Quantum Sensor Networks

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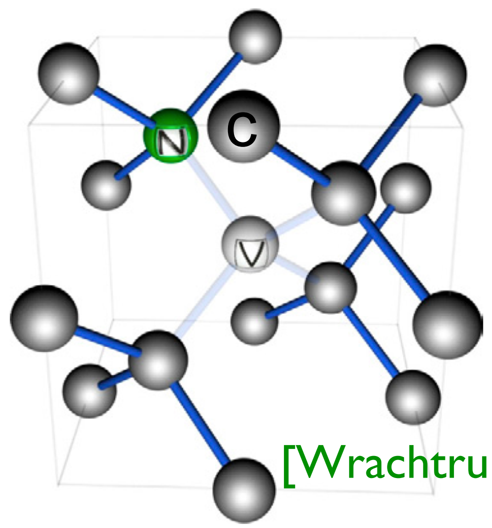
NP-AMO QIS Workshop
UMass Boston
January 15, 2025

Quantum sensors

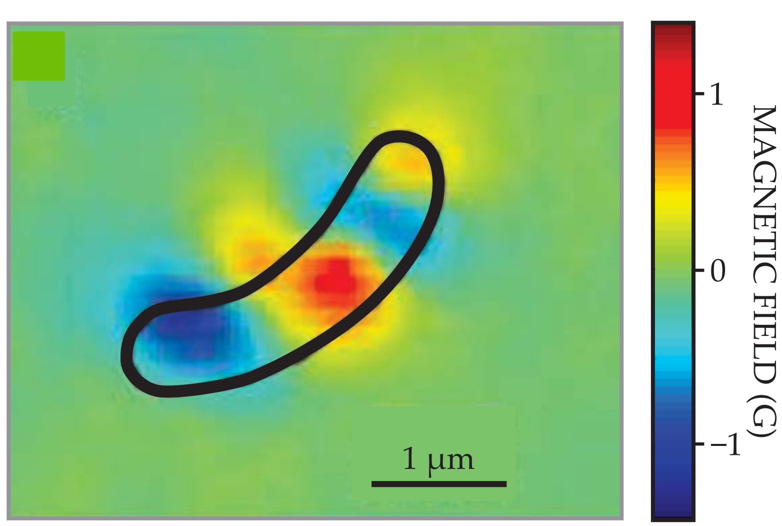
- combine high spatial resolution and high precision

NV-center magnetometer & thermometer



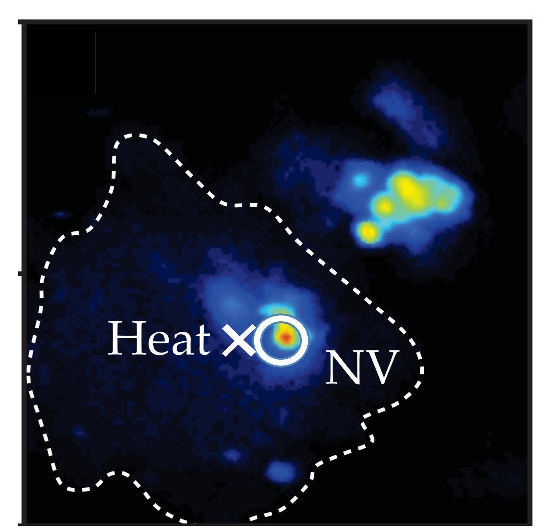
[Wrachtrup et al.]

magnetic imaging of live bacteria



[Le Sage, Walsworth et al, 2013]

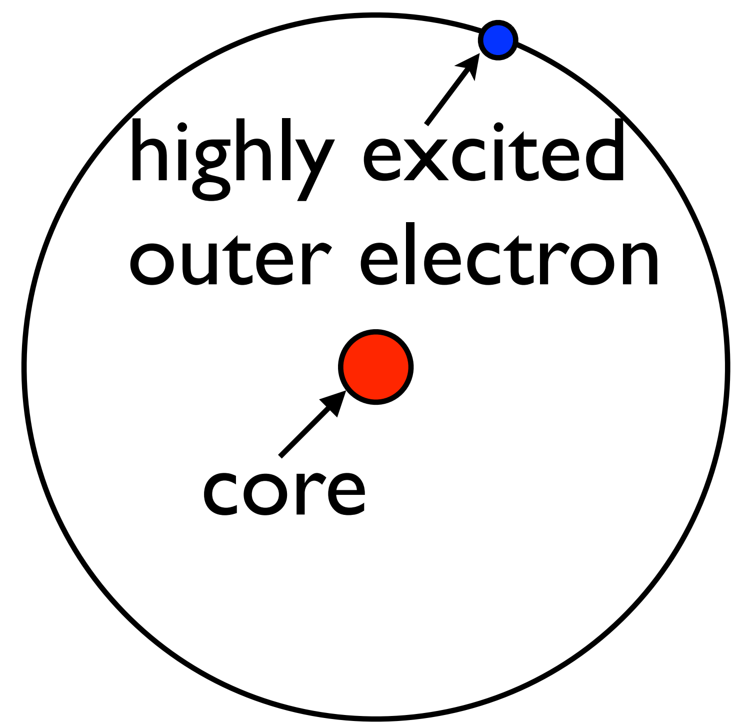
nanoscale thermometry of live human cells



[Kucsko, Lukin et al, 2013]

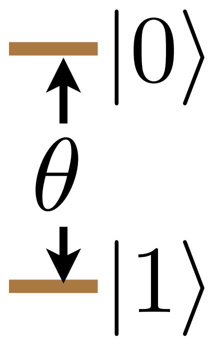
Rydberg-atom electrometer

[Holloway et al, 2014; Facon, Haroche et al, 2016]



see also talks by
Ronald Fernando Garcia Ruiz,
Kyle Leach, Soonwon Choi,
Akira Sone, David Moore

Quantum sensors



$$\hat{H} = \frac{1}{2}\theta\hat{Z}$$

parameter of interest

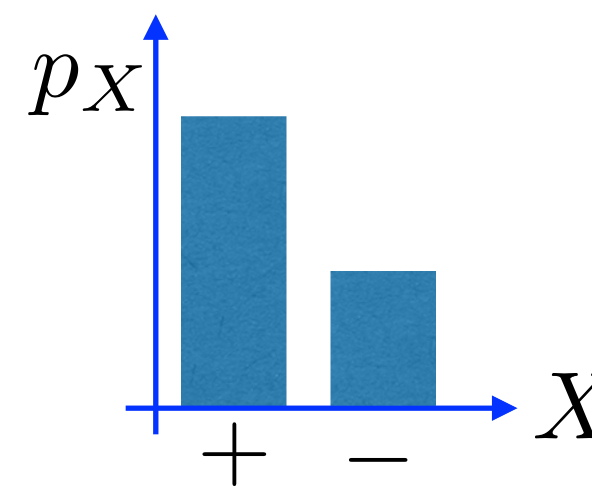
$$|\psi(t=0)\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle = (e^{-i\theta t/2}|0\rangle + e^{i\theta t/2}|1\rangle)/\sqrt{2}$$

- measure $\hat{X}$. Eigenstates $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$

$$p_+ = |\langle + |\psi(t)\rangle|^2 = \cos^2(\theta t/2)$$

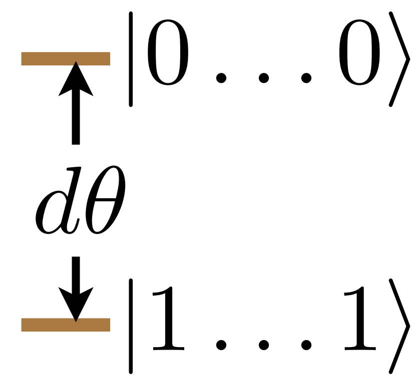
$$p_- = |\langle - |\psi(t)\rangle|^2 = \sin^2(\theta t/2)$$



- sample random variable X whose distribution depends on θ
- Fisher info = info about θ in X: $F = \sum_{X=\pm} p_X \left(\frac{\partial \ln p_X}{\partial \theta}\right)^2 = t^2$
- Cramér-Rao bound: $\Delta\theta \geq \frac{1}{\sqrt{F}} = \frac{1}{t}$

d sensors

$$\hat{H} = \frac{1}{2}\theta \sum_{i=1}^{d} \hat{Z}_i$$



Used independently: $|\psi(0)\rangle \propto (|0\rangle + |1\rangle) \otimes \cdots \otimes (|0\rangle + |1\rangle)$

$$\Delta\theta = \frac{1}{t\sqrt{d}}$$ standard quantum limit

Using entanglement: $|\psi(0)\rangle \propto |0\ldots0\rangle + |1\ldots1\rangle$ "cat" state (GHZ state)

$$|\psi(t)\rangle \propto |0\ldots0\rangle + e^{id\theta t}|1\ldots1\rangle$$

$$\Delta\theta = \frac{1}{td}$$ Heisenberg limit - best possible measurement

[Caves, Wineland, Holland, etc… '90s]

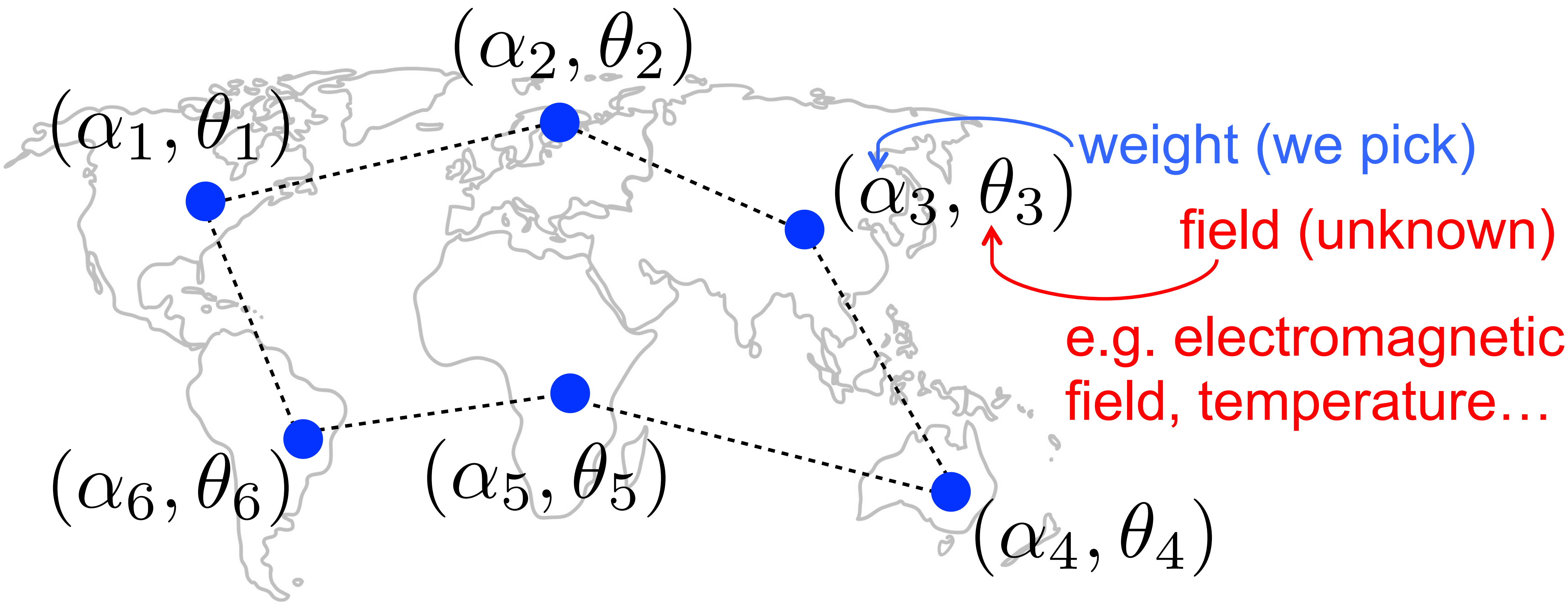
- contributions to quantum noise from each sensor conspire to cancel

entanglement advantage

Quantum sensor network

$$\hat{H} = \frac{1}{2}\sum_{i=1}^{d}\theta_i \hat{Z}_i \qquad\qquad Q = \sum_{i=1}^{d}\alpha_i\theta_i$$

- measure a desired linear combination of fields at the sensors



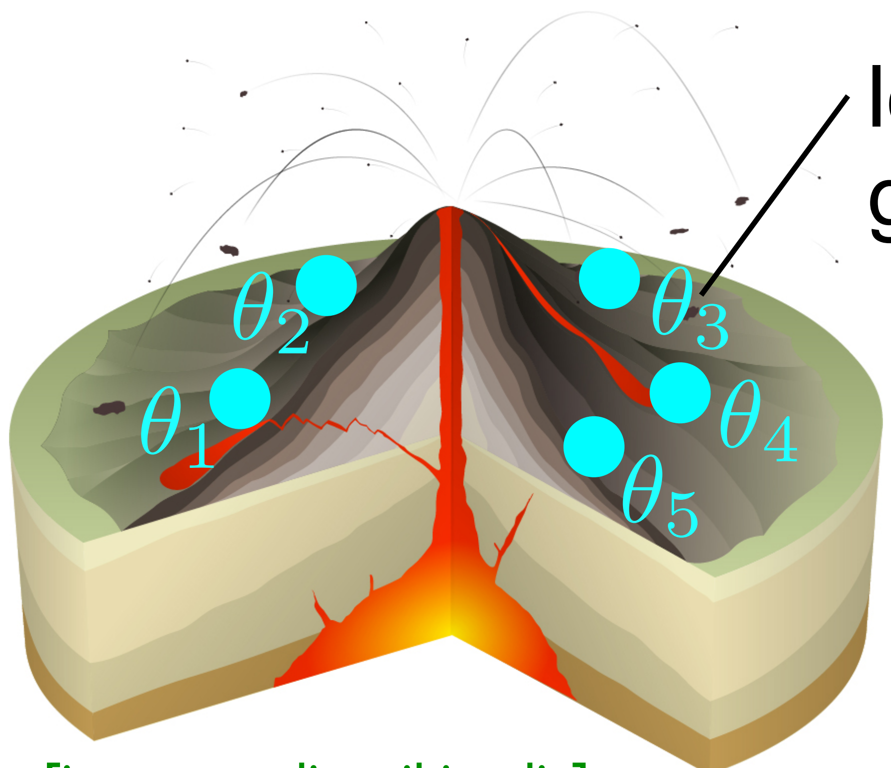
- make an aggregate query and learn a collective property
- target spatial profile of desired signal (e.g. Fourier mode or spherical harmonic)

Eldredge, Foss-Feig, Gross, Rolston, AVG, PRA 97, 042337 (2018); US Patent 10,007,885

Quantum sensor network

$$\hat{H} = \frac{1}{2}\sum_{i=1}^{d} \theta_i \hat{Z}_i \qquad\qquad Q = \sum_{i=1}^{d} \alpha_i \theta_i$$

- measure a desired linear combination of fields at the sensors
- found optimal entanglement-enhanced protocol (use GHZ states)
  Eldredge et al (AVG), PRA 97, 042337 (2018); US Patent 10,007,885
- generalized to measuring any analytic function $q(\theta_1, \ldots, \theta_d)$
  Qian et al (AVG), PRA 100, 042304 (2019); US Patent 11,562,049
- generalized to the case where θ_i are correlated



[image credit: wikipedia]

local gravitational potential or local gravitational acceleration

- determined by a few independent variables (e.g. density of magma in magma chamber)

Qian et al (AVG), PRA 103, L030601 (2021);
US Patent Application 17978420

Quantum sensor network

• found fast protocols for measuring multiple functions simultaneously

Bringewatt, Boettcher, Niroula, Bienias, AVG, PRR 3, 033011 (2021)
US Patent Application 18136257
related prior work: Rubio et al, J. Phys. A 53, 344001 (2020)

• minimized amount of entanglement used

Ehrenberg, Bringewatt, AVG, PRR 5, 033228 (2023) ; US Prov. Patent App. 63/397546
prior related work: Eldredge, Foss-Feig, Gross, Rolston, AVG, PRA 97, 042337 (2018)
Gross, Caves, J. Phys. A: Math. Theor. 54, 014001 (2020)

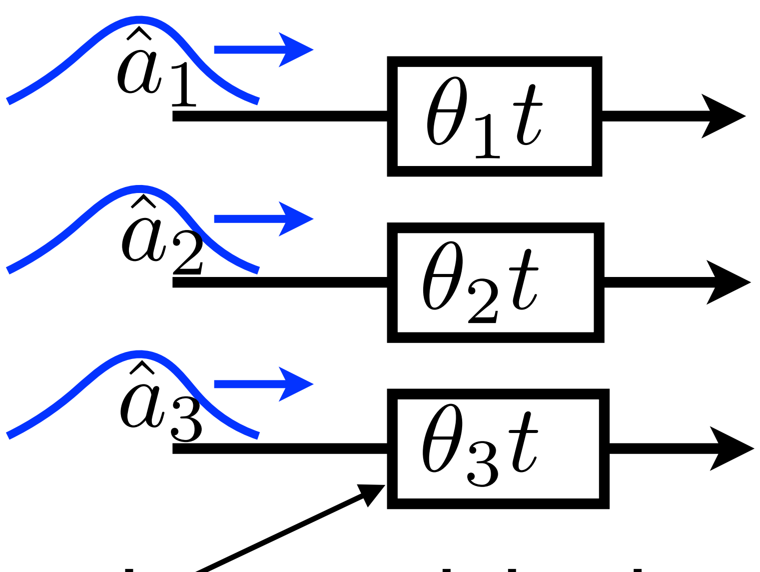
Photons or phonons as sensors

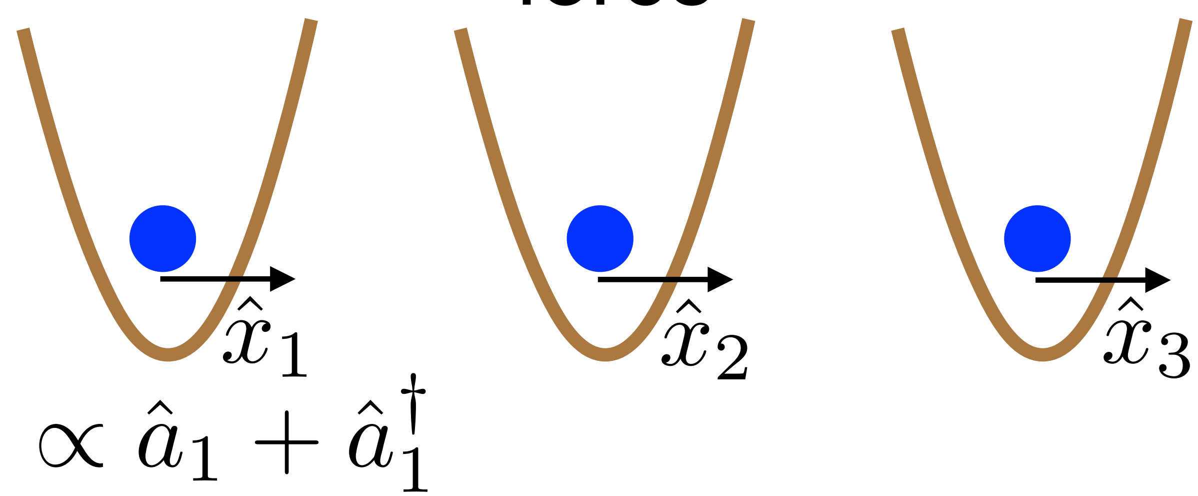
• same ideas apply to oscillators (e.g. photons or phonons) as sensors:

$$\hat{H} = \sum_i \theta_i \underbrace{\hat{a}_i^\dagger \hat{a}_i}_{\text{photon number}}$$

$$\hat{H} = \sum_i \theta_i \underbrace{(\hat{a}_i + \hat{a}_i^\dagger)}_{\text{displacement}}$$

force





e.g. phases picked up by photons

e.g. forces on oscillators

- fix $\sum_i \hat{a}_i^\dagger \hat{a}_i$

- fix $\left\langle \sum_i \hat{a}_i^\dagger \hat{a}_i \right\rangle$

Proctor et al, arXiv:1702.04271; Ge et al, PRL 121, 043604 (2018); Zhuang et al, PRA 97, 032329 (2018); Qian et al (AVG), PRA 100, 042304 (2019); Xia et al, 124, 150502 (2020); Guo et al, Nature Phys. 16, 281 (2020); Liu et al, Nature Photon. 15, 137 (2021); Hong et al, Nature Commun. 12, 5211 (2021); Malia et al, Nature 612, 661 (2022); Bringewatt et al (AVG), PRR 6, 013246 (2024),…

Reviews: Quantum Sci. Technol. 6, 043001 (2021); AVS Quantum Sci. 2, 024703 (2020)

Photons or phonons as sensors

$$\hat{H} = \sum_{i=1}^{d} \theta_i \hat{a}_i^\dagger \hat{a}_i + \hat{H}_c(t)$$

- fix $\sum_i \hat{a}_i^\dagger \hat{a}_i = N$ and total evolution time t
- want to measure $Q = \sum_{i=1}^{d} \alpha_i \theta_i$ (assume $\alpha_i \geq 0$) $\quad s = \sum_i \alpha_i$

Optimal protocol uses “proportionally weighted NOON state”:

$$e^{-i\phi} \left| N\frac{\alpha_1}{s}, \ldots, N\frac{\alpha_d}{s}, 0 \right\rangle + |0, \ldots, 0, N\rangle$$

reference mode

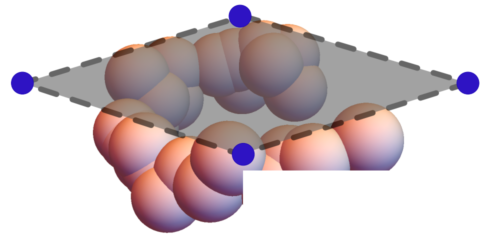
$$\phi = \sum_i \theta_i N \frac{\alpha_i}{s} t = QNt/s$$

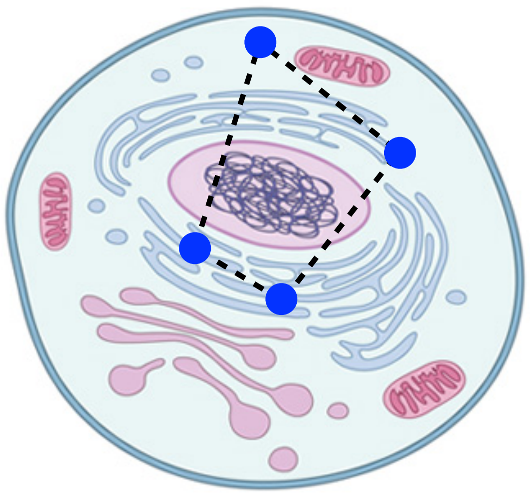
- we proved optimality
- entanglement gives uncertainty reduction $\sim 1/\sqrt{d}$
- 2-mode entanglement sufficient if allow for control Hamiltonian

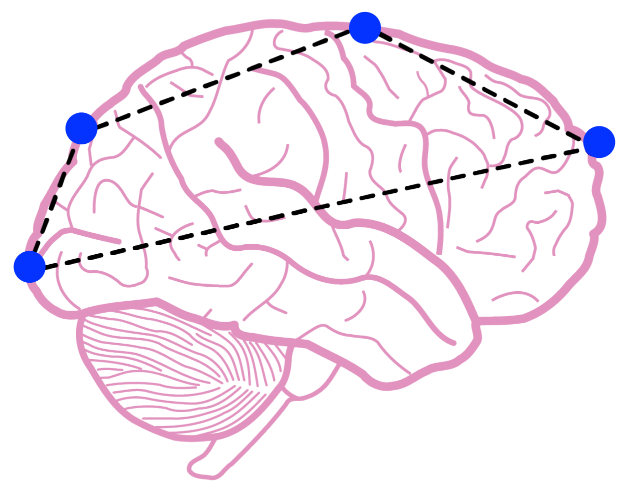
Proctor et al, arXiv:1702.04271, Proctor et al, PRL 120, 080501 (2018),
Bringewatt et al (AVG), PRR 6, 013246 (2024).

Applications

Small & intermediate scale



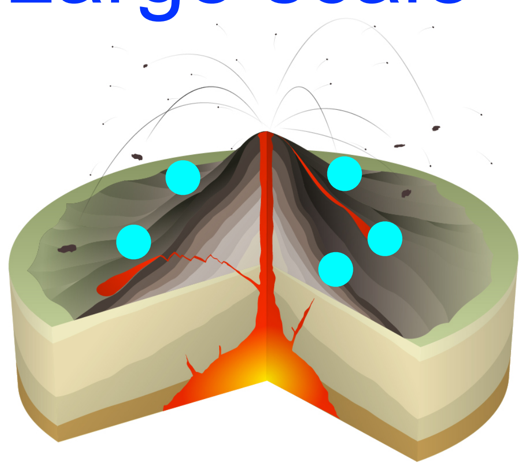




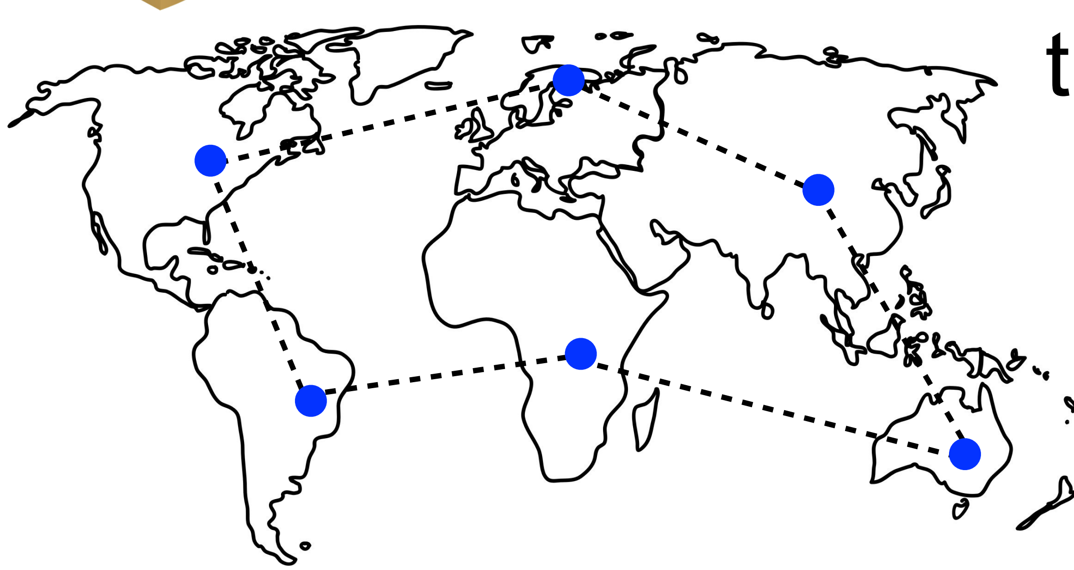
- chemistry, biology, medicine (magnetic fields, electric fields, temperature)

nuclear physics applications?

Large scale



[image credit: wikipedia]



- geodesy & geophysics (earthquake/volcano prediction)
- e.g. magnetometry, electrometry, thermometry, gravimetry, etc…

Summary

- entanglement improves measurements of properties of spatially varying fields
- need to distribute entanglement (e.g. GHZ-like $|0\ldots0\rangle + |1\ldots1\rangle$)
- entanglement gives at most a factor of $1/\sqrt{\text{number of sensors}}$ reduction in measurement uncertainty

Outlook

- other sets of commuting generators, non-commuting generators
- verified and/or encrypted measurements, including differential privacy [Spencer, Shi, Khabiboulline, Alagic, AVG, in prep];
- discrete parameters
- using squeezed states instead of GHZ-like states
- error correction/mitigation [e.g. Zhou et al, Nat. Commun. 9, 78 (2018)]
- allow for movement of sensors during measurement
- limited amount of data
- nuclear physics applications?
- sensing correlated noise

Can one obtain entanglement advantage for sensing noise? Yes!

"Exponential entanglement advantage in sensing correlated noise"

Wang, Bringewatt, Seif, Brady, Oh, AVG, arXiv:2410.05878

"Correlated noise estimation with quantum sensor networks"

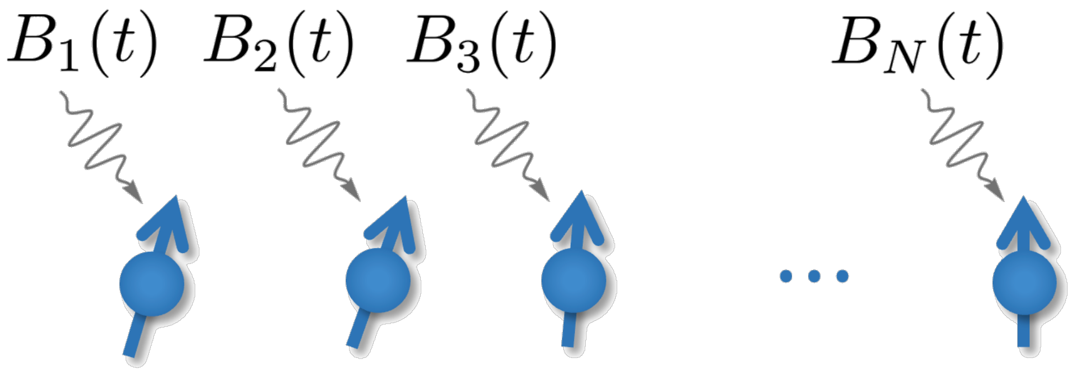
Brady, Wang, Albert, AVG, Zhuang, arXiv:2412.17903

see also talk by Joonhee Choi on noise learning

see also Rovny et al (Kolkowitz, De Leon), Science (2022); Cambria et al, arXiv:2408.11715; Cheng et al, arXiv:2408.11666; Huxter et al, arXiv:2407.19576; Ji et al (Du), Nature Photonics (2024); Delord et al, Nano Letters (2024)

Noise sensing

- N qubits undergoing Gaussian Markovian dephasing



$$\hat{H}_{\rm sys}(t) = \sum_{\ell=1}^{N} \frac{\hat{Z}_\ell}{2}[\omega_\ell + B_\ell(t)]$$

- assume noise known except for one unknown parameter ξ:

$$\overline{B_j(t)B_\ell(t')} = C_{j\ell}(\xi)\cdot\frac{\gamma}{2}\delta(t-t') \qquad C_{j\ell}(\xi) = C_{j\ell}(0) + \xi\Delta C_{j\ell}$$

- sensor dynamics given by Lindblad master equation:

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}_\xi[C(\xi)]\hat{\rho} = \frac{\gamma}{2}\sum_{j,\ell=1}^{N} C_{j\ell}(\xi)\left(\hat{Z}_\ell\hat{\rho}\hat{Z}_j - \frac{1}{2}\{\hat{Z}_j\hat{Z}_\ell,\hat{\rho}\}\right)$$

- quantum environment can also lead to such dephasing
- motivation for sensing spatially correlated noise:
  - trapped ion qubits coupled to collective motion or to common phase reference
  - atomic array qubits driven by a global laser beam
  - nuclear physics applications?

Noise sensing

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}_\xi[C(\xi)]\hat{\rho} = \frac{\gamma}{2}\sum_{j,\ell=1}^{N} C_{j\ell}(\xi)\left(\hat{Z}_\ell\hat{\rho}\hat{Z}_j - \frac{1}{2}\{\hat{Z}_j\hat{Z}_\ell, \hat{\rho}\}\right) \qquad C_{j\ell}(\xi) = C_{j\ell}(0) + \xi\Delta C_{j\ell}$$

- question: does an entangled initial state lead to an improvement in sensing ξ?

- $\mathcal{F}_Q$ = quantum Fisher information (QFI) = classical Fisher information maximized over all possible measurements

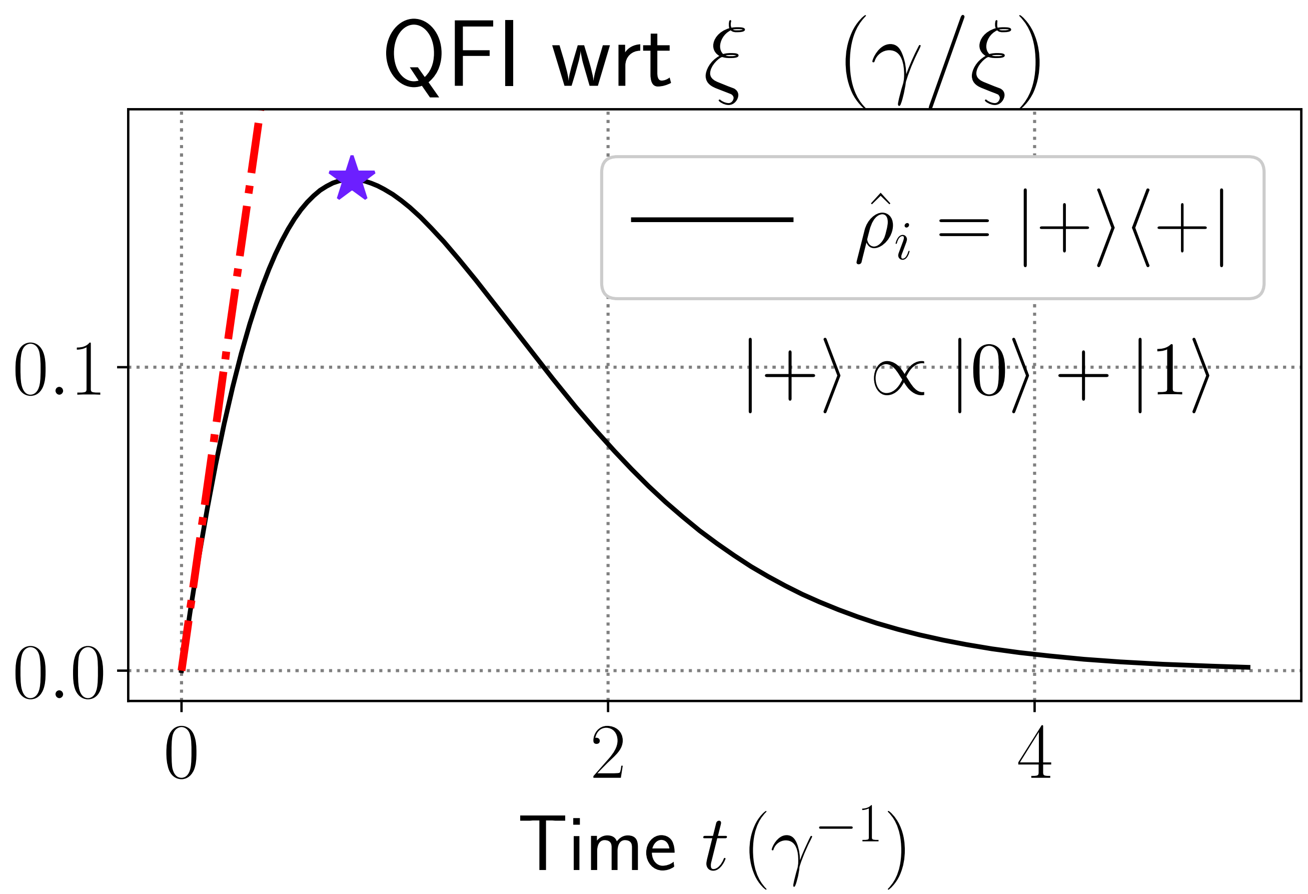
- can we get $\mathcal{F}_Q^{(ent)}/\mathcal{F}_Q^{(sep)} > 1$?

- the answer may depend on whether total evolution time is fixed or number of shots is fixed

  - if time for reset and measurement is negligible, we are time limited
  - if time for reset and measurement dominates, we are shot limited

N=1 qubit

$$\frac{d\hat{\rho}}{dt} = \frac{\gamma}{2}\xi\mathcal{D}[\hat{Z}]\hat{\rho} = \frac{\gamma}{2}\xi(\hat{Z}\hat{\rho}\hat{Z} - \hat{\rho})$$



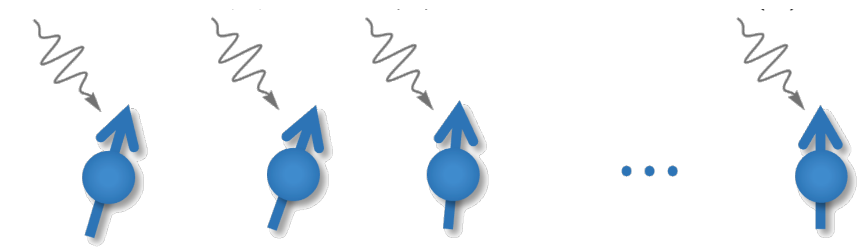
- when fixing total time, the optimal strategy is to do fast repetitive resets

- when fixing number of shots, the optimal strategy is to evolve for a finite time

Uncorrelated noise

- simplest example: uncorrelated dephasing noise

$$\mathcal{L}_\xi \hat{\rho} = \frac{\gamma}{2} \sum_{j,\ell=1}^{N} C_{j\ell}(\xi) \left(\hat{Z}_\ell \hat{\rho} \hat{Z}_j - \frac{\{\hat{Z}_j \hat{Z}_\ell, \hat{\rho}\}}{2} \right)$$



$$C_{j\ell}(\xi) = \xi \delta_{j\ell}$$

- there is no entanglement advantage e.g. Pirandola & Lupo PRL 2017
- intuition 1: GHZ state ⇒ faster dephasing, but reduced signal
- intuition 2: dephasing = random unitaries

N=1: $\mathcal{L}_\xi \hat{\rho} = \frac{\gamma}{2} \xi \mathcal{D}[\hat{Z}] \hat{\rho}$ ➡ $e^{\mathcal{L}_\xi t} \hat{\rho} = \frac{1 + e^{-\xi \gamma t}}{2} \hat{\rho}(0) + \frac{1 - e^{-\xi \gamma t}}{2} \hat{Z} \hat{\rho}(0) \hat{Z}$

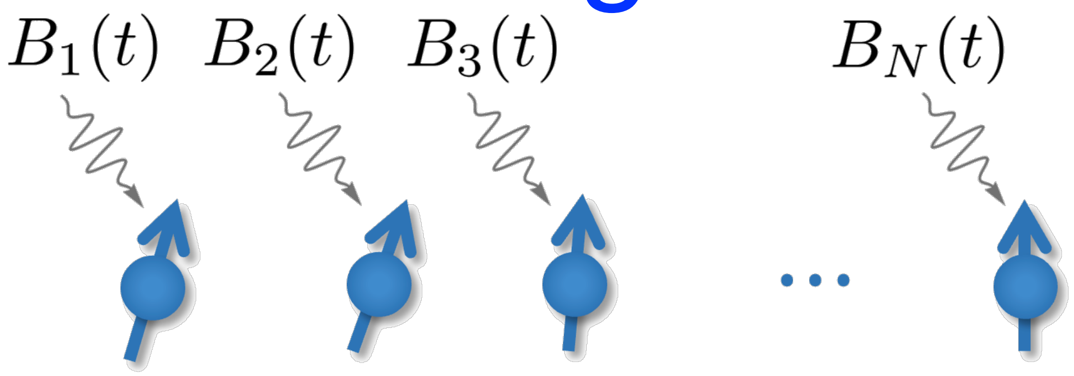
QFI ≤ F_1 ≡ classical FI of $p_\pm(\xi) = \left(1 \pm e^{-\xi\gamma t}\right)/2$

N>1: QFI ≤ classical FI of a joint distribution of N identically distributed independent random variables = N*F_1

$\rightarrow$ scales as N (not N^2) and can be saturated with product initial states

We ask if there is entanglement advantage for sensing *spatially correlated* noise and answer affirmatively

Sensing maximally correlated noise



$$\hat{H}_{\text{sys}}(t) = \sum_{\ell=1}^{N} \frac{\hat{Z}_\ell}{2}[\omega_\ell + B_\ell(t)]$$

$$B_1(t) = B_2(t) = \cdots = B_N(t)$$

$$C_{jl}(\xi) = \xi$$

$$\mathcal{L}_\xi \hat{\rho} = \frac{\gamma}{2} \sum_{j,\ell=1}^{N} C_{j\ell}(\xi) \left(\hat{Z}_\ell \hat{\rho} \hat{Z}_j - \frac{\{\hat{Z}_j \hat{Z}_\ell, \hat{\rho}\}}{2} \right)$$

(know noise form but not strength)

- when fixing number of shots, no entanglement advantage
- when fixing total time, fast resets are optimal and get entanglement advantage:

$$\mathcal{F}_Q^{(sep)} \propto N \quad \text{(SQL)} \qquad \mathcal{F}_Q^{(ent)} \propto N^2 \quad \text{(Heisenberg)}$$

initial state: $|\psi_+\rangle \propto |0\ldots0\rangle + |1\ldots1\rangle$

measurement: $M_0 = |\psi_+\rangle\langle\psi_+| \quad M_1 = 1 - M_0$

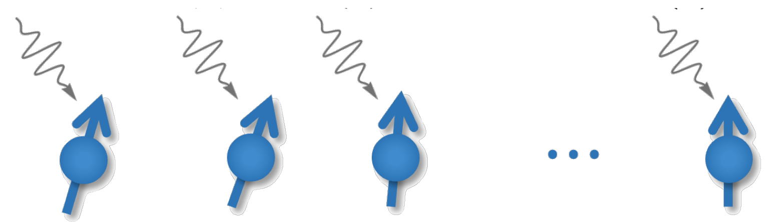
- undo GHZ preparation unitary & measure in computational basis

- intuition: amplitudes of correlated noise processes add constructively to give boost

Brady, Wang, Albert, AVG, Zhuang, arXiv:2412.17903

Sensing deviations from maximally correlated noise

N = 2 qubits



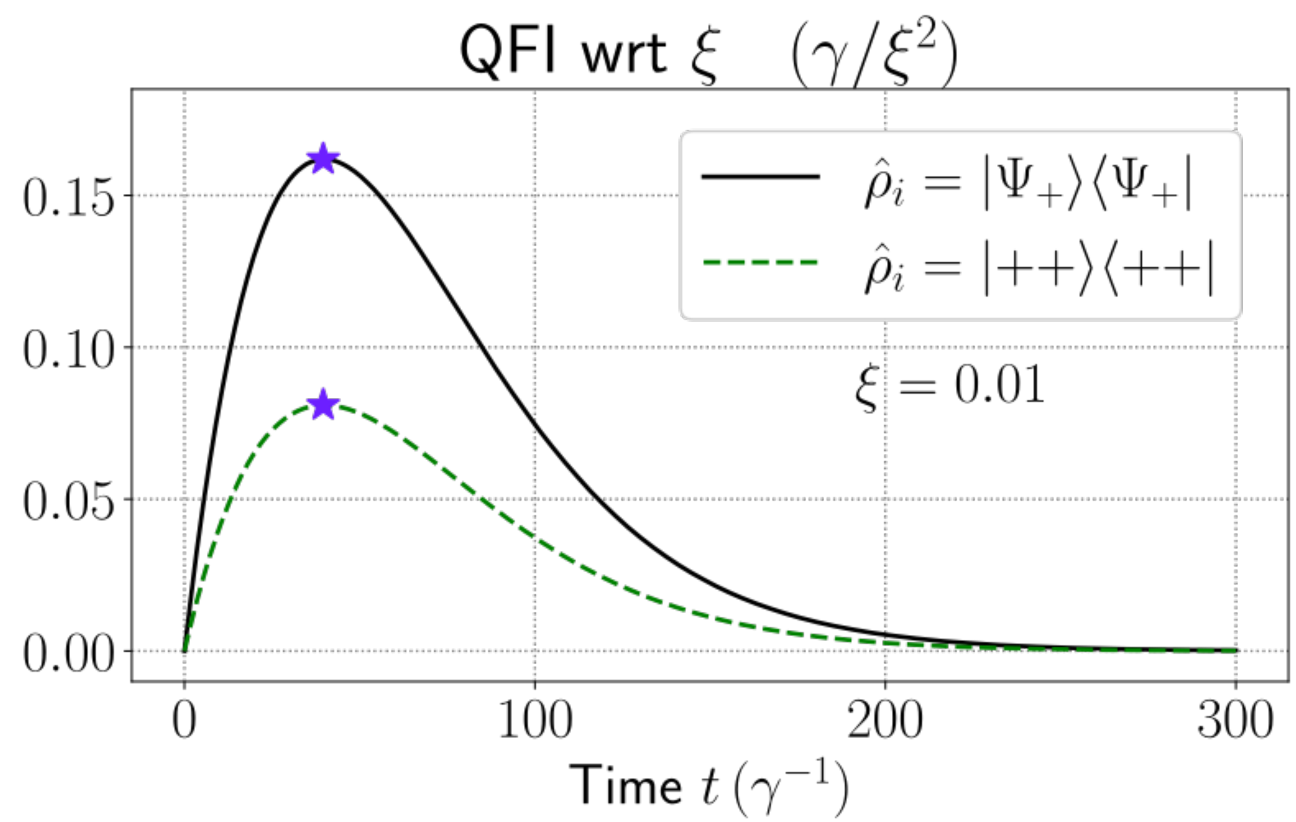
$$C_{2\mathrm{qb}}(\xi) = \begin{pmatrix} 1 & \xi - 1 \\ \xi - 1 & 1 \end{pmatrix} \quad \xi \ll 1 \qquad \mathcal{L}_\xi \hat{\rho} = \frac{\gamma}{2} \sum_{j,\ell=1}^{N} C_{j\ell}(\xi) \left(\hat{Z}_\ell \hat{\rho} \hat{Z}_j - \frac{\{\hat{Z}_j \hat{Z}_\ell, \hat{\rho}\}}{2} \right)$$

- ξ = deviation from perfect (anti-)correlation

- both for fixing total time and for fixing number of shots, get

$$\mathcal{F}_Q^{(ent)} / \mathcal{F}_Q^{(sep)} = 2$$

- intuition: at ξ = 0, $|\psi_+\rangle$ doesn't dephase
  - at small ξ, it is hidden from strong dephasing, but senses ξ



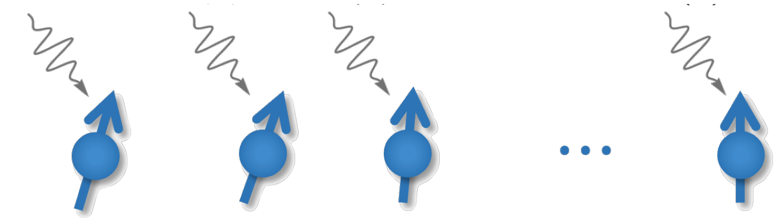
$$|\psi_+\rangle \propto |0\ldots0\rangle + |1\ldots1\rangle$$

Wang, Bringewatt, Seif, Brady, Oh, AVG, arXiv:2410.05878

Sensing deviations from maximally correlated noise

general N

• idea: construct correlated dephasing such that a single GHZ state is hidden from dephasing at $\xi = 0$



$$\mathcal{L}_{\xi}\hat{\rho} = \frac{\gamma}{2}\sum_{j,\ell=1}^{N} C_{j\ell}(\xi)\left(\hat{Z}_\ell\hat{\rho}\hat{Z}_j - \frac{\{\hat{Z}_j\hat{Z}_\ell, \hat{\rho}\}}{2}\right)$$

$$\mathcal{L}_{N\mathrm{qb}} = \frac{\gamma}{2}\left(\sum_{k\neq 0}\mathcal{D}[\hat{Z}_k] + \xi\mathcal{D}[\hat{Z}_{k=0}]\right) \qquad \hat{Z}_k = \frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{i\frac{2\pi k}{N}j}\hat{Z}_j \qquad \xi \ll 1$$

• possible origin: approximate symmetry

$$[C_{N\mathrm{qb}}(\xi)]_{j\ell} = \delta_{j\ell} - \frac{1-\xi}{N}$$

• when fixing total time, Heisenberg-type entanglement advantage:

$$\mathcal{F}_Q^{(ent)}/\mathcal{F}_Q^{(sep)} = N$$

• when fixing number of shots, exponential entanglement advantage: $\mathcal{F}_Q^{(ent)}/\mathcal{F}_Q^{(sep)} = 2^{N-1}$

• intuition: product states are either not sensitive to ξ at all or have exponentially small overlap with the GHZ state

Wang, Bringewatt, Seif, Brady, Oh, AVG, arXiv:2410.05878

Summary

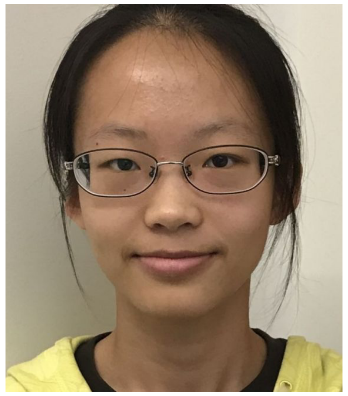
- while there is no entanglement advantage for sensing spatially uncorrelated noise, found entanglement advantage for sensing spatially correlated noise
- found a regime where this advantage is exponential
- new mechanism for entanglement advantage in sensing

Outlook

- focused on qubits, but similar results hold for bosons and fermions [Brady et al, arXiv:2412.17903]
- have some results for multiple unknown parameters [Brady et al, arXiv:2412.17903], but still many open questions
- have some results on sensing non-Markovian environments [Wang et al, arXiv:2410.05878], but still many open questions
- have some results on evaluating the impact of deleterious decoherence [Brady et al, arXiv:2412.17903], but still many open questions
- nuclear physics applications?

Thank You

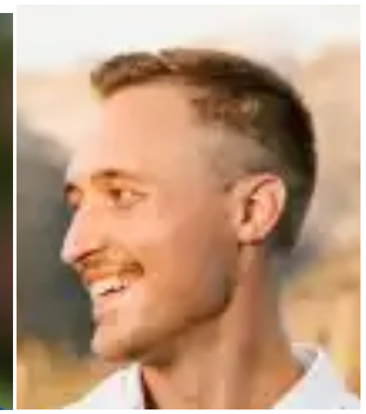
Exponential entanglement advantage in sensing correlated noise



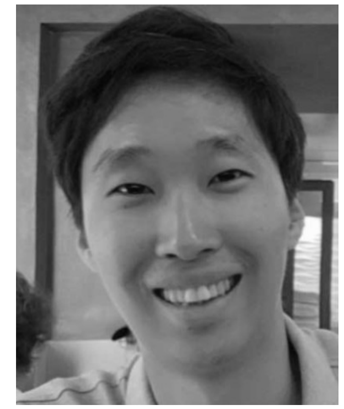
Yu-Xin Wang

Jake Bringewatt (→Harvard)

Alireza Seif (IBM)



Anthony Brady

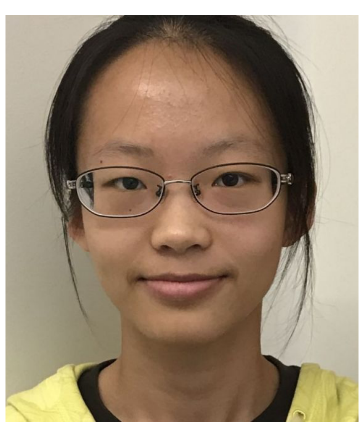


Changhun Oh (KAIST)

arXiv:2410.05878

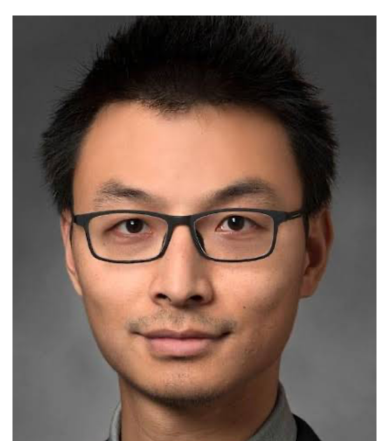
Correlated noise estimation with quantum sensor networks

Anthony Brady



Yu-Xin Wang

Victor Albert



Quntao Zhuang (USC)

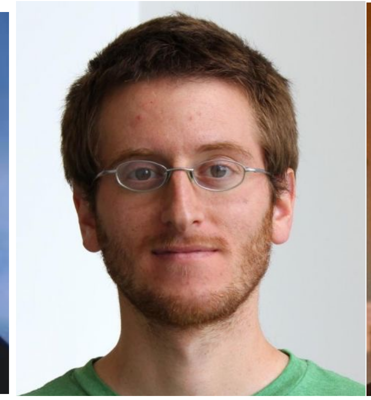
arXiv:2412.17903

Thank You

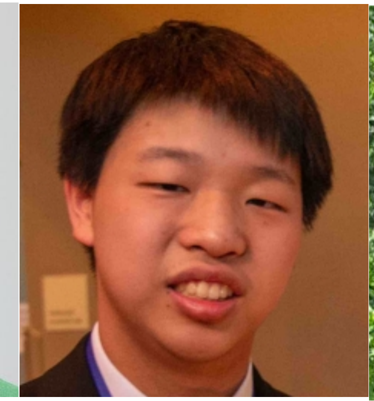
Zachary Eldredge (→DoE)

Jake Bringewatt (→Harvard)

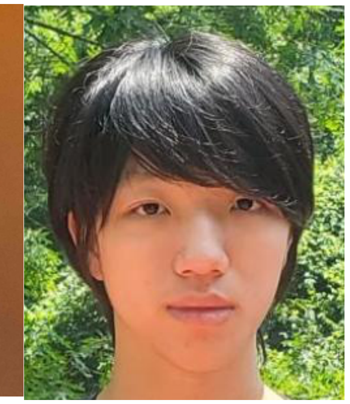
Adam Ehrenberg



Michael Foss-Feig (→Honeywell)



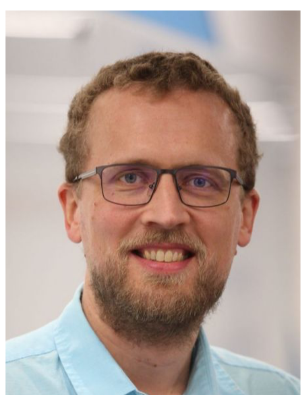
Kevin Qian (Montg. Blair High →MIT)



Tim Qian (Montg. Blair High →MIT)

Pradeep Niroula (→JP Morgan)

Wenchao Ge (→U Rhode Island)



Carl Miller

Yusuf Alnawakhtha

Trey Porto

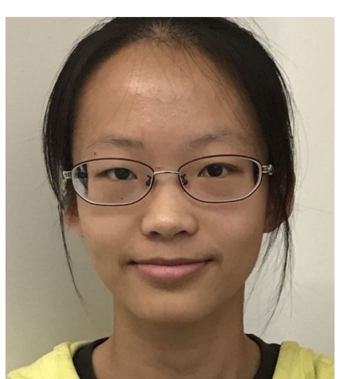
Steve Rolston

Emil Khabiboulline

Sean Muleady

Paul Lett

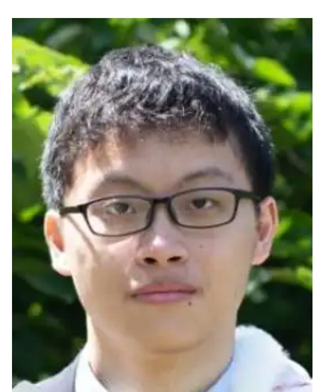
Zhifan Zhou



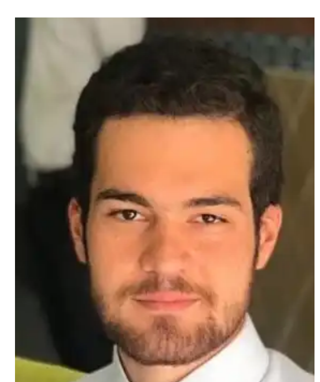
Yu-Xin Wang

Anthony Brady

Dan Spencer



Zhenning Liu



Erfan Abbasgholinejad

Kurt Jacobs (ARL)

Michael Gullans

Saikat Guha

Thank You

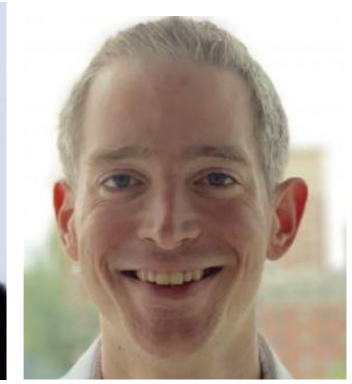
Shimon Kolkowitz (Berkeley)

Jeff Thompson (Princeton)

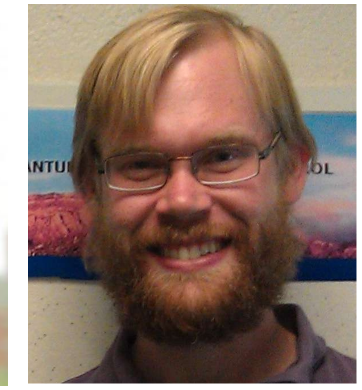
Kevin Cox (ARL)

Jack Dolde (Wisconsin)

Xin Zheng (Wisconsin)



Przemek Bienias (→AWS)



Jonathan Gross (Google)

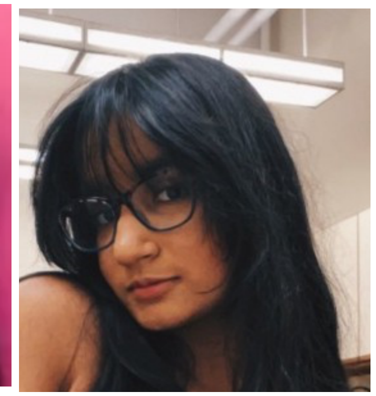
Igor Boettcher (→Alberta)

Guido Pagano (→Rice)

Chris Monroe (→Duke)

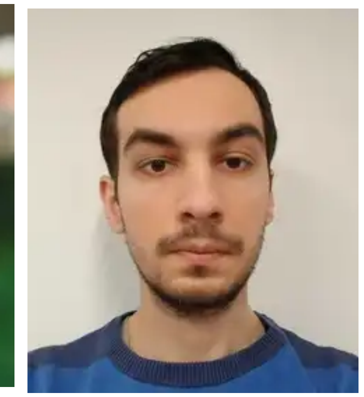
Jean Lautier-Gaud (WeLinQ)

Tarushi Goel (MIT)



Akshita Gorti (Cornell)

Luispe Garcia-Pintos (→ LANL)



Alex Cojocaru (→Edinburgh)

Ana Maria Rey (Boulder)

Rafael Kaubruegger (Boulder)

Kaiyan Shi

Gorjan Alagic

Thank You

PRA 97, 042337 (2018) [US Patent 10,007,885]

PRA 100, 042304 (2019) [US Patent 11,562,049]

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PRA 103, L030601 (2021) [US Patent Application 17978420]

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PRR 5, 033228 (2023) [U.S. Patent Application 18232890]

PRL 133, 080801 (2024)

PRR 6, 013246 (2024)

arXiv:2410.05878

arXiv:2412.17903

$$$: AFOSR, ARO, DoE, NSF, DARPA

Thank You

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Abhinav Deshpande → Caltech
Yidan Wang → Harvard
Minh Tran → MIT
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Fangli Liu → QuEra
Andrew Guo → Sandia
Ron Belyansky → Chicago
Pradeep Niroula → J.P. Morgan
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Jake Bringewatt → Harvard
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Jeet Shah

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Zhe-Xuan Gong → Asst. Prof. @ Colorado School of Mines
Sergey Syzranov → Asst. Prof. @ UC Santa Cruz
Paraj Titum → Applied Physics Lab at Johns Hopkins
Igor Boettcher → Asst. Prof. @ U Alberta
Rex Lundgren → NSA Laboratory for Physical Sciences
Zhicheng Yang → Asst. Prof. @ Peking U
Chris Baldwin → Asst. Prof. @ Michigan State
Oles Shtanko → IBM
James Garrison → IBM
Przemek Bienias → AWS
Lucas Brady → NASA QuAIL
Yaroslav Kharkov → AWS
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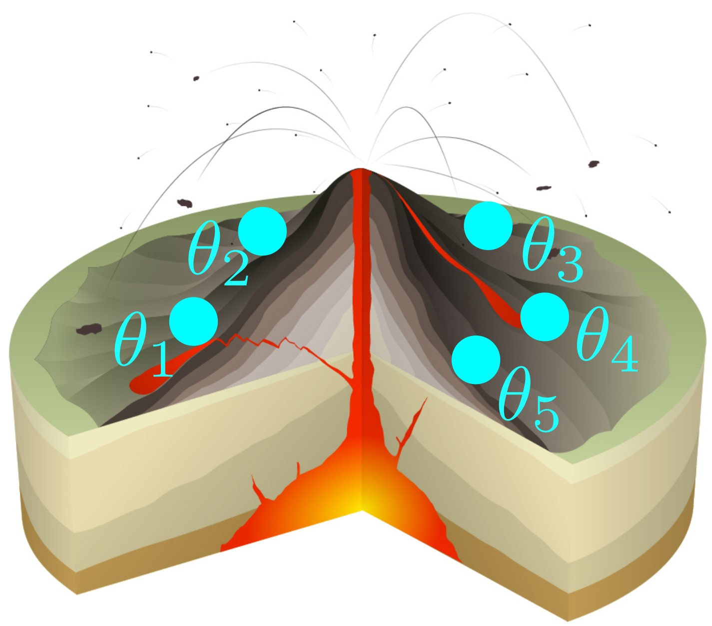
Jacob Lin
Sean Muleady
Yu-Xin Wang
Yan-Qi Wang
Yifan Hong
Anthony Brady

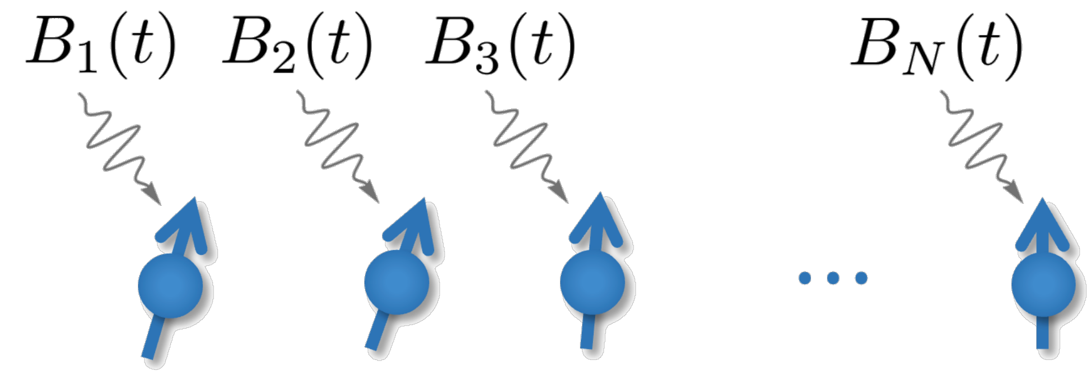
$$$: AFOSR, ARO, DoE, NSF, DARPA

Thank You



$$\hat{H} = \frac{1}{2}\sum_{i=1}^{d} \theta_i \hat{Z}_i$$

$$Q = \sum_{i=1}^{d} \alpha_i \theta_i$$



$$\frac{d\hat{\rho}}{dt} = \mathcal{L}_\xi[C(\xi)]\hat{\rho} = \frac{\gamma}{2}\sum_{j,\ell=1}^{N} C_{j\ell}(\xi)\left(\hat{Z}_\ell\hat{\rho}\hat{Z}_j - \frac{1}{2}\{\hat{Z}_j\hat{Z}_\ell, \hat{\rho}\}\right)$$

"Correlated noise estimation with quantum sensor networks"

Brady, Wang, Albert, AVG, Zhuang, arXiv:2412.17903

$$C_{jl}(\xi) = \xi \qquad \mathcal{F}_Q^{(ent)}/\mathcal{F}_Q^{(sep)} = N$$

"Exponential entanglement advantage in sensing correlated noise"

Wang, Bringewatt, Seif, Brady, Oh, AVG, arXiv:2410.05878

$$\mathcal{L}_{N\mathrm{qb}} = \frac{\gamma}{2}\left(\sum_{k\neq 0}\mathcal{D}[\hat{Z}_k] + \xi\mathcal{D}[\hat{Z}_{k=0}]\right) \qquad \hat{Z}_k = \frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{i\frac{2\pi k}{N}j}\hat{Z}_j$$

$$\mathcal{F}_Q^{(ent)}/\mathcal{F}_Q^{(sep)} = 2^{N-1} \qquad [C_{N\mathrm{qb}}(\xi)]_{j\ell} = \delta_{j\ell} - \frac{1-\xi}{N}$$