Hydrodynamics in phenomenology of heavy ion and proton-ion collisions

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Strong interactions beyond simple factorization:
Collectivity at high energy from initial to final state
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Foreword
Feynman: Scattering of protons on protons is like colliding Swiss watches to find out how they are built.
Studying the hydrodynamics of water by shooting at a watermelon!

- What is the equation of state, viscosity . . .?
- What was the shape before destruction?
Little bangs
Three stages of the "Standard Model" of Little Bangs

- partons
- hydrodynamization
- quark-gluon plasma
- freeze-out
- hadrons

Time:
- $\sim 1 \text{ fm/c}$
- $\sim 10 \text{ fm/c}$

These lectures focus on the intermediate (hydro) phase and its phenomenological implications.
Introduction
At high temperatures the thermal motion is so high, that also the momenta transferred are large. An early expectation was that weakly-interacting quark-gluon plasma (QGP) should be formed. It is not really the case at accessible temperatures!

\[ \text{QGP} \rightarrow \text{sQGP} \text{ – strongly interacting QGP} \]

Reminder: the Stefan-Boltzmann law

With the grand-canonical ensemble

\[-pV = \Omega(T, V, \mu) = V \gamma T \int \frac{d^3k}{(2\pi)^3} \log \left(1 \pm e^{-(E(k) - \mu)/T}\right)\]

+ fermions, – bosons, \(E(k) = \sqrt{m^2 + k^2}\), \(V\) - volume, \(T\) - temperature, \(\mu\) - chemical potential, \(\gamma\) - degeneracy factor.

\(m = 0\) and \(\mu = 0\): \(p = \gamma \frac{\pi^2}{90} T^4\) for bosons and \(p = \gamma \frac{7}{8} \frac{\pi^2}{90} T^4\) for fermions, whereas \(\epsilon \equiv E/V = 3p\), \(s = 4p/T\).

QGP (gluons and quarks+antiquarks)

\(p/T^4 = 8 \times 2\) (color \times spin) + \(7/8 \times 2 \times 3 \times 2 \times N_f ([q + \bar{q}] \times \text{color} \times \text{spin} \times \text{flavor})\)

\(N_f = 2\) and \(3\): \(p/T^4 \approx 4.06\) and \(\approx 5.21\), respectively (+ bag constant in some models)

\(s \approx 14/\text{fm}^3\) for \(T = 175\) MeV

massive pions at \(T \to 0\)

\(p = \gamma_{\pi} e^{-m_{\pi}/T} \frac{m_{\pi}^3/2T^{5/2}}{4\sqrt{2\pi}^{3/2}}\), with \(\gamma_{\pi} = 3\)

A dramatic growth of the number of degrees of freedom, as seen on the lattice!
“Our criteria for the discovery of QGP are

1. **matter at energy densities so large that simple degrees of freedom are quarks and gluons. This energy density is that predicted from lattice gauge theory for the existence of a QGP in thermal systems, and is about 2 GeV/fm$^3$**

2. **the matter must be to a good approximation thermalized**

3. **the properties of the matter . . . must follow QCD computations based on hydrodynamics, lattice gauge theory results, and perturbative QCD for hard processes such as jets.**

All of the above are satisfied from the published data at RHIC . . . This leads us to conclude that the matter produced at RHIC is a strongly coupled QGP (sQGP) contrary to original expectations that were based on weakly coupled plasma estimates.”
Phase diagram of QCD
Thermal ideas

Strong (soft) interactions $\rightarrow$ so complicated that things become simple again!

It is easier for a system to reach fast the thermal equilibrium when the interactions are strong – shorter mean free path $\rightarrow$ more collisions. How to achieve this from QCD-based approaches is a topic of current active research.

- **Isotropization** puzzle: how are the pressures in the longitudinal and transverse directions equilibrated
- **Early thermalization** puzzle: how is thermal equilibrium achieved?
- Relaxed to **early hydrodynamization**

It is needed that at a short time of the order of 1 fm/c approximate thermal equilibrium in the fireball is reached.

[thermal approach: Fermi 1950, Landau 1953]
What is collectivity?

Groups of objects (particles) move in a similar way

Collectivity = \(n\)-body correlations with large \(n\)

2-body: \(C_2(x_1, x_2) \equiv f_2(x_1, x_2) - f_1(x_1)f_1(x_2)\)

3-body: \(C_3(x_1, x_2, x_3) \equiv f_3(x_1, x_2, x_3)\)

\[-f_1(x_1)C_2(x_2, x_3) - f_1(x_2)C_2(x_1, x_3) - f_1(x_3)C_2(x_1, x_2)\]

\[-f_1(x_1)f_1(x_2)f_1(x_3)\]

...and so on

The genuine correlated distribution \(C_n\) in nonzero only if there exists a direct physical mechanism correlating \(n\) or more particles

Examples:

- 2-body resonance decays give rise to \(C_2(\vec{p}_1, \vec{p}_2)\), and not \(C_3\).
- Bose-Einstein correlations involve all identical bosons in the system
Flow $\rightarrow$ collectivity

A prominent source of momentum correlations is flow

In the intermediate stage system treated as gas/fluid (see later) - Quark-Gluon Plasma
No container! $\rightarrow$ the fireball expands and cools down, inevitability of flow

$$p_\parallel \rightarrow p_\parallel \cosh \zeta + E \sinh \zeta, \text{ with rapidity } \zeta = \text{arctanh}(v/c)$$
Doppler effect $\rightarrow$ emission of hadrons form a moving (boosted) source
Boosting the distribution

Same space (points) and momentum (arrows) distribution of thermal pions ($T = 160$ MeV) in a fluid element at rest, and moving to the right at a velocity $v$.
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Many elements in modeling, “circumstantial evidence”
Many elements in modeling, “circumstantial evidence”

Fireball

We start with the end of the evolution to show how thermal ideas work
An ALICE event

In relativistic heavy-ion collisions thousands of particles are formed in a single collision.
Multiplicities

Growth with $\sqrt{s_{NN}}$, not superposition of p+p

$\sqrt{s_{NN}}$ – energy per nucleon pair in their center-of-mass (CM) frame

- pseudorapidity $\eta = \frac{1}{2} \log \frac{p_\parallel + p_\perp}{p_\parallel - p_\perp} = -\log[tg(\theta/2)]$, rapidity $y = \frac{1}{2} \log \frac{E + p_\perp}{E - p_\perp}$

- $N_{part}$ - number of participating nucleons

[Aamodt et al. (ALICE) PRL 105(2010)252301]
Kinematic range in rapidity is \( y_{\text{beam}} = \arccosh\left[\frac{\sqrt{s_{NN}}}{2m_N}\right] \)

\(\approx 8 \text{ at 2.76 TeV, } \approx 5.4 \text{ at 200 GeV} \)

\[ y = \frac{1}{2} \log \left( \frac{\sqrt{p_T^2 \cosh^2(\eta) + m^2} + p_T \sinh(\eta)}{\sqrt{p_T^2 \cosh^2(\eta) + m^2} - p_T \sinh(\eta)} \right) \leq \eta, \quad \frac{dy}{d\eta} = \frac{p_T \cosh(\eta)}{\sqrt{m^2 + p_T^2 \cosh^2(\eta)}} \leq 1 \]

For identified particles \( y \) is typically used. Note that \( \eta \) distributions are wider and lower. The central dip is of kinematic origin: \( dN/d\eta = dy/d\eta \cdot dN/dy \)

1600 of charged hadrons per unit of \( \eta \)! (\( \approx 2400 \) for all hadrons)
To a very good approximation, in large systems

$$c \sim \frac{\pi b^2}{\sigma_{AB}^{inel}} \sim \frac{b^2}{(R_A + R_B)^2},$$

where $R_A$ and $R_B$ are the radii of the nuclei.

[ALICE, arXiv:1306.3130]

Archery competition

probability $\sim 2\pi b db \rightarrow$ cumulative distribution function:

$$c(b) = \int_0^b P(b') db' = \frac{b^2}{b_{\text{max}}^2} = \frac{b^2}{(R_A + R_B)^2}$$
Statistical (thermal) model of hadronization

[Fermi, Pomeranchuk, Hagedorn, Kapusta, Koch, Muller, Rafelski, Sollfrank, Heinz, Becattini, Braun-Munzinger, Stachel, Redlich, Cleymans, Gazdzicki, ...]

Large multiplicities $\rightarrow$ statistical description – the higher collision energies, the better!

By counting all the particles we cannot obtain the temperature $T$, as we do not know the volume $V$. Idea: look at identified hadron multiplicities and take ratios to divide out $V$.

For the simplified case of the Boltzmann distribution ($\hbar = k_B = c = 1$)

$$N = V \int \frac{d^3p}{(2\pi)^3} e^{-(E-\mu)/T} = V e^{\mu/T} \int \frac{d^3p}{(2\pi)^3} e^{-\sqrt{m^2+p^2}/T} = \frac{VT^3}{2\pi^2} e^{\mu/T} \left( \frac{m}{T} \right)^2 K_2 \left( \frac{m}{T} \right)$$

In chemical equilibrium

$$\mu = B\mu_B + S\mu_S + I_3\mu_{I_3}$$
Bessel functions

Modified Bessel function of the second kind

![Graph of \((m/T)^2 K_2(m/T)\) vs. \(m/T\)]

- higher \(m\) → lower yield of a species
For boost-invariant systems (approximately satisfied at midrapidity) the ratio of abundances of species $i$ and $j$ is

$$
\frac{dN_i}{dN_j} = \frac{N_i}{N_j} \approx \frac{2J_i + 1}{2J_j + 1} e^{(\mu_i - \mu_j)/T} \frac{m^2_i K_2(m_i/T)}{m^2_j K_2(m_j/T)}
$$

For instance

$$
\frac{p}{\bar{p}} = e^{2\mu_B/T}, \quad \frac{K^+}{K^-} = e^{2\mu_S/T}, \quad \frac{\pi^+\pi^-}{p\bar{p}} = \left(\frac{1}{2} \frac{m^2_\pi K_2(m_\pi/T)}{m^2_p K_2(m_p/T)}\right)^2
$$

3 equations allow to find the thermal parameters $T$, $\mu_B$, $\mu_S$.

In practice $\mu_S$ and $\mu_{I_3}$ are determined by requiring that the strangeness of the system is zero, and the ratio of the baryon number to the electric charge densities is the same as in the colliding nuclei $\rightarrow$ solve overdetermined system for many ratios in the $\chi^2$ sense

($V$ should be treated as an independent parameter [Becattini, arXiv:0707.4154])
Sensitive thermometer

\(\mu\)-independent combination

\[ (N(p)N(\bar{p}))N(\pi^+)^{1/2} \]

- lower \( T \) \( \rightarrow \) more difference between species
very important: $\sim 75\%$ of pions come from resonance decays (!)

SHARE, THERMUS - publicly available codes carrying out statistical hadronization with the decays of all resonances from Particle Data Tables
RHIC success

Strangeness production/enhancement

data from NA57

see, e.g., a recent review by Koch, Müller, Rafelski, Int. J. Mod. Phys. A32 (2017) 1730024
LHC

Yield $dN/dy$

Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV, 0-10% centrality

$\pi^+ \pi^-$

$K^+ K^- K_s^0$

$\phi$

$p \bar{p} \Lambda \bar{\Lambda}$

$\Xi^- \Xi^+$

$\Omega^- \Omega^+$

d $\bar{d}$

$^3\text{He}^3\text{He}$

$^3\text{H}^3\text{H}$

$^4\text{He}^4\text{He}$

Data, ALICE

Statistical Hadronization

Data/Model

[0.5, 1, 1.5, 2]

Yields of light nuclei

9 orders of magnitude!

- Fundamentally not possible to understand the production of the light nuclei (albeit described) in the statistical hadronization model. Too weakly bound to achieve thermal equilibrium during the fireball’s lifetime. Too large compared to the inter-particle spacing.


Open problem!
$T-\mu_B$ diagram

- Quark-Gluon Matter
- Hadronic Matter
- Band: Lattice QCD, $T_c$
- Points: Statistical Hadronization, $T_{CF}$

Hadron resonance gas vs LQCD

Other effects

- To satisfy the baryon number and strangeness conservation laws → canonical ansatz
- To satisfy the energy conservation → microcanonical ansatz - relevant for systems with small numbers of particles
- Short-range repulsion, excluded volume
- Incomplete equilibrium (Rafelski’s fugacity factors)
- Hierarchy of freeze-outs, based on hierarchy of cross sections
Off mid-rapidity

$\mu_i$ depend on the spatial rapidity $\alpha_\parallel = \frac{1}{2} \log \left( \frac{t+z}{t-z} \right)$

[B. Biedroń, WB, PRC 75(2007)054905]
Summary of thermal approach

- Dense system with numerous collisions
- Estimate: after freeze-out typically one collision per particle (as it should be)
- Thermal and chemical equilibrium (at FO) explain the hadron abundances
- The embarrassing success of light (hyper)nuclei production
- Resonances crucial, HRG
- HRG compares reasonably well to LQCD

The system (at least near the end of the evolution) is close to thermal and chemical equilibrium
Expansion and flow

The key concept of the approach to collectivity

Flow (and jet quenching) are the two major discoveries of the ultra-relativistic heavy-ion program!
Inevitability of expansion

No container! → the fireball expands (and cools down)
Think in terms of fluid - dense medium, short mean-free path, multiple rescattering

Flow is generic to a system with copious rescattering: hydro, transport, ... 

Obviously, the expansion affects the momentum spectra, as the velocity of the fluid element yields the Doppler effect
Frye-Cooper formula

One needs to collect particles (hadrons) produced from various fluid elements. For a single element of volume $V$ at rest

$$\frac{d^3 N_i}{d^3 p} = V f_i(E)$$

Rewrite invariantly: $u^\mu = \frac{1}{\sqrt{1-v^2}}(1, \vec{v})$, at rest $u^\mu = (1, 0, 0, 0)$

$E = p^0 \rightarrow p \cdot u$, $E/d^3p$ - Lorentz invariant $\rightarrow$

$$\frac{E d^3 N_i}{d^3 p} = \int d^3 \Sigma_\mu(x) p^\mu f_i[p \cdot u(x)]$$

where $d\Sigma_\mu(x)$ describes the element of a 3D freeze-out hypersurface on the 4D coordinate space
Collecting along pseudorapidity:
Hypersurface

\[ d^3 \Sigma_\mu(x) = \epsilon_{\alpha\beta\gamma} \frac{\partial x^\alpha}{\partial p} \frac{\partial x^\beta}{\partial q} \frac{\partial x^\gamma}{\partial r} dp \, dq \, dr \]

- coordinates in space-time, \( p, q, r \) - parameters of a 3-dim. hypersurface

Examples:

- \( x = p, \; y = q, \; z = r \) \( \rightarrow \) \( d^3 \Sigma_\mu(x) = dx \, dy \, dz \)

- Boost-inv. freeze-out \([\text{Schnedermann, Sollfrank, Heinz, PRC48 (1993) 2462}]\)
  \( x^\mu = (t, x, y, z) = (\tau(\zeta) \cosh \alpha||, \rho(\zeta) \cos \phi, \rho(\zeta) \sin \phi, \tau(\zeta) \sinh \alpha||) \) \( \rightarrow \)
  \[ d^3 \Sigma^\mu = \left( \frac{d\rho}{d\zeta} \cosh \alpha||, \frac{d\tau}{d\zeta} \cos \phi, \frac{d\tau}{d\zeta} \sin \phi, \frac{d\rho}{d\zeta} \sinh \alpha|| \right) \rho(\zeta) \tau(\zeta) d\zeta d\alpha|| d\phi \]

With a complementary hypothesis for \( u^\mu \) one may obtain model results without running hydro

Hydrodymanics provides \( d^3 \Sigma^\mu \) and \( u_\mu \) when a freeze-out condition is met (typically, \( T = T_f \)) as a numerical output

Freeze-out from one-shot perfect hydrodynamics

boost-inv. case for RHIC@200 GeV, $r$ - transverse radius, $t$ - time, labels - $v/c$

Effects on the $p_T$ spectra

- **thermal**: pion spectrum from a static fireball
- **thermal+decays**: initial and secondary pions, which lead to a decrease of the inverse slope
- **Bjorken**: pure longitudinal expansion $\rightarrow$ redshift, as all fluid elements move away from the observer $\rightarrow$ cooling of the spectrum.
- **our model**: transverse flow added, hence some fluid elements move towards the observer $\rightarrow$ blueshift

Radial flow $\rightarrow$ **blueshift** and **redshift** $\rightarrow$ convex

[WB, W. Florkowski, PRL 87(2001)272302 ]
Example $p_T$ spectra @130 GeV

$T_f = 165$ MeV, $\mu_B = 41$ MeV \quad [WB, W. Florkowski, PRL 87(2001)272302 ]

- mass hierarchy (from thermal motion and from transverse flow)
$p_T$ spectra at the LHC

More flow with increasing energy!
Mean transverse momenta

- Thermal component
- Radial flow component

**Blast wave model:** → enhancement of the mass hierarchy

\[
\frac{dN}{dy d^2 p_T} = \text{const} \times m_T I_0 \left( \frac{p_T \sinh \alpha}{T} \right) K_1 \left( \frac{m_T \cosh \alpha}{T} \right), \quad v_r/c = \tanh \alpha
\]
Initial geometry

Au+Au collision at RHIC (view along the beam)

1. Participants determine the geometry of the overlap region
2. Initial entropy distribution in more microscopic approaches (IP Glasma) also follows the geometry of the overlap region
3. Strong radial flow
4. Initial eccentricity → anisotropic flow of hadrons [Ollitrault 1992]
Initial geometry

**Initial geometry**

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**Au+Au collision at RHIC**
(view along the beam)
Rescattering/collectivity essential

In each event, define the harmonic flow coefficients and event-plane angles:

\[
dN/d\phi \propto 1 + 2 \sum_n v_n \cos[n(\phi - \Psi_n)]
\]

[ALICE]
Fluctuations

Collapse of the nuclear wave function → each Little Bang different

Higher Fourier components appear
Odd harmonics also show up, **triangular flow**
Fluctuations dominant for central A+A and for *small systems*, such as p+A (see later on)

New thinking since [Miller and Snellings 2003]
Collectivity: shape/size – flow transmutation

Any rescattering will do!

smaller $\rightarrow$ faster
Collimation from the Doppler effect

- Emission from a fast moving element of fluid
- Collimation of hadrons (increasing with mass)

Multi-particle correlations in the azimuth are used in the cumulant or other methods to extract the flow coefficients without the non-flow contamination (from jets, resonance decays, . . .)

[Borghini, Ollitrault 2001]
Features of harmonic flow

1. Mass ordering of harmonic flow coefficients $v_n$
2. Higher harmonics suppressed
3. Near-side ridge (discussed later on) - considered the “proof” of harmonic flow

### ALICE 40-50% Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV

- $\pi^\pm$
- $K^\pm$
- $K^0$
- $p+\overline{p}$
- $\phi$
- $\Lambda+\overline{\Lambda}$
- $\Xi+\overline{\Xi}$
- $\Omega+\overline{\Omega}$

$\nu_2$ vs $p_T$ (GeV/c)
Features of harmonic flow

1. Mass ordering of harmonic flow coefficients $v_n$
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$v_2$ vs $\sqrt{s_{NN}}$
At the LHC the differential elliptic flow is the same as at RHIC, but “sampling” is at higher $p_T$.
Hydrodynamics
- Perfect hydro
- Viscous hydro
- Initial conditions
- Anisotropic hydro

Correlations
- Paradigms
- $p_T$ fluctuations
- Flow fluctuations

Modeling in rapidity
- Ridges
- Fluctuating strings
- Torque decorrelation
- $\eta_1-\eta_2$ correlations

Small systems
- $p$-A and $d$-A
- Other small systems
- Polarized $d$-A
- $\alpha$ clusterization
Flow (radial and harmonic) leads to correct phenomenology of the $p_T$ spectra and $v_n$, with proper dependence on the geometry (shape-flow transmutation), collision energy, and mass hierarchy.

Hydrodynamics

What produces the flow (collectivity)?

Flow (and jet quenching) are the two major discoveries of the ultra-relativistic heavy-ion program!
**Basics**

- **Fluid** ≡ substance that cannot resist any shear force (gas, liquid, plasma), continuously deforms
- size of particles ≪ fluid element ≪ size of the system
- **Knudsen number**: \( Kn = \frac{\lambda}{L} \), \( \lambda \) mean free path, \( L \) - system’s size
- \( Kn \ll 1 \) → fluid description

![Diagram showing different flow regimes](Wikipedia)
Perfect hydrodynamics (no viscosity)

Local thermal equilibrium at point $x$: $T^{\mu \nu}(x) = \int \frac{d^3p}{p_0} p^\mu p^\nu f_{eq}(x, u \cdot p; T, \mu)$

Landau definition of the four-velocity of the fluid

$$T^{\mu \nu}(x) u_\mu(x) = \lambda(x) u^\nu(x)$$

$$u_\mu u^\mu = 1, \quad u^\mu = \gamma(1, v_x, v_y, v_z) = \frac{1}{\sqrt{1 - v^2}}(1, v_x, v_y, v_z)$$

The perfect hydro form follows ($u^\mu$ and $g^{\mu \nu}$ for disposal):

$$T^{\mu \nu} = (\varepsilon + P) u^\mu u^\nu - Pg^{\mu \nu} \quad (\lambda = \varepsilon)$$

In the fluid element’s rest frame $u^\mu = (1, 0, 0, 0)$

and $T^{\mu \nu} = \begin{pmatrix}
\varepsilon & 0 & 0 & 0 \\
0 & P & 0 & 0 \\
0 & 0 & P & 0 \\
0 & 0 & 0 & P
\end{pmatrix}$
The perfect hydro equations

Energy-momentum conservation \(\rightarrow\)

\[\partial_\mu T^{\mu\nu}(x) = 0,\]

4 equations for 5 unknown functions: \(\vec{v}, \varepsilon, P\) – need the equation of state linking \(\varepsilon\) and \(P\) to close the system

- Example: massless particles \(\rightarrow\) \(\varepsilon = 3P\)

Entropy is conserved

\[\partial_\mu (sw^\mu) = 0\]

Similarly for conserved charges \(\partial_\mu (nu^\mu) = 0\)
Sound velocity

Consider perturbation on a static background

\[ \varepsilon(x) = \varepsilon_0 + \delta \varepsilon(x), \quad P(x) = P_0 + \delta P(x) \]

and a small velocity \( u^\mu = (1, \delta v_x, \delta v_y, \delta v_z) \) (recall the 5 variables). To first order

\[
T^{\mu \nu} = \begin{pmatrix}
\varepsilon_0 + \delta \varepsilon & (\varepsilon_0 + P_0) \delta v_x & (\varepsilon_0 + P_0) \delta v_y & (\varepsilon_0 + P_0) \delta v_z \\
(\varepsilon_0 + P_0) \delta v_x & P_0 + \delta P & 0 & 0 \\
(\varepsilon_0 + P_0) \delta v_y & 0 & P_0 + \delta P & 0 \\
(\varepsilon_0 + P_0) \delta v_z & 0 & 0 & P_0 + \delta P
\end{pmatrix}
\]

\[
\partial_0 T^{00} + \partial_i T^{i0} \rightarrow \partial_t \delta \varepsilon + (\varepsilon_0 + P_0) \vec{\nabla} \cdot \vec{\delta \vec{v}}
\]

\[
\partial_0 T^{0j} + \partial_i T^{ij} \rightarrow (\varepsilon_0 + P_0) \delta \partial_t v^j + \nabla^j \delta P
\]

Combining,

\[
\partial_t^2 \delta \varepsilon - \nabla^2 \delta P = 0
\]

For zero chemical potentials there is only one thermodynamic parameter \( T \). Then

\[ \delta P = \frac{dP}{d\varepsilon} \delta \varepsilon = c_s^2(T) \delta \varepsilon. \]

We thus arrive at the wave equation

\[
\partial_t^2 \delta \varepsilon - c_s^2 \nabla^2 \delta \varepsilon = 0
\]

with \( c_s \) being the sound velocity, dependent on \( T \) (or \( \varepsilon \)).
A simple form hydro for $\mu = 0$

In the case of vanishing chemical potentials one may rewrite the perfect hydro equations in the form, e.g.,

$$s \frac{du^\nu}{d\tau} = c_s^2(s)(g^{\mu\nu} - u^\mu u^\nu)\partial_\mu s, \quad \tau^2 = t^2 - \bar{x}^2$$

- no first order phase transition
- no shock or rarefaction waves (!)
- laminar flow

[M. Chojnacki, W. Florkowski 2007]
Digression on hadronization

As the system cools down, quarks and gluons are gradually replaced with hadrons.

- **Hadronization** is conveniently carried over “behind the back”, hidden in the eq. of state.
- Fluid changed into particles via the Frye-Cooper mechanism.
Bjorken flow

Purely longitudinal expansion \( u^\mu = \frac{1}{\tau}(t, 0, 0, z) \), assumed boost invariance involves dependence on the proper time \( \tau = \sqrt{t^2 + z^2} \) only
\[
\partial_\mu u^\mu = \frac{1}{\tau}, \quad \partial_\mu \tau = u_\mu
\]

\[
0 = \partial_\mu (su^\mu) = \frac{ds(\tau)}{d\tau} + \frac{s(\tau)}{\tau} \quad \rightarrow \quad s(\tau) = s(\tau_0) \frac{\tau_0}{\tau}
\]

Thermodynamic relations for \( \mu = 0 \): \( \varepsilon + P = Ts \), \( d\varepsilon = T ds \), \( dP = s dT \), from where (for ultra-relativistic particles, where \( P = c_s^2 \varepsilon \))
\[
\varepsilon(\tau) = \varepsilon(\tau_0) \left( \frac{\tau_0}{\tau} \right)^{1+c_s^2}, \quad T(\tau) = T(\tau_0) \left( \frac{\tau_0}{\tau} \right)^{c_s^2}
\]

\( \tau_0 \) estimates based on entropy conservation per unit of rapidity. From known experimental hadronic yields one infers \( \varepsilon_{\text{QGP}}(\tau_0) \approx 4 \) GeV/fm\(^3\)
Relativistic 2+1D perfect hydro

central (0-20%)
Relativistic 2+1D perfect hydro

non-central (40-60%)
Kinetic arguments for viscosity

\[ F/A = \eta \partial_y v_x \]

\[ Re = \frac{\rho v L}{\eta} \]

Navier-Stokes equations:

\[ \rho \left( \partial_t v_i + \bar{v} \cdot \nabla v_i \right) = -\nabla_i P + \eta \nabla^2 v_i \]

one of Millennium Problems!
Various materials

<table>
<thead>
<tr>
<th>material</th>
<th>$\eta$ [Pa s]</th>
<th>$\eta/s [\hbar/k_B]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>$3 \times 10^{-4}$</td>
<td>8</td>
</tr>
<tr>
<td>honey</td>
<td>1000</td>
<td>$5 \times 10^7$</td>
</tr>
<tr>
<td>superfluid $^4$He</td>
<td>$10^{-6}$</td>
<td>2</td>
</tr>
<tr>
<td>ultra-cold $^6$Li</td>
<td>$&lt; 10^{-15}$</td>
<td>$&lt; 0.3$</td>
</tr>
<tr>
<td>QGP</td>
<td>$&lt; 2 \times 10^{11}$</td>
<td>$&lt; 0.4$</td>
</tr>
<tr>
<td>pitch</td>
<td>$2 \times 10^{11}$</td>
<td>$10^{16}$</td>
</tr>
</tbody>
</table>

U. of Queensland, 8 drops since 1927, Ig Nobel prize
Bounds on shear viscosity

dilute gas: $\eta = \frac{1}{3} np l$ (density $\times$ momentum $\times$ mean free path)

Quantum limit
Heisenberg uncertainty principle: $pl \geq \hbar$ and $s \sim k_B n \rightarrow \eta/s \geq \hbar/k_B$
[P. Danielewicz and M. Gyulassy, PRD 31 (1985) 53]

KSS bound based on AdS/CFT: $\eta/s \geq \frac{1}{4\pi} \hbar/k_B$

- $l = \frac{1}{n \sigma} \rightarrow \eta = \frac{p}{3\sigma_{el}}$ – counterintuitive!
Shear and bulk

shear viscosity $\eta$ – resistance to deformation
bulk viscosity $\zeta$ – resistance to expansion (volume change)

Bulk viscosity

Shear viscosity

Karsh&Kharzeev&Tuchin
Noronha&Noronha&Greiner

Hirano&Gyulassy

[from G. Denicol]
Adding viscosities into relativistic hydro


Israel-Stewart second-order hydro: perfect fluid

\[ T_0^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} \]

+ stress corrections from shear \( \pi \) (traceless) and bulk \( \Pi \) viscosities

\[ T^{\mu\nu} = T_0^{\mu\nu} + \pi^{\mu\nu} + \Pi \Delta^{\mu\nu} \]
\[ \partial_\mu T^{\mu\nu} = 0 \]

The viscous corrections are solutions of 6 additional equations:

\[ \Delta^{\mu\alpha} \Delta^{\nu\beta} u^\gamma \partial_\gamma \pi_{\alpha\beta} = \frac{2\eta\sigma^{\mu\nu} - \pi^{\mu\nu}}{\tau_\pi} - \frac{4}{3} \pi^{\mu\nu} \partial_\alpha u^\alpha \]
\[ u^\gamma \partial_\gamma \Pi = \frac{-\zeta \partial_\gamma u^\gamma - \Pi}{\tau_\Pi} - \frac{4}{3} \Pi \partial_\alpha u^\alpha \]
\[ \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu , \nabla^\mu = \Delta^{\mu\nu} \partial_\nu \]
\[ \sigma_{\mu\nu} = \frac{1}{2} \left( \nabla_\mu u_\nu + \nabla_\nu u_\mu - \frac{2}{3} \Delta_{\mu\nu} \partial_\alpha u^\alpha \right) \]

The relaxation time is taken as \( \tau_\pi = \tau_\Pi = \frac{3\eta}{T_s} \)
Quenching of flow

- Quenching of flow with viscosity
- Increasing with the Fourier rank
- Sets limits on viscosity, which is close to the KSS bound $\eta/s = 1/4\pi$
- ... but many other model parameters

Figure:
[Bazow, Heinz, Strickland 2016]
Damping of flow

Effect of bulk viscous pressure

- Red: Bulk only, $\eta=0$
- Blue: Shear only, $\zeta=0$

- $n=2 \rightarrow$ almost same effect
- $n>2 \rightarrow$ more damped by shear

[from G. Denicol]
3D numerics

[B. Schenke https://quark.phy.bnl.gov/~bschenke]

[other codes]
Initial conditions

- Initial value problem for partial differential equations → need to choose initial conditions for the functions on a time-like hypersurface, e.g., with constant \( \tau = \sqrt{t^2 - z^2} \)
- These conditions fluctuate event-by-event ...
- ... and are carried over to freeze-out approximately deterministically
- \( \tau \) must be short (a fraction of fm) for sufficient flow to develop

However, on the general grounds of the fluctuation-dissipation theorem, hydro must also bring in some fluctuations

L. Yan, H. Grönqvist, JHEP 1603 (2016) 121 – Gubser flow:
“...the effect of hydrodynamical noise on flow harmonics is found to be negligible, especially in the ultra-central Pb-Pb collisions ...”]
Glasma initial conditions

Glauber model

wounded + binary: $N \sim (1 - \alpha)N_w/2 + \alpha N_{\text{bin}}, \alpha \sim 0.14$

soft – wounded (a nucleon gets wounded only once)
hard – binary

[Białas, Błeszyński, Czyż, NPB 111 (1976) 461]

[D. Kharzeev, M. Nardi, PLB 507 (2001) 121]
Proportionality of flow to eccentricity

\[ v_n = \kappa_n \varepsilon_n, \quad (n = 2, 3) \]

- \( \kappa_n \) depend on the collision energy, multiplicity, viscosity . . .
- Approximate linearity allows us to build scale-less combinations independent of the response coefficient \( \kappa_n \) (see later)

[Niemi, Denicol, Holopainen, Huovinen 2012]

“Hydro without hydro” – linearity of the shape-flow transmutation

\[ c(\varepsilon_2, \varepsilon_2) = 0.996 \]
\[ C_2 = 0.153 \]
\[ c(\varepsilon_3, \varepsilon_3) = 0.973 \]
\[ C_3 = 0.093 \]
Isotropization in Color Glass Condensate (with $SU_c(2)$)

$T^{\mu\nu}/Q_s^4$

$\tau$ [fm/c]

$P_L/P_T = 0.70$

$\tau^{-1.26}$

[Epelbaum, Gelis, arXiv:1307.2214]

Longitudinal-transverse anisotropy

[Florkowski, Ryblewski, 2008]
Anisotropic hydro

[see also Babak Kasmaei’s talk]

One can obtain satisfactory phenomenology in approaches without isotropization, where $P_T \geq P_L$

[Alqahtani, Nopoush, Ryblewski, Strickland, PRL 119 (2017) 042301]
That things are nontrivial...

The Crooks radiometer

Which way will it turn?

Figure 1 - Radiometer Adaptation

Hanging Radiometer
Epoxy Seal Around Brass Tube
Glass Pump-Out Tube - Grind off end
5/16 Inch Diameter Brass Tube
KF25 Adapted to Brass Tube

Copyright 1964-1995, see Bell Lab
That things are nontrivial...

The Crooks radiometer

Figure 1 - Radiometer Adaptation

Which way will it turn?

- Not the light pressure!
- Not Navier-Stokes
- The Kortweg equations (capillarity) do it

https://www.quantamagazine.org/famous-fluid-equations-are-incomplete-20150721/
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Up to now:

- thermal equilibrium at freeze-out $\rightarrow$ species ratios
- radial flow $\rightarrow \langle p_T \rangle$, mass hierarchy, shape of $p_T$ spectra
- initial anisotropy, shape-flow transmutation from copious rescattering $\rightarrow$ harmonic flow
- viscosity $\rightarrow$ smoothing effect
- early thermalization $\rightarrow$ early hydrodynamization
Up to now:

- thermal equilibrium at freeze-out $\rightarrow$ species ratios
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- early thermalization $\rightarrow$ early hydrodynamization

Correlations
Where predominantly generated?

- At the early gluonic stage?
- In hydro/rescattering phase?
- All over?
- Are the early fluctuations destroyed?
Initial fluctuations in the Glauber approach

Two typical configurations of wounded nucleons in the transverse plane generated with GLISSANDO, isentropes at $s = 0.05, 0.2, \text{ and } 0.4 \text{ GeV}^{-3}$

Random fluctuations in Color Glass
Size – radial flow transmutation

smaller size $\rightarrow$ stronger flow
larger size $\rightarrow$ weaker flow

[WB, Chojnacki, Obara 2009]
Transverse momentum fluctuations in Au+Au@200GeV

- Measure removes trivial fluctuations from finite sampling
- Model overshoots the data by about 50% for most central collisions, need to decrease initial fluctuations
- Hydro response not modified by viscosity, freeze-out temperature, source smearing, total momentum conservation, ... \( \frac{\Delta \langle p_T \rangle}{\langle p_T \rangle} \approx 0.4 \frac{\Delta \langle r \rangle}{\langle r \rangle} \)
Transverse momentum fluctuations with wounded quarks

Wounded quark model as implemented in [Bożek, WB, Rybczyński 2016]: more participants → less fluctuation
Transverse momentum fluctuations with wounded quarks

Nontrivial dependence on multiplicity

Excludes independent production from sources (would be flat)
Size – flow anti-correlation

Very strong e-by-e anti-correlation of size and $\langle p_T \rangle$

- This is the mechanism for $p_T$ fluctuations!
Flow fluctuations

Recall $v_n = \kappa_n \varepsilon_n \rightarrow \sigma(v_n)/\langle v_n \rangle = \sigma(\varepsilon_n)/\langle \varepsilon_n \rangle$

\[ \left( \frac{v_n}{h} \right) = \frac{\langle \varepsilon_n \rangle}{\langle \varepsilon_n \rangle} \]

\[ \left( \frac{v_i^2}{h} \right) = \frac{\langle \varepsilon_i^2 \rangle}{\langle \varepsilon_n \rangle} \]

\[ \left( \frac{v_i^3}{h} \right) = \frac{\langle \varepsilon_i^3 \rangle}{\langle \varepsilon_n \rangle} \]

[ WB, Rybczyński 2016]
Flow fluctuations

\[ F_n = \sqrt{\frac{\varepsilon_n \{2\}^2 - \varepsilon_n \{4\}^2}{\varepsilon_n \{2\}^2 + \varepsilon_n \{4\}^2}} \]

\[ \varepsilon_n \{2\} = \langle \varepsilon_2^2 \rangle^{1/2}, \quad \varepsilon_n \{4\} = 2 \left( \langle \varepsilon_n^2 \rangle - \langle \varepsilon_n^4 \rangle \right)^{1/4} \]
Flow fluctuations

\[ F_n = \sqrt{\frac{\varepsilon_n \{2\}^2 - \varepsilon_n \{4\}^2}{\varepsilon_n \{2\}^2 + \varepsilon_n \{4\}^2}} \]

\[ \varepsilon_n \{2\} = (\varepsilon_2^2)^{1/2}, \quad \varepsilon_n \{4\} = 2 \left( (\varepsilon_n^2)^2 - (\varepsilon_n^4) \right)^{1/4} \]
Higher cumulants

(a) ALICE Pb-Pb at 5.02 TeV and 2.76 TeV, Hydrodynamics from Ref. [27].

(b) Hydrodynamics, Ref. [25].

(c) Ratio of experimental data to hydrodynamic predictions.

\[ \epsilon_2 \{m\} \]

Centrality percentile vs. \( N_{\text{part}} \).
IP-Glasma initial conditions

\[ \langle v^2 \rangle_{1/2} \]

\[ \eta/s = 0.12 \]

\[ \eta/s(T) \]

[\text{Gale, Jeon, Schenke, Venugopalan, PRL 110 (2013) 012302}]
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Modeling in rapidity
2D two-particle correlations

\[ R_2(\Delta \eta, \Delta \phi) \equiv C(\Delta \eta, \Delta \phi) = \frac{\langle N_{\text{phys}}(\Delta \eta, \Delta \phi) \rangle}{\langle N_{\text{mixed}}(\Delta \eta, \Delta \phi) \rangle} \]

"Tent":

\[
\int_{-\eta_a}^{\eta_a} d\eta_1 \int_{-\eta_a}^{\eta_a} d\eta_2 \delta[\Delta \eta - (\eta_1 - \eta_2)] = \text{triangle in } \Delta \eta \text{ from } -2\eta_a \text{ to } 2\eta_a
\]

\[
\int_{0}^{2\pi} d\phi_1 \int_{0}^{2\pi} d\phi_2 \delta[\Delta \phi - (\phi_1 - \phi_2)] = \text{flat in } \Delta \phi
\]
2D two-particle correlations

\[ R_2(\Delta \eta, \Delta \phi) \equiv C'(\Delta \eta, \Delta \phi) = \frac{\langle N_{\text{pairs}}^{\text{phys}}(\Delta \eta, \Delta \phi) \rangle}{\langle N_{\text{pairs}}^{\text{mixed}}(\Delta \eta, \Delta \phi) \rangle} \]

- free of detector acceptance bias
Near-side ridge indicates collectivity

Total surprise in p-p!
Factorization of the transverse and longitudinal distributions

left-moving participants  strings  right-moving participants
Factorization of the transverse and longitudinal distributions

left-moving participants  strings  right-moving participants

Approximate (up to fluctuations) alignment of F and B event planes
Collimation of flow at very distant longitudinal separations \(\rightarrow\) ridges!
Glasma tubes
Surfers - the near-side ridge

Collimated even if separated by a mile!
p+p – high multiplicity only!

(a) CMS MinBias, $p_T > 0.1 \text{GeV/c}$

(b) CMS MinBias, $1.0 \text{GeV/c} < p_T < 3.0 \text{GeV/c}$

(c) CMS $N \geq 110$, $p_T > 0.1 \text{GeV/c}$

(d) CMS $N \geq 110$, $1.0 \text{GeV/c} < p_T < 3.0 \text{GeV/c}$
2 particles from the same jet $\to$ central peak ($\Delta \phi \sim 0$, $\Delta \eta \sim 0$)

from the opposite jets $\to$ away ridge ($\Delta \phi \sim \pi$, $\Delta \eta$ - washed out)
Other sources of correlations
Flow measures with rapidity gap

The flow vector in rapidity bin $\eta$:

$$q_n(\eta) = \frac{1}{m} \sum_{k=1}^{m} e^{in\phi_k}$$

$$V_{n\Delta} = \langle q_n(\eta_1)q_n^*(\eta_2) \rangle = \langle \langle \cos n(\phi_1 - \phi_2) \rangle \rangle$$

with bins at $\eta_1$ and $\eta_2$ sufficiently separated

$$\frac{dN_{\text{pair}}}{d\Delta\phi} \sim 1 + 2 \sum_{n} V_{n\Delta} \cos n\Delta\phi$$
Triangles and fluctuating strings

Extracted from the d-Au collisions at RHIC:

Source emits mostly in its own forward hemisphere

[Białas, Czyż 2004]
Triangles and fluctuating strings
[... Bierlich, Gustafson, Lönnblad 2016, Monnai, Schenke 2015, Schenke, Schlichting 2016 ... Brodsky, Gunion, Kuhn, 1977]
String models 1970’s

Dual Parton Model (Capella et al.)

Lund model (Anderson et al.)

Basis of many successful codes (Pythia, HIJING, AMPT, EPOS, ...)

Fig. 1.2. Dominant two-chain diagram describing multiparticle production in high energy proton-proton collisions. The two quark-diquark chain structure results from an e-
Strings are spatial objects

AMPT [Wu et al. 2018]

\[
\sqrt{s_{\text{NN}}} = 5.02\text{TeV}
\]

\[
\sqrt{s_{\text{NN}}} = 2.76\text{TeV}
\]

\[
\sqrt{s_{\text{NN}}} = 200\text{GeV}
\]

String end-points fluctuate in (here: space-time rapidity) \(\eta\), uniform production of particles along the string (same thickness)
Fluctuating strings
Torque effect (event-by-event)

Transverse sections with triangles

- Both e-by-e fluctuations and longitudinal asymmetry of the emission profile needed

[prediction in PB, WB, Moreira 2010 & PB, WB, Olszewski 2015, PB, WB 2016]
Torque in Pb+Pb

\[ r_n(\eta_a, \eta_b) = \frac{\langle \cos(n[\phi_i(-\eta_a) - \phi_j\eta_b]) \rangle}{\langle \cos(n[\phi_i(\eta_a) - \phi_j\eta_b]) \rangle} \]

thin - triangles
thick - string breaking

\(v_2\) and \(v_3\)

(b) \(c=20-30\%\) Pb+Pb@2.76TeV \(4.4 < \eta_b < 5\)
Torque in p-Pb

String breaking essential to describe torque in p-Pb

With triangles:
Slope of $r_n$

- Fair description of mid-central collisions
- Way too much decorrelation in central collisions
- $F_4 \approx 4F_2$
\( \eta_1 - \eta_2 \) correlations and \( a_{nm} \) coefficients


\[
a_{nm} = \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} \frac{1}{N_C} C(\eta_1, \eta_2) T_n \left( \frac{\eta_1}{Y} \right) T_m \left( \frac{\eta_2}{Y} \right)
\]

$\eta_1 - \eta_2$ correlations and $a_{nm}$ coefficients


\[
a_{nm} = \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} \frac{1}{N_C} C(\eta_1, \eta_2) T_n \left( \frac{\eta_1}{Y} \right) T_m \left( \frac{\eta_2}{Y} \right)
\]
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Small systems
Snapshots of initial Glauber condition in central $p$-Pb

Typical transverse-plane configuration of centers of the participant nucleons in a $p$+Pb collision generated with GLISSANDO
5% of collisions have more than 18 participants, rms $\sim 1.5$ fm – large!
Snapshot of peripheral Pb+Pb

Most central values of $N_w$ in p-Pb would fall into the 60-70% or 70-80% centrality class in Pb+Pb

Pb+Pb: $c=60-70\% \equiv 22 \leq N_w \leq 40$, $c=70-80\% \equiv 11 \leq N_w \leq 21$
$d$ has an intrinsic dumbbell shape with a large deformation: \( \text{rms} \approx 2 \text{ fm} \)

Initial entropy density in a $d$-Pb collision with $N_{\text{part}} = 24$  

[Bożek 2012]
$d$ has an intrinsic dumbbell shape with a large deformation: \( \text{rms} \approx 2 \text{ fm} \)

Initial entropy density in a $d$-$Pb$ collision with $N_{\text{part}} = 24$ [Bożek 2012]

Resulting large elliptic flow confirmed with the later RHIC analysis (geometry + fluctuations)
Size of the $p$-Pb fireball

Isotherms at freeze-out $T_f = 150$ MeV
(for two sections in the transverse plane)

Evolution lasts about 4 fm/c – shorter but more rapid than in A+A
Mass hierarchy in $p$-$A$

![Graph showing mass hierarchy in $p$-$A$](image)

P. Bożek, WB, G. Torrieri, PRL 111 (2013) 172303
Mass hierarchy in $p$-$A$

$\pi$, $K$, $p$

$[P. \, Bożek, \, WB, \, G. \, Torrieri, \, PRL \, 111 \, (2013) \, 172303]$
Harmonic flow in $p$-$A$

no geometry, only fluctuations

![Graph showing harmonic flow](image)

[Bożek, P., WB, PRC 88 (2013) 014903]
Harmonic flow in $p$-$A$

no geometry, only fluctuations

\[v_3(\Delta \eta > 2, N_{\text{track}} < 20 \text{ sub.})\]

\[v_3(\Delta \eta > 2, N_{\text{track}} > 20 \text{ sub.})\]

3+1D Hydro

[C. Božek, WB, PRC 88 (2013) 014903]
Ridge in p-Pb, ATLAS

ATLAS \( p+Pb, \sqrt{s_{NN}} = 5.02 \text{ TeV} \)

- \( \Sigma E_T^{Pb} < 20 \text{ GeV} \)
- \( \Sigma E_T^{Pb} > 80 \text{ GeV} \)

\( L = 1 \mu b^{-1} \)

- \( 0.5 < p_T^{ab} < 4 \text{ GeV} \)

- \( c = 0 - 3.4\% \), \( 0.5 < p_T < 4 \text{ GeV} \)
Near-side ridge, $2 \leq |\Delta \eta| \leq 5$

$$Y(\Delta \phi) = \frac{\int B(\Delta \phi)d(\Delta \phi)}{N}C(\Delta \phi) - b_{ZYAM}$$

c=0-3.4%

$0.5 < p_T < 4.0$ GeV

two variants of the Glauber model:
red – $< R^2 >^{1/2} = 1.5$ fm, blue – $< R^2 >^{1/2} = 0.9$ fm, dots – ATLAS

see also CGC-based calculation: [K. Dusling, R. Venugopalan, PRD 87 (2013) 094034]
Ridge in $^3$He-Au at RHIC

(see on both pseudorapidity sides)
Flow hierarchy in small systems

[PHENIX, 2018]
Color Glass Condensate

independent sources in $d+A \rightarrow \nu_2$ in $d+A$ would be smaller than in $p+A$, contrary to experiment.

[Mace, Skokov, Tribedy, Venugopalan, 2018]: high multiplicity events have larger saturation scales and specific orientation of the deuteron, with one nucleon behind the other

Questioned in [Nagle, Zajc 2018] → controversy
Polarized d+A collisions

\[ j_3 = \pm 1 \quad \Rightarrow \quad v_2(\Phi_p) < 0 \]

\[ j_3 = 0 \quad \Rightarrow \quad v_2(\Phi_p) > 0 \]

[\text{P. Bożek, WB, PRL 121 (2018) 202301}]
Predictions

\[
\frac{dN}{d\phi} \propto 1 + 2v_2 \cos[2(\phi - \Phi_P)]
\]

\(\Phi_P\) fixed!

\[v_2 \approx k\epsilon_2, \; k \sim 0.2\]

For \(j = 1\) nuclei the tensor polarization is

\[
P_{zz} = n(1) + n(-1) - 2n(0)
\]

\[
v_2\{\Phi_P\} \approx k\epsilon_2^{j_3=\pm 1}\{\Phi_P\} P_{zz}
\]

\[-0.5\% \lesssim v_2\{\Phi_P\} \lesssim 1\%\]

One-particle distribution - can be measured precisely!

Prospects for AFTER@LHC
$^{12}\text{C-Pb} - \text{role of } \alpha \text{ clusters}$

Nuclear structure from ultra-relativistic collisions!
Probe to what degree $^{12}\text{C}$ is made of three $\alpha$’s

Specific features of the $^{12}\text{C}$ collisions with a “wall”:

The cluster plane parallel or perpendicular to the transverse plane:

- Higher multiplicity
- Higher triangularity
- Lower ellipticity

- Lower multiplicity
- Lower triangularity
- Higher ellipticity
Ellipticity and triangularity vs multiplicity

$RDS$

$n=2$  
$n=2$, uniform

$n=3$  
$n=3$, uniform

$\nu_n\{4\}/\nu_n\{2\}$

$N_W$

$^{12}C + ^{208}Pb$

10%  
1%  
0.1%
Ellipticity and triangularity vs multiplicity

\[ \frac{v_{n\{4\}}}{v_{n\{2\}}} \]

\( RDS \)

- \( n=2 \)
- \( n=2, \) uniform
- \( n=3 \)
- \( n=3, \) uniform

\( 16O + 208Pb \)

\( N_W \)

10\% 1\% 0.1\%
Idea picked up in [Lim, Carlson, Loizides, Lonardoni, Lynn, Nagle, Orjuela Koop, Ouellette, PRC 99 (2019) 044904] with exp. prospects
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Summary
Circumstantial evidence

- Multiplicities $\rightarrow$ thermal parameters
- $p_T$ spectra $\rightarrow$ radial flow
- Harmonic flow $\rightarrow$ initial geometry and fluctuations
- Fluctuations of $\langle p_T \rangle$ $\rightarrow$ fluctuations of the initial size
- Ridge $\rightarrow$ flow
- Interferometry $\rightarrow$ size and flow (not covered)
Conclusions

The approach with hydro (copious rescattering in the intermediate stage) works

- **Collectivity** from rescattering in A+A commonly accepted
- Explanation of the near-side ridge
- Mechanism for $p_T$ fluctuations
- Torque (event-plane angle decorrelation)

- Small systems ($p$-Pb, $d$-Pb) not so small
- Torque in p-Pb $\rightarrow$ longitudinal fluctuations (string breaking)
- Shape-flow transmutation in small systems
- Polarized deuteron
- Clustered small nuclei
Jet quenching by the medium
Early probes
Femtoscopy
Chiral magnetic effect
Vorticity and $\Lambda$ polarization
...
Recommended literature (and references therein)

- *Phenomenology of Ultra-Relativistic Heavy-Ion Collisions*, Wojciech Florkowski, World Scientific 2010 (with exercises!)


- ...
THANKS!