Radiative corrections to $e^+e^- \to \pi^+\pi^-\gamma$ and the Standard Model predictions of the anomalous magnetic moment of the muon

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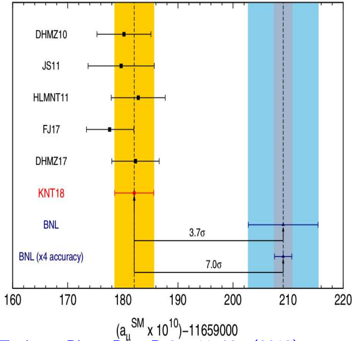
Outline

1. Motivation
2. Radiative corrections to the reaction $e^+e^- \to \pi^+\pi^-\gamma$
3. Results
4. Conclusions

Anomalous magnetic moment of the muon a_μ

$$a_\mu^{SM} = 11659182.04 \pm 3.56$$

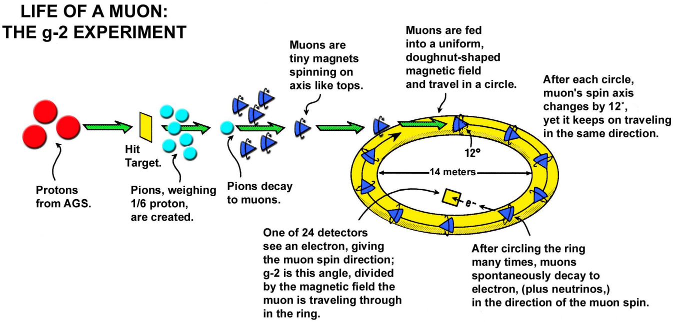
$$a_\mu^{exp} = 11659208.9 \pm 5.4 \pm 3.3$$



A.Keshavarzi, D.Nomura and T.Teubner, Phys. Rev. D 97, 114025 (2018)
Muon g-2 Collaboration (G.W. Bennett et al.), Phys. ReV. D 73, 072003 (2006) [hep-ex/0602035]

Experimental measurement of a_μ

$$\vec{\mu} = g\left(\frac{q}{2m}\right)\vec{S}, \qquad a_\mu = \frac{g-2}{2}$$



Idealized measurement:

$$\vec{\omega}_c = -\frac{q\vec{B}}{m\gamma}, \qquad \vec{\omega}_s = \frac{gq\vec{B}}{2m} - (1-\gamma)\frac{q\vec{B}}{m\gamma}$$

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -a_\mu \frac{q\vec{B}}{m}$$

In reality:

$$\vec{\omega}_a = -\frac{q}{m}\Big(a_\mu \vec{B} - \Big(a_\mu - \frac{1}{\gamma^2-1}\Big)\frac{\vec{\beta}\times\vec{E}}{c}\Big)$$

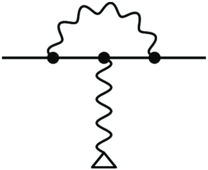
for $\gamma = 29.3$ one has to measure ω_a and B.

Muon g-2 Collaboration (G.W. Bennett et al.), Phys. ReV. D 73, 072003 (2006) [hep-ex/0602035]

Theoretical prediction of a_μ

Schwinger term:

$$a_\mu^{(QED,LO)} = \alpha/2\pi \approx 1.16 \cdot 10^{-3}$$



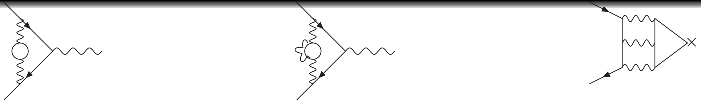
$$a_\mu^{exp} - a_\mu^{SM} = 27.06 \pm 7.26$$

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$$

$$a_\mu^{QED} = 11658471.8971 \pm 0.007$$

$$a_\mu^{EW} = 15.36 \pm 0.10$$

A.Keshavarzi, D.Nomura and T.Teubner, Phys. Rev. D 97, 114025 (2018),
Muon g-2 Collaboration (G.W. Bennett et al.), Phys. ReV. D 73, 072003 (2006) [hep-ex/0602035].



$$a_\mu^{had} = a_\mu^{had,LO} + a_\mu^{had,NLO} + a_\mu^{had,LBL}$$

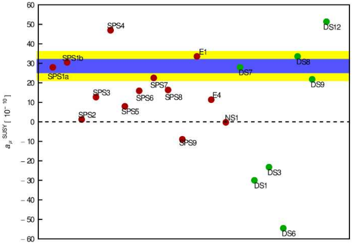
$$\begin{aligned} a_\mu^{had,NLO} &= -9.82 \pm 0.04 \\ a_\mu^{had,LO} &= 684.68 \pm 2.42 \\ a_\mu^{had,LBL} &= 9.8 \pm 2.6 \end{aligned}$$

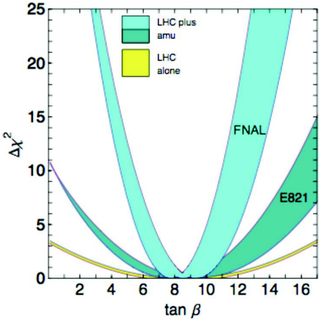
$$a_\mu^{had,LO} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma_0}$$

$a_\mu^{had,LBL}$ - effective models

Why do we need g-2?





M. Bach, D. Stöckinger, H. Stöckinger-Kim and J. H. Park, Acta Phys. Polon. B **46** (2015) no.11, 2243,

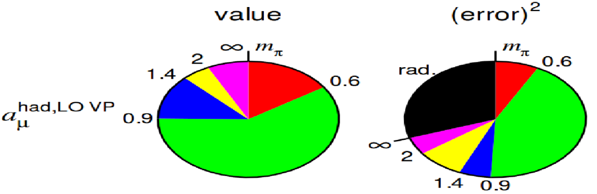
B. C. Allanach *et al.*, Eur. Phys. J. C **25** (2002) 113, [hep-ph/0202233], C. Adam, J. L. Kneur, R. Lafaye,

Radiative corrections to the reaction $e^+e^- \to \pi^+\pi^-\gamma$

F. Campanario, H. Czyz, J. Gluza, T. Jelinski, G. Rodrigo, S. Tracz and D. Zhuridov, arXiv:1903.10197 [hep-ph]

Contributions to HVP

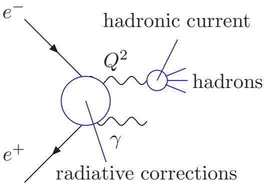
$$a_\mu^{had,LO} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{ds}{s}K(s)R(s)$$



A.Keshavarzi, D.Nomura and T.Teubner, Phys. Rev. D 97, 114025 (2018)

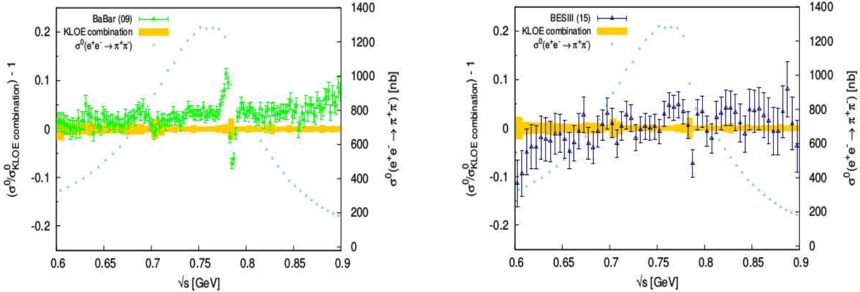
Radiative return method

$$d\sigma(e^+e^- \to hadrons+\gamma_{isr}) = H(Q^2,\theta_\gamma)d\sigma(e^+e^- \to hadrons)(Q^2)$$



- measurement of $R(s)$ over the wide range of energies, from threshold up to $\sqrt{s}$
- large luminosity from factories compensate α/π from photon radiation
- precise measurement involves radiative corrections
- FSR contribution has to be subtracted
- Monte Carlo generators needed (Phokhara)

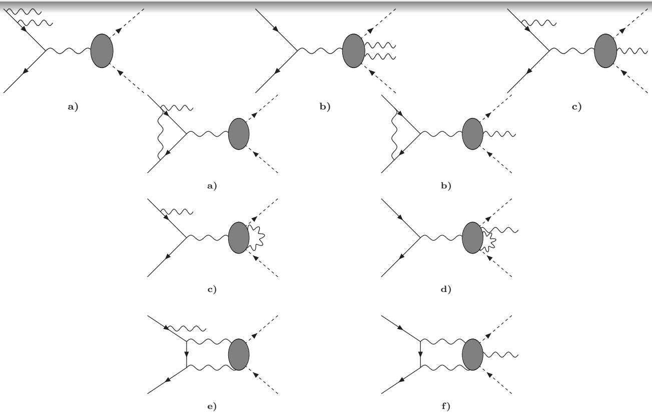
BaBar, BES and KLOE data



Data analysis used approximate radiative corrections from MC PHOKHARA.
<u>Full NLO radiative corrections were added</u>

A. Anastasi *et al.* [KLOE-2 Collaboration], JHEP **1803** (2018) 173

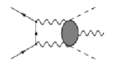
NLO radiative corrections for $e^+e^- \to \pi^+\pi^-\gamma$



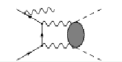
A. Denner and S. Dittmaier, Nucl. Phys. B **734** (2006) 62, T. Binoth, J. P. Guillet, G. Heinrich, E. Pilon and

Modeling pion-photon interaction

-Factorization of the form factor:



$$= \quad \mathbf{F_\pi(s) \times sQED}$$



$$= \quad \mathbf{F_\pi(q^2) \times sQED}$$

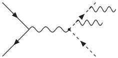
-Real emission proportional to the form factor

-Form factor:

$$F_\pi(q^2) = \sum_n c^\pi_{\rho_n} BW_{\rho_n}(q^2)$$

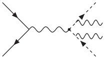
F. Campanario, H. Czyz, J. Gluza, T. Jelinski, G. Rodrigo, S. Tracz and D. Zhuridov, arXiv:1903.10197 [hep-ph]
H. Czyz, A. Grzelinska and J. H. Kuhn, Phys. Rev. D **81** (2010) 094014

Infrared divergences

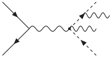


$\mathcal{M}_1$

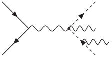
$\mathcal{M}_2$



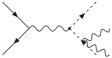
$\mathcal{M}_3$



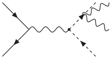
$\mathcal{M}_4$



$\mathcal{M}_5$



$\mathcal{M}_6$



$\mathcal{M}_7$

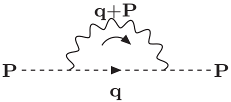
$$\sigma_{1h,1s} = \sigma_{1h}\frac{-\alpha}{4\pi}\int_{0\leq|k|\leq E_{max}}\frac{d^3k}{E_k}\left(\frac{p_1}{p_1\cdot k}-\frac{p_2}{p_2\cdot k}+\frac{q_1}{q_1\cdot k}-\frac{q_2}{q_2\cdot k}\right)^2,$$

Dimensional regularization vs photon mass regulator scheme:

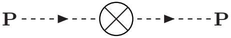
$$\log\left(\frac{\lambda^2}{s}\right)\to\Delta=\frac{(4\pi)^\epsilon}{\epsilon\Gamma(1-\epsilon)}\left(\frac{\mu^2}{s}\right)^\epsilon$$

Counter-terms for virtual FSR corrections

self energy correction:

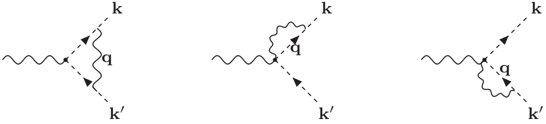


counter-term:

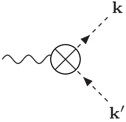


$$
\begin{aligned}
\bar{\mathcal{A}}_{1PI}|_{P^2=m_\pi^2} &= 0, \\
\frac{d\bar{\mathcal{A}}_{1PI}}{dP^2}|_{P^2=m_\pi^2} &= 0.
\end{aligned}
$$

vertex corrections:

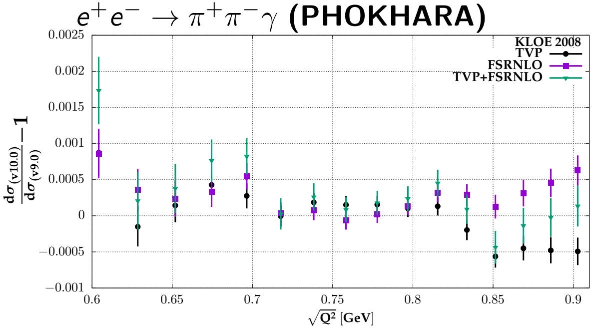


counter-term:



$$\bar{\mathcal{A}}_v|_{s=0} = 0.$$

- $\sqrt{s} = 1.02$ GeV
- Pion tracks: $50^o < \theta_{\pi^\pm} < 130^o$, $|p_{z_{\pi^\pm}}| > 90$ MeV
- Missing photon angle: $|\cos\theta_\gamma| > \cos 15^o$
- Track mass: $m_{trk} > 130$ MeV
- $q^2 \in (0.35, 0.95)$



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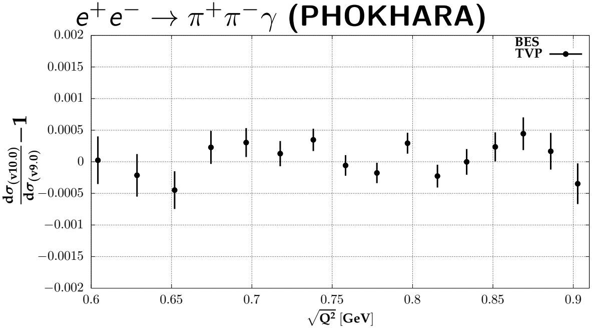
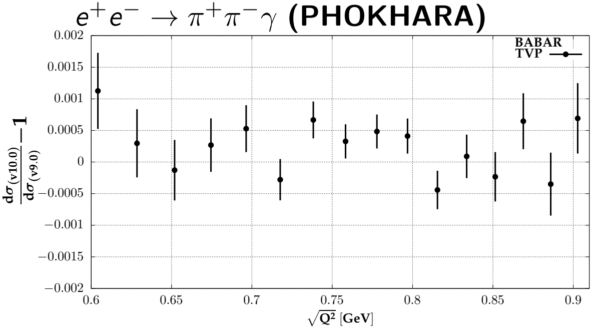
- $\sqrt{s} = 3.773$ GeV
- Pion tracks: $22.9^o < \theta_{\pi^\pm} < 157.1^o$, $|p_{T_{\pi^\pm}}| > 300$ MeV
- Minimal photon energy: $E_\gamma > 400$ MeV
- Missing photon angle: $|\cos\theta_\gamma| < 0.8$ or $0.86 < |\cos\theta_\gamma| < 0.92$
- $q^2 \in (0.35, 0.95)$



F. Campanario, H. Czyz, J. Gluza, T. Jelinski, G. Rodrigo, S. Tracz and D. Zhuridov, arXiv:1903.10197 [hep-ph]

- $\sqrt{s} = 10.56$ GeV
- Pion tracks: $20^o < \theta_{\pi^\pm} < 160^o$, $|p_{T_{\pi^\pm}}| > 300$ MeV
- Minimal photon energy: $E_\gamma > 3$ GeV
- Missing photon angle: $20^o < \theta_\gamma < 160^o$
- $q^2 \in (0.35, 0.95)$, $|q_1| > 1$ GeV (π^-) and $|q_2| > 1$ GeV (π^+)



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Conclusions

- Discrepancy between BABAR and KLOE data for $e^+e^- \to \pi^+\pi^-\gamma$ cannot be explained by missing radiative corrections. The source of the difference can be only of the experimental origin.
- As a consequence, these corrections cannot be the origin of the discrepancy between the experimental measurement and the SM prediction of the muon anomalous magnetic moment.

What's next ?

- **Declared precision of Phokhara: 0.5 %**