Toward quantum simulations for nuclear physics with qubits and qumodes

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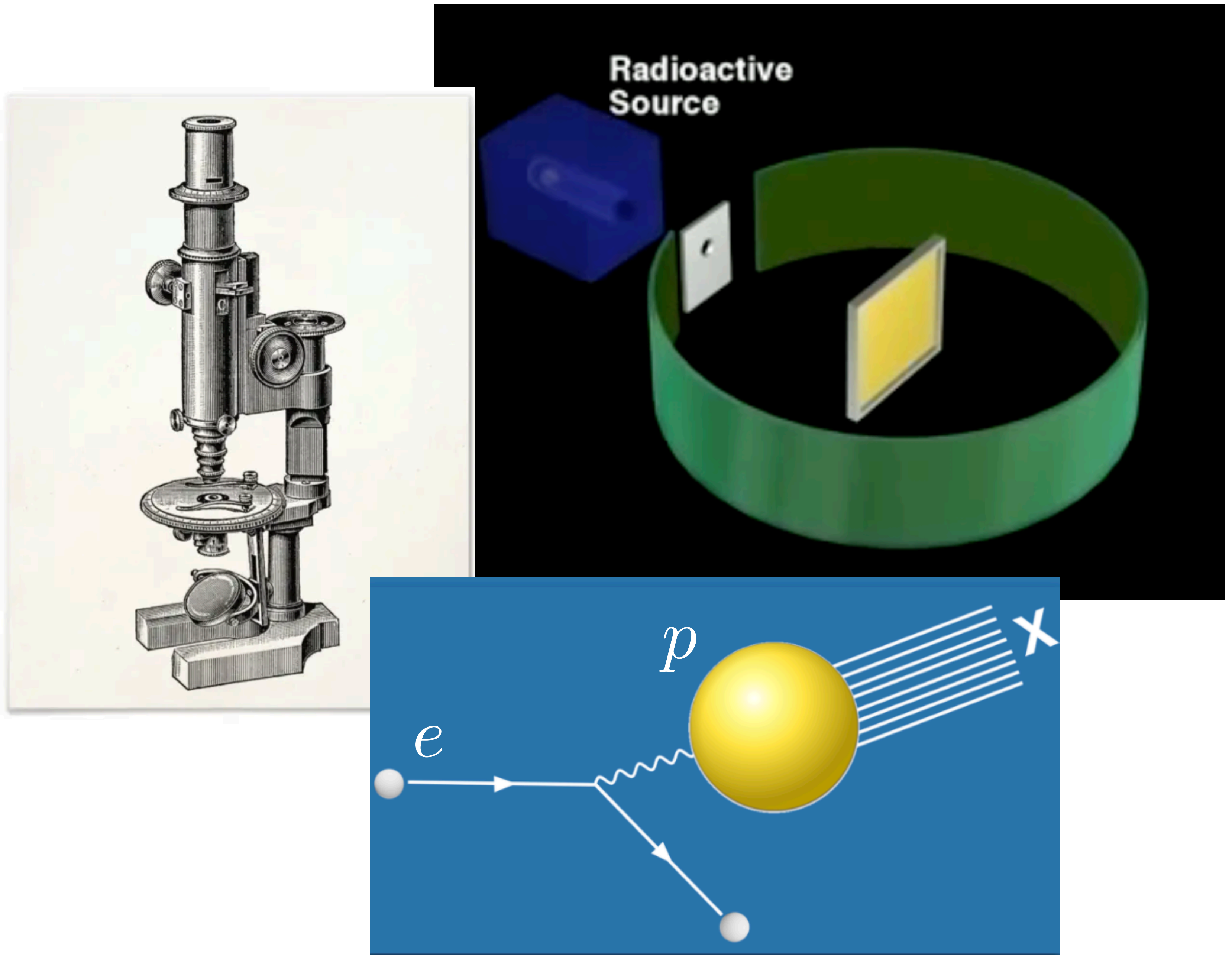
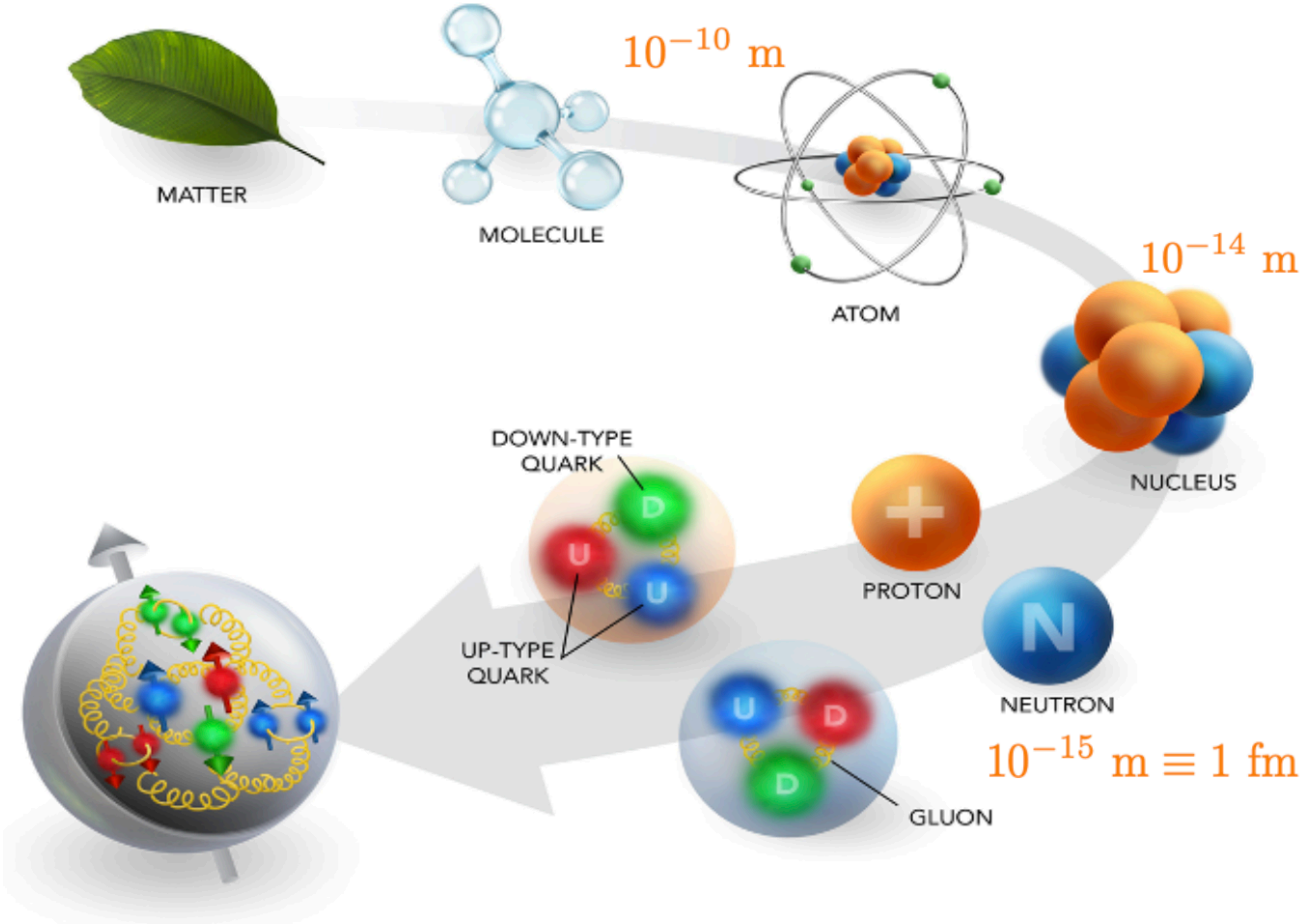
Quantum Information Science on the Intersection of
Nuclear and AMO Physics, UMass Boston, 01/13/24

Stony Brook University

Probing the fundamental structure of matter

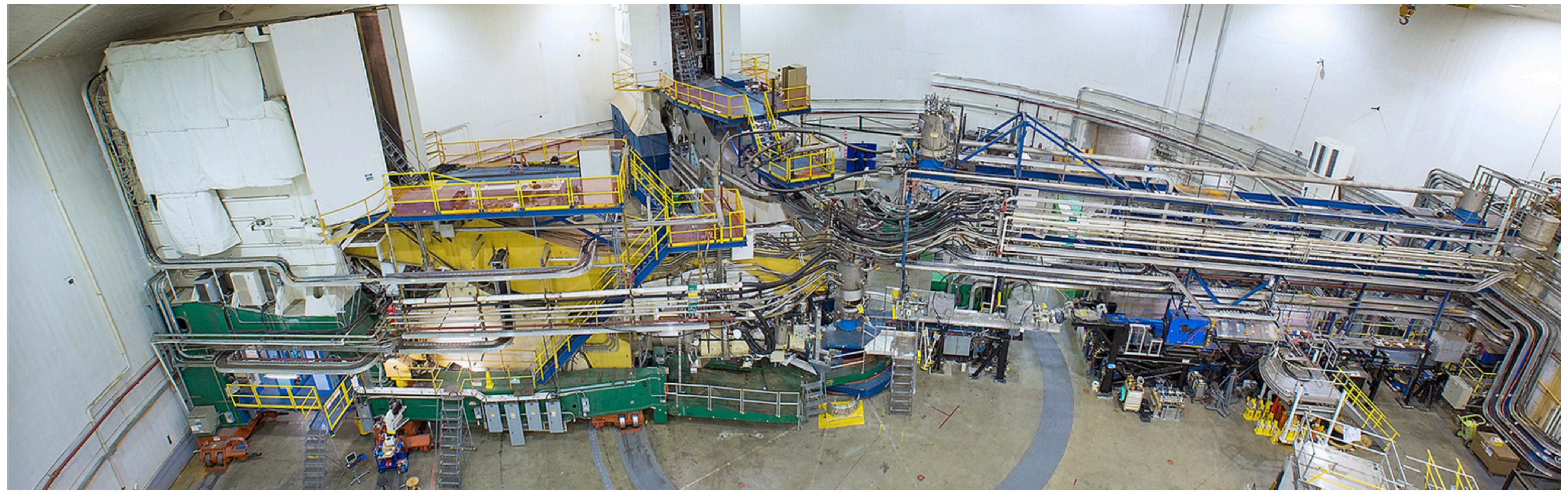


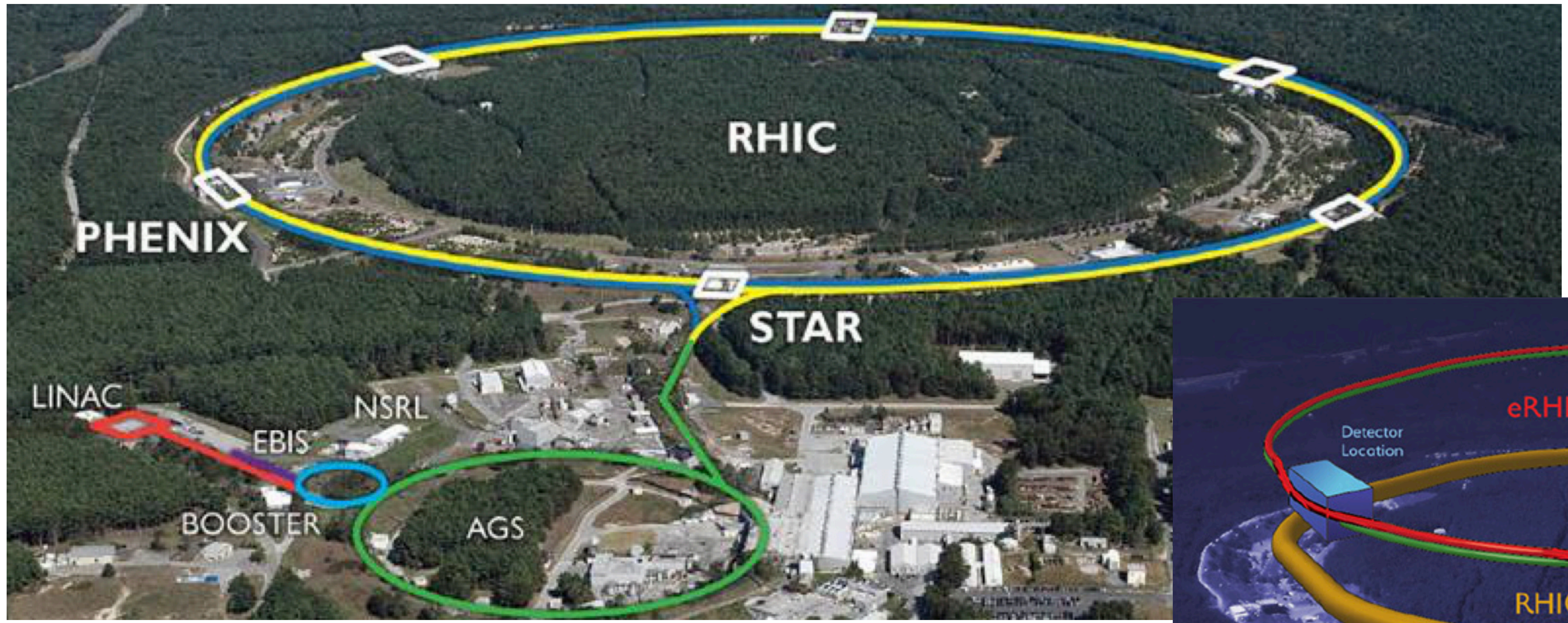


Quantum chromodynamics (QCD)

Large Hadron Collider CERN

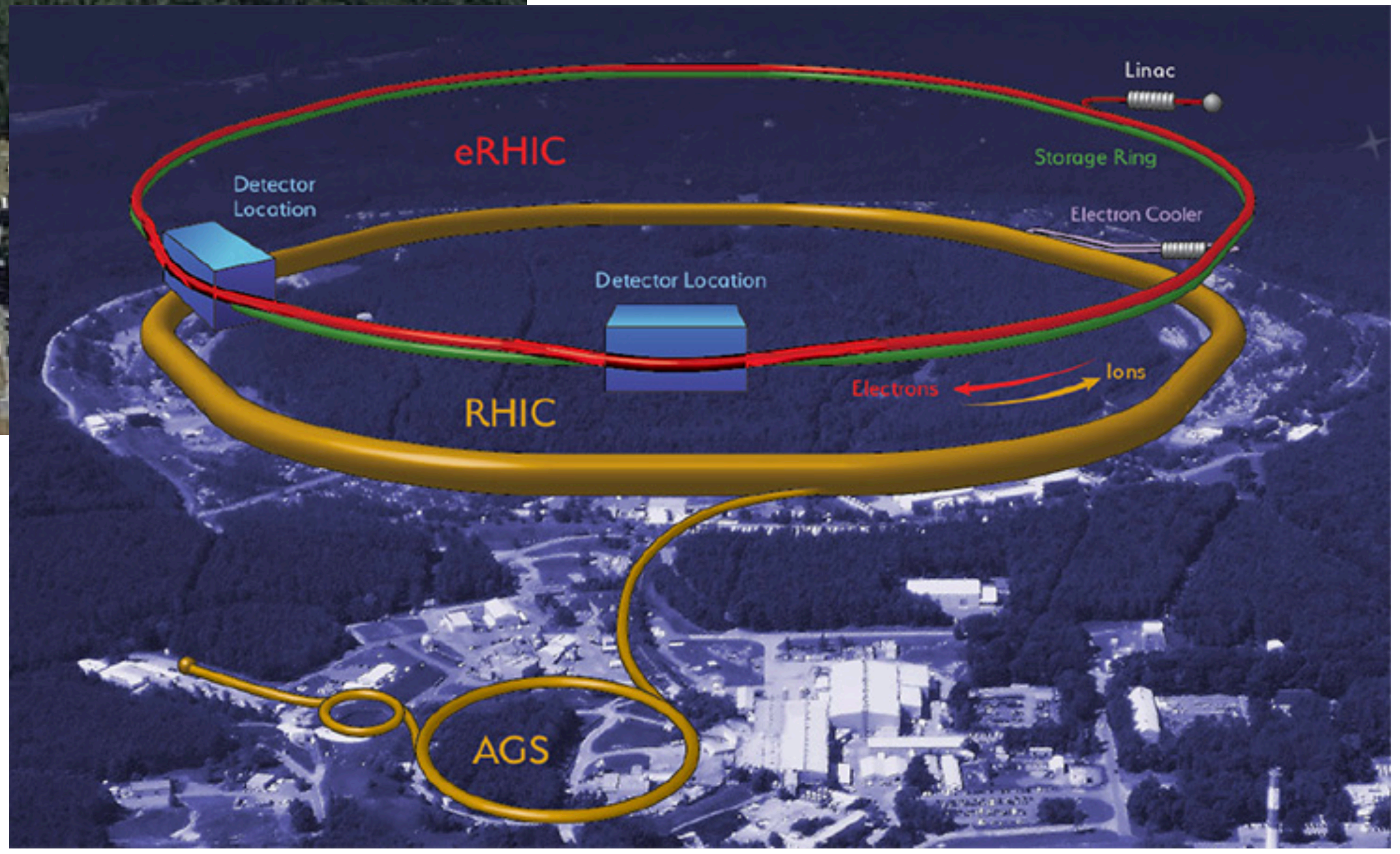
CEBAF JLab





RHIC & EIC

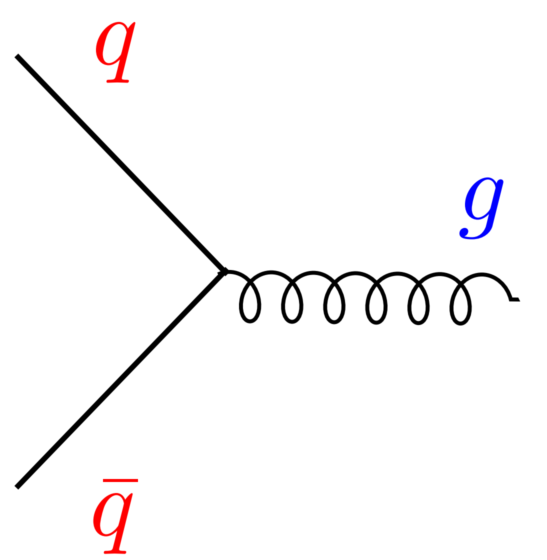
BNL

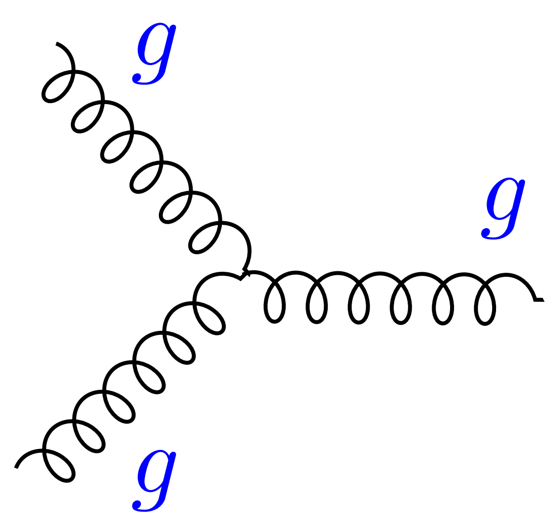


High-energy collider experiments

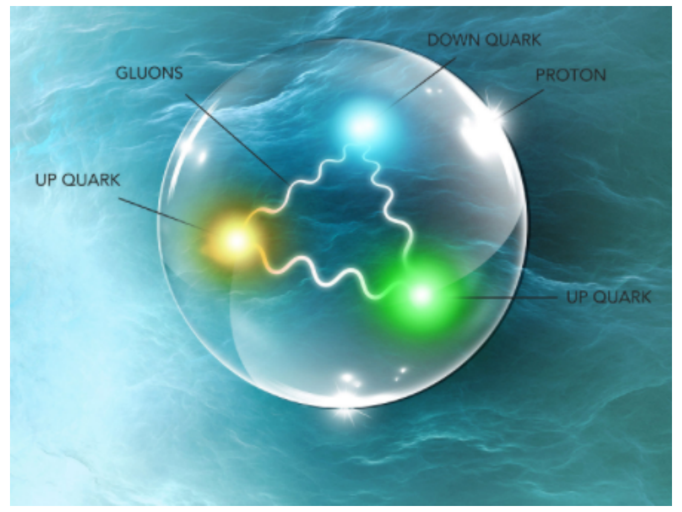
Quantum chromodynamics

- Theory of the strong interaction between quarks and gluons
- The Lagrangian $\mathcal{L} = \bar{\psi}\left(i\partial^\mu\gamma_\mu - m\right)\psi + g_s\bar{\psi}\gamma^\mu T_a\psi A^a_\mu - \frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}$



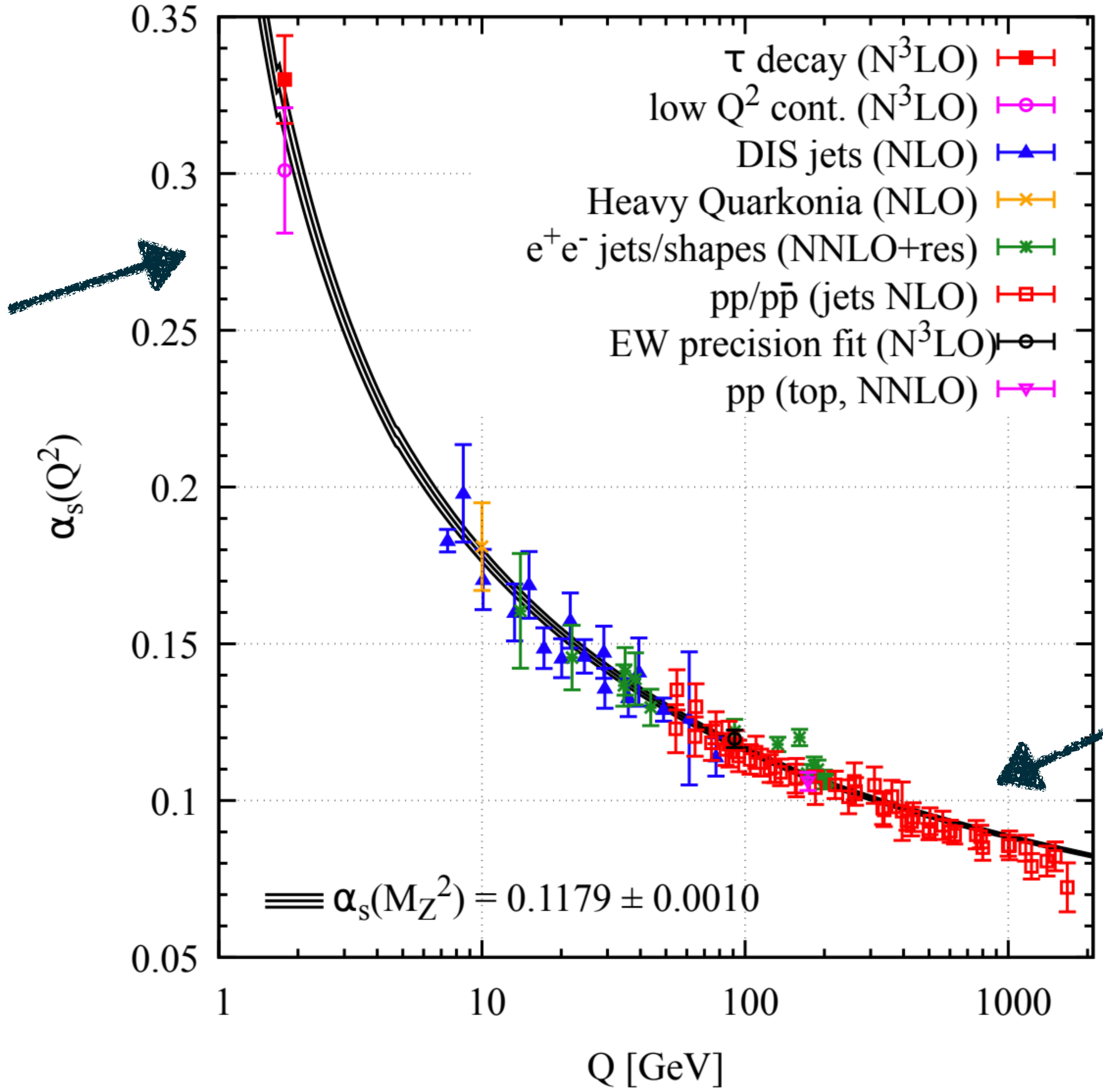


Confinement



- QCD coupling constant $\alpha_s = \frac{g_s^2}{4\pi}$

Nobel Prize 2004



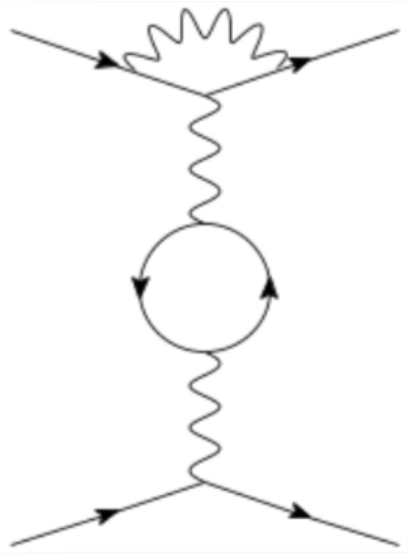
Asymptotic freedom

Huston, Rabbertz, Zanderighi (PDG)

Quantum chromodynamics

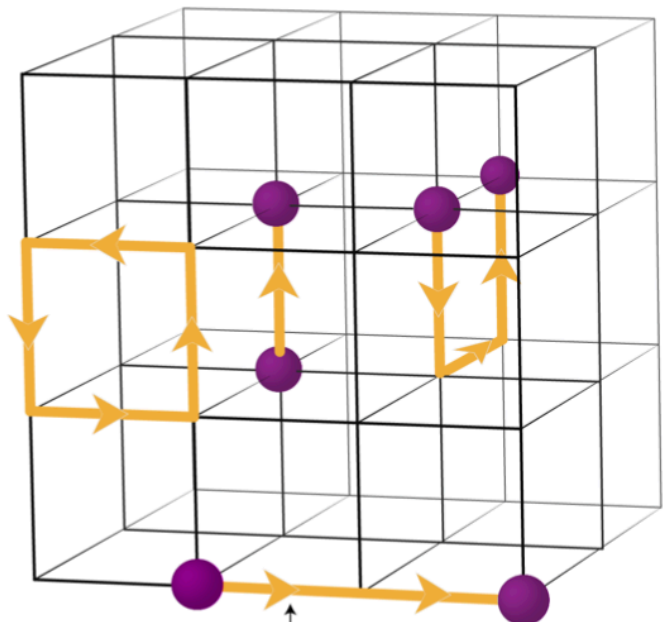
1. Perturbative QCD at high energies

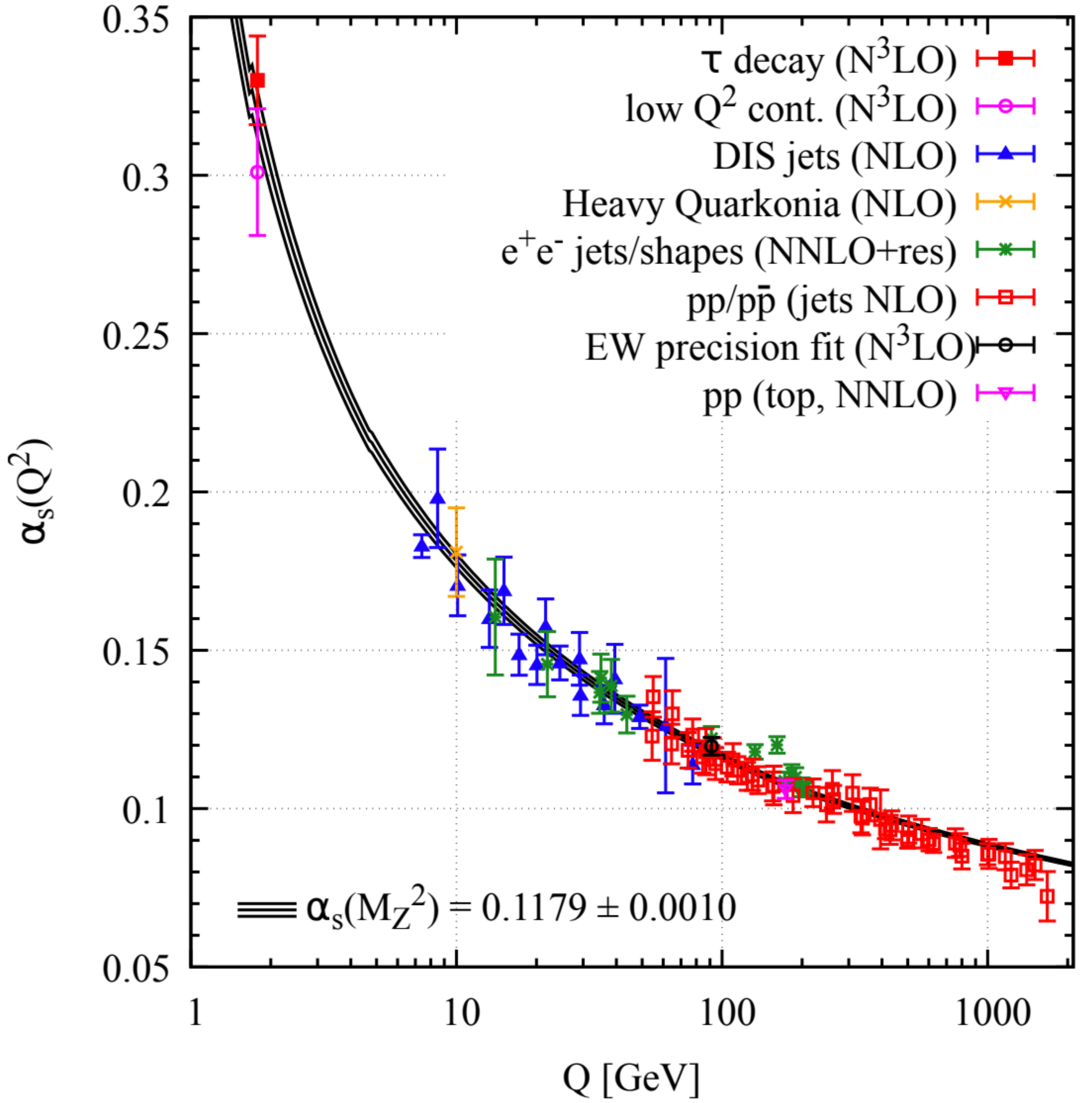
$$\sigma = \sigma^{(0)} + \alpha_s\,\sigma^{(1)} + \alpha_s^2\,\sigma^{(2)} + \ldots$$



2. Nonperturbative at low energies

- Simulations using a lattice discretization
- Path integral vs. Hamiltonian formulation (imaginary vs. real time)
- Quantum computing for real-time dynamics?





Huston, Rabbertz, Zanderighi (PDG)

Wilson; Kogut, Susskind `70s

Toward simulating scattering processes

- Need to simulate classically intractable real-time dynamics

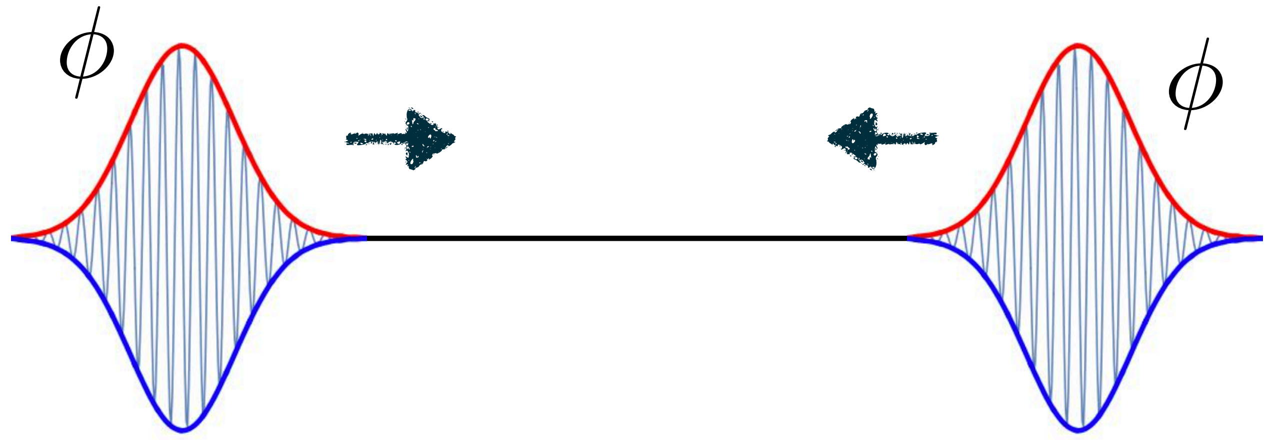
- Parton distribution functions, in-medium correlation functions or hadronization

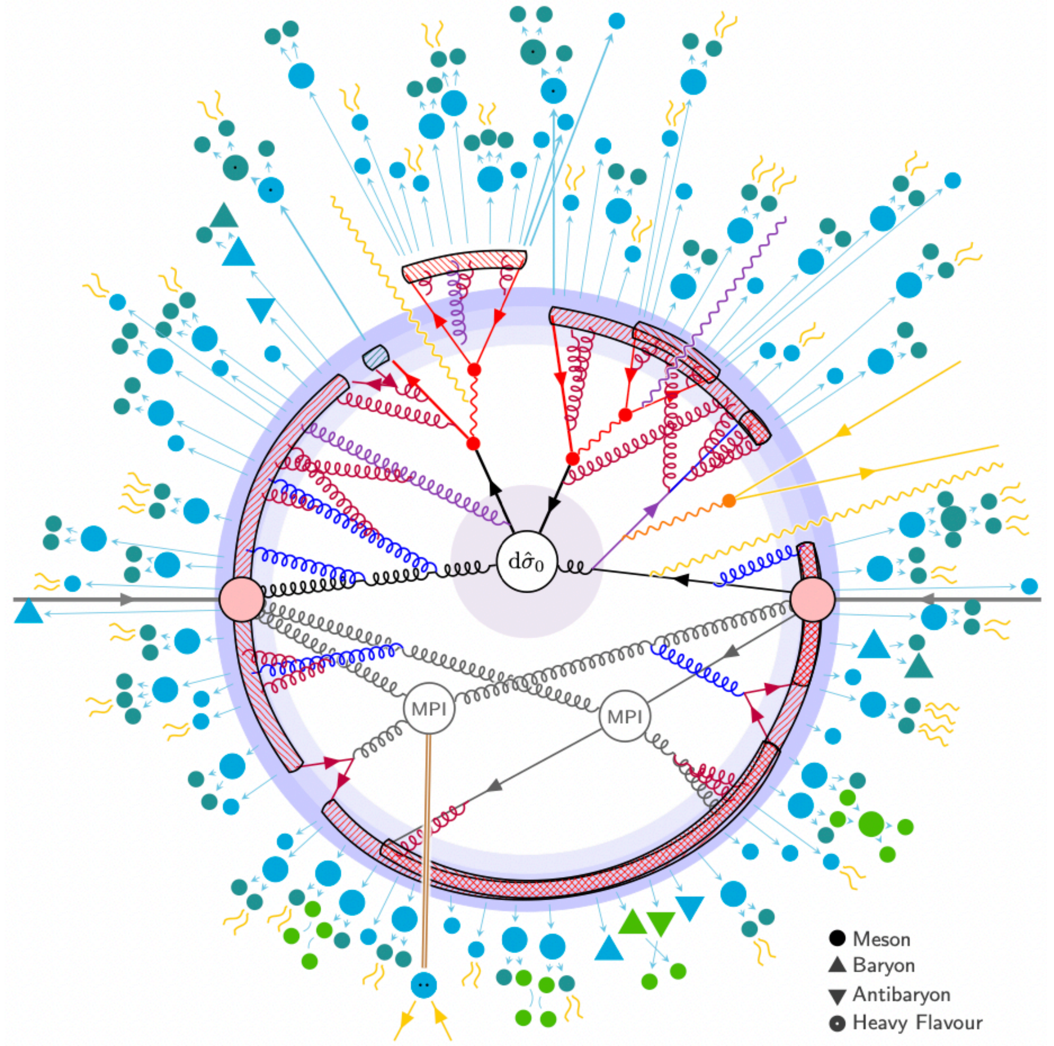
$$\sim \int \mathrm{d}t\, \mathrm{d}\vec{x}\, e^{iq\cdot x} \langle P_f | \mathcal{T}[\mathcal{O}(t,\vec{x})\mathcal{O}(0)] | P_i \rangle$$

QCD factorization

- Scalar field theory $|\langle X | U(t,t_0) | \phi\phi\rangle|^2$

Jordan, Lee, Preskill `10-`17





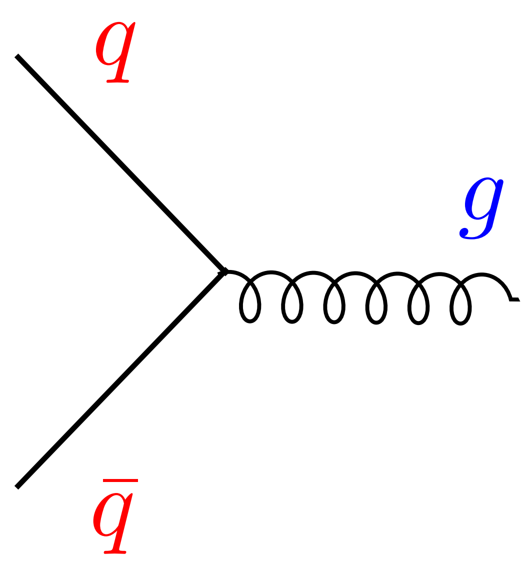
Outline

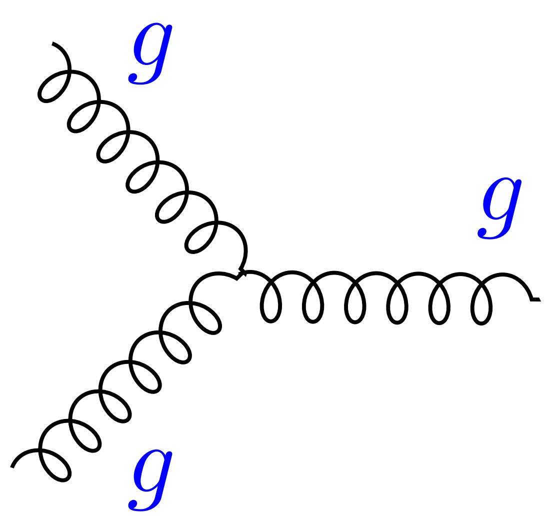
Qubits & qumodes with trapped ions

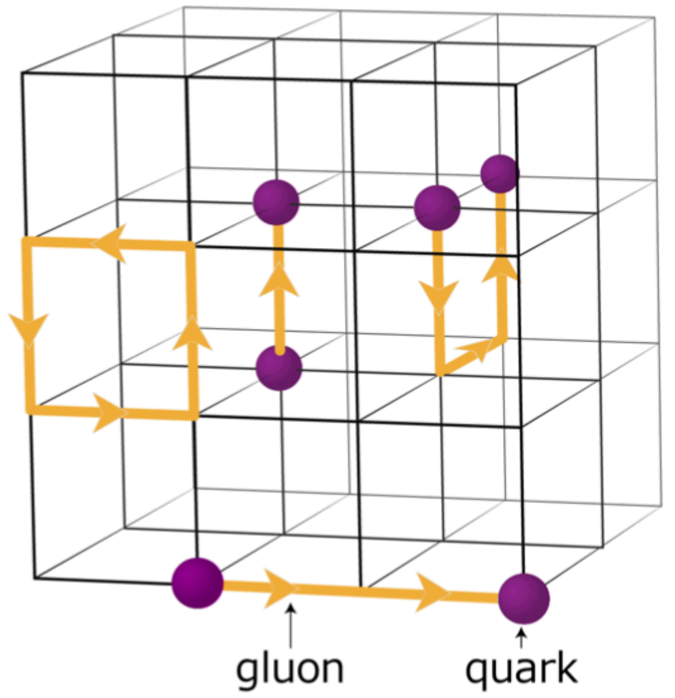
Simulation of spin-boson lattice models

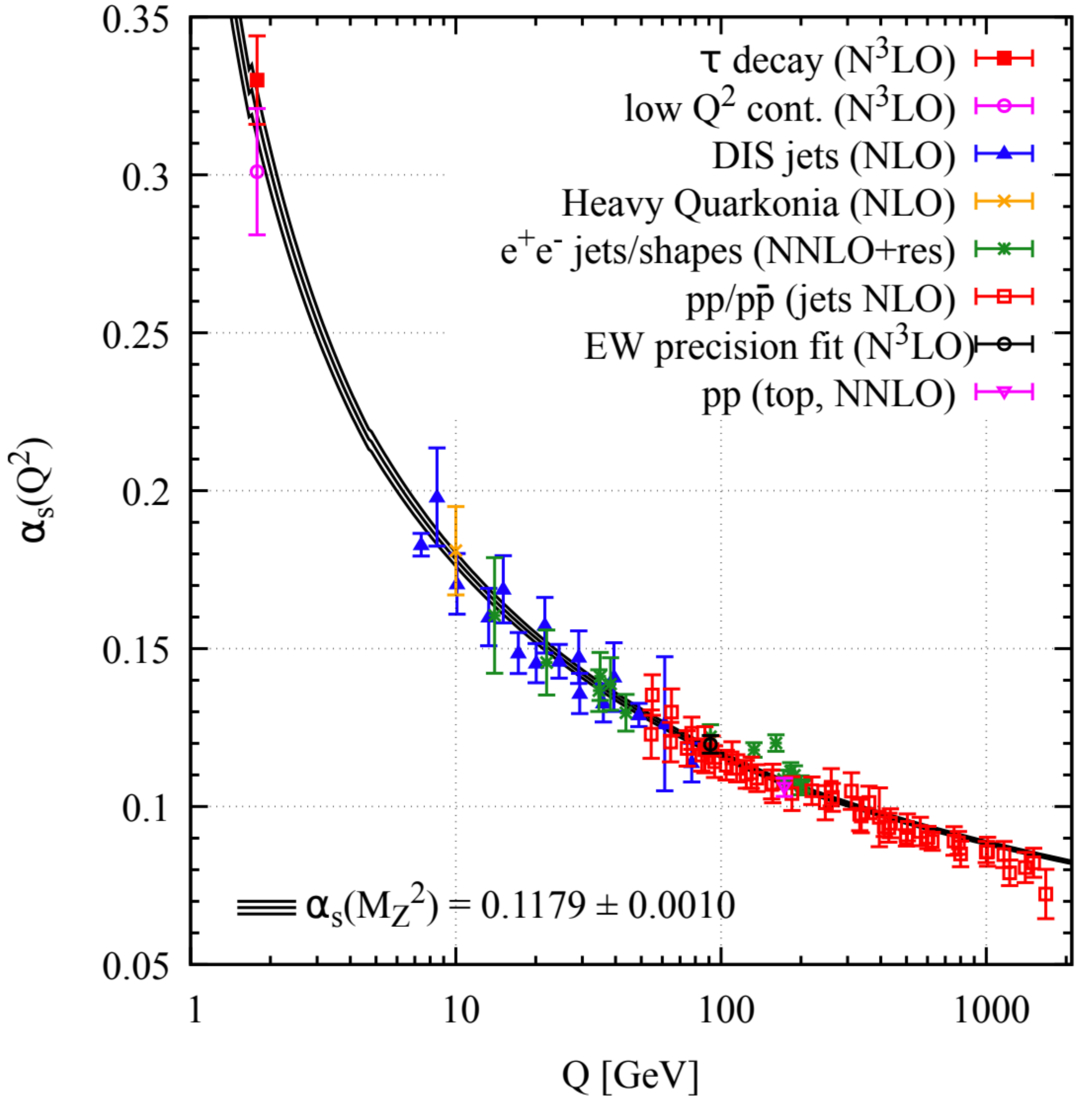
Quantum chromodynamics

- Theory of the strong interaction between quarks and gluons
- The Lagrangian $\mathcal{L} = \bar{\psi}\left(i\partial^{\mu}\gamma_{\mu} - m\right)\psi + g_s\bar{\psi}\gamma^{\mu}T_a\psi A^a_{\mu} - \frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}$









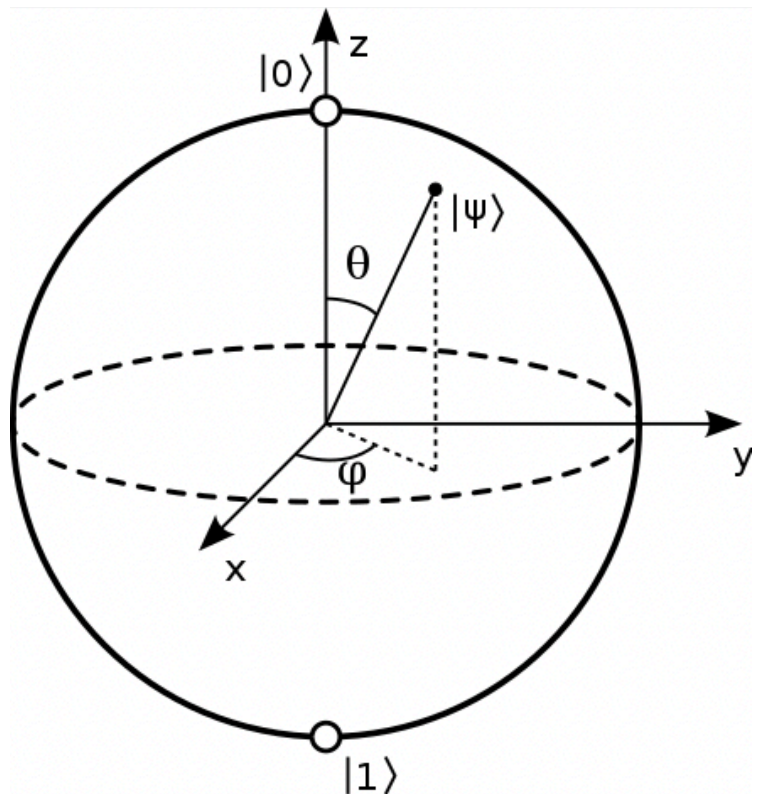
Huston, Rabbertz, Zanderighi (PDG)

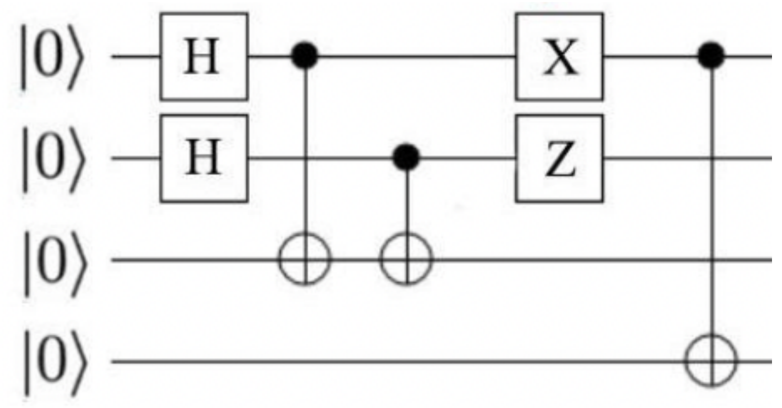
Simulate quarks and gluons with different quantum resources?

Qubits, qudits and qumodes

Elementary units for computing

- **Qubits**
  - Superconducting circuits, cold atoms, trapped ions, topological qubits
  - Digital gate-based computing





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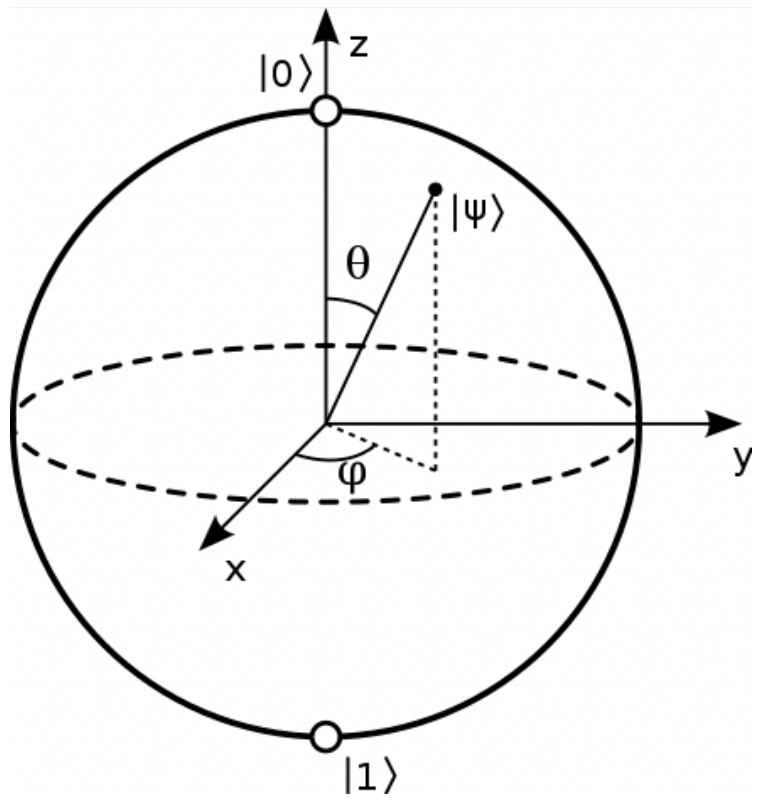
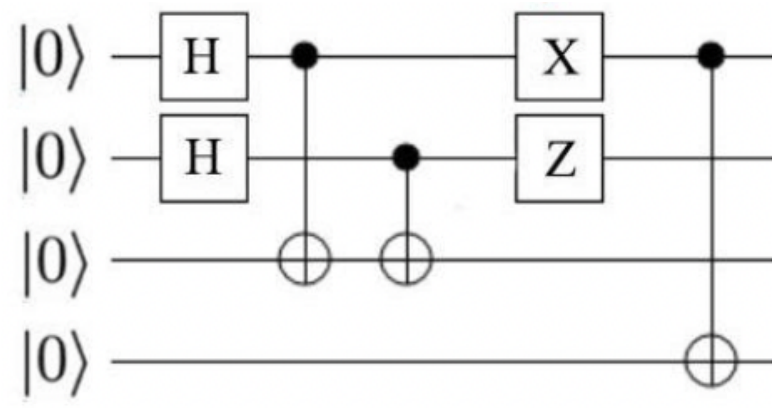
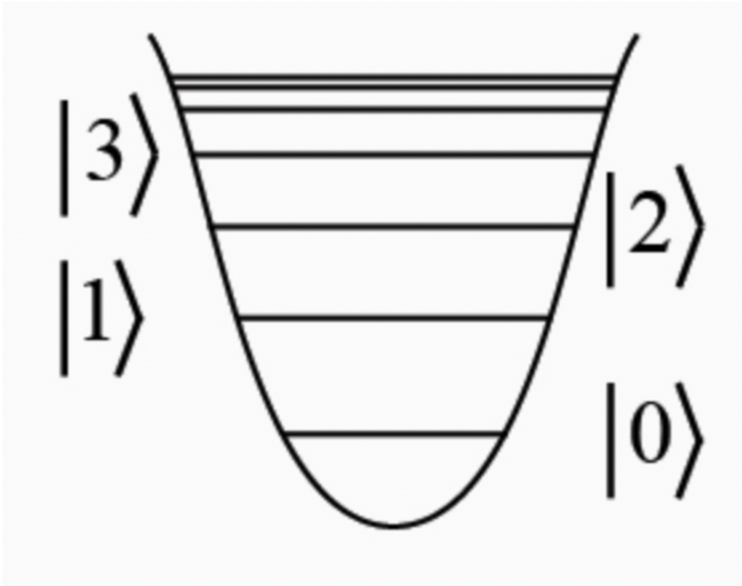




- **Qudits**
  - Multi-level $d > 2$ computational units
  - Gates e.g. $X_d,\ Z_d,\ C_2[R_d]$
  - Various hardware platforms



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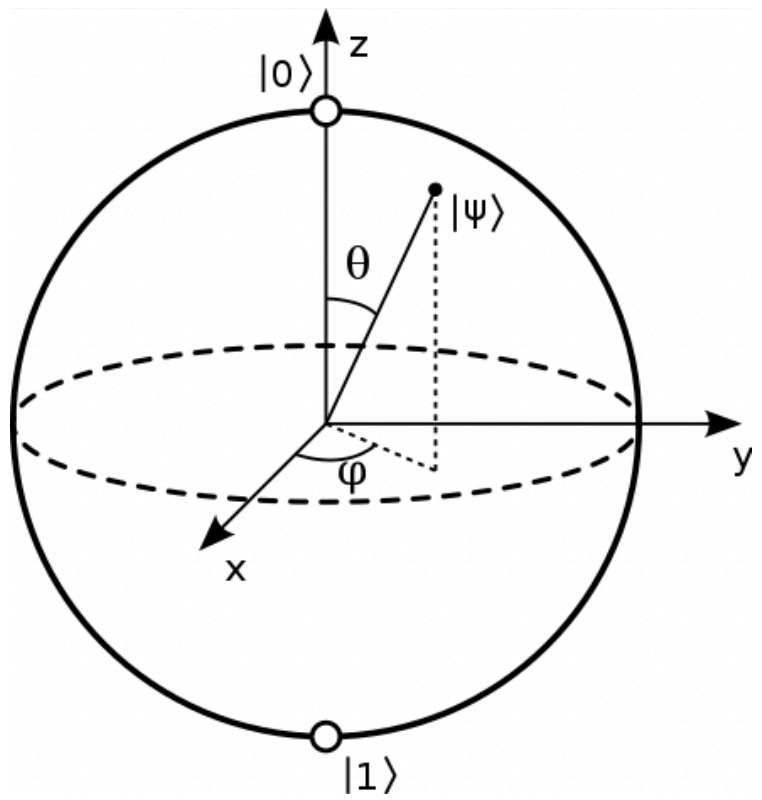
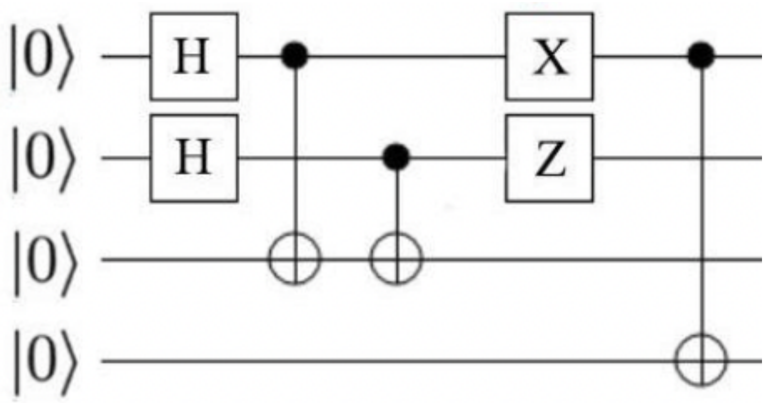
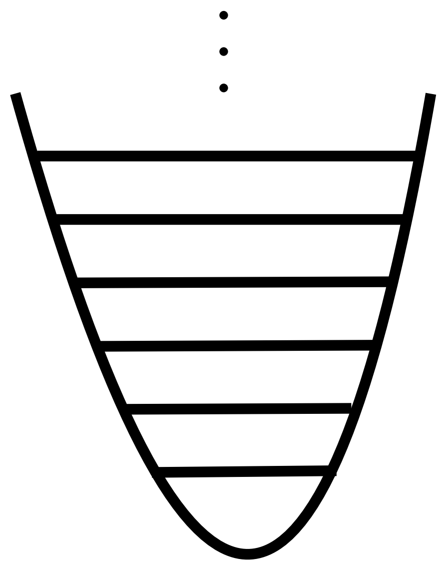
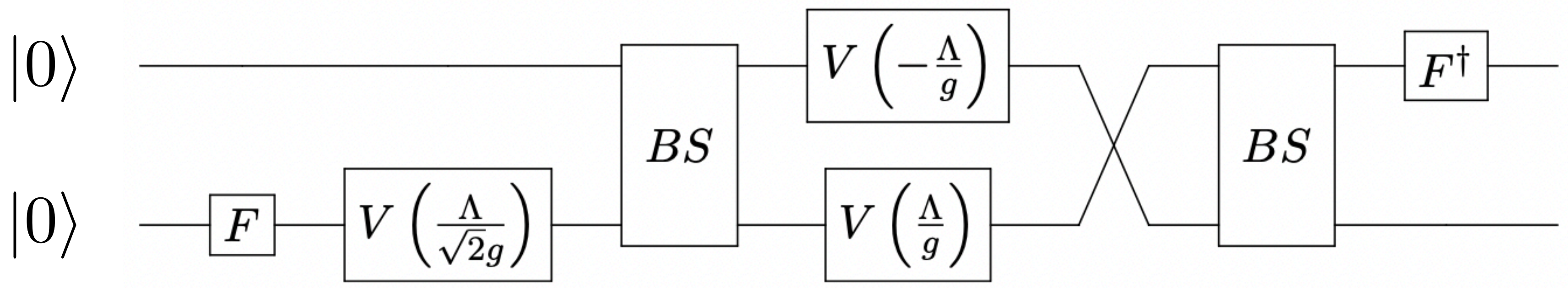




- **Qumodes**
  - Photonics, trapped ions, superconducting circuits
  - Infinite-dimensional Hilbert space
  - Gate based but with continuous variables



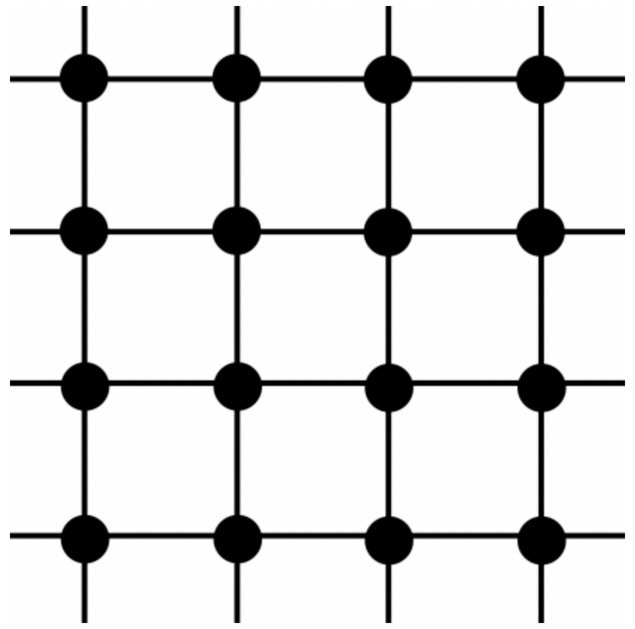


Quantum simulations for collider physics

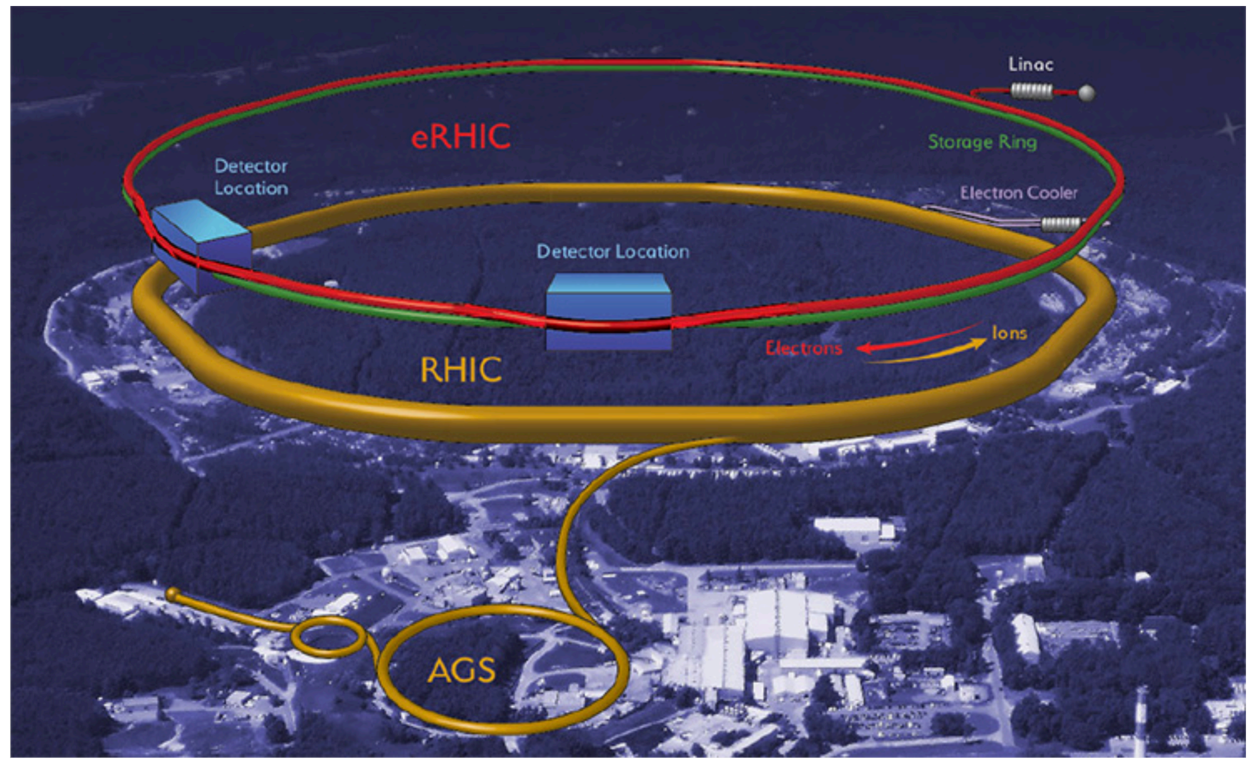
Kogut, Susskind `70s, Jordan, Lee, Preskill `11-`17

- Use the Hamiltonian formulation instead

$$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}\left(\nabla\phi\right)^2 + \frac{1}{2}m_0^2\phi^2 + \frac{\lambda}{4!}\phi^4$$



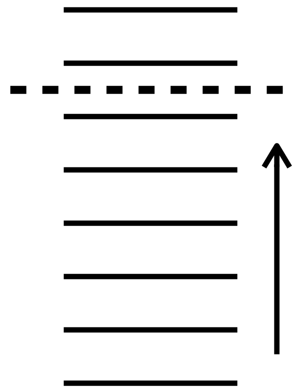
$\phi(x)$



- Discretize spatial component $x = an\,, \quad n = 0, ..., L-1$

- Truncate the local Hilbert space

Λ



Time-dependent n-point correlation functions $\langle \mathcal{O}(t_1)\mathcal{O}(t_2)\rangle$

- Take limits $L, \Lambda \to \infty\,,\ a \to 0$ + finite volume formalism

Toward quantum simulations for QCD

- Qumodes or continuous variables well suited for bosonic modes, scalar ϕ^4, and gauge fields

Marshall, Pooser, Siopsis, Weedbrock `15
Briceno, Edwards, Eaton, Gonzalez, Pfister, Siopsis `23
Abel, Spannowsky, Williams `24

ϕ, t continuous, $x = an$ discretized on a lattice

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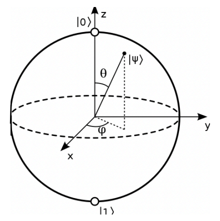
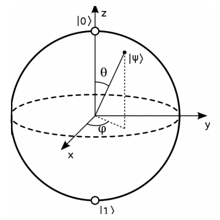
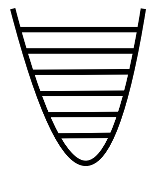
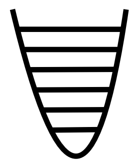
- QCD Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\partial^\mu\gamma_\mu - m)\,\psi + g_s\bar{\psi}\gamma^\mu T_a\psi A^a_\mu - \frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}$$

Qubits 

Qubit-qumode coupling  

Qumodes 

See George Siopsis' talk

→ Develop a hybrid qubit/qumode approach

$$\mathcal{H}^m_{\text{qumode}} \otimes \mathcal{H}^n_{\text{qubit}}$$

Quantum simulations with qumodes

see Lloyd, Braunstein `99

- Bosonic raising/lowering operators $\hat{a}, \hat{a}^\dagger$ and Fock states

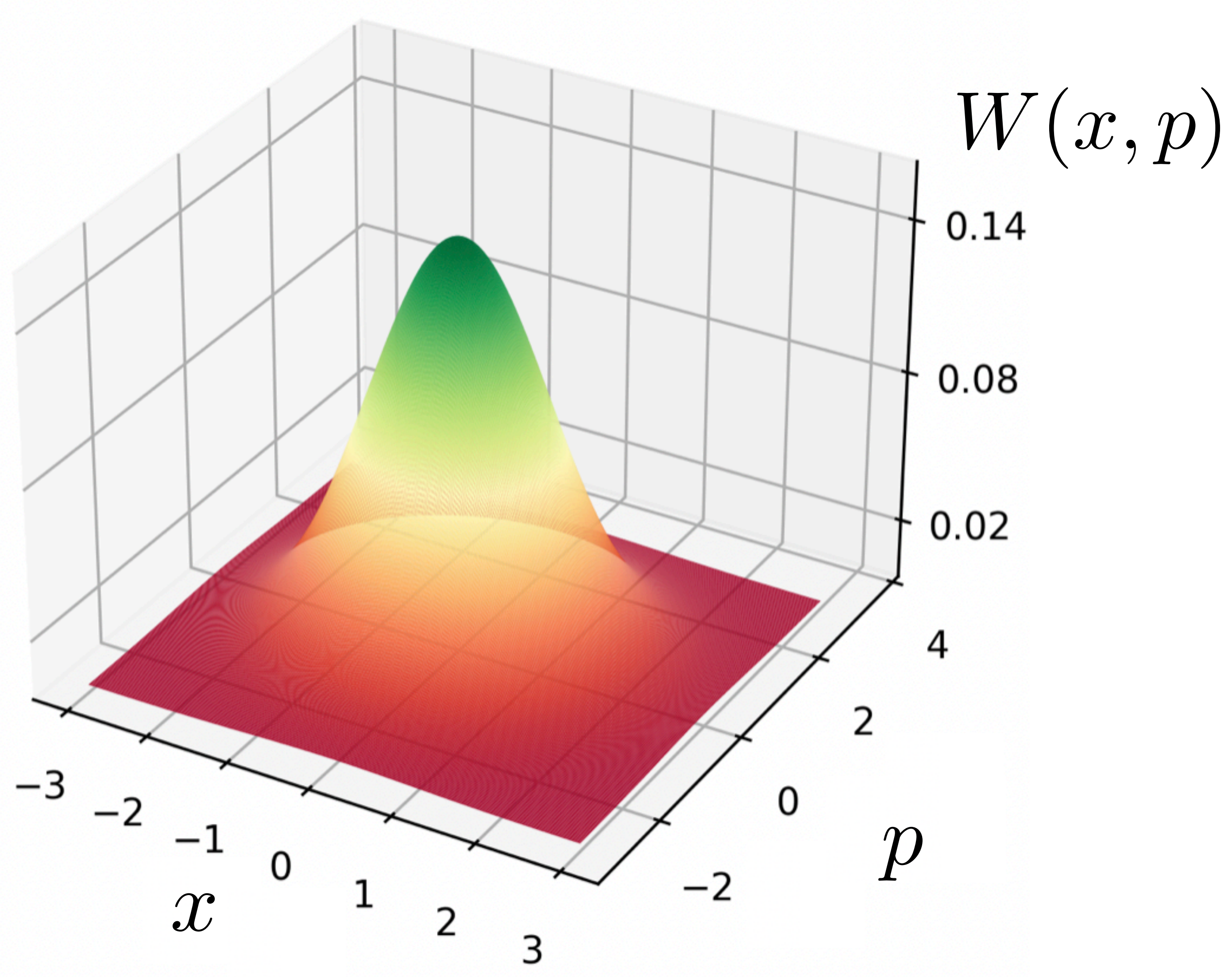
$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle$$

- Quadratures $\hat{X}, \hat{P}$ with $[\hat{X}, \hat{P}] = i\hbar$ and state

$$|\psi\rangle = \int_{\mathbb{R}} \psi(x)\,|x\rangle\,\mathrm{d}x$$

- Visualize using Wigner functions

$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \psi^*(x+y)\psi(x-y)e^{2ipy/\hbar}\mathrm{d}y$$



Ground state (in the Fock basis) $|0\rangle$

Quantum simulations with qumodes

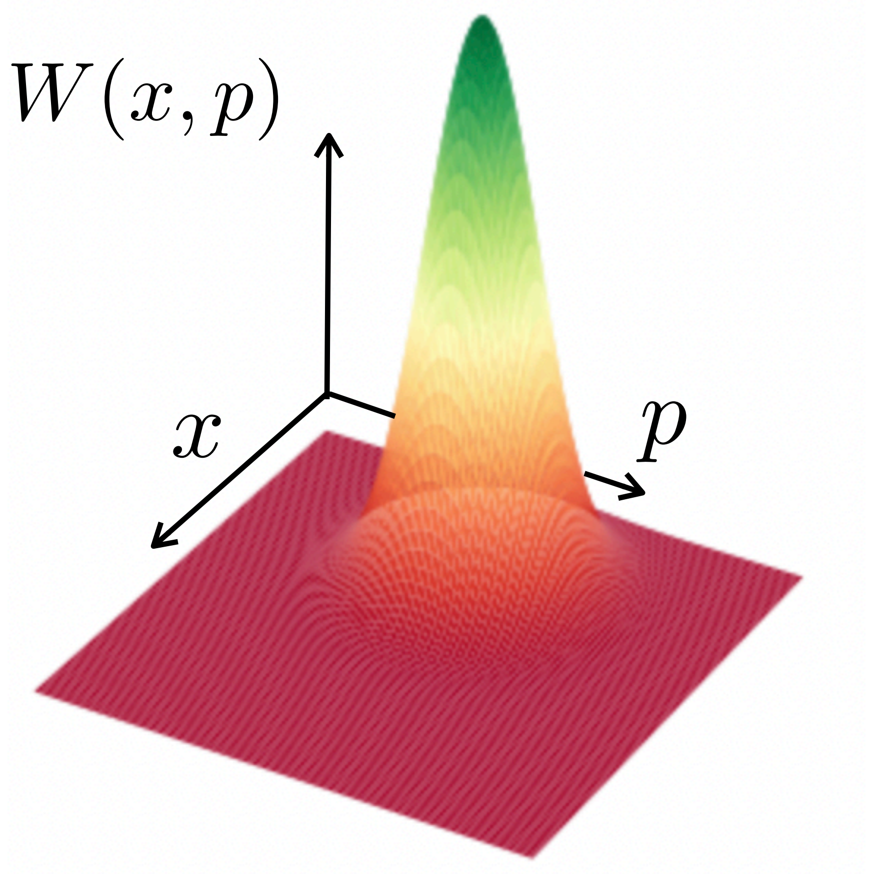
see Lloyd, Braunstein `99

Universal gate set

- Displacement $D(z) = e^{z\hat{a}^\dagger - z^*\hat{a}}$
- Rotation $R(\theta) = e^{i\theta\hat{a}^\dagger\hat{a}}$
- Two-qumode beam splitter

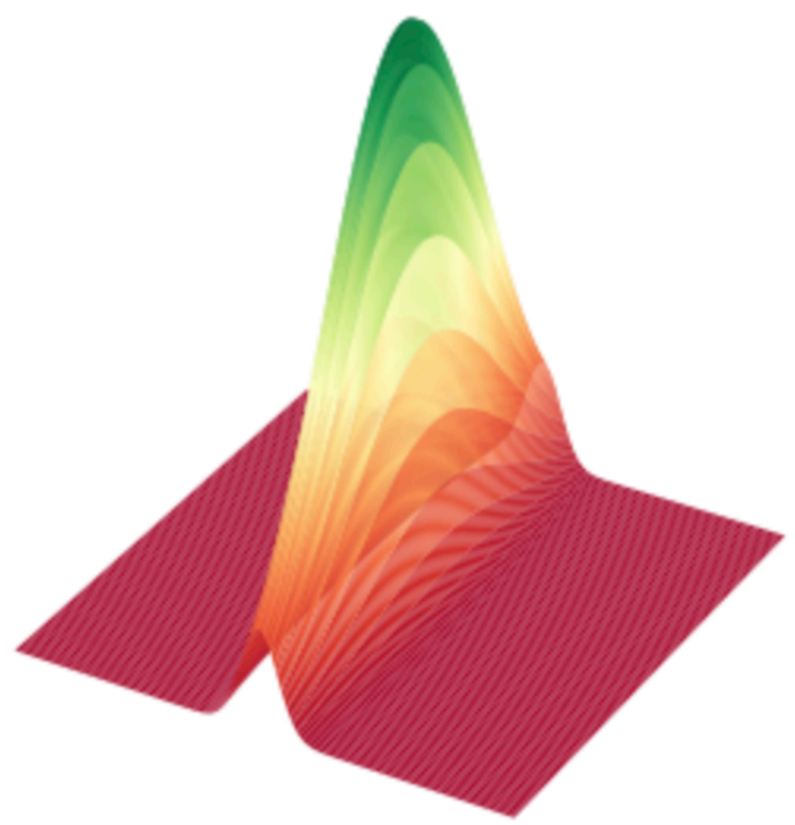
$$U_{\rm bs}(z) = e^{z\hat{a}\hat{b}^\dagger - z^*\hat{a}^\dagger\hat{b}}$$

$$z = \theta e^{i\phi}$$



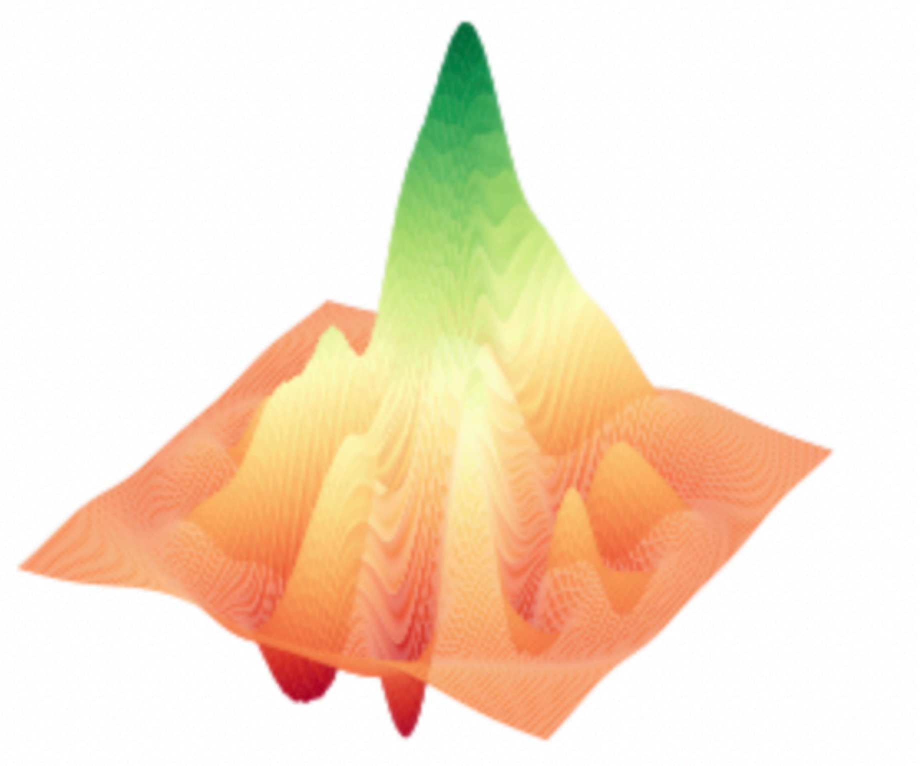
- Squeezing

$$S(z) = e^{\frac{1}{2}(z^*\hat{a}^2 - z\hat{a}^{\dagger 2})}$$



- Non-Gaussian operations

e.g. Kerr gate

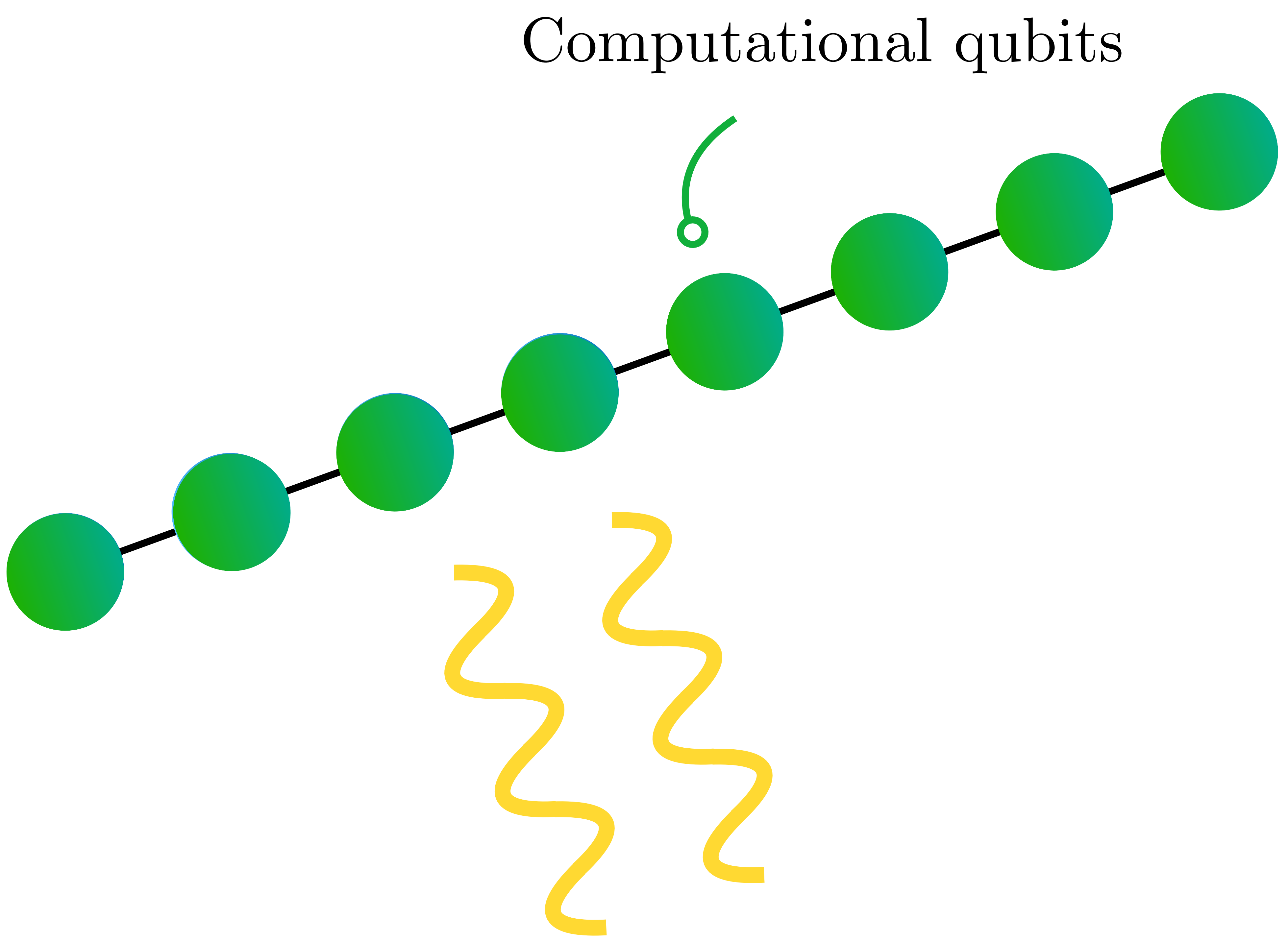


Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

See also Kenneth Brown's talk

- **Qubits:** electronic states of the trapped ion



Circuit QED, see Bosonic Qiskit, Girvin, Wiebe et al. `22

Qubit & qumodes with trapped ions

- **Qubits:** electronic states of the trapped ion

- **Qumodes:** collective vibrational modes of the ion chain (phonons)

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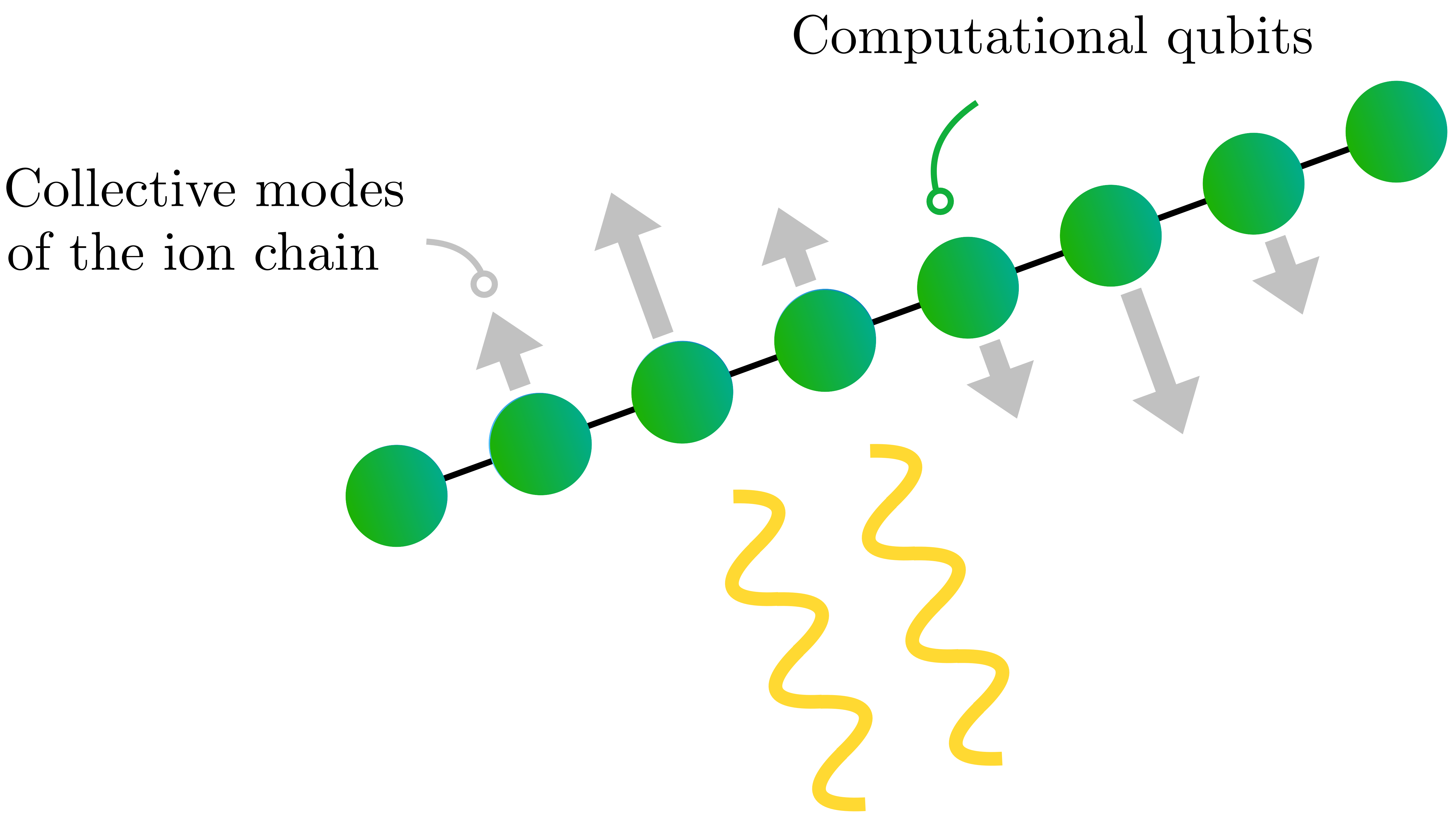

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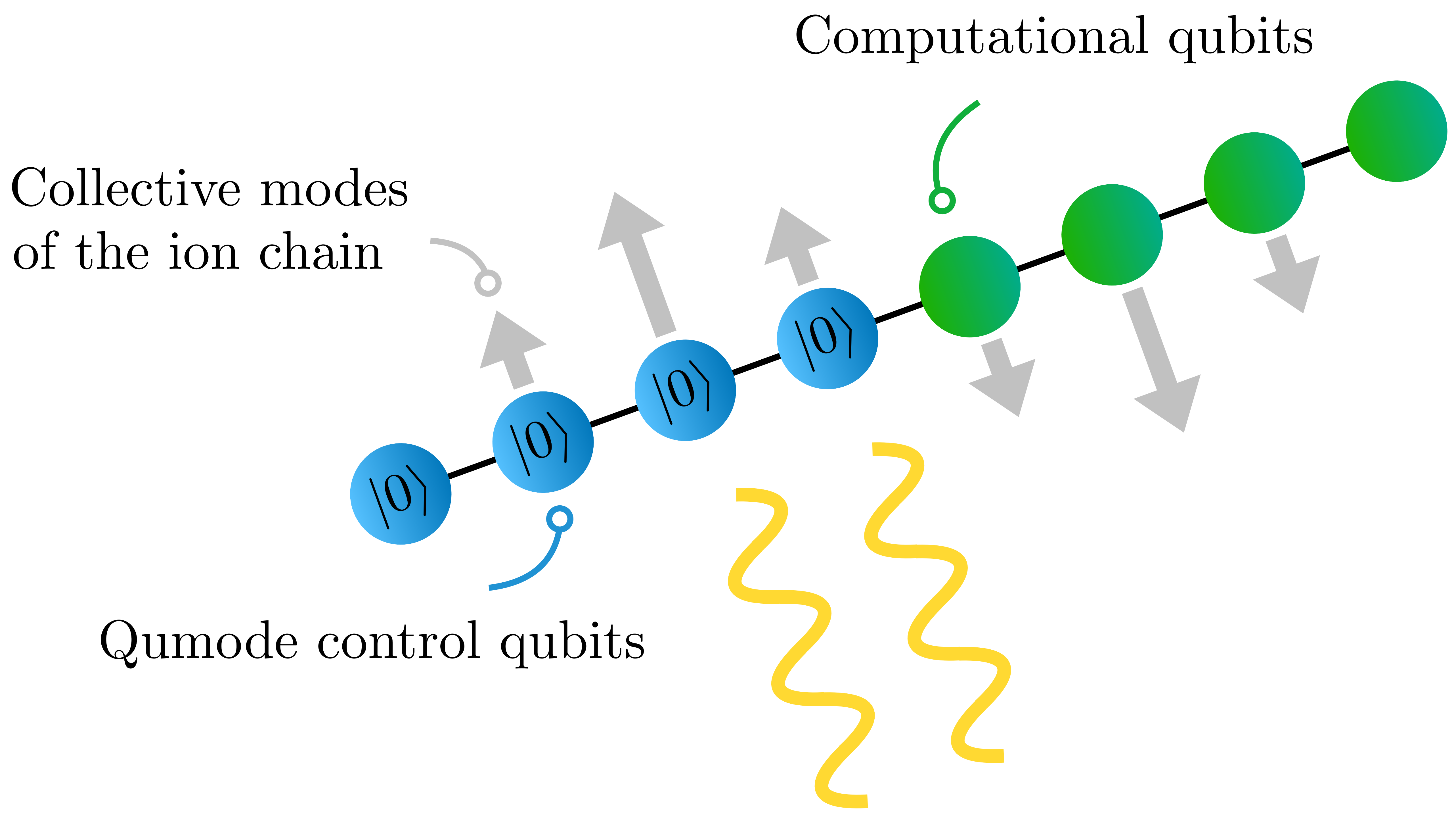
- **Qubits:** electronic states of the trapped ion
- **Qumodes:** collective vibrational modes of the ion chain (phonons)
- Laser interacts with qubits

→ Need several qubits to control and readout qumodes



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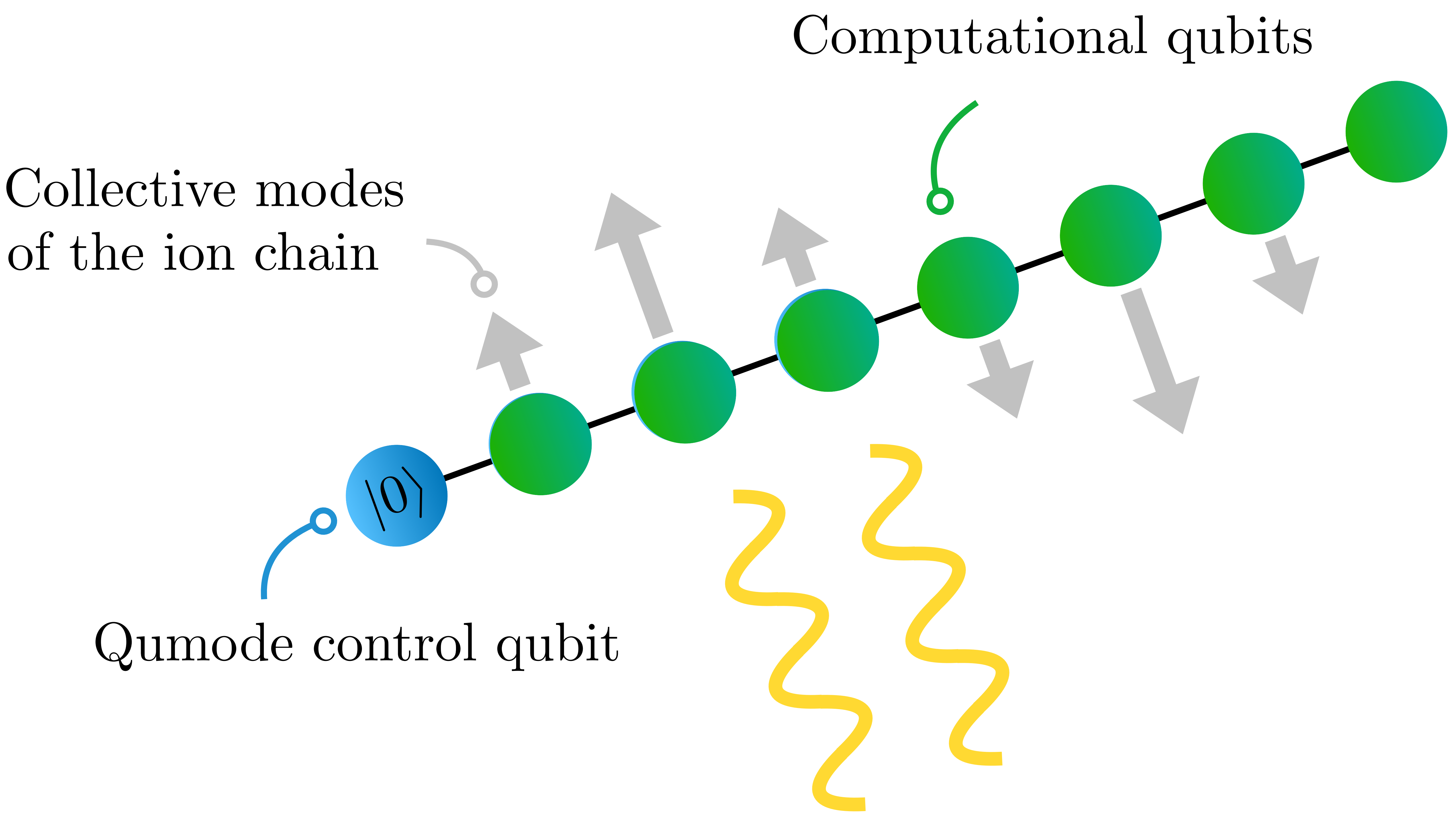
- Laser interacts with qubits

→ Feasibility studies with a minimal number of control qubits

Araz, Grau, Montgomery, FR `24

See also Kenneth Brown's talk



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Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

Type	Operation	Short	Operator	Estimated gate time	Estimated fidelity	Ref.
Qubit gates	Pauli operators		σ^i	2 µs	99.999%	[92]
	Rotation	$R_i(\theta)$	$e^{i\theta\sigma^i/2}$	2 µs	99.999%	[92]
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1-\sigma_1^z)(\mathbb{I}_2-\sigma_2^x)}$	30 µs	99.9%	[93]
Qumode gates	Rotation	$R(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$	200 µs*	99%*	[94]
	Displacement	$D(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$	10 µs	99%	[95]
	Single-mode squeezing	$S(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$	3 µs	98%	[96]
	Beam splitter	$BS(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$	250 µs	99%	[68]
	Kerr	$K(z)$	$e^{i\theta(\hat{a}^\dagger\hat{a})^2}$	10 ms*	95%*	[97]
	Cross-Kerr	$CK(z)$	$e^{i\theta\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$	800 µs	97%	[98]
Hybrid gates	Red sideband	$RSB(z)$	$e^{iz\hat{a}\sigma^++iz^*\hat{a}^\dagger\sigma^-}$	200 µs	99.9%	[68]
	Blue sideband	$BSB(z)$	$e^{iz\hat{a}^\dagger\sigma^++iz^*\hat{a}\sigma^-}$	200 µs	99.9%	[68]
	Controlled rotation	$CR(\theta)$	$e^{i\theta\sigma^z\hat{a}^\dagger\hat{a}}$	200 µs*	99%*	[13]
	Controlled displacement	$CD(z)$	$e^{\sigma^z(z\hat{a}^\dagger-z^*\hat{a})}$	800 µs	95%*	[99]
	Controlled squeezing	$CS(z)$	$e^{\sigma^z(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$	120 µs*	99%*	[13]
	Controlled beam splitter	$CBS(z)$	$e^{\sigma^z(z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger)}$	250 µs	99%	[68]
Measurements	Qubit Pauli strings		σ^i	145 µs	99.99%	[100]
	Average phonon number		$\hat{N}$	200 µs	97%	[101]
	Qumode PNR		$\lvert n\rangle\langle n\rvert$	400 µs	99%	[102]
	Qumode homodyne		$\hat{X}, \hat{P}$	200 µs	95%*	[102]
	Hybrid		$\sigma^i\hat{X},\ \sigma^i\hat{P}$	200 µs†	95%†	

$^{171}Yb^+$ ions in a linear Paul trap

where $z = \theta e^{i\phi}$

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	Cross-Kerr	$\mathrm{CK}(z)$	$e^{i\theta\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$	800 µs	97%	[98]
Hybrid gates	Red sideband	$\mathrm{RSB}(z)$	$e^{iz\hat{a}\sigma^+ + iz^*\hat{a}^\dagger\sigma^-}$	200 µs	99.9%	[68]
	Blue sideband	$\mathrm{BSB}(z)$	$e^{iz\hat{a}^\dagger\sigma^+ + iz^*\hat{a}\sigma^-}$	200 µs	99.9%	[68]
	Controlled rotation	$\mathrm{CR}(\theta)$	$e^{i\theta\sigma^z\hat{a}^\dagger\hat{a}}$	200 µs*	99%*	[13]
	Controlled displacement	$\mathrm{CD}(z)$	$e^{\sigma^z(z\hat{a}^\dagger - z^*\hat{a})}$	800 µs	95%*	[99]
	Controlled squeezing	$\mathrm{CS}(z)$	$e^{\sigma^z(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$	120 µs*	99%*	[13]
	Controlled beam splitter	$\mathrm{CBS}(z)$	$e^{\sigma^z(z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger)}$	250 µs	99%	[68]
Measurements	Qubit Pauli strings		σ^i	145 µs	99.99%	[100]
	Average phonon number		$\hat{N}$	200 µs	97%	[101]
	Qumode PNR		$\|n\rangle\langle n\|$	400 µs	99%	[102]
	Qumode homodyne		$\hat{X},\hat{P}$	200 µs	95%*	[102]
	Hybrid		$\sigma^i\hat{X},\ \sigma^i\hat{P}$	200 µs†	95%†	

Araz, Grau, Montgomery, FR `24

$^{171}Yb^+$ ions in a linear Paul trap

where $z = \theta e^{i\phi}$

Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

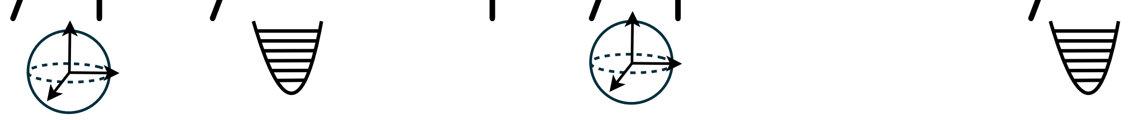
See also Kenneth Brown's talk

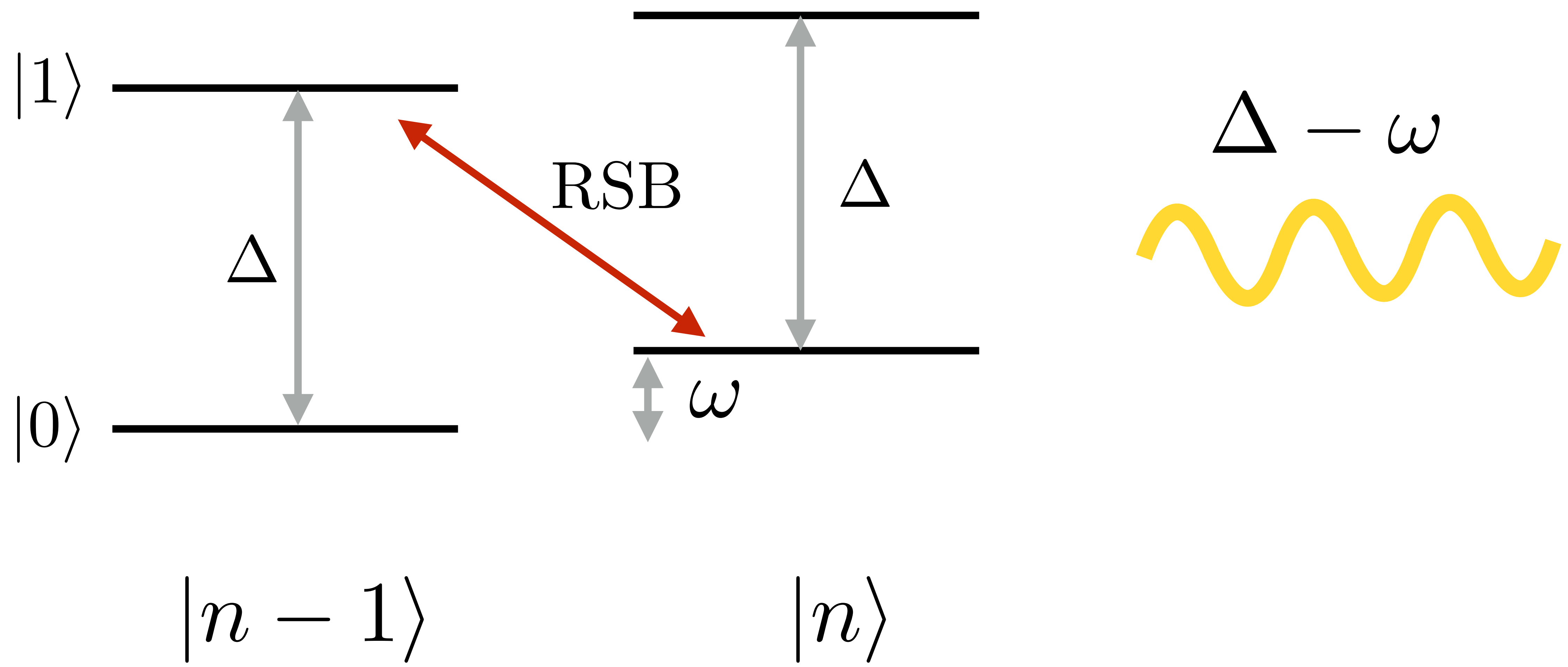
- Hybrid gate example

Red sideband gate $U_{\text{rsb}}(z) = e^{iz\hat{a}\sigma^+ + iz^*\hat{a}^\dagger\sigma^-}$

Red sideband detuned laser

$$|0\rangle\,|n\rangle \leftrightarrow |1\rangle\,|n-1\rangle$$





- Preserves the total number of excitations
- Beam splitter using two detuned red sidebands

Qubit & qumodes with trapped ions

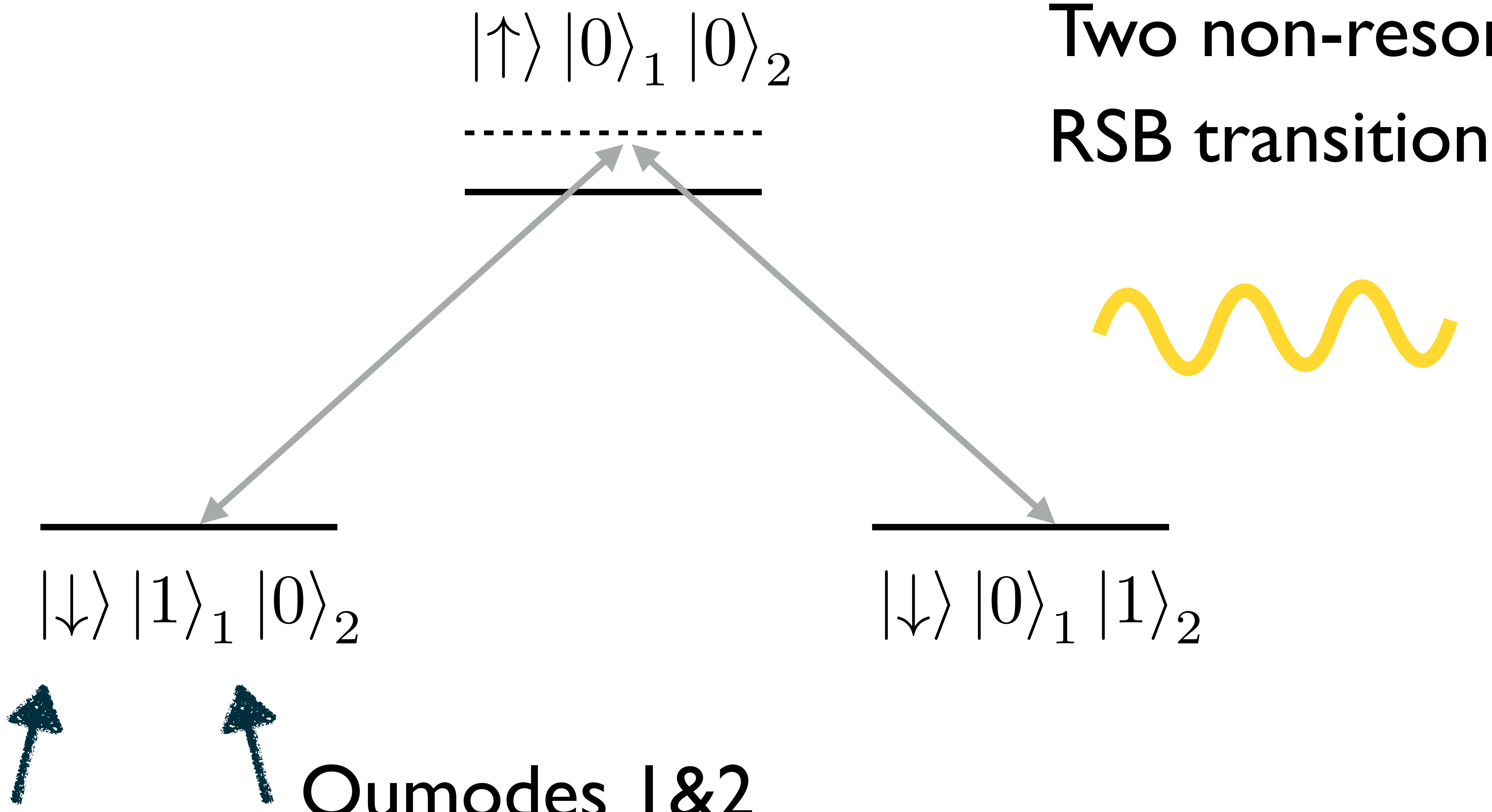
Araz, Grau, Montgomery, FR `24

See also Kenneth Brown's talk

- Two-qumode beam splitter *Katz, Monroe `22, Kim et al. `23*

- Laser does not interact with the two modes directly

$$U_{\rm bs}(z) = e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$$



Two non-resonant RSB transitions

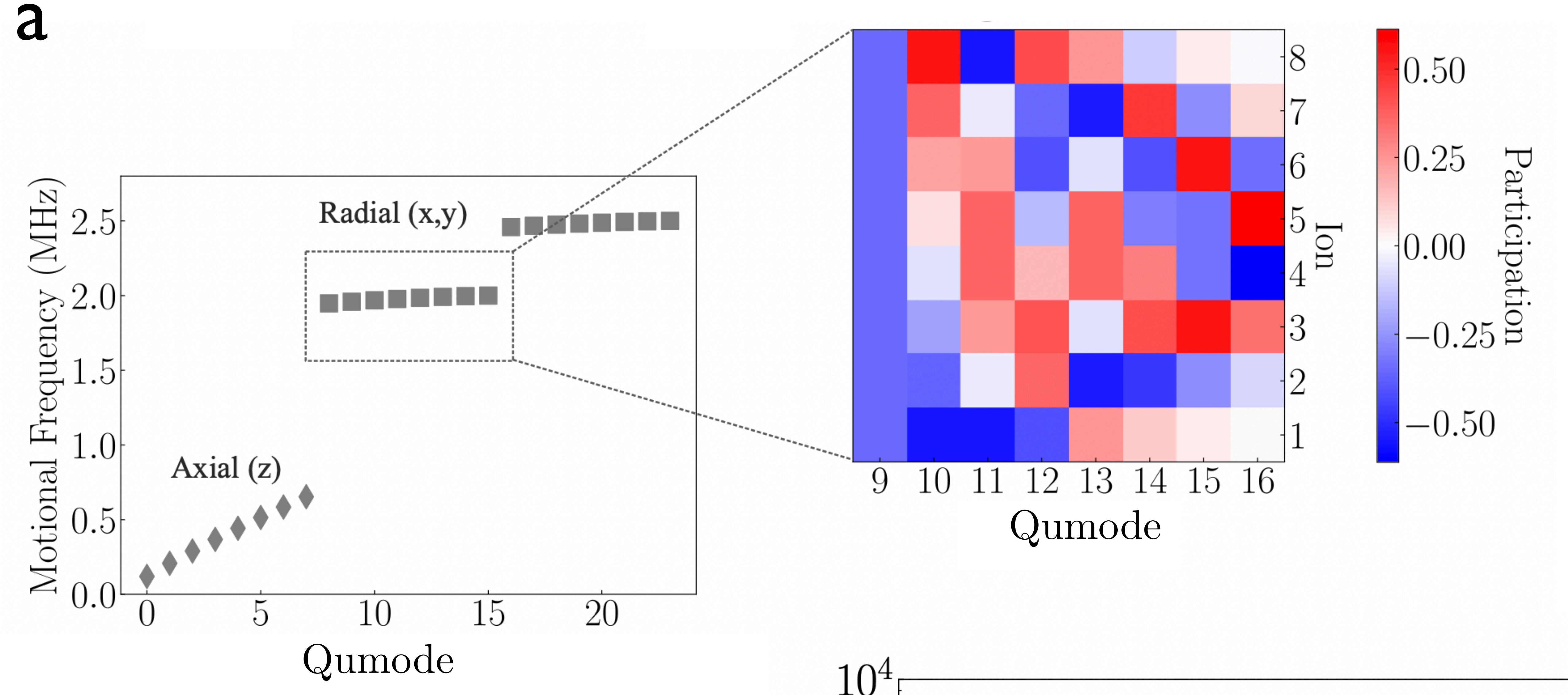
Control qubit

Qumodes 1&2

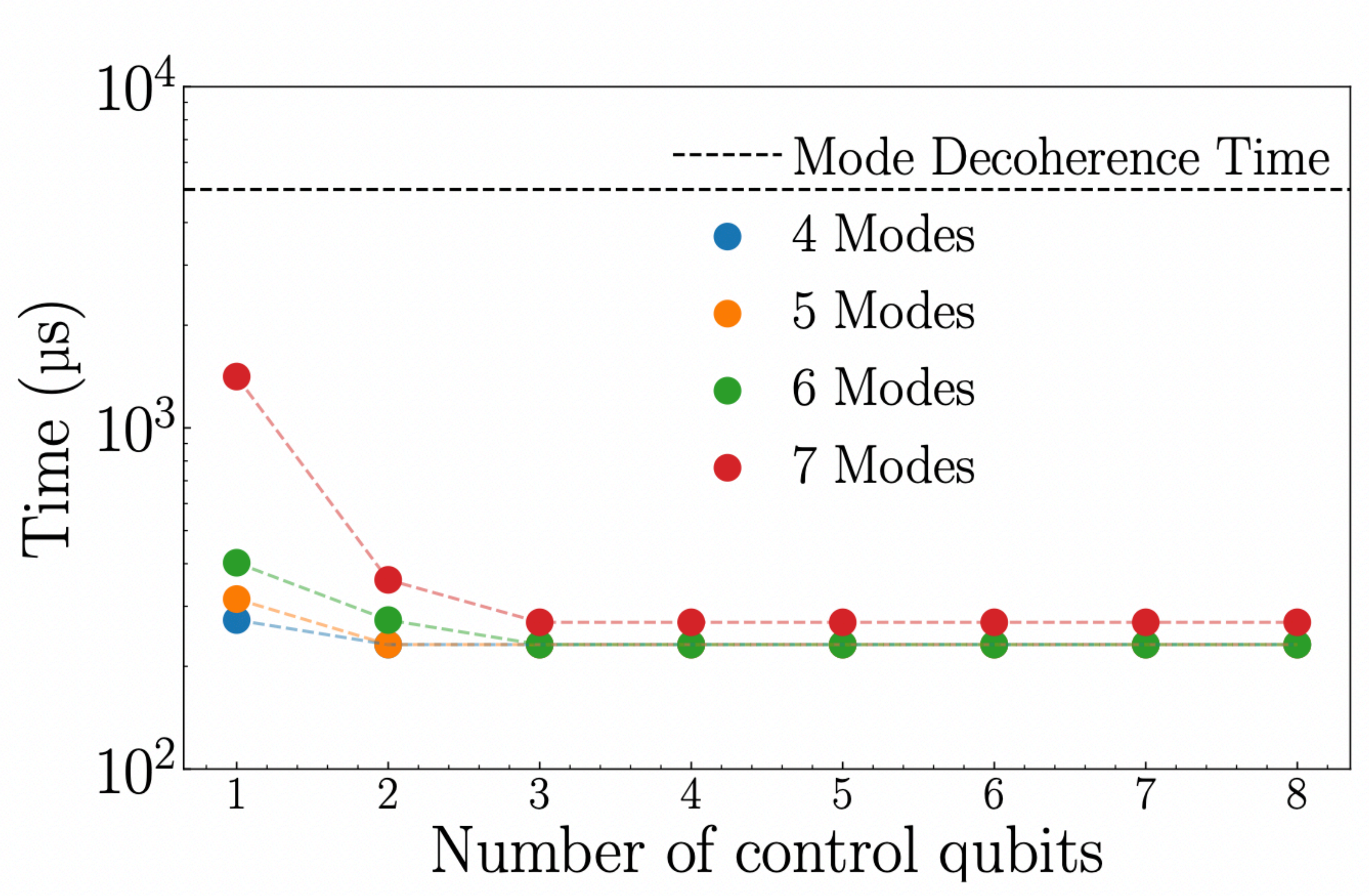
Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

- Eight $^{171}Yb^{+}$ ions in a linear Paul trap



- E.g. two-qumode beam splitter $U_{\rm bs}(z) = e^{z\hat{a}^\dagger\hat{b} - z^*\hat{a}\hat{b}^\dagger}$



→ Expect to be feasible with current platforms

Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

Type	Operation	Short	Operator	Estimated gate time	Estimated fidelity	Ref.
Qubit gates	Pauli operators		σ^i	2 μs	99.999%	[92]
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$	2 μs	99.999%	[92]
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1-\sigma_1^z)(\mathbb{I}_2-\sigma_2^x)}$	30 μs	99.9%	[93]
Qumode gates	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$	200 μs*	99%*	[94]
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$	10 μs	99%	[95]
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$	3 μs	98%	[96]
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$	250 μs	99%	[68]
	Kerr	$\mathrm{K}(z)$	$e^{i\theta(\hat{a}^\dagger\hat{a})^2}$	10 ms*	95%*	[97]
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{i\theta\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$	800 μs	97%	[98]
Hybrid gates	Red sideband	$\mathrm{RSB}(z)$	$e^{iz\hat{a}\sigma^++iz^*\hat{a}^\dagger\sigma^-}$	200 μs	99.9%	[68]
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$^{171}Yb^+$ ions in a linear Paul trap

where $z=\theta e^{i\phi}$

Qubit & qumodes with transmons

Native transmon gates

Gate Name	Gate Operation
Mode gates	
$\mathrm{R}_i(\theta)$	$\exp\left(-i\theta\hat{n}_i\right)$
$\mathrm{D}_i(\alpha)$	$\exp\left(\alpha\hat{a}_i^\dagger - \alpha^*\hat{a}_i\right)$
$\mathrm{BS}_{i,j}(\varphi,\theta)$	$\exp\left(-i\theta\left(e^{i\varphi}\hat{a}_i^\dagger\hat{a}_j + e^{-i\varphi}\hat{a}_i\hat{a}_j^\dagger\right)\right)$
Transmon gates	
$\mathrm{R}_i^z(\theta)$	$\exp\left(-i\frac{\theta}{2}\hat{Z}_i\right)$
$\mathrm{R}_i^\varphi(\theta)$	$\exp\left(-i\frac{\theta}{2}\hat{\sigma}_i^\varphi\right)$ $\hat{\sigma}_i^\varphi = \hat{X}_i\cos\varphi + \hat{Y}_i\sin\varphi$
Transmon-Mode gates	
$\mathrm{CR}_{i,j}(\theta)$	$\exp\left(-i\frac{\theta}{2}\hat{Z}_i\hat{n}_j\right)$
$\mathrm{C\Pi}_{i,j}$	$\exp\left(-i\frac{\pi}{2}\hat{Z}_i\hat{n}_j\right)$
$\mathrm{CD}_{i,j}(\alpha)$	$\exp\left(\hat{Z}_i\left(\alpha\hat{a}_j^\dagger - \alpha^*\hat{a}_j\right)\right)$
$\mathrm{SNAP}_{i,j}(\vec{\theta})$	$\exp\left(-i\hat{Z}_i\sum_n\theta_n\|n\rangle\langle n\|_j\right)$
$\mathrm{SQR}_{i,j}(\vec{\theta},\vec{\varphi})$	$\sum_n \mathrm{R}_i^{\varphi_n}(\theta_n)\otimes\|n\rangle\langle n\|_j$

Liu, Singh, Smith et al. `24
Crane, Smith, Tomesh et al. `24

- Different native gate set
- Connectivity
- Coherence times

Trapped ions:

Qubits $\mathcal{O}(s)$, qumodes $\mathcal{O}(ms)$

Transmons:

Qubits $\mathcal{O}(ms)$, qumodes $\mathcal{O}(s)$

Outline

Qubits & qumodes with trapped ions

Simulation of spin-boson lattice models

The many-body Jaynes-Cummings model

Araz, Grau, Montgomery, FR `24

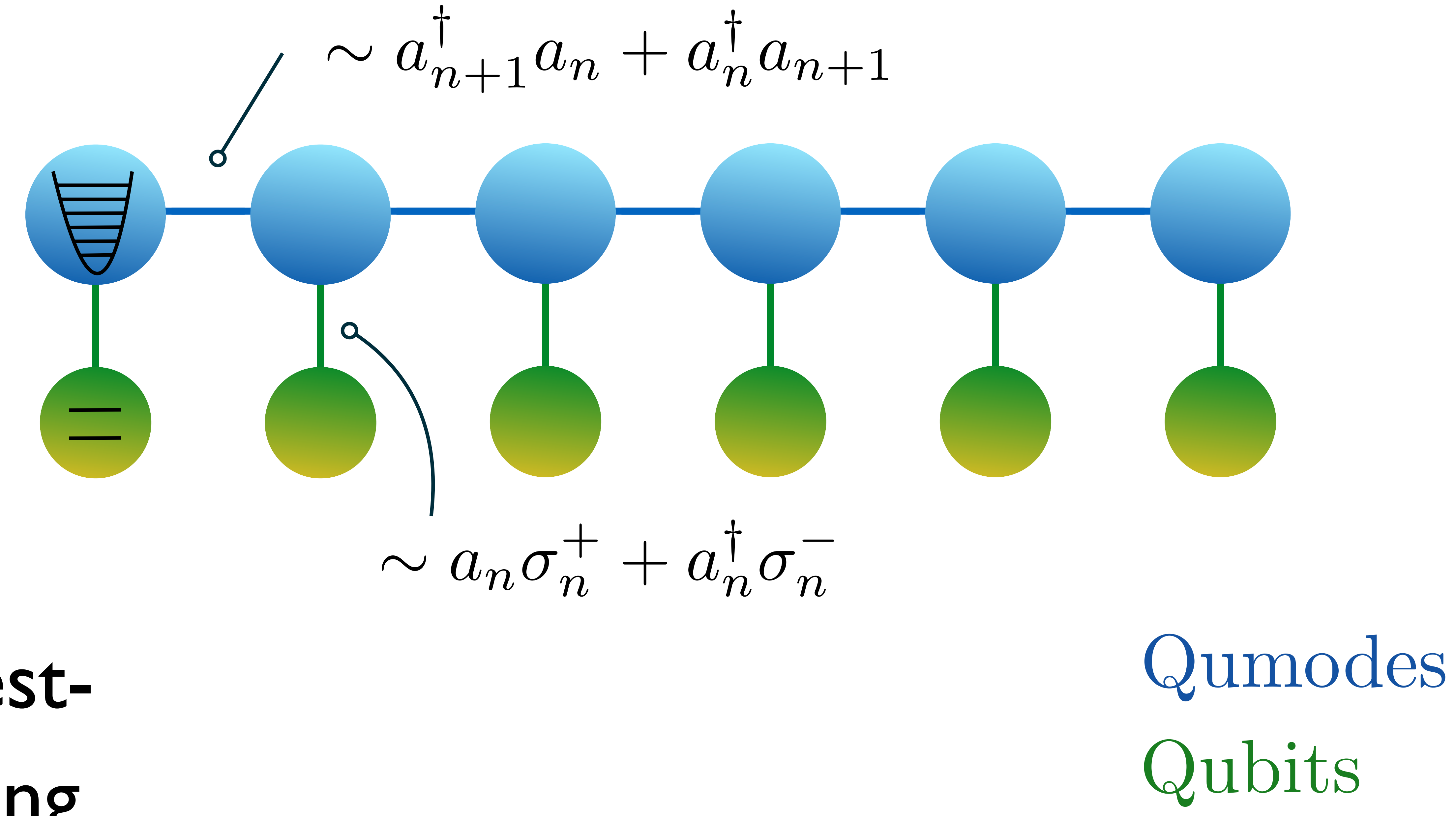
- 1-dimensional Hamiltonian with N lattice sites

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n$$ → N qumodes

$$+ \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^-$$ → N qubits

$$+ \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right)$$ → Qumode nearest-neighbor hopping

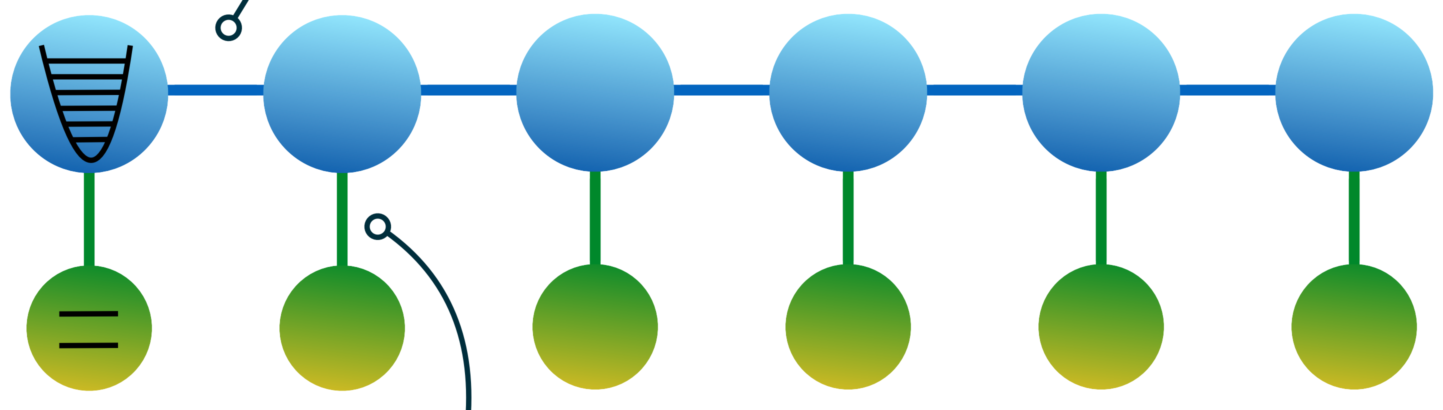
$$+ \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$ → Onsite qumode-qubit interaction



The many-body Jaynes-Cummings model

- 1-dimensional Hamiltonian

Araz, Grau, Montgomery, FR `24



$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n$$

$$+ \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^-$$

$$+ \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right)$$

$$+ \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$

- The total number of excitations is conserved

$$[\hat{H}, \hat{N}_{\text{tot}}] = 0 \quad \text{where} \quad \hat{N}_{\text{tot}} = \sum_{n=1}^{N} a_n^\dagger a_n + \sum_{n=1}^{N} \sigma^+ \sigma^-$$

- Related to the Schwinger model, U(1) gauge theory + fermions

The many-body Jaynes-Cummings model

- Real-time evolution

Araz, Grau, Montgomery, FR `24

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n$$ → Qumode rotation

$$+ \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^-$$ → Qubit $\sim \mathbb{I} + \sigma_z$

$$+ \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right)$$ → Qumode beam splitter

$$+ \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$ → Red sideband

Trotter decomposition

$$U_1(t) = \prod_j e^{-iH_j t}$$

The many-body Jaynes-Cummings model

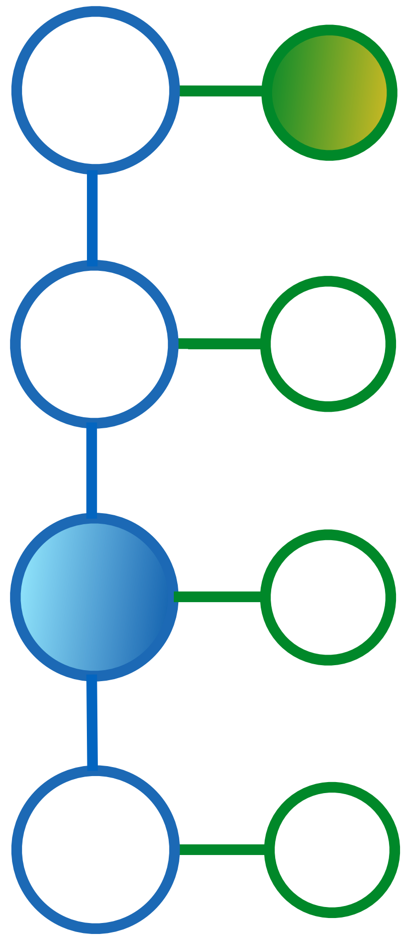
Araz, Grau, Montgomery, FR `24

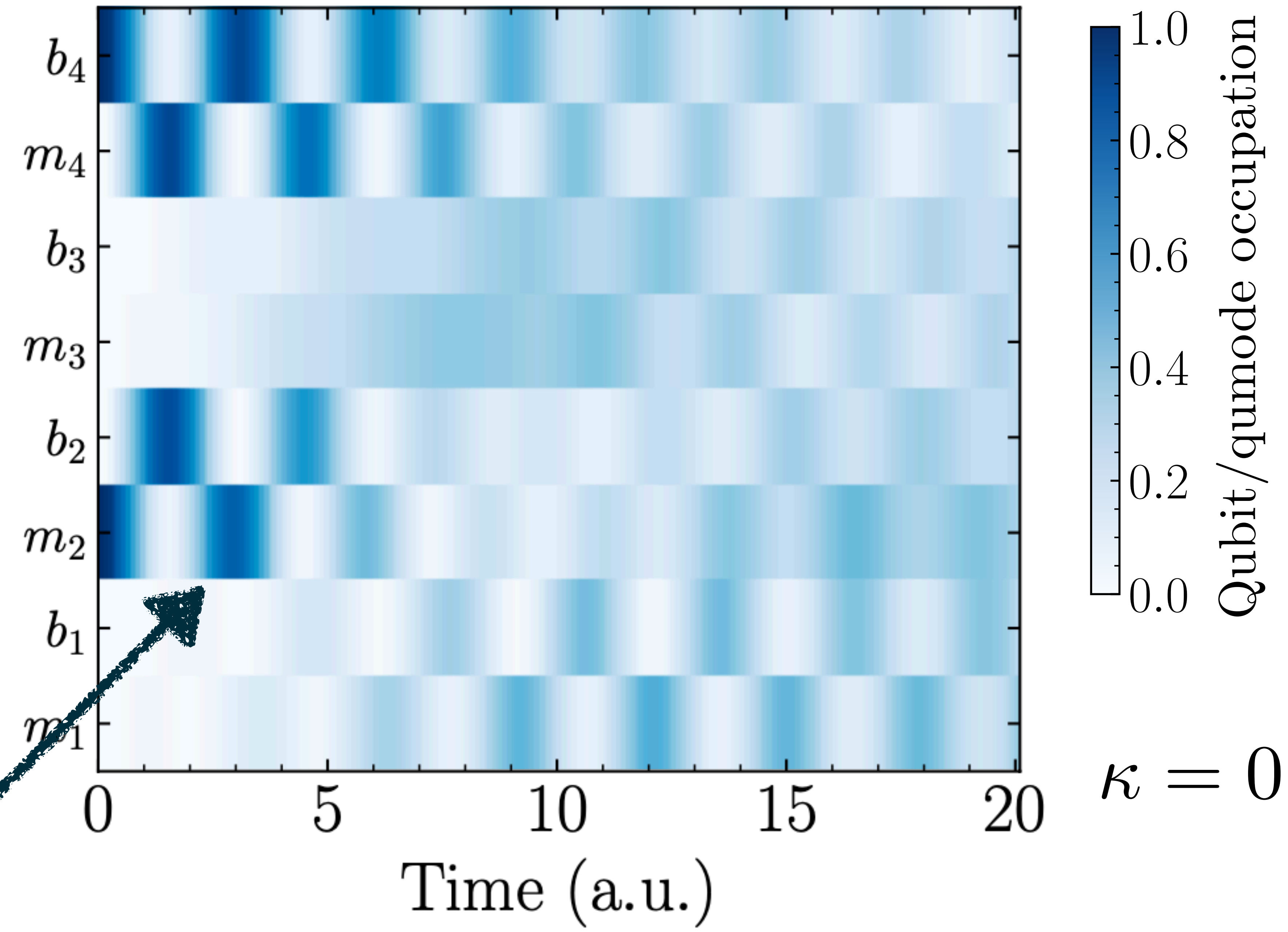
- Real-time evolution

$$H=\sum_{n=1}^{N}\omega_c a_n^\dagger a_n+\sum_{n=1}^{N}\omega_a\sigma_n^+\sigma_n^-+\kappa\sum_{n=1}^{N}\left(a_{n+1}^\dagger a_n+a_n^\dagger a_{n+1}\right)+\eta\sum_{n=1}^{N}\left(a_n\sigma_n^++a_n^\dagger\sigma_n^-\right)$$

- Lattice sites

Initial configuration





$\kappa=0.2,\ \eta=1$

Excitation initially localized

The many-body Jaynes-Cummings model

Araz, Grau, Montgomery, FR `24

- Ground state preparation

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n$$

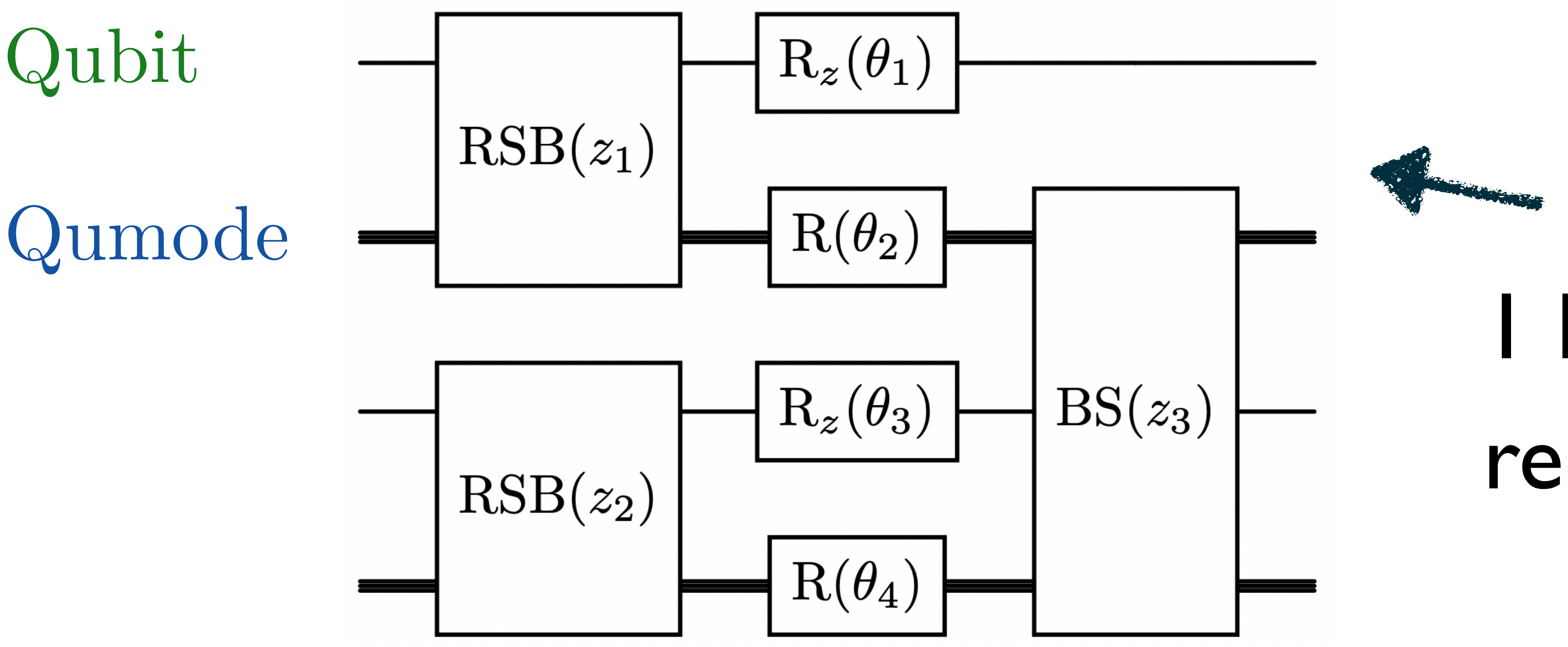
$$+ \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^-$$

$$+ \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right)$$

$$+ \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$

cf. Lloyd et al. `18, Siopsis et al. `23

- Hamiltonian-based VQA



1 layer with non-Gaussian red sideband gate

The many-body Jaynes-Cummings model

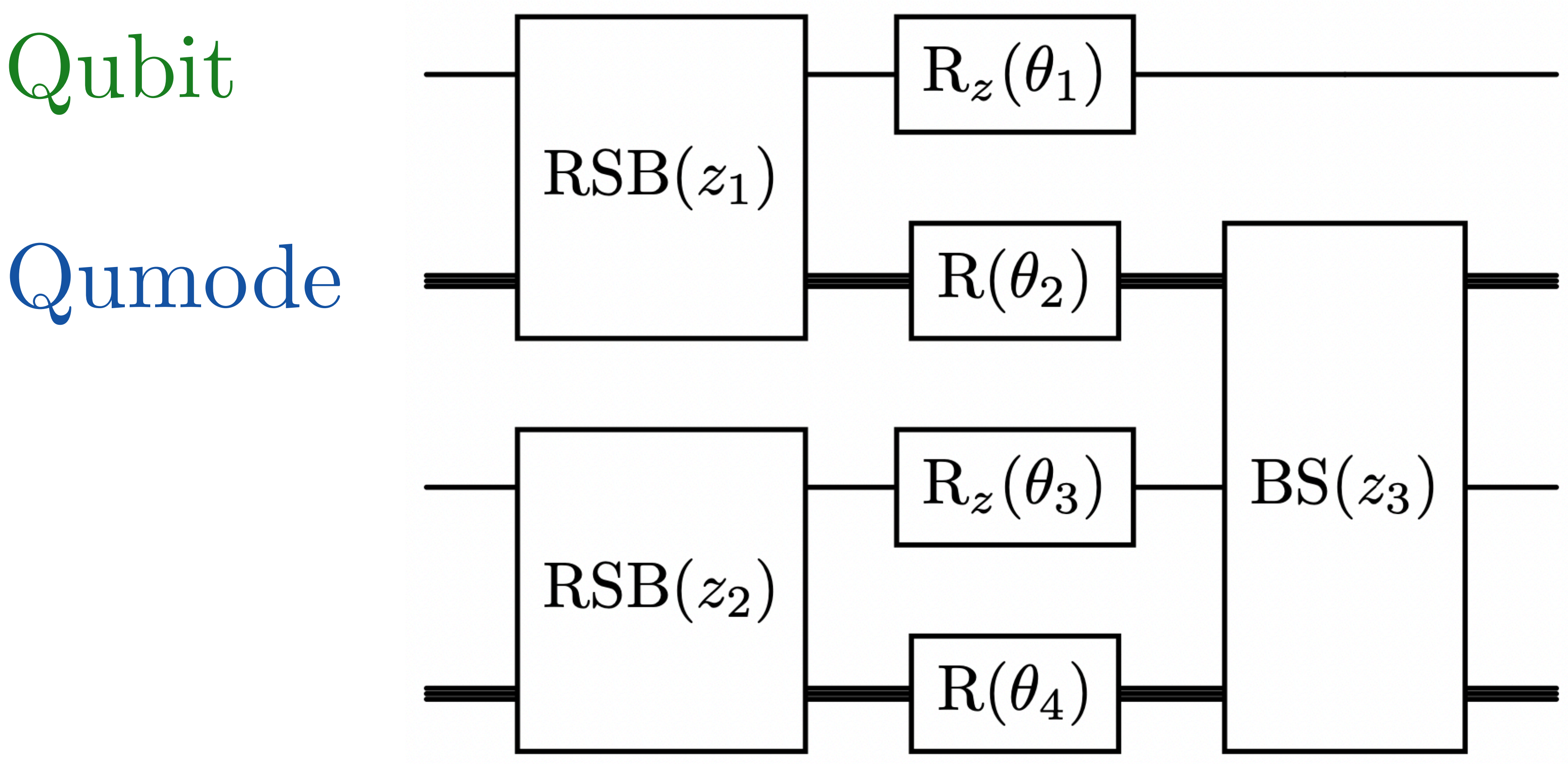
Araz, Grau, Montgomery, FR `24

- Ground state preparation

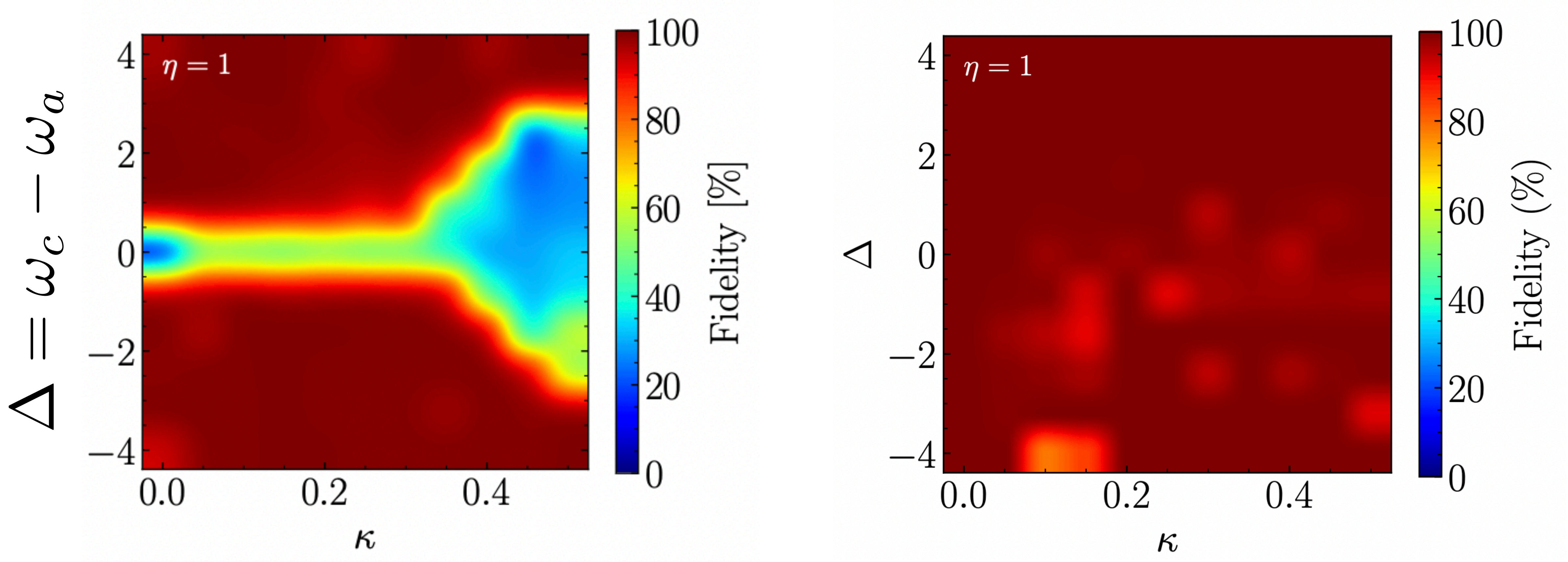
$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n + \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- + \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$

cf. Lloyd et al. `18, Siopsis et al. `23

- Hamiltonian-based VQA



1 layer with non-Gaussian red sideband gate



Fixed vs. variable $\langle N_{\text{tot}} \rangle$

$|\langle \psi_{\text{VQA}} | \psi \rangle|^2$

Outline

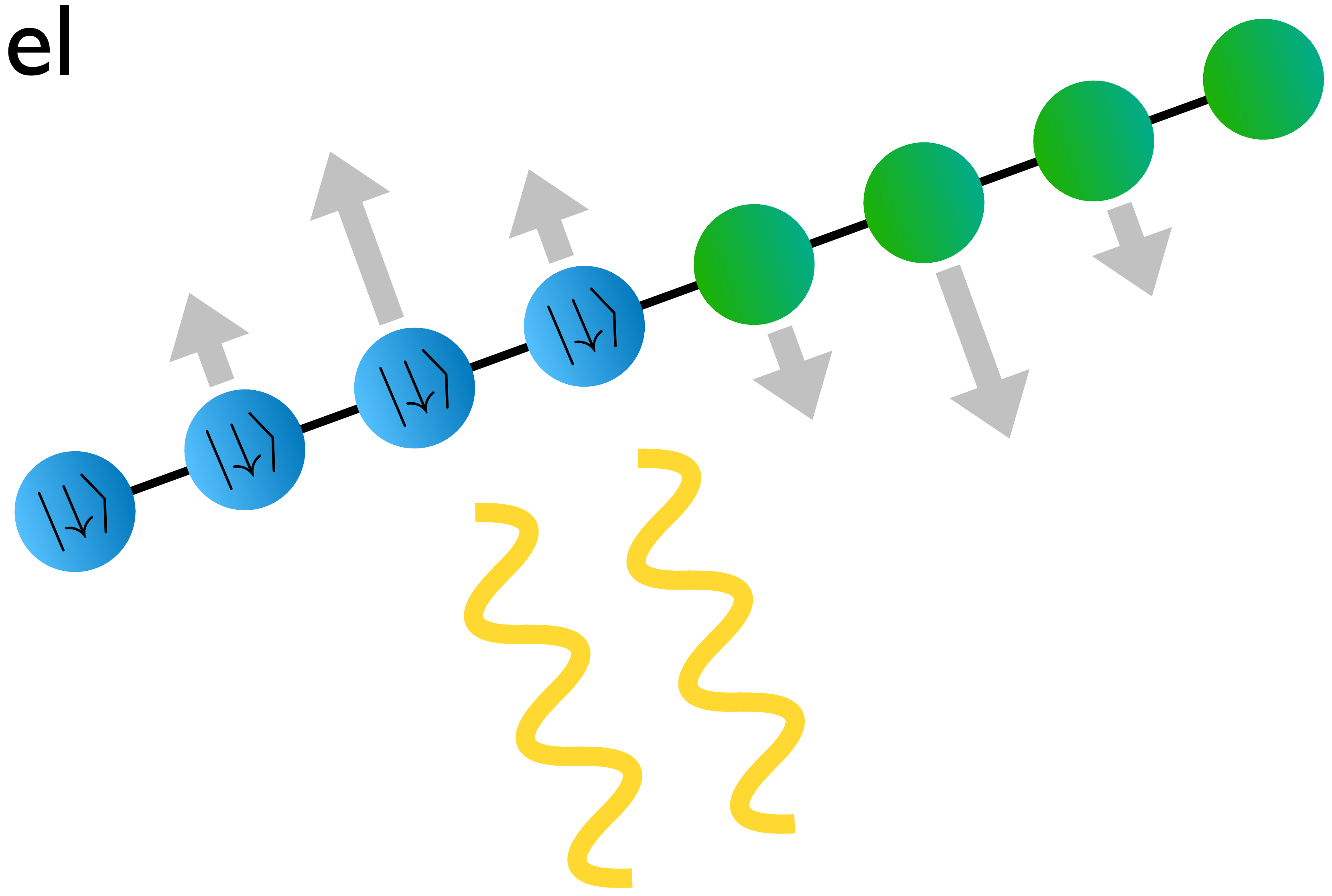
Qubits & qumodes with trapped ions

Simulation of spin-boson lattice models

Conclusions

- Quantum computing well-suited to address challenges in fundamental physics
- Exploration of qubits, qudits, qumodes *see George Siopsis' talk*

- Small-scale simulations
- Building up toward simulations of the Standard Model



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Conclusions

- Quantum computing well-suited to address challenges in fundamental physics
- Exploration of qubits, qudits, qumodes *see George Siopsis' talk*

- Small-scale simulations
- Building up toward simulations of the Standard Model

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Matt Grau

Jake Montgomery

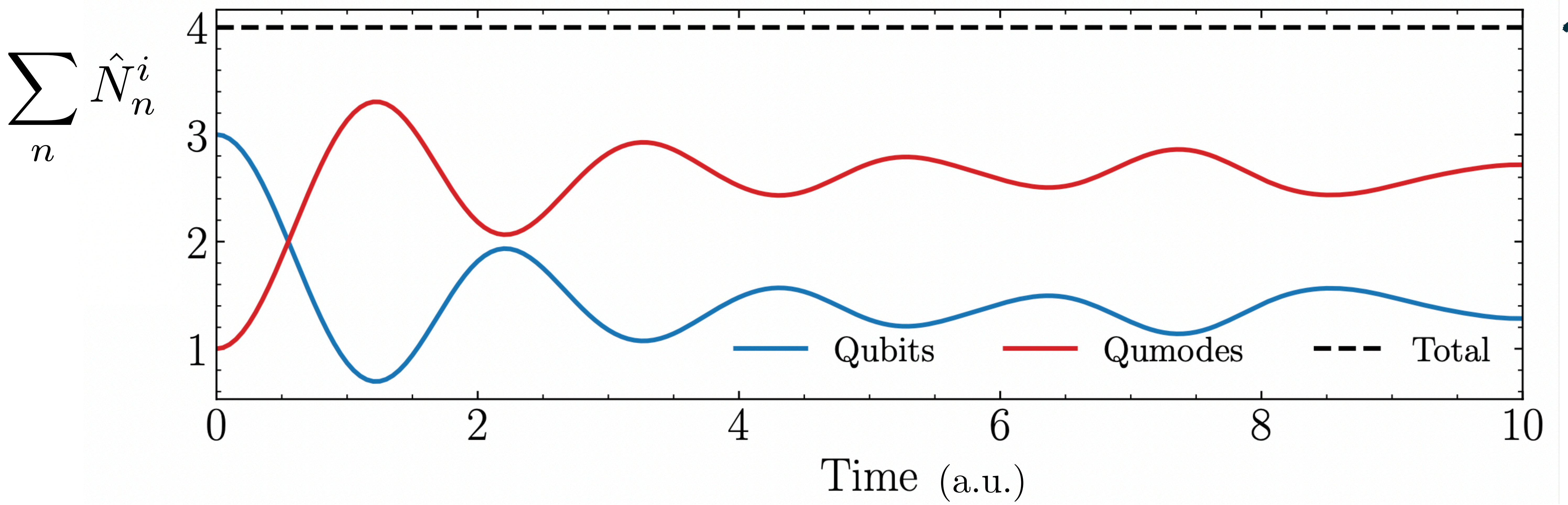
Jack Araz

The many-body Jaynes-Cummings model

Araz, Grau, Montgomery, FR `24

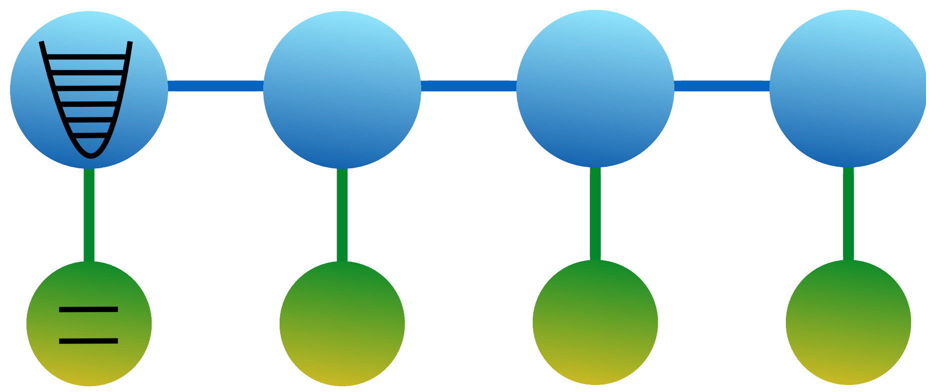
- Real-time evolution

$$H = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n + \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- + \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$



As expected for

$$[\hat{H}, \hat{N}_{\text{tot}}] = 0$$



- Open boundary conditions
- 4 sites and qumodes truncated to 4 levels for the classical simulation

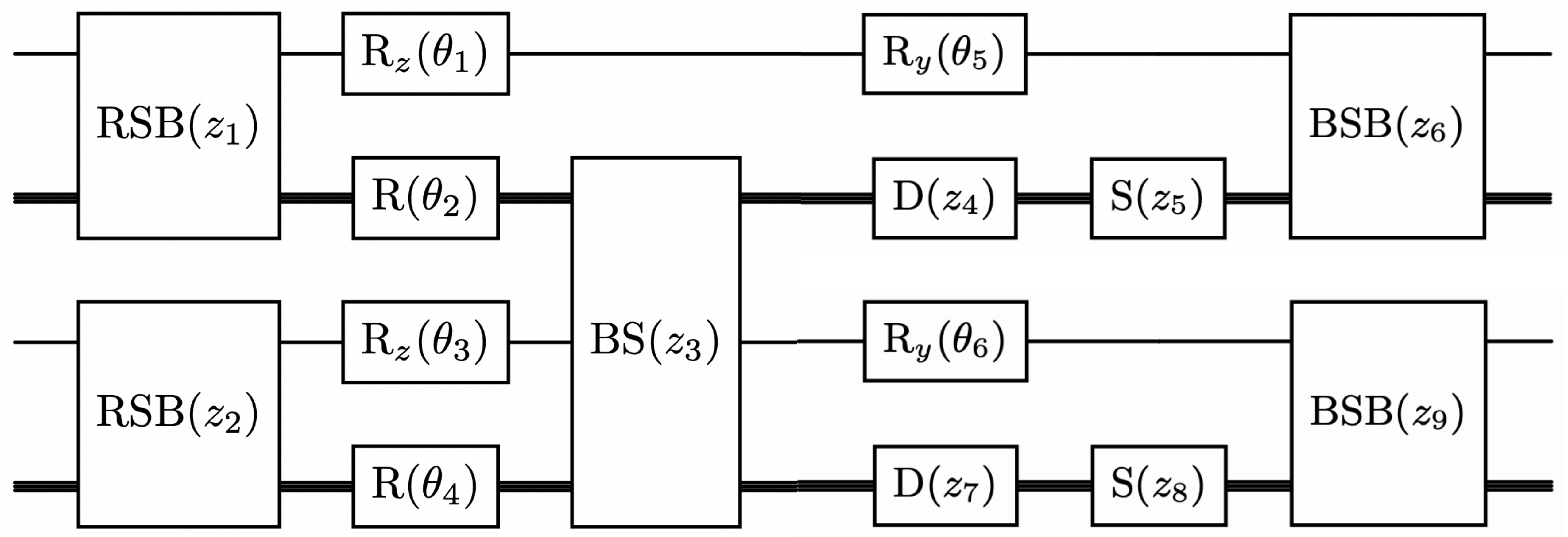
The many-body Jaynes-Cummings model

Araz, Grau, Montgomery, FR `24

- Ground state preparation

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n + \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- + \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$

- VQA ansatz
- Include additional number-nonconserving gates



Hamiltonian ansatz

Can achieve variable $\langle \hat{N}_{\text{tot}} \rangle$