Flavor dependence of GPDs in the Large-N_c limit

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Outline

- Chiral-even GPDs
- Chiral-odd GPDs
- Bag Model
- Chiral-odd GPDs in Bag Model
- Large-N_c expansion in Bag Model
- Phenomenological implications

Chiral-even GPDs

- Chiral-even GPDs parametrize the following off-forward matrix elements of quark operators at a light-like separation
D.Müller, D.Robaschik, B.Geyer, F.-M.Dittes, J.Hořejši, Fortsch. Phys. **42**, 101 (1994)
X.D.Ji, PRL **78**, 610 (1997); PRD **55**, 7114 (1997).

A.V.Radyushkin, PLB **380**, 417 & **385**, 333 (1996).

$$P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{z}{2}) \gamma^+ \psi(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+=0, \vec{z}_T=0}$$
$$= \bar{u}(p', \lambda') \left[H^q \gamma^+ + E^q \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} \right] u(p, \lambda)$$

$$P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{z}{2}) \gamma^+ \gamma_5 \psi(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+=0, \vec{z}_T=0}$$
$$= \bar{u}(p', \lambda') \left[\tilde{H}^q \gamma^+ \gamma_5 + \tilde{E}^q \frac{\gamma_5 \Delta^+}{2m} \right] u(p, \lambda)$$

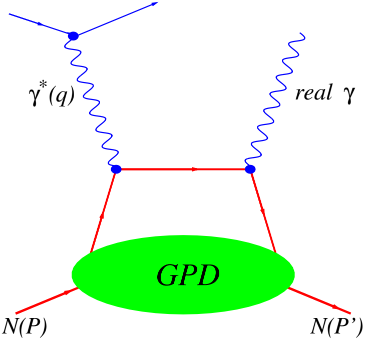
- GPDs depend on three parameters

$$x, \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+}, \qquad t = \Delta^2$$

where $\Delta = p' - p$ *and* $P^+ = (p' + p)/2$.

Chiral-even GPDs

- Chiral-even GPDs are accessible through Deeply Virtual Compton Scattering

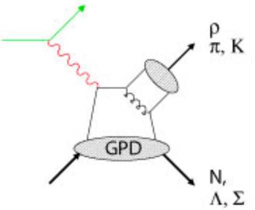


Chiral-odd GPDs

- There are four chiral-odd GPDs $H_T, \tilde{H}_T, E_T, \tilde{E}_T$ at leading twist

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\, i\sigma^{+i}\,\psi(\tfrac{1}{2}z)\;|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\Big[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\,\frac{P^+\Delta^i - \Delta^+P^i}{m^2} + E_T^q\,\frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q\,\frac{\gamma^+P^i - P^+\gamma^i}{m}\Big]\,u(p,\lambda).$$

  where $i = 1, 2$ is the transversity index [Diehl '03]
- Accessible through exclusive meson production processes



Properties of GPDs

- In the forward limit $\Delta \to 0$, certain GPDs are related to PDFs

$$H^q(x,0,0) = f_1^q(x)$$
$$\tilde{H}^q(x,0,0) = g_1^q(x)$$
$$H_T^q(x,0,0) = h_1^q(x).$$

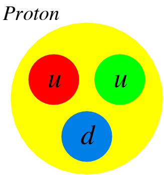
- It follows from the time reversal invariance that under $\xi \to -\xi$

$$F^q(x,\xi,t) = F^q(x,-\xi,t) \text{ for } F^q = H^q, E^q, \tilde{H}^q, \tilde{E}^q, H_T^q, \tilde{H}_T^q, E_T^q$$
$$F^q(x,\xi,t) = -F^q(x,-\xi,t) \text{ for } F^q = \tilde{E}_T^q$$

- Related to form factors via

$$\int_{-1}^{1} \left\{H, E, \tilde{H}, \tilde{E}\right\}(x,\xi,t)dx = F_1(t), F_2(t), G_A(t), G_P(t)$$
$$\int_{-1}^{1} \left\{H_T, \tilde{H}_T, E_T\right\}(x,\xi,t)dx = H_T(t), \tilde{H}_T(t), E_T(t)$$
$$\int_{-1}^{1} \left\{\tilde{E}_T\right\}(x,\xi,t)dx = 0$$

MIT Bag Model Chodos, Jaffe, Johnson, Thorn, Weisskopf, PRD9 (1974) 3471



- Quarks are constrained inside of a finite size "bag"
- Quarks are free inside the bag (mimics asymptotic freedom), however are subject to sharp boundary conditions on the surface (mimics confinement).
- The Lagrangian density of the system for massless quarks is given by

$$\mathcal{L} = \left(i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - B\right)\Theta_V - \frac{1}{2}\bar{\psi}\psi\delta_S$$

where Θ_V is the volume inside the bag and δ_S is a δ-function at the bag surface. The constant B is the energy needed to create the perturbative vacuum inside the bag.

MIT Bag Model

- Quarks inside the bag satisfy the Dirac equation
- Equations of motion of the system further asserts that

$$\eta_\mu j^\mu = \eta_\mu \bar{\psi}\gamma^\mu\psi = 0 \text{ (conservation of current)}$$
$$\partial_\mu T^{\mu\nu} = 0 \text{ (conservation of EMT)}$$

- Bag model has been used to obtain the first estimations for chiral-even GPDs [Ji, Melnitchouk, Song '97]
- We use the Bag model to evaluate the off-forward matrix elements in the Breit frame

$$p^\mu = (\bar{m}, -\vec{\Delta}/2) \quad \text{and} \quad p'^\mu = (\bar{m}, \vec{\Delta}/2)$$

where $\bar{m}^2 = P^2$.

Chiral-odd GPDs in Bag Model

- The momentum space wave function in the bag is given by

$$\varphi(\vec{k}) = \sqrt{4\pi} N R^3 \begin{pmatrix} t_0(k)\chi_m \\ \vec{\sigma}\cdot\hat{k} \quad t_1(k)\chi_m \end{pmatrix}$$

  where N is the normalization constant, R is the bag radius and

$$t_0(k) = \frac{j_0(w_0)\cos(kR) - j_0(kR)\cos(w_0)}{w_0^2 - \vec{k}^2 R^2}$$

$$t_1(k) = \frac{j_0(kR) j_1(w_0) kR - j_0(w_0) j_1(kR) w_0}{w_0^2 - \vec{k}^2 R^2}$$

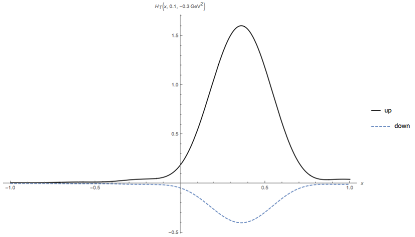
- Use this wave function to evaluate the correlators on the left hand side;

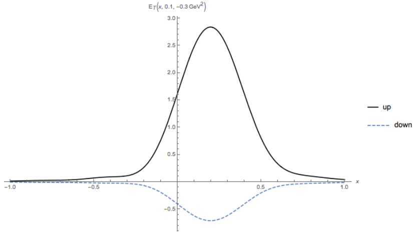
$$\varphi^\dagger(k') S(\Lambda_{-\vec{v}}) \gamma^0 \Gamma S(\Lambda_{\vec{v}}) \varphi(k)$$

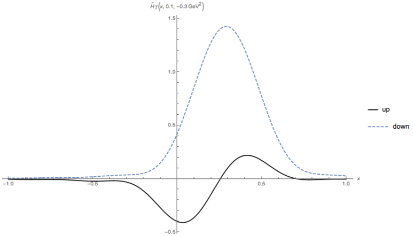
  where $\Gamma = i\sigma^{+i}$ and $S(\Lambda_{\vec{v}})$ is the Lorentz boost transformation
- We have 2 equation (for $i = 1, 2$) and 4 unknowns; project on different spin components to obtain 4 equations with 4 unknowns

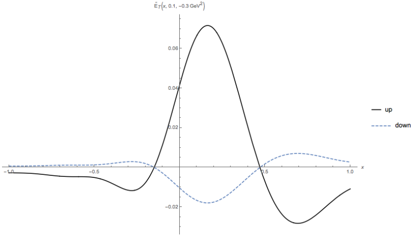
Chiral-odd GPDs in Bag Model

- Chiral-odd GPDs in Bag Model at $\xi = 0.1, t = -0.3GeV^2$









Large-N_c expansion

- Usually once we can not solve a problem analytically, we tend to use perturbation theory; anharmonic oscillator in QM, ϕ^4 theory in QFT, ect.
- In QCD, however, the coupling constant g is high at low energies. Hence is not a good expansion parameter.
- The only known expansion parameter valid in all regions in QCD is $1/N_c$ obtained by generalizing the color gauge group $SU(3) \to SU(N_c)$ [t'Hooft '74]
- As $N_c \to \infty$, QCD simplifies significantly and can be approached nonperturbatively; with an expansion parameter $1/N_c$

Large-N_c expansion

- In this picture, baryons appear as infinitely massive ($\sim N_c$) in the background of weakly interacting mesons [Witten '79]
- Large-N_c results can be checked in various ways: Diagrammatic techniques, chiral soliton model, Large-N_c quark model
- We use Bag Model as a tool to investigate model independent (N_c-scaling) results of GPDs in the Large-N_c framework

Large-N_c expansion in Bag Model

- In Large-N_c limit, the nucleon mass scales with N_c while retaining a stable size

$$M_N \sim N_c$$
$$R \sim N_c^0$$

Hence the nucleon becomes denser and denser.

- On the other hand

$$\Delta^0 \sim N_c^{-1} \quad \text{and} \quad \vec{\Delta} \sim N_c^0$$
$$x \sim N_c^{-1}$$
$$\xi \sim N_c^{-1}$$
$$t \sim N_c^0$$

Large-N_c expansion in Bag Model

- In the Large-N_c framework, a GPD G can be expressed in the form

$$G(x,\xi,t) \sim N_c^k \times F(N_c x, N_c \xi, t)$$

where $k \in \mathbb{Z}_+$ and F is a model dependent function.

- Here k depends on the GPD in question and the function F depends on the dynamical model used
- The leading flavor combinations of chiral-even GPDs in $1/N_c$ expansion is found to be [Goeke,Polyakov,Vanderhaeghen '01]

$$H^{u+d} \sim N_c^2, \quad E^{u-d} \sim N_c^3$$
$$\tilde{H}^{u-d} \sim N_c^2, \quad \tilde{E}^{u-d} \sim N_c^4.$$

Large-N_c expansion in Bag Model

- By using Bag Model, we obtain the following scaling properties of chiral-odd GPDs

$$\begin{aligned} H_T^q &\sim N_c^2 \\ E_T^q &\sim N_c^4 \\ \tilde{H}_T^q &\sim N_c^4 \\ \tilde{E}_T^q &\sim N_c^3. \end{aligned} \tag{1}$$

- Here we note that among chiral-odd GPDs there is a special linear combination, $\bar{E}_T^q = E_T^q + 2\tilde{H}_T^q$, which shows a cancellation of leading order scalings in the Large-N_c expansion

$$\bar{E}^q \sim N_c^3.$$

Large-N_c expansion in Bag Model

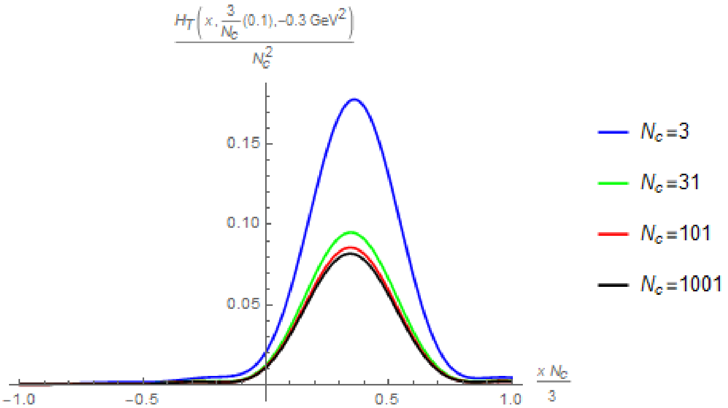


Figure: N_c-scaling of the chiral-odd GPD H_T^u as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3 GeV^2$.

Large-N_c expansion in Bag Model

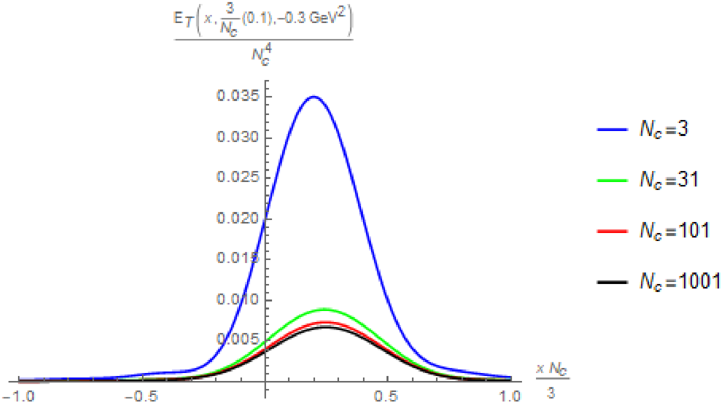


Figure: N_c-scaling of the chiral-odd GPD E_T^u as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3 GeV^2$.

Large-N_c expansion in Bag Model

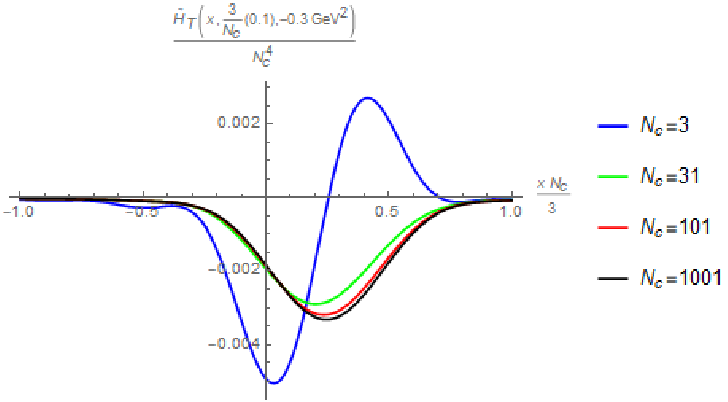


Figure: N_c-scaling of the chiral-odd GPD $\tilde{H}_T^u$ as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

Large-N_c expansion in Bag Model

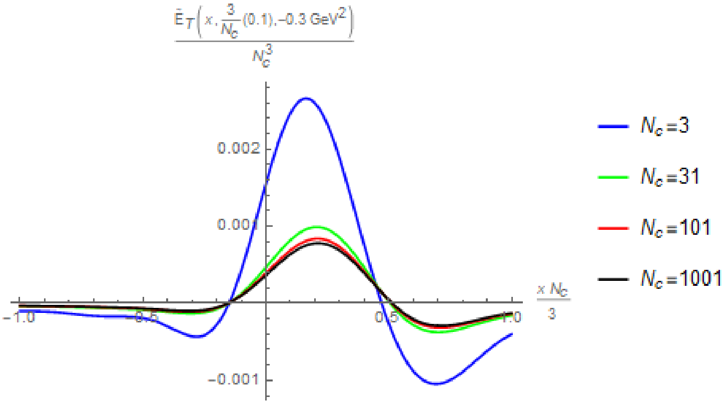


Figure: N_c-scaling of the chiral-odd GPD $\tilde{E}_T^u$ as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

Large-N_c expansion in Bag Model

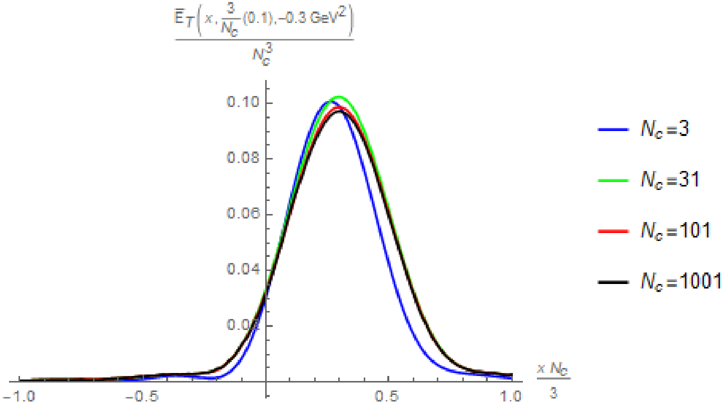


Figure: N_c-scaling of the chiral-odd GPD $\bar{E}^u_T$ as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

Large-N_c expansion in Bag Model

- On the other hand, dominant isospin combinations of chiral-odd GPDs in the Large-N_c limit appear as

$$\begin{aligned} H_T^{u-d}(x,\xi,t) &\sim N_c^2 \\ E_T^{u+d}(x,\xi,t) &\sim N_c^4 \\ \tilde{H}_T^{u+d}(x,\xi,t) &\sim N_c^4 \\ \tilde{E}_T^{u-d}(x,\xi,t) &\sim N_c^3 \\ \bar{E}_T^{u+d}(x,\xi,t) &\sim N_c^3. \end{aligned} \tag{2}$$

- Whereas, opposite isospin combinations are suppressed by order one
- The N_c scaling behaviors of the isospin combinations of chiral-odd GPDs: $\bar{E}_T, H_T$ and $\tilde{E}_T$ were discussed by [Schweitzer, Weiss '16] using a solitonic field with known symmetry properties. The results are confirmed in the bag model

Phenomenological implications

- What are the phenomenological implications of our findings?
- Since we have Large-N_c relations among flavor-singlet and flavor-nonsinglet components of GPDs, this order among them predicts the relative sign of flavor decomposed GPDs
- For instance, dominance of flavor-nonsinglet $(u - d)$ component of the GPD H_T in the Large-N_c limit implies a sign difference in the flavor decomposition of H_T
- Similarly for $\bar{E}_T$, flavor-singlet $(u + d)$ component is dominant in the Large-N_c limit. This implies that the flavor decomposition is expected to have the same sign

- Preliminary π^0, η electroproduction data at JLab confirms our predictions for H_T and $\bar{E}_T$

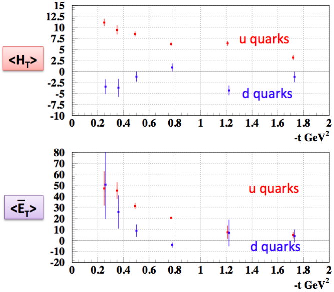


Figure: Preliminary [Kubarovsky '15], talk given at EMP and Short Range Hadron Structure. $Q^2 = 1.8 GeV^2$

where $< ... >$ denotes a convolution of the associated GPDs with a subprocess as introduced in $Goloskokov - Kroll$ model.

Conclusions

- Chiral-odd GPDs at leading twist are evaluated in the MIT Bag Model
- In the Large-N_c limit, scaling properties of GPDs and their isospin combinations were analyzed
- Phenomenological results supports the Large-N_c expectations.