40

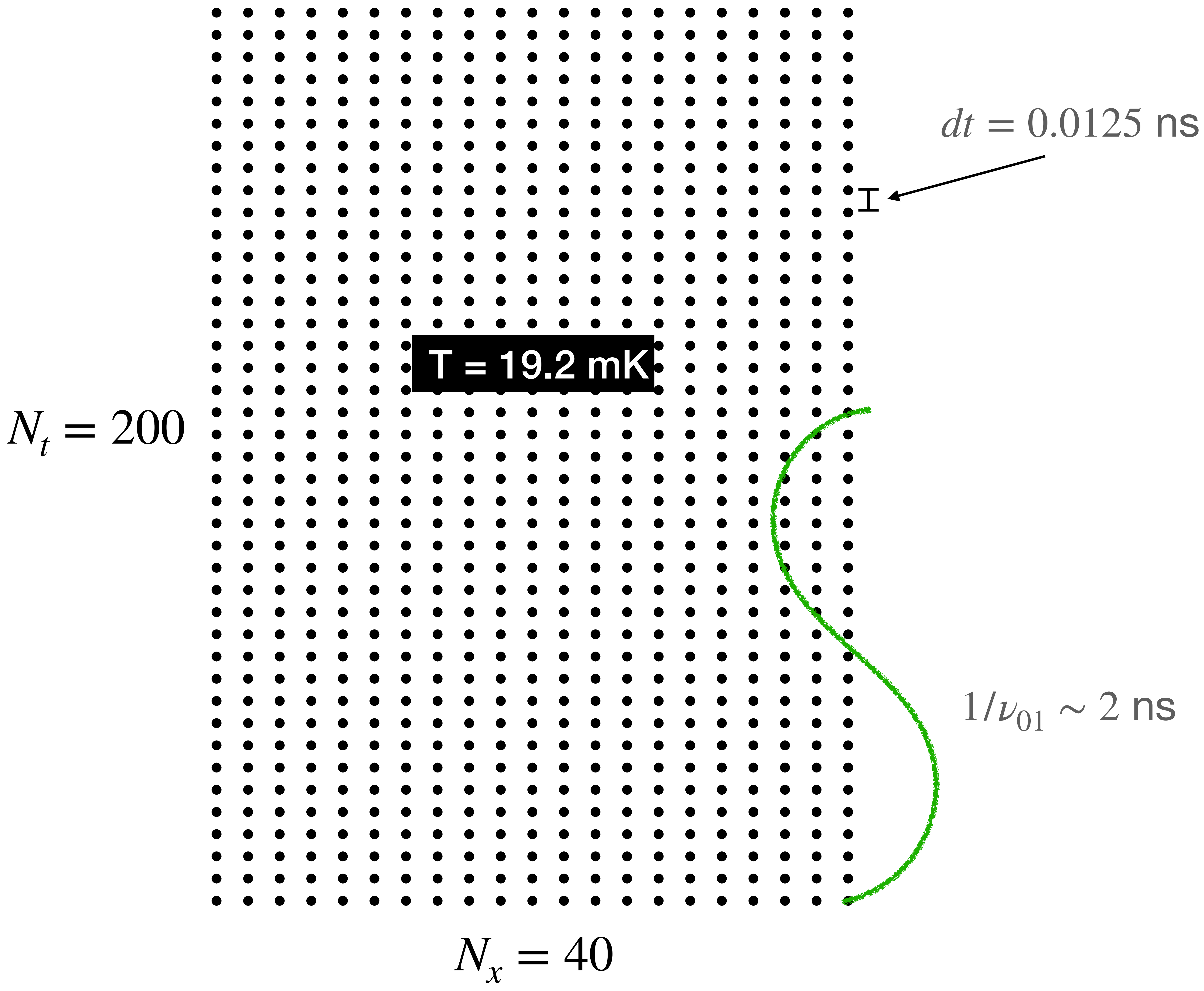


di Paolo et. al. npj Quantum Information volume 7, 11 (2021)

Parameters: CJb=7.5 fF, EJb = 8.9 h GHz, ω_p/2π = 12.5 h GHz, and Cg = 0

Lattice simulation:





Small junction:

$C_b = 7.5$ fF, $E_J^b = 8.9$ h GHz

Array:

N=40

$z = 0.14,\ \hbar\omega_{pl} = 12.5$ h GHz

$C_g = 0$ fF

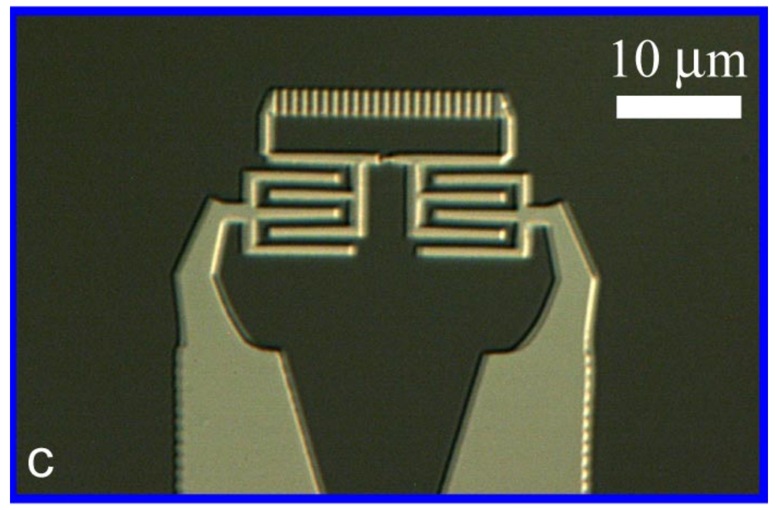
Sims:

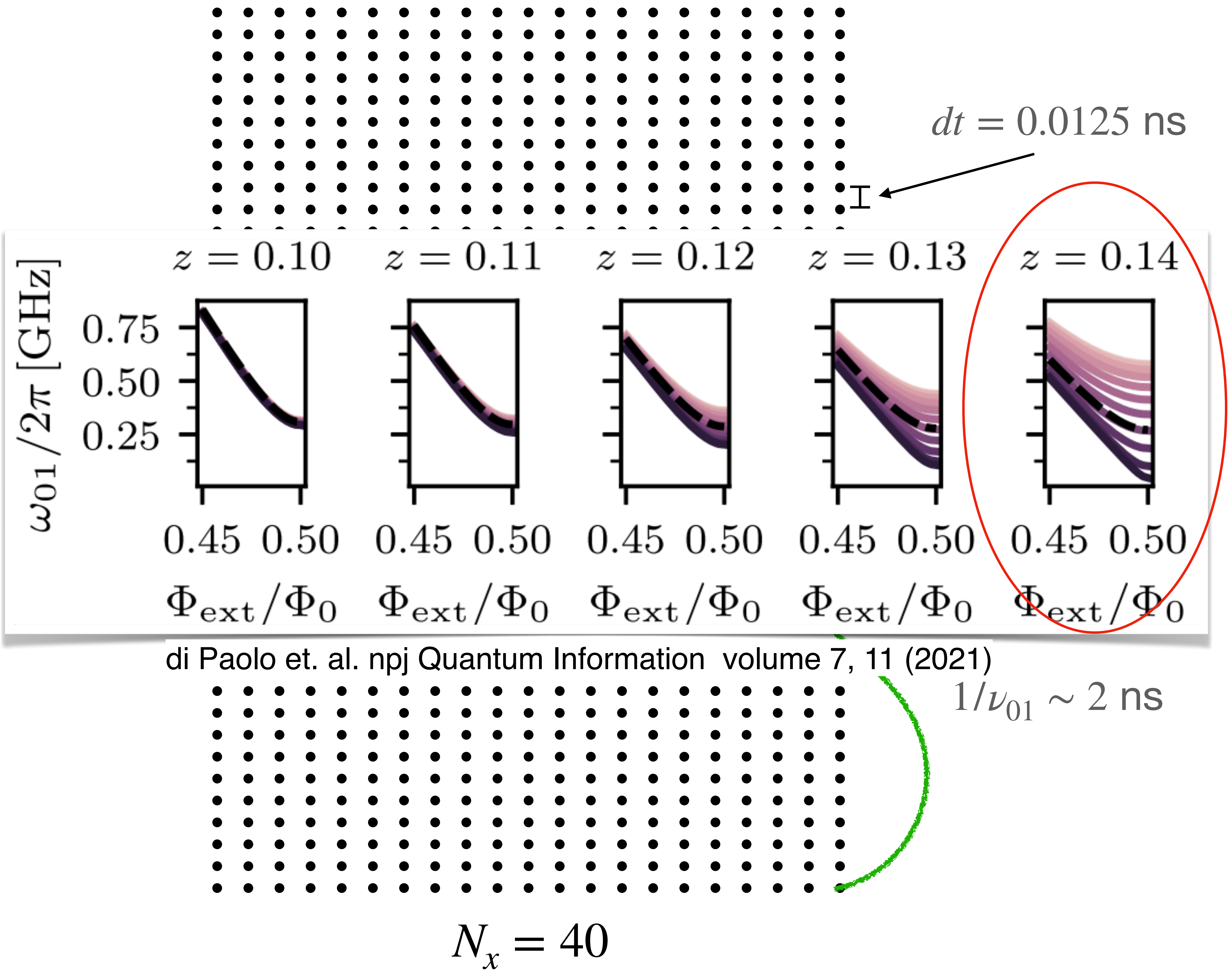
24 hours

1 node with 8 a100 GPUs

3.7 million measurements (that's a lot)

Lattice simulation:





di Paolo et. al. npj Quantum Information volume 7, 11 (2021)

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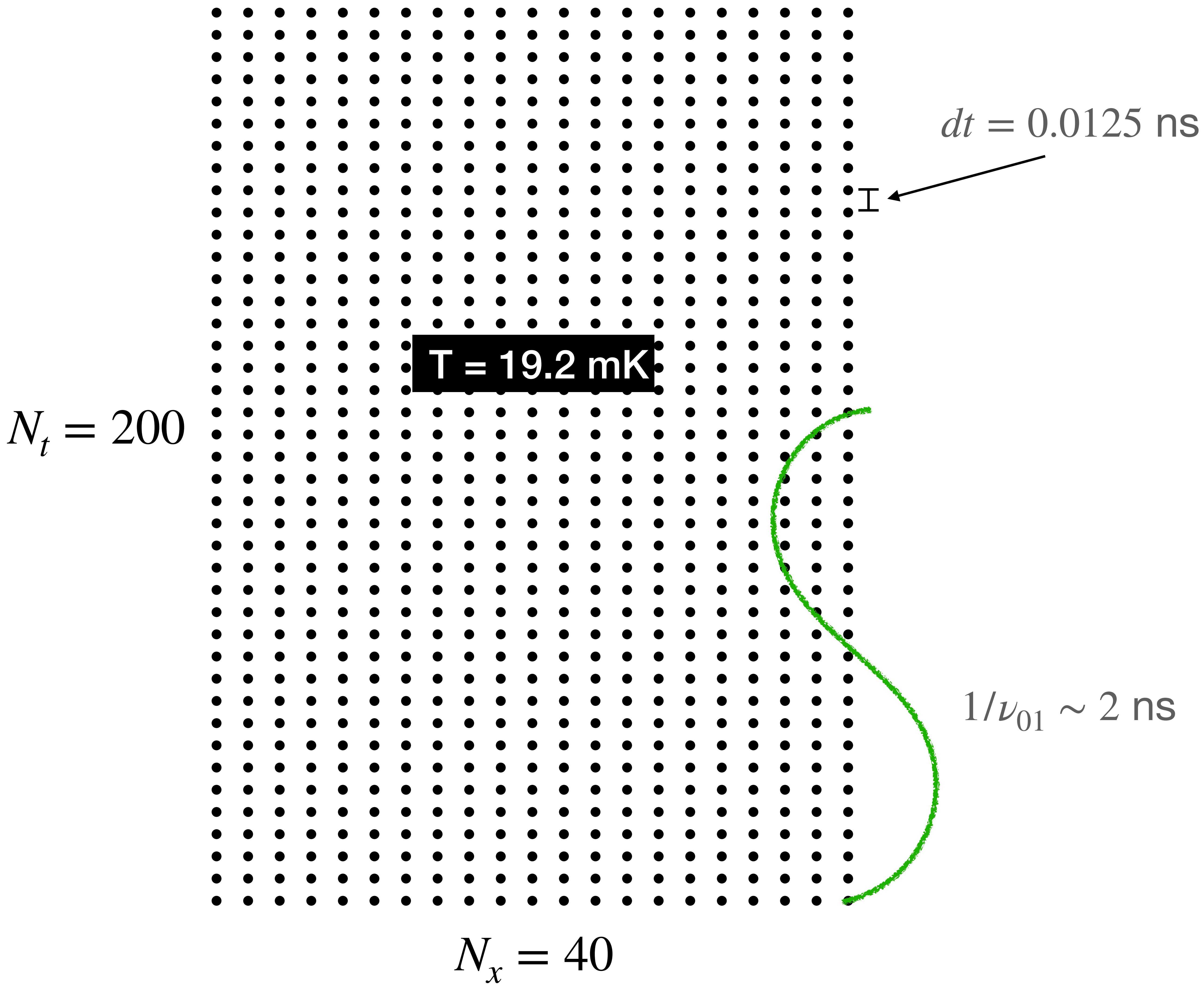
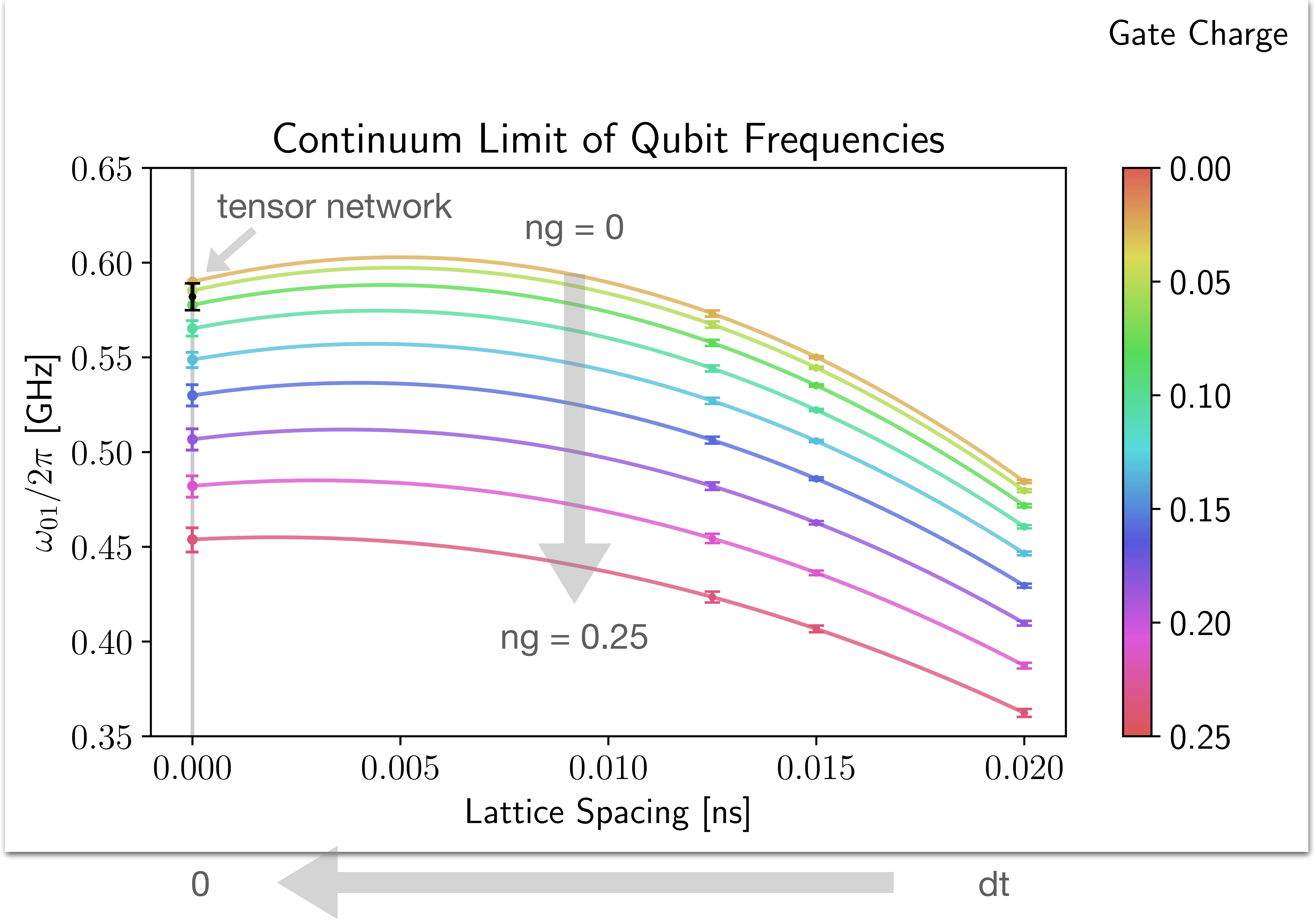
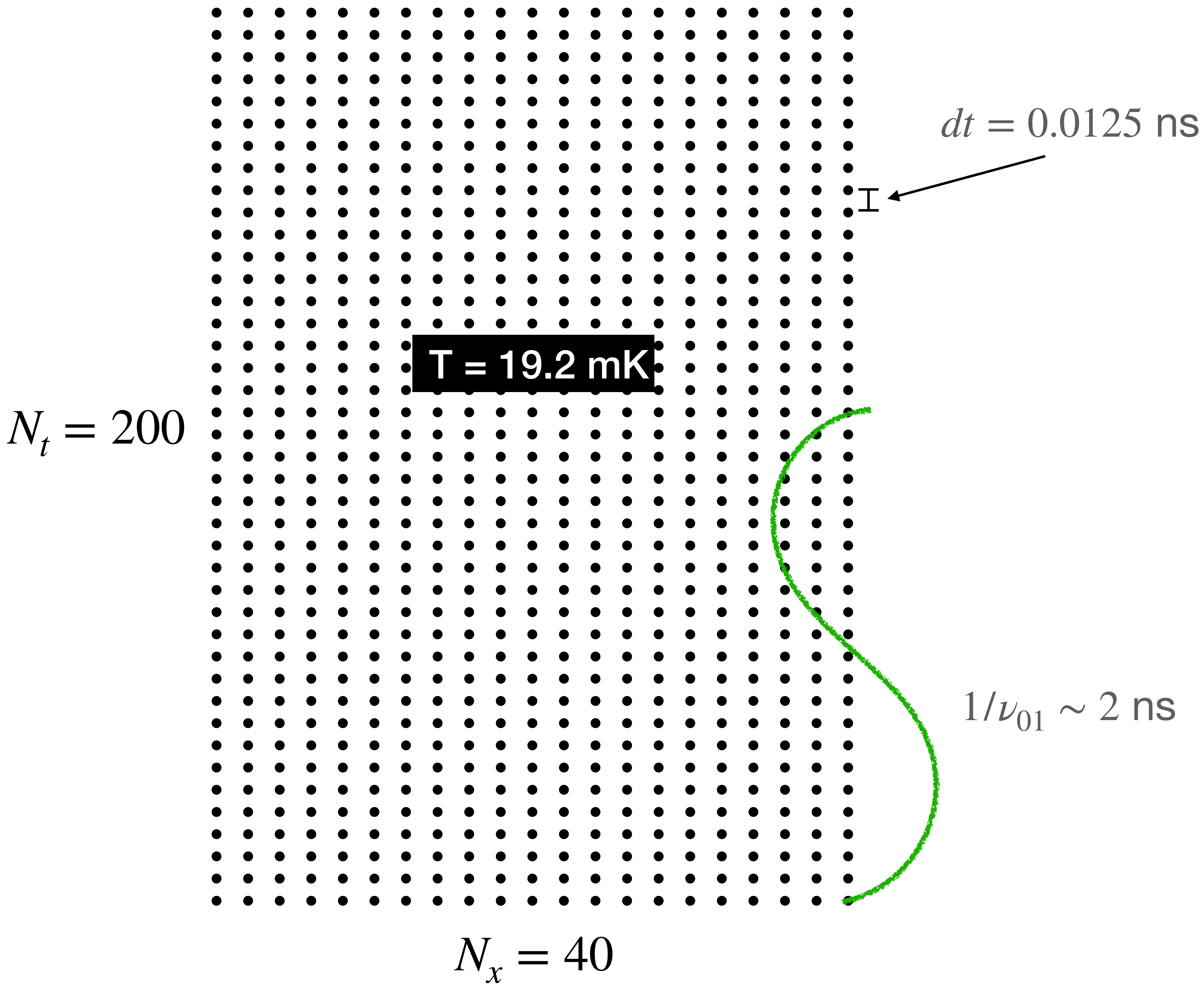
Lattice simulation:

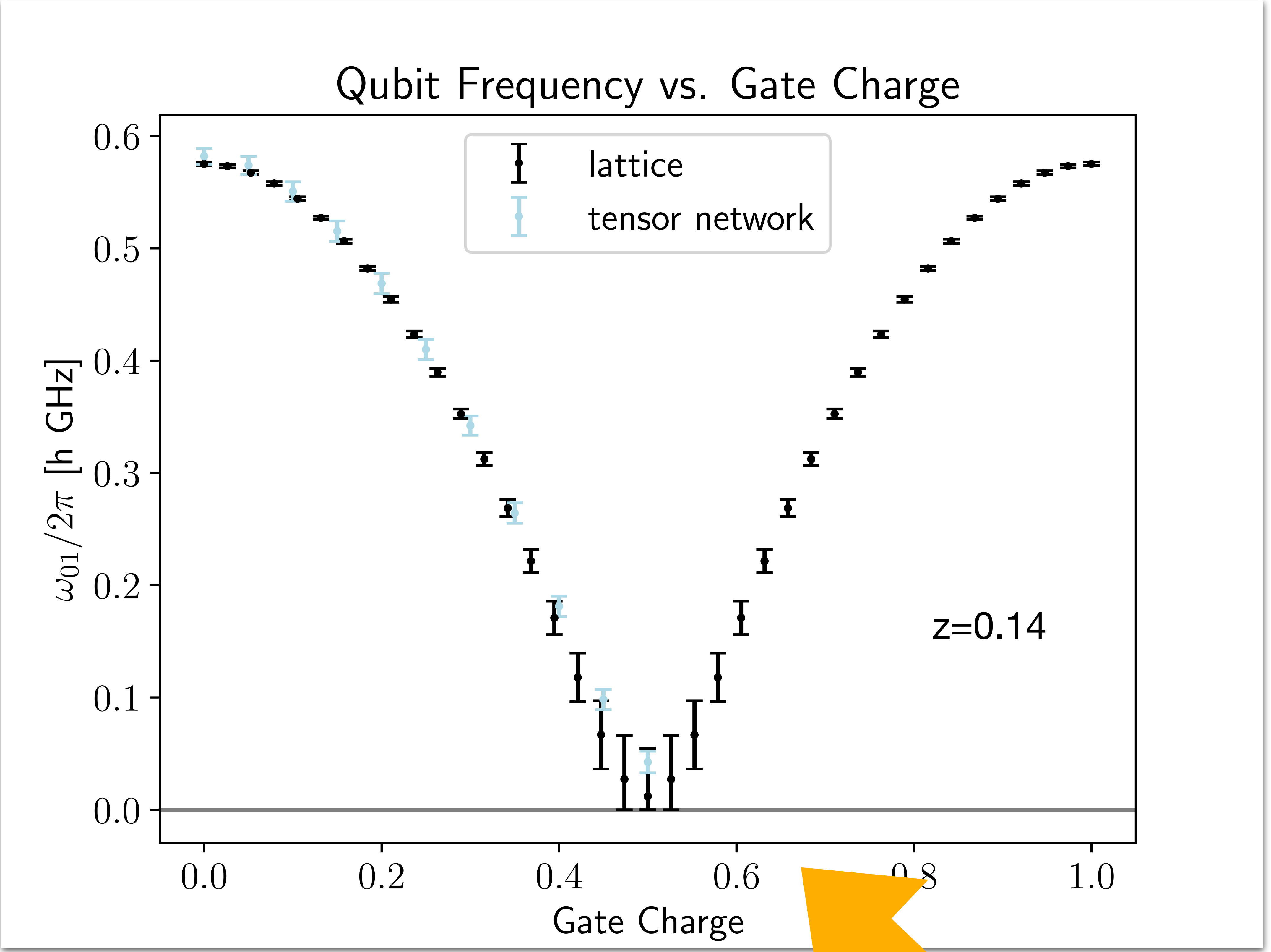


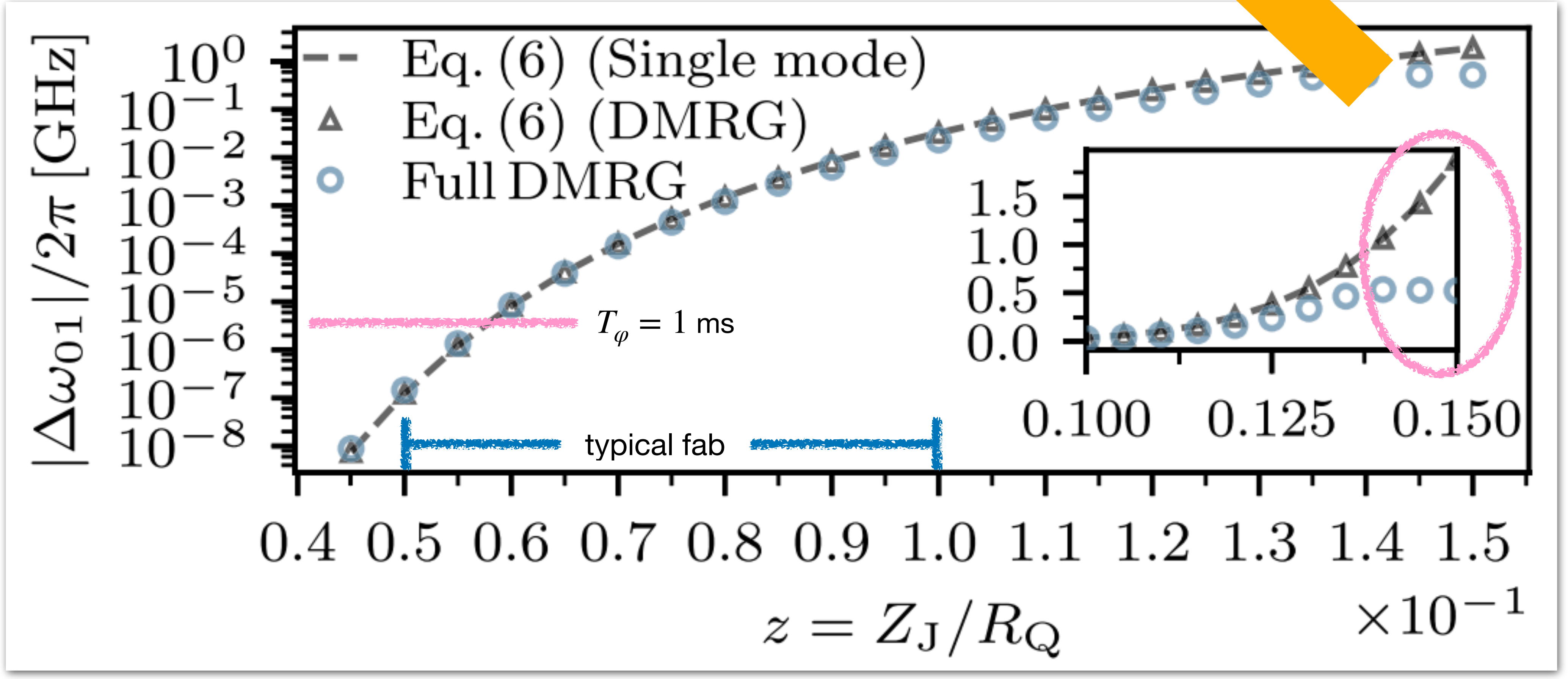


- $C(t) = \langle \mathcal{O}(t)\mathcal{O}(0)\rangle \sim e^{-(E_1-E_0)t}|\langle 1|\mathcal{O}|0\rangle|^2$
- **"Interpolating operator"** $\mathcal{O} = \sum_x \sin\theta_x$
- $f(\Delta t) = a + b\Delta t + c\Delta t^2$
- **Total error budget is 3%**

Lattice simulation:







Gate charge & the θ angle:

Gate charges produce a topological term:

$$S(n_g) - S(0) = i\int_0^\beta \mathrm{d}t \left(\frac{1}{2e}\right)^2 D^T\dot{\theta} = 2\pi i n_g^T N_{\text{instanton}}, \quad N_{\text{instanton}} := \theta(t=\beta) - \theta(t=0)$$

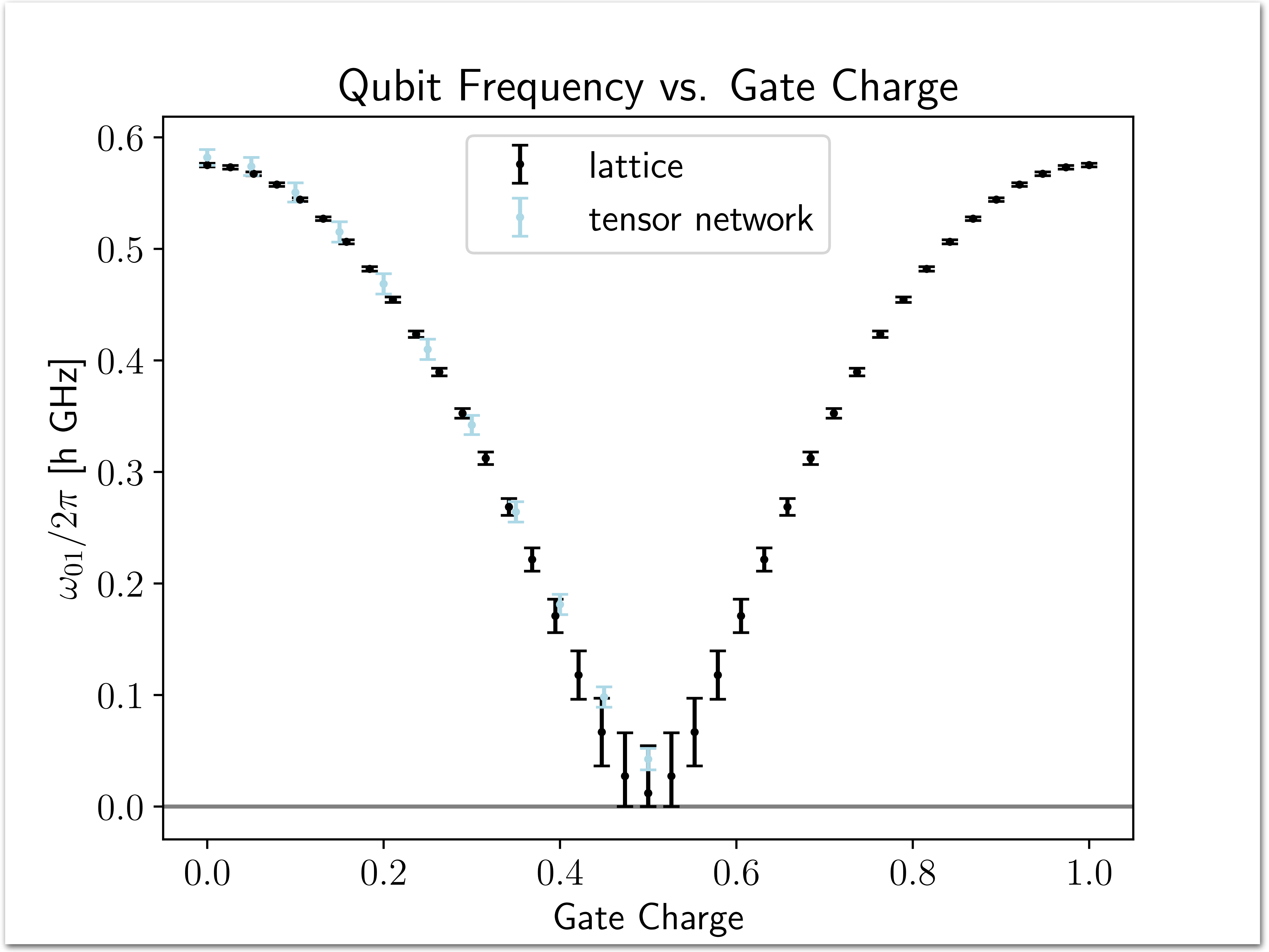
Theta angle produces a topological term:

$$S(\theta) - S(0) = i\frac{g^2}{32\pi^2}\int d^4x\ \mathrm{tr}F_{\mu\nu}\widetilde{F}^{\mu\nu} = i\theta Q_{\text{top}}$$

Izubuchi et. al PoS Lattice [0802.1470] (2008)

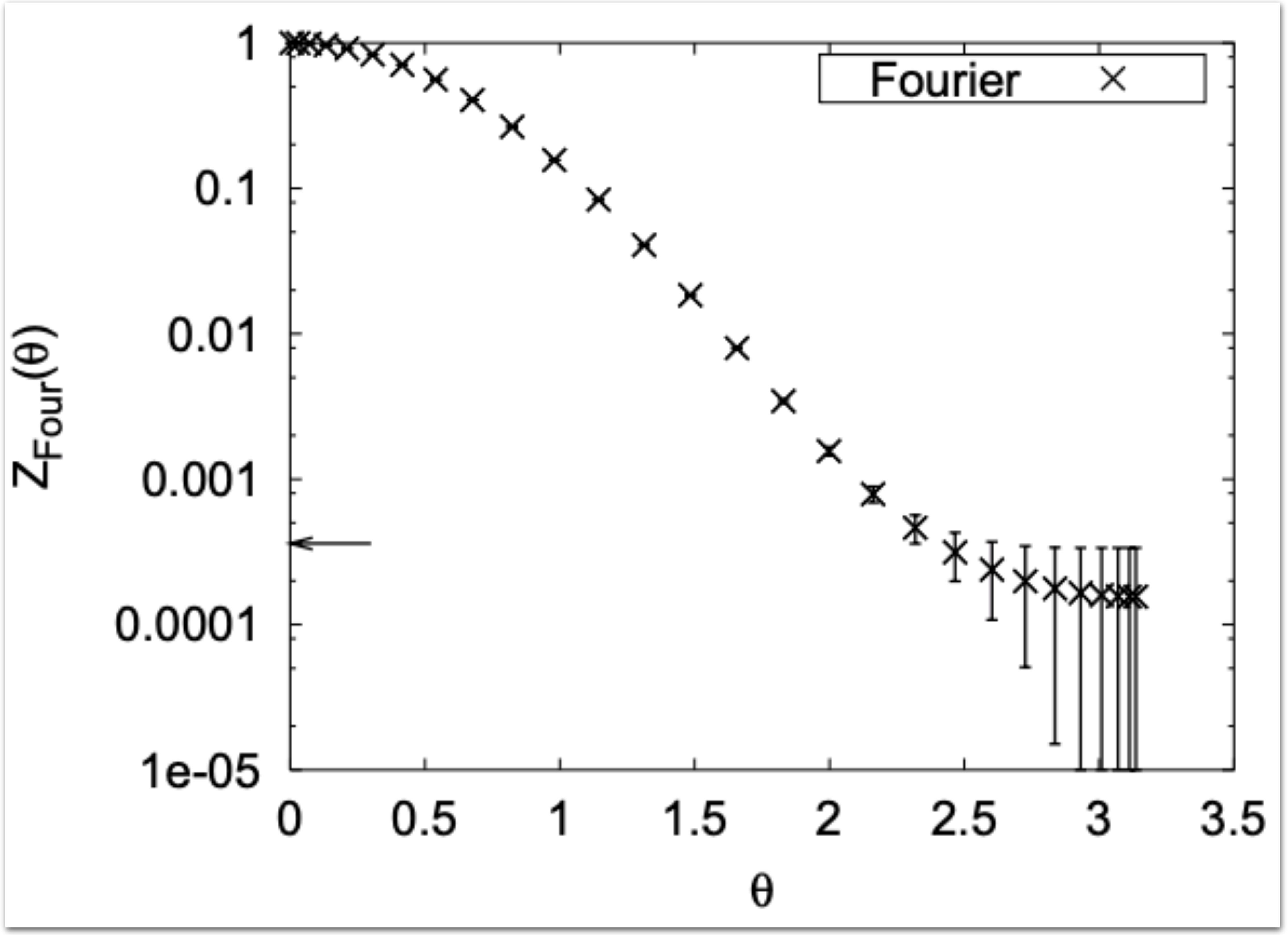
(Note both are imaginary)

Gate charge & the θ angle:



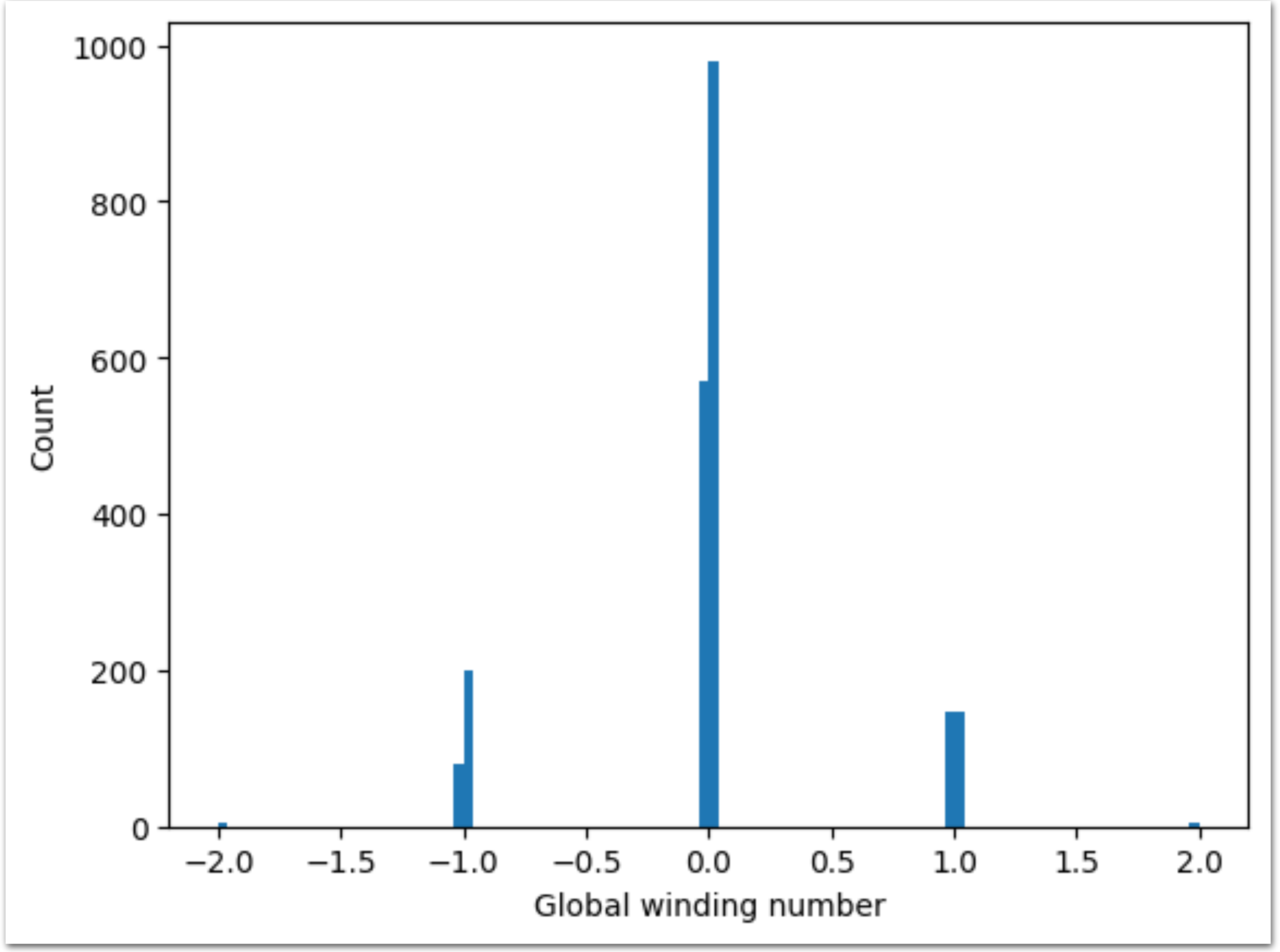
Fluxonium

Imachi Prog.Theor.Phys. 115 (2006) 931-949



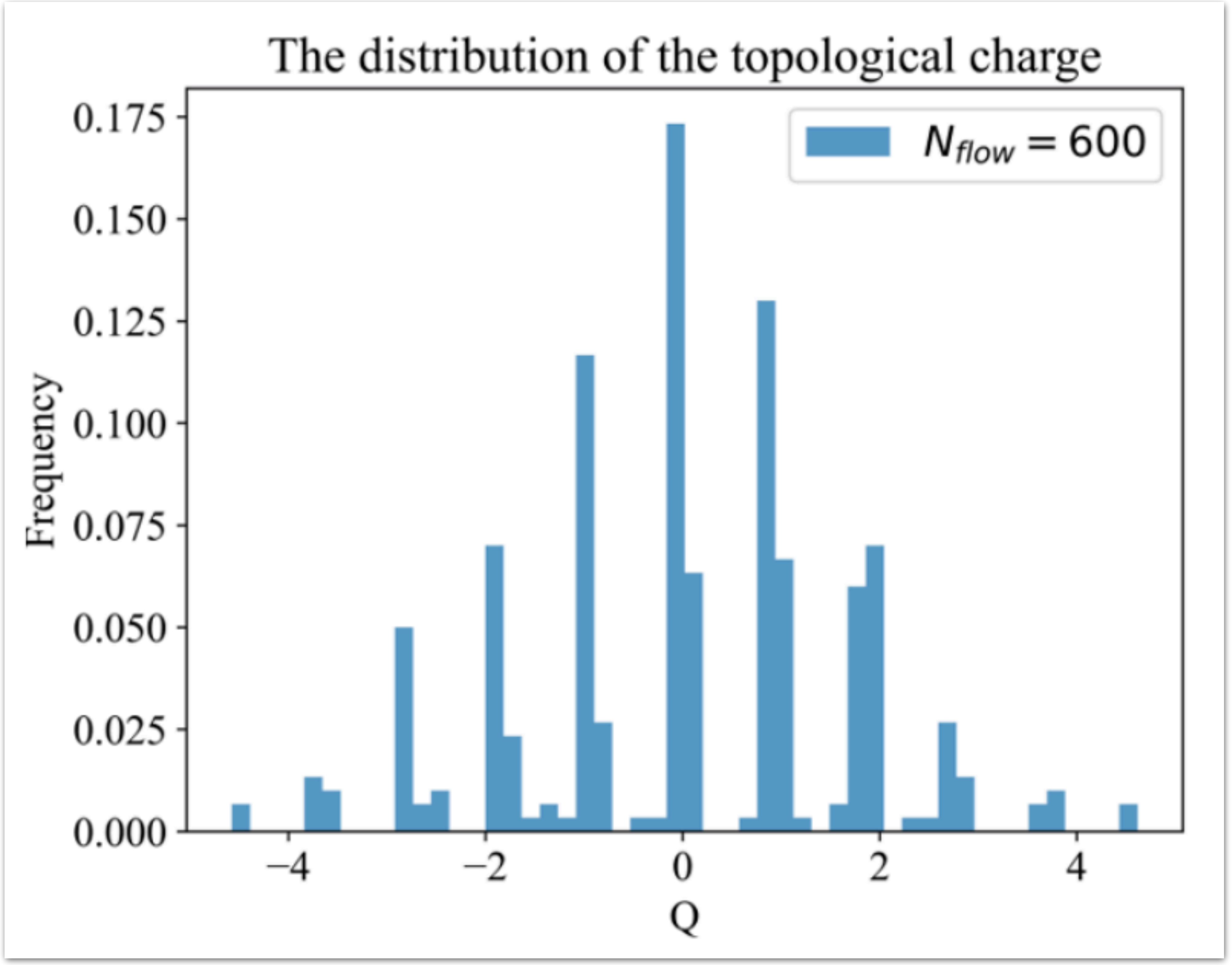
Lattice QCD with θ term

Gate charge & the θ angle:



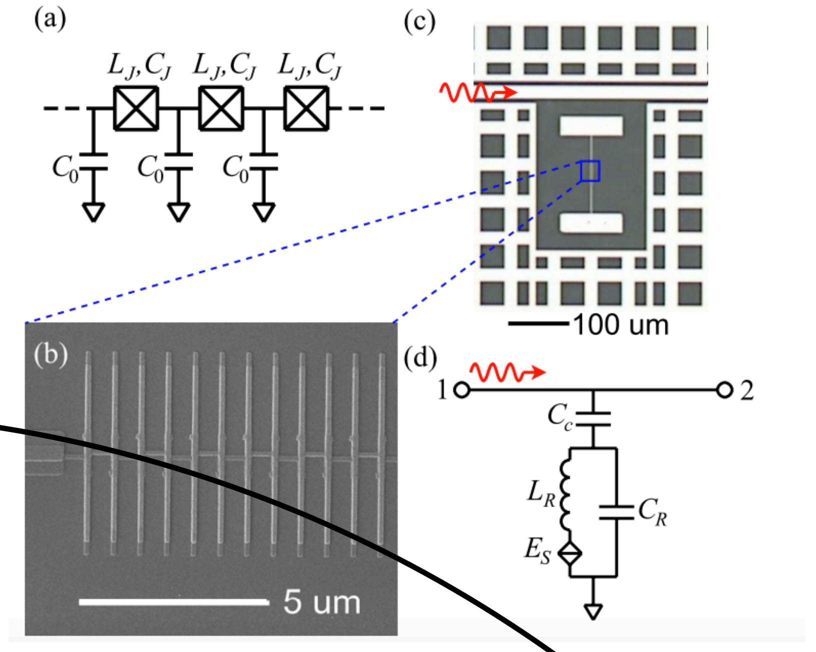
Fluxonium

Gao et. al. Phys. Rev. D **109**, 074509

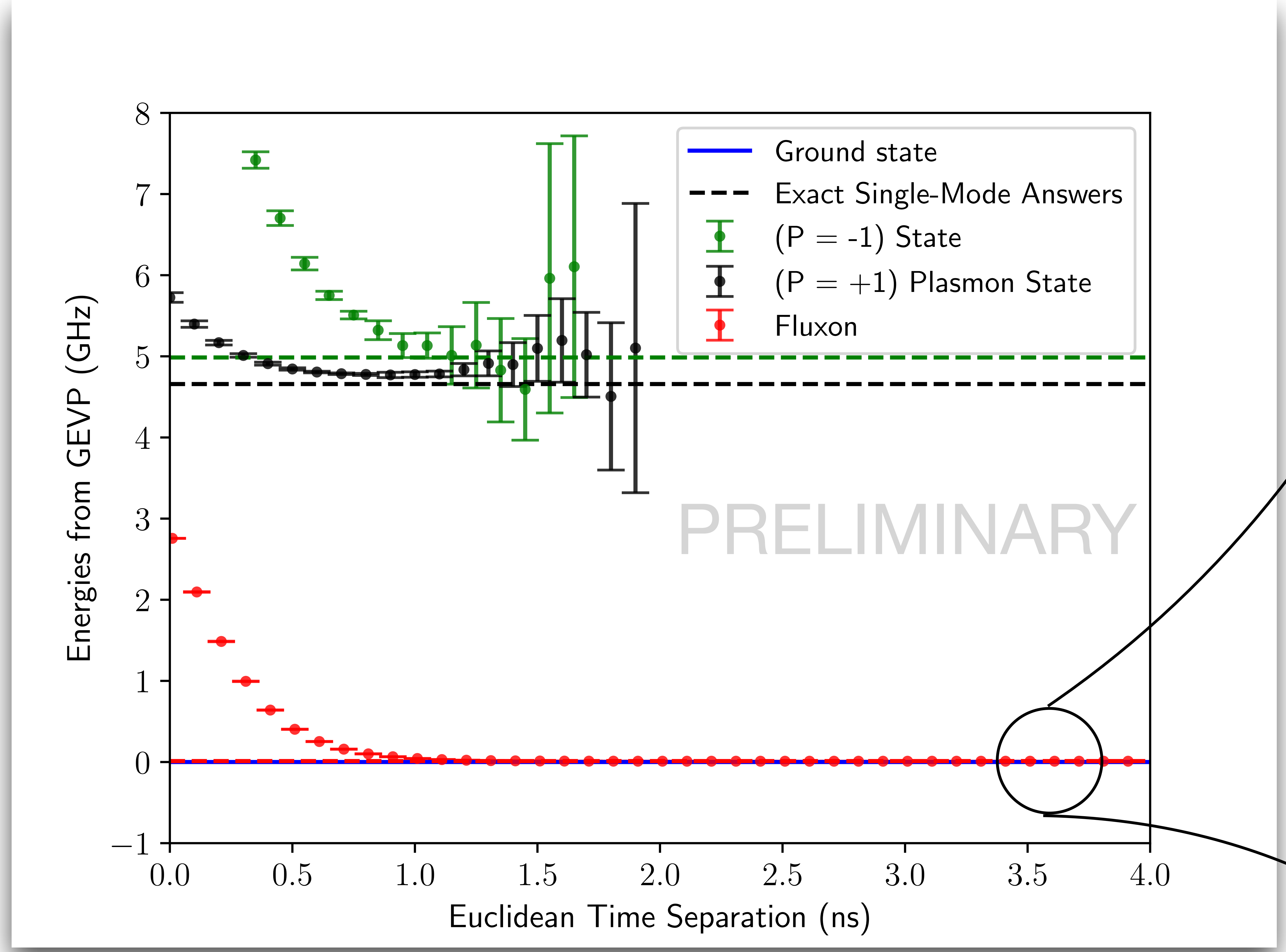


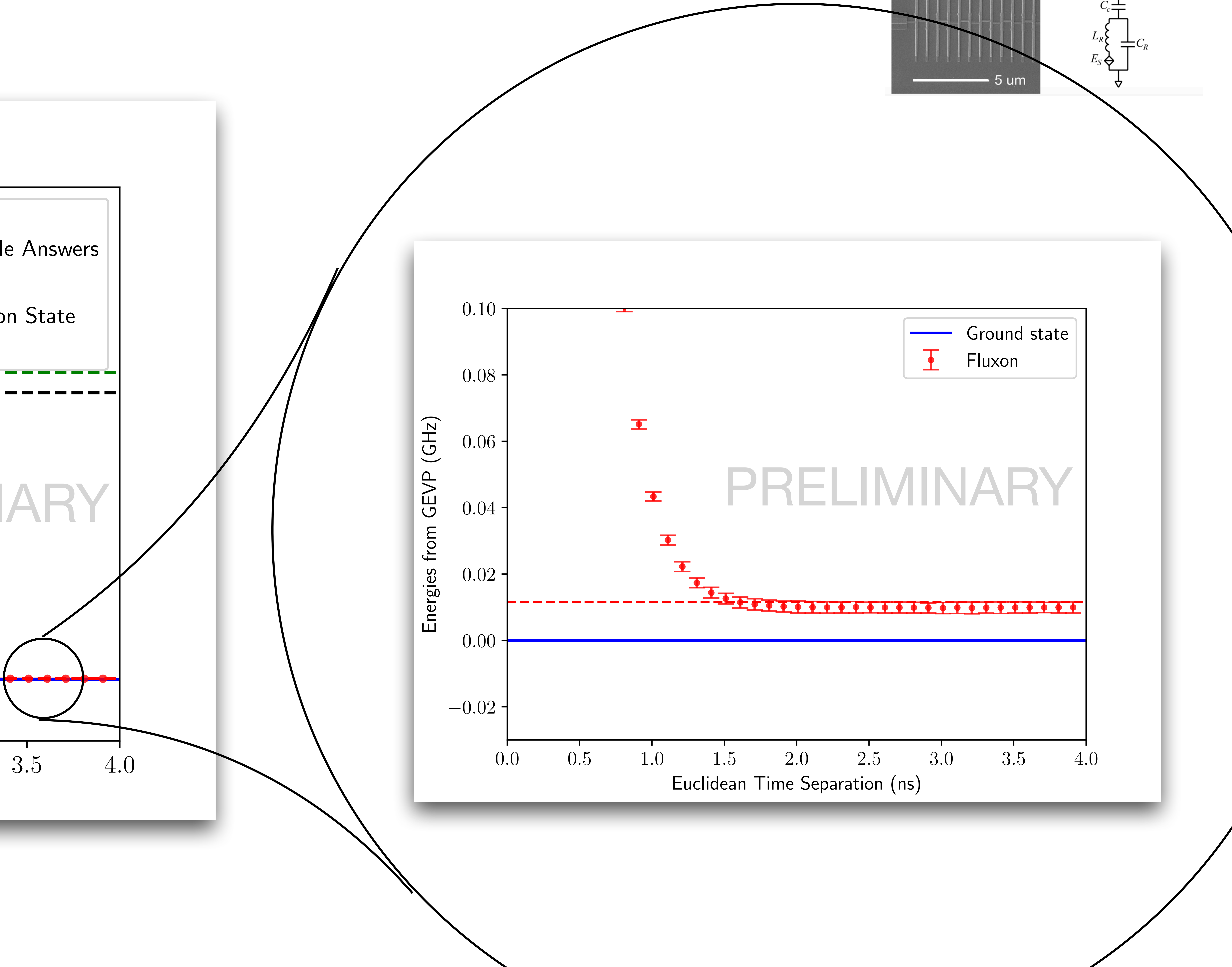
Lattice QCD with θ term

Masluk et. al. Phys. Rev. Lett. **109**, 137002 (2012)



N=120:



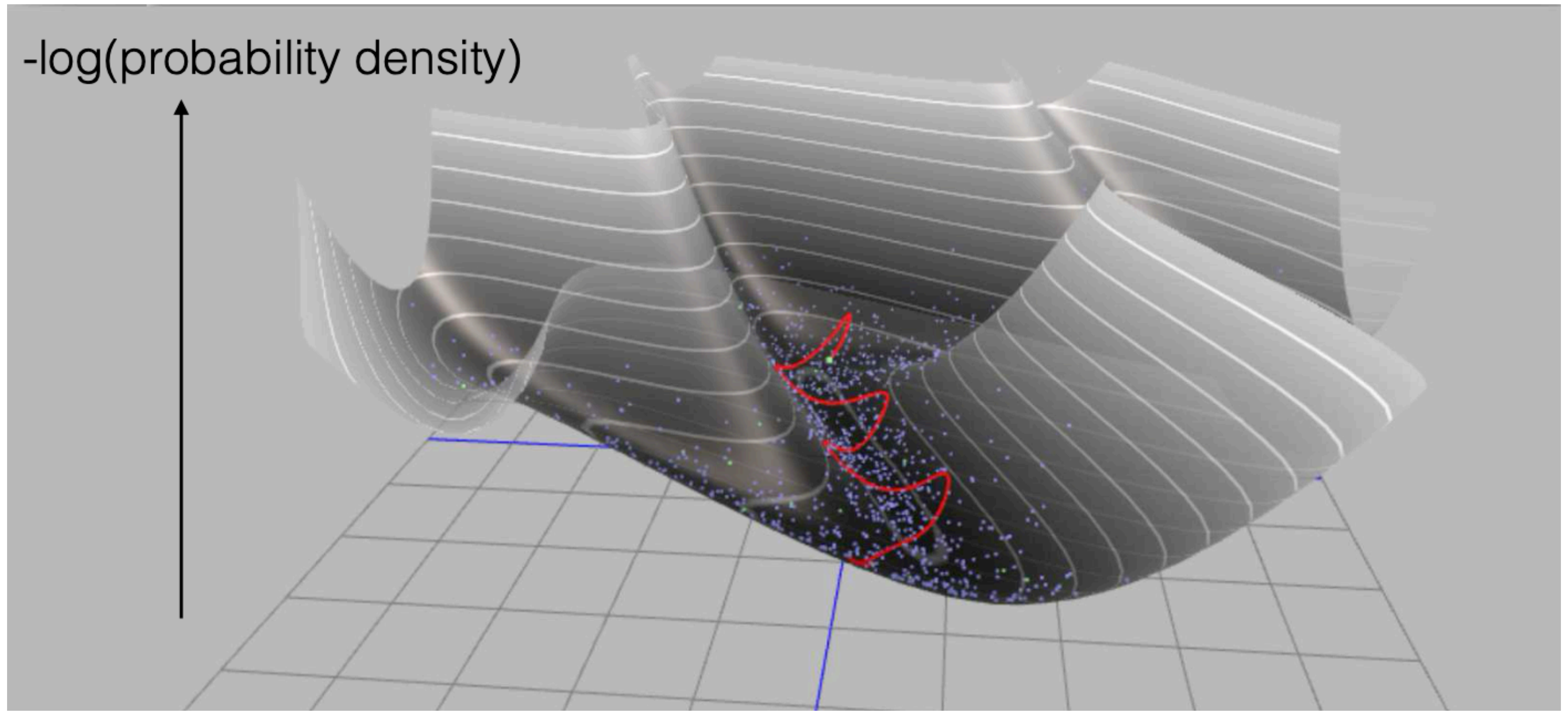



Qubit A
Fluxonium
5 μm

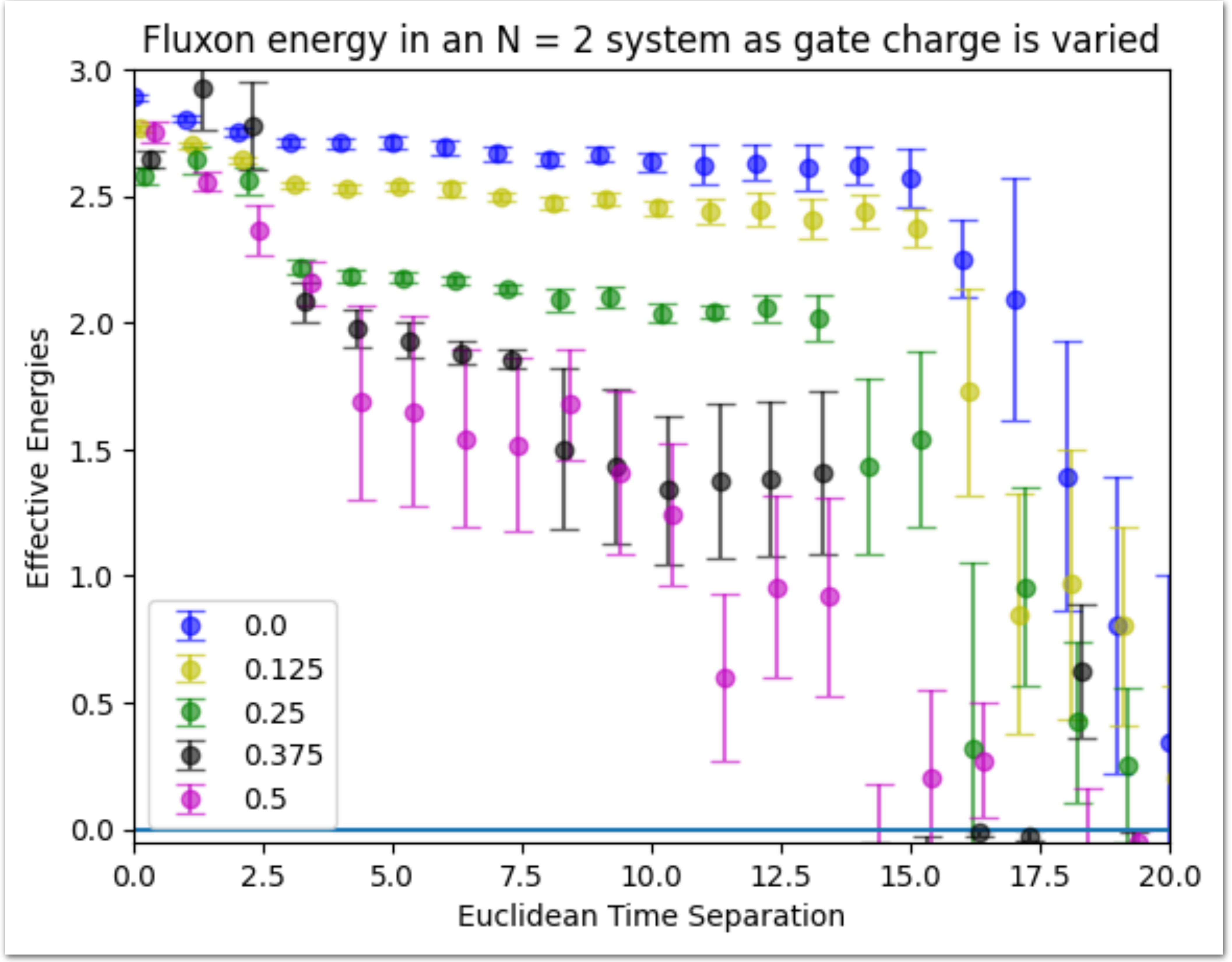
Thank you

Backup slides

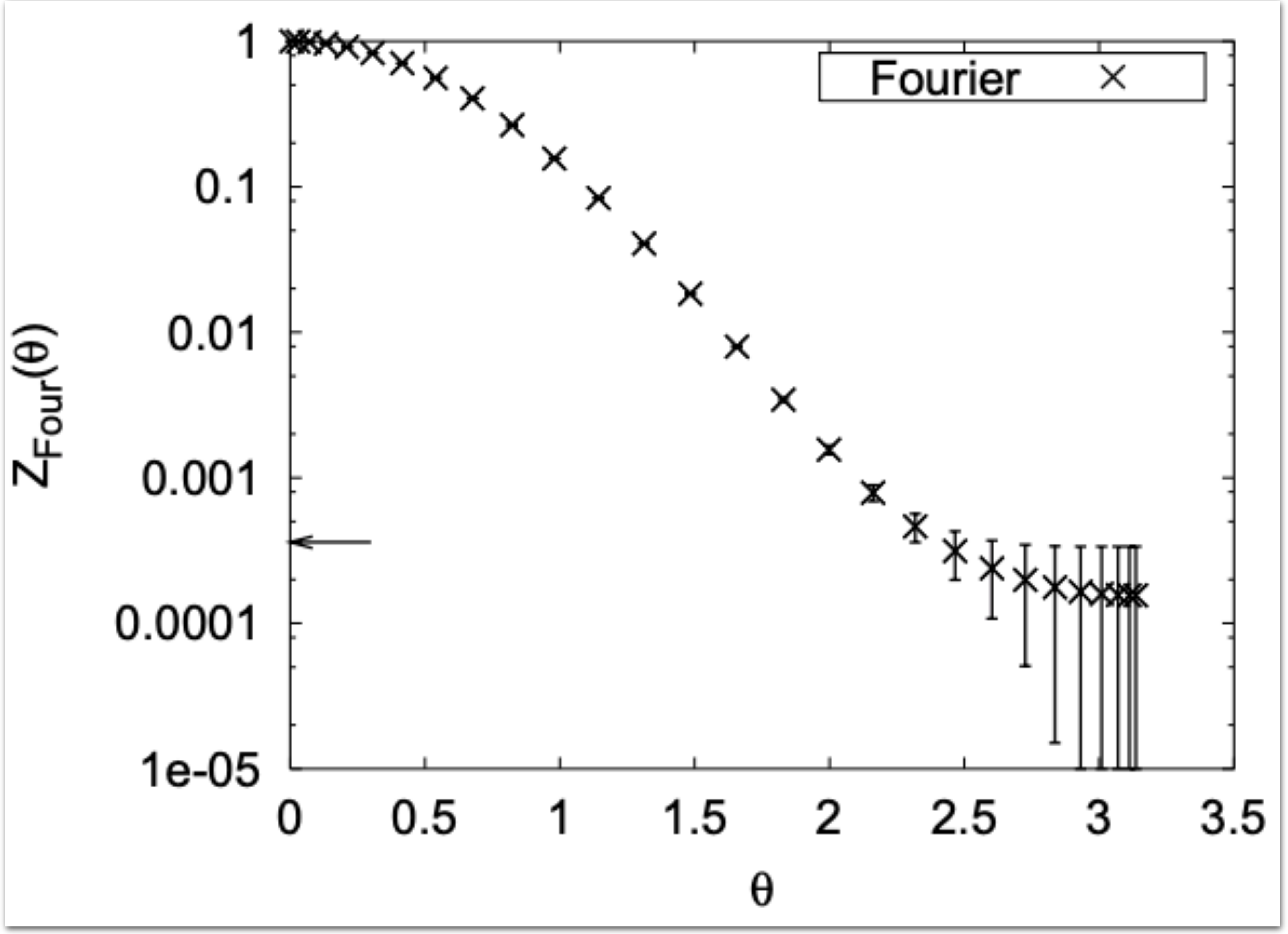
142 lines



Gate charge & the θ angle:

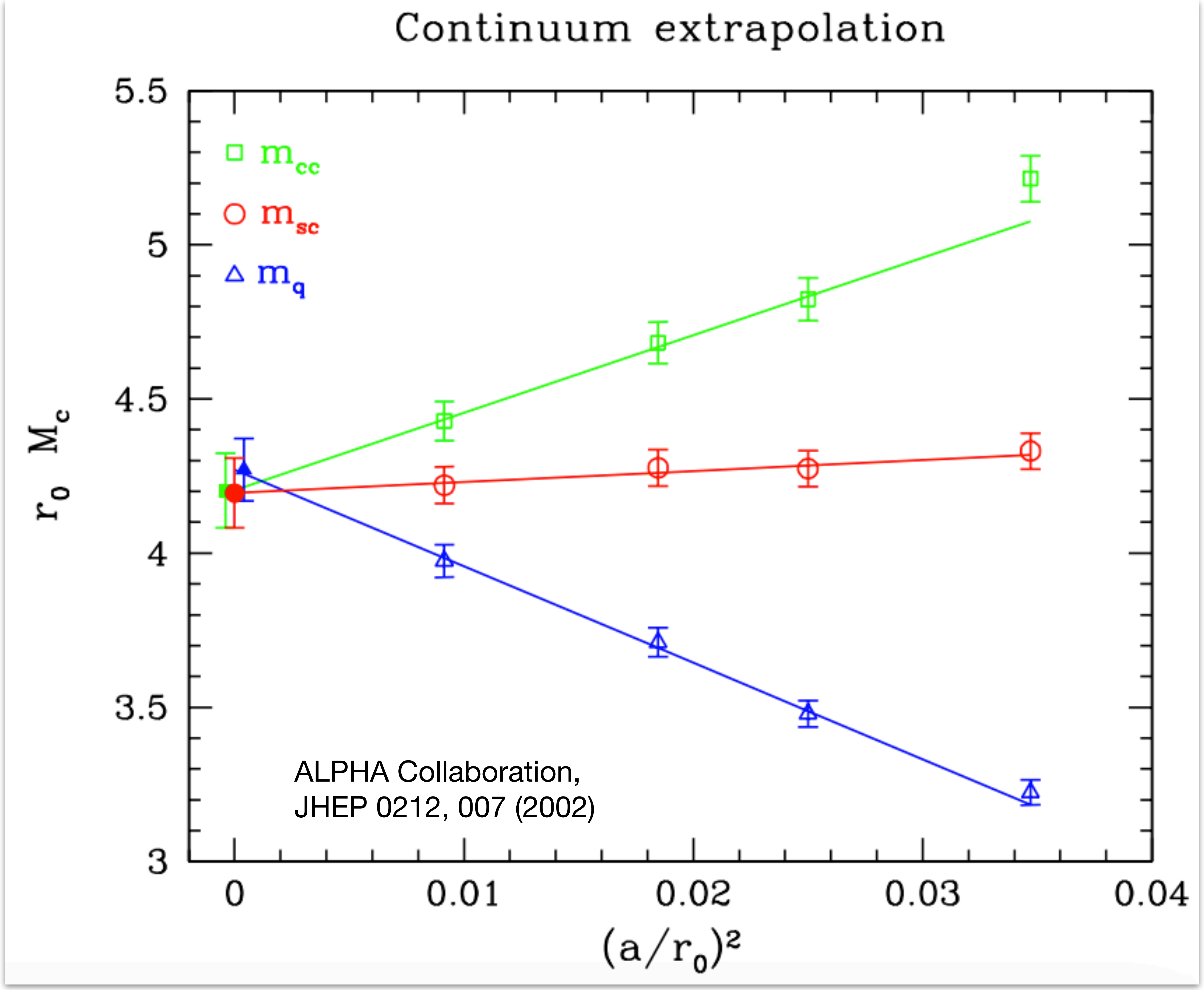


Fluxonium



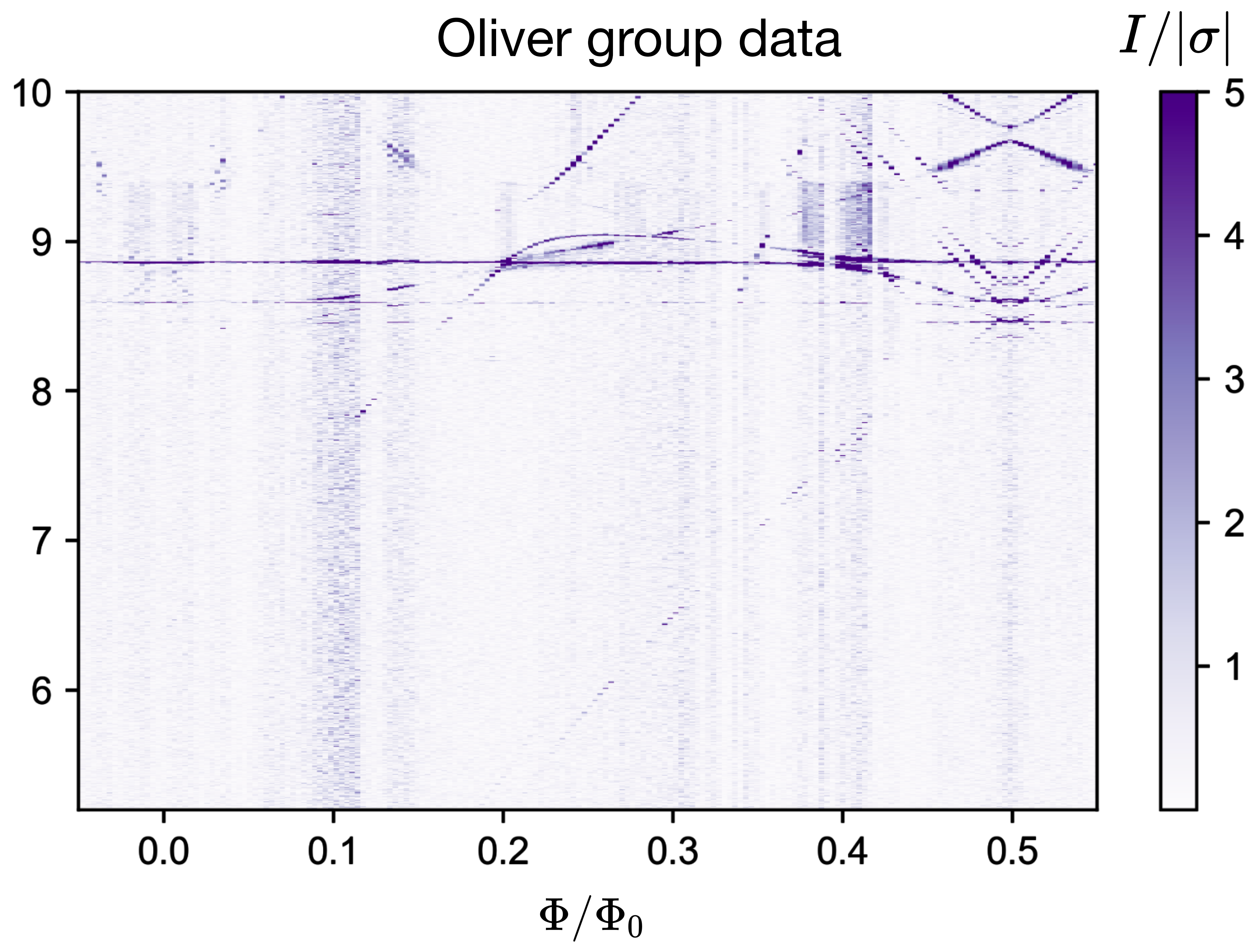
Lattice QCD with θ term

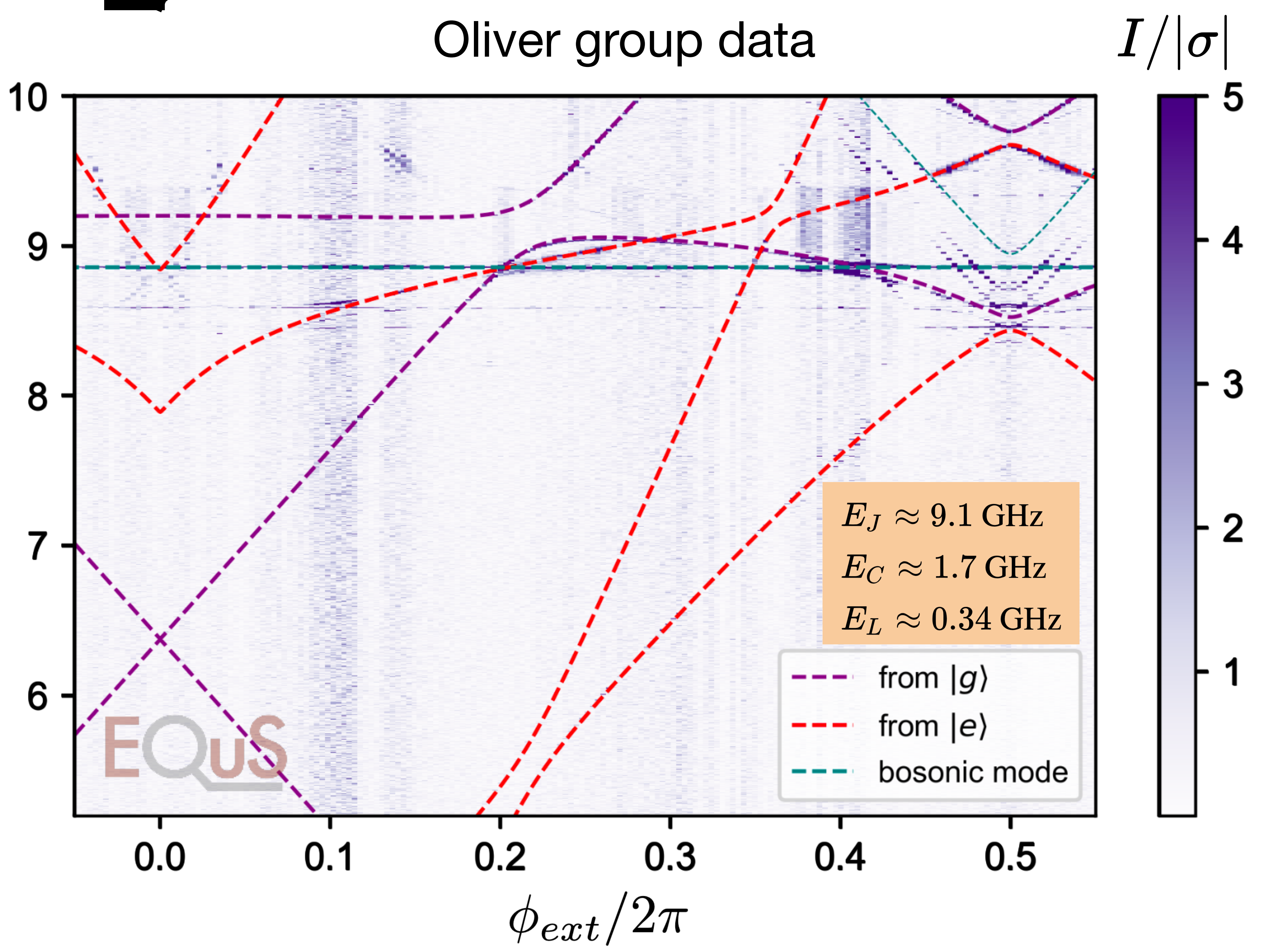
Continuum limit:



Two-tone spectroscopy:

$$H(\varphi) = -4E_C\frac{\partial^2}{\partial\varphi^2} + \frac{1}{2}E_L\varphi^2 - E_J\cos(\varphi - \varphi_{\text{ext}})$$





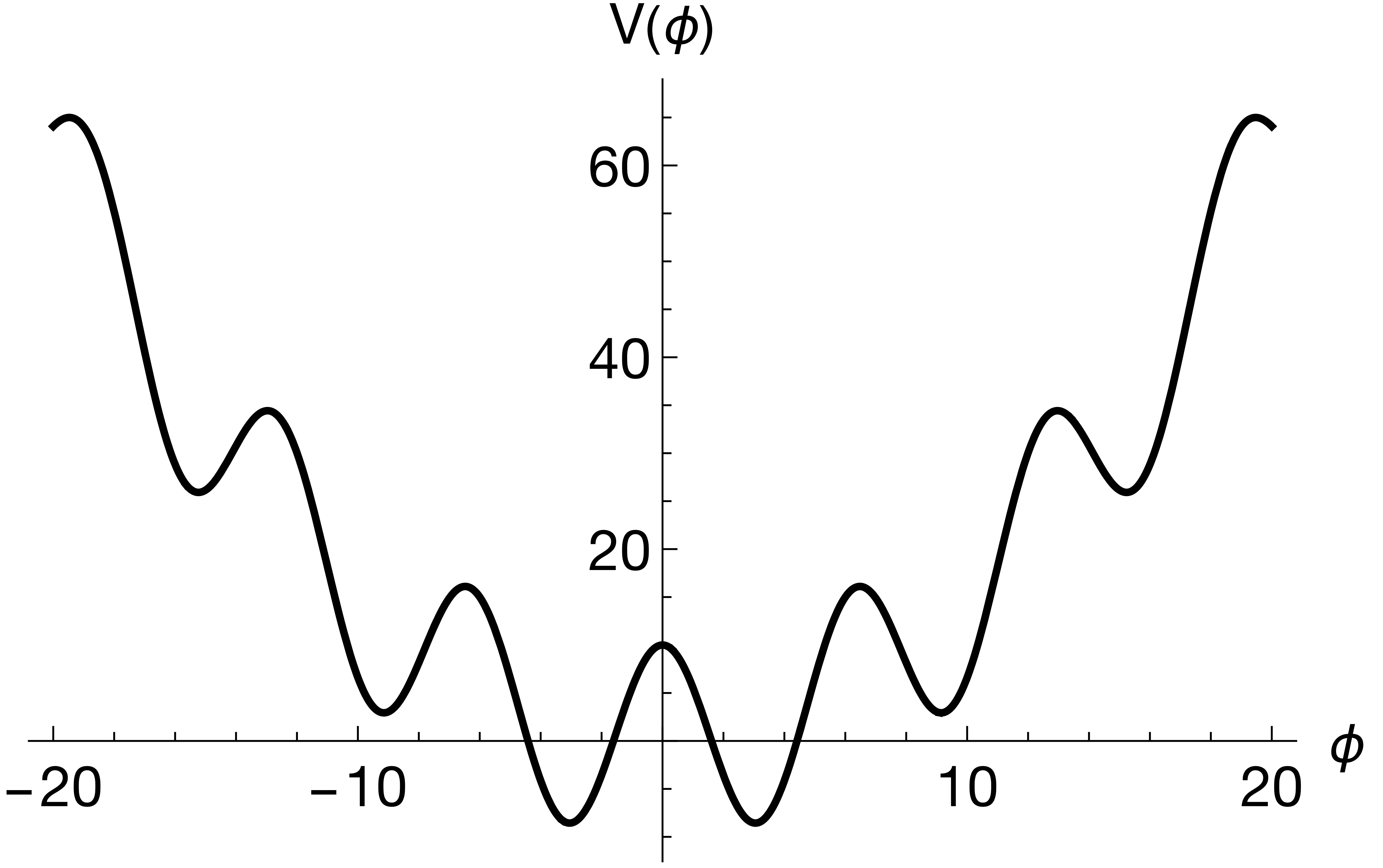
One variable model:

$$H = (2e)^2 n^T C^{-1} n + U(\theta)$$

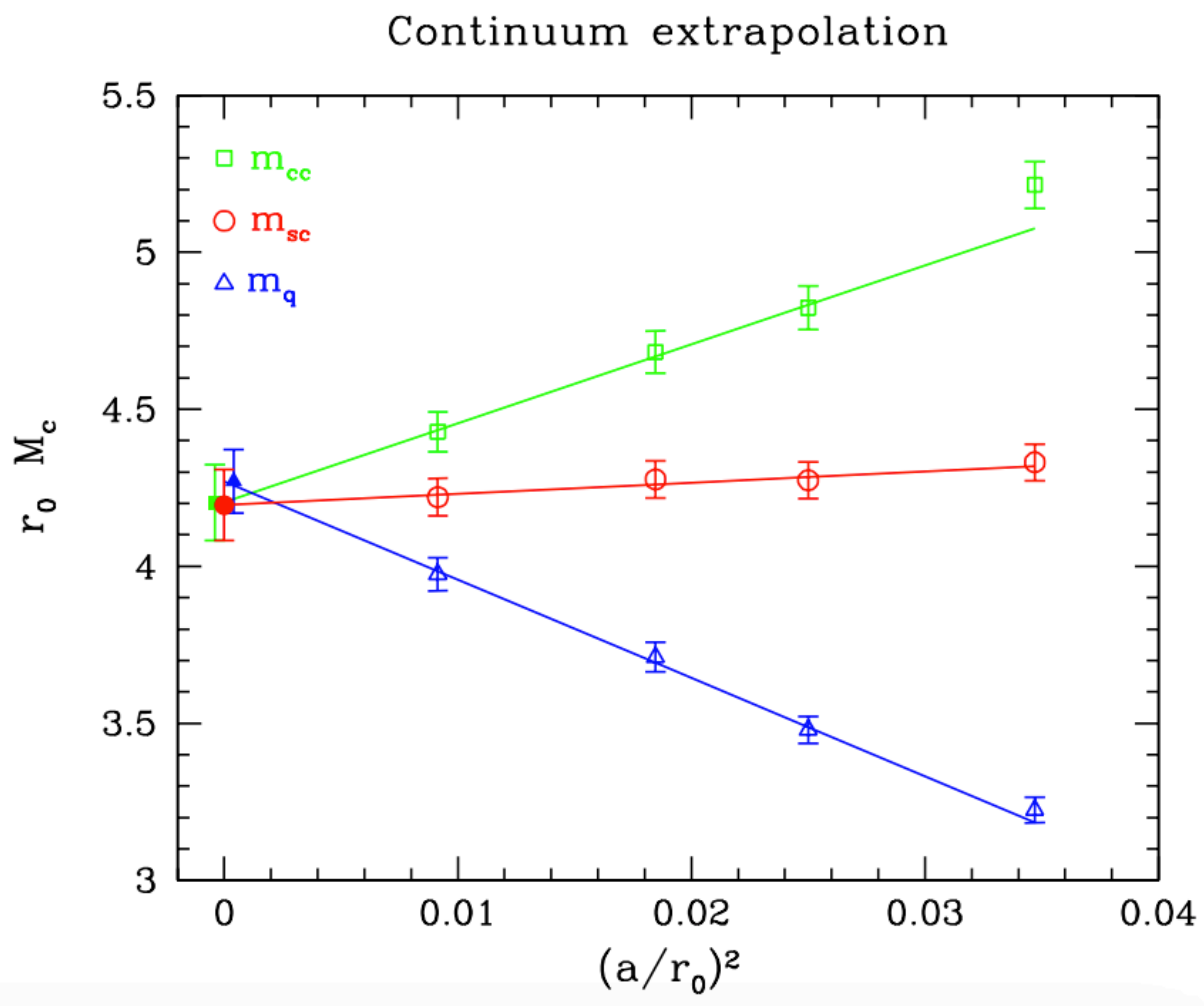
$$= H(\varphi) + \delta H(\varphi, \xi)$$

where

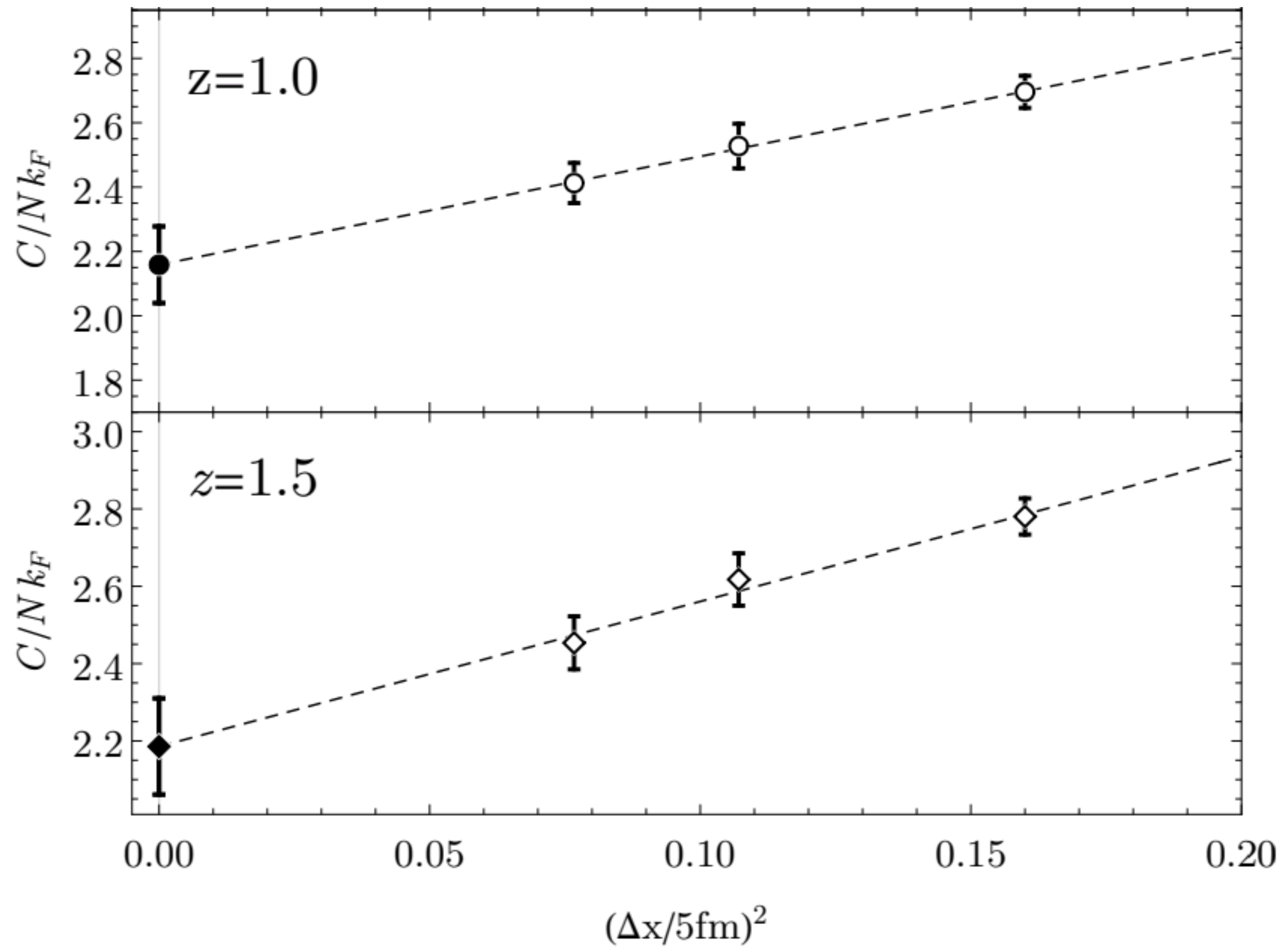
$$H(\varphi) = -4E_C \frac{\partial^2}{\partial \varphi^2} + \frac{1}{2} E_L \varphi^2 - E_J \cos(\varphi - \varphi_{\text{ext}})$$



An analogy:



ALPHA Collaboration, JHEP 0212, 007 (2002)



Warrington et. al. PRL 126, 132701 (2021)