On HVP contribution to $g-2$ of μ versus e

arXiv: 2606.08329 [hep-ph]

9th Plenary Workshop of the Muon g-2 Theory Initiative

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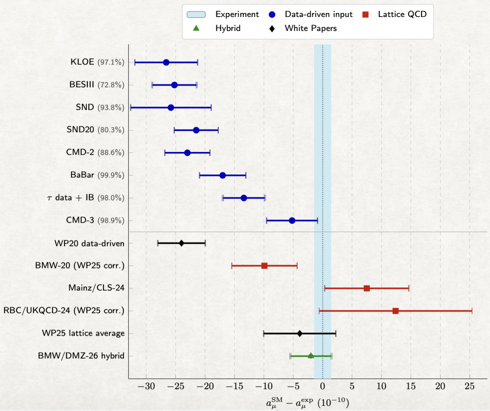
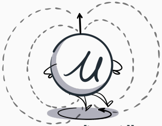
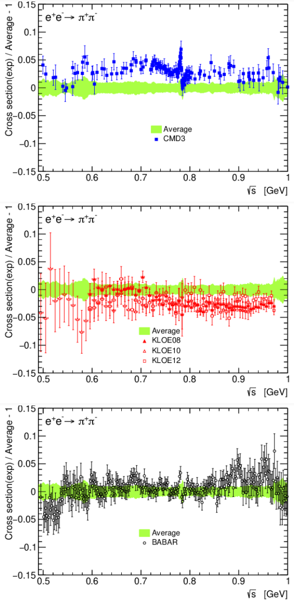
$a_\mu^{\mathrm{HVP}} = (g-2)_\mu^{\mathrm{HVP}}$ Landscape



[V. Pascalutsa's talk]



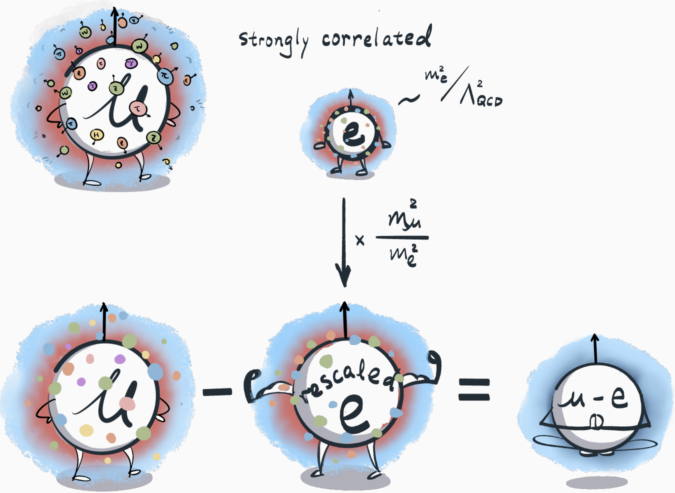


[Eur.Phys.J.C 84 (2024) 7, 721, arXiv:2312.02053 [hep-ph]]
[B. Malaescu, muon g-2 workshop 2024]

$(g-2)^{\mathrm{HVP}}$: muon & electron

Given an accurate enough α, [H. Wittig's talk]
can a_e be used to constrain HVP,
and therefore improve the uncertainty of a_μ?

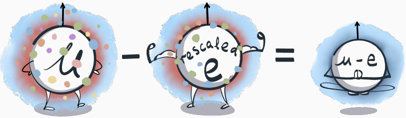
$(g-2)^{\mathrm{HVP}}$: muon & electron



$$a_{\mu-e} \equiv a_\mu - \left(m_\mu^2/m_e^2\right) a_e$$ $\Rightarrow$ 2606.08329 [hep-ph]

$(g-2)^{\mathrm{HVP}}$: muon versus electron

*see more data in Pascalutsa's talk



LO HVP	$a_\mu\ (10^{-10})$	$a_e\ (10^{-14})$	$a_{\mu-e}\ (10^{-10})$	
lattice QCD [ETMC][1]	692.1(16.3)	185.5(4.2)	$-102.3(1.66)$	$\Rightarrow R_{e/\mu}$[5]
lattice QCD [WP25]	713.2(6.1)[2]	189.3(8.2)[3]	$-103.8(5.0)$	
data-driven[4]	692.8(2.4)	186.10(66)	$-102.8(4)$	
discrepancy [ETMC]	0.68(16.48)	0.28(4.25)	0.5(24.6)	
discrepancy [WP25]	20.4(6.6)	3.2(8.2)	1.1(7.0)	

1. D. Giusti and S. Simula, PoS LATTICE2019 (2019) 104, arXiv:1910.03874 [hep-lat].
2. [WP25] R. Aliberti et al., Phys. Rept. 1143, 1 (2025), arXiv:2505.21476 [hep-ph].
3. [BMW] S. Borsanyi et al., Phys. Rev. Lett. 121, 022002 (2018), arXiv:1711.04980 [hep-lat].
4. A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020), arXiv:1911.00367 [hep-ph].
5. D. Giusti and S. Simula, Phys. Rev. D 102, 054503 (2020), arXiv:2003.12086 [hep-lat].

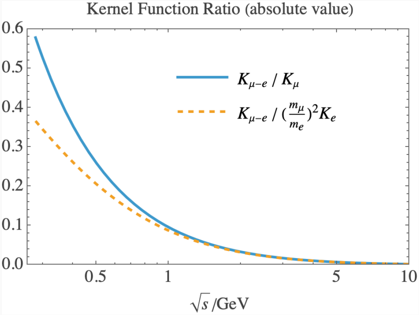
Dispersion Relation & Kernel function

◇ the dispersion relation for lepton ℓ

$$a_\ell^{\mathrm{HVP}} = \frac{\alpha}{\pi^2}\int_{s_0}^{\infty} \mathrm{d}s\, \frac{1}{s}\, \mathrm{Im}\,\Pi(s) K_\ell(s)$$

◇ where the kernel function

$$K_\ell(s) = \frac{m_\ell^2}{s}\int_0^1 \mathrm{d}x \frac{x^2(1-x)}{1-x+x^2(m_\ell^2/s)}$$



$\Rightarrow$ high-s suppression

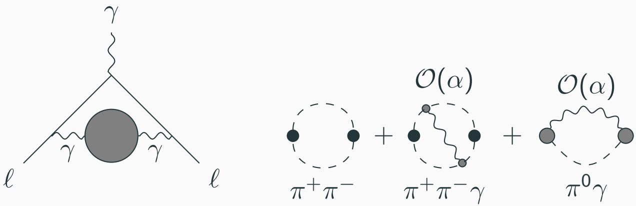
◇ with electron approximation
$m_e^2/s \to 0,\ K_e(s) \simeq m_e^2/3s$

$$K_{\mu-e}(s) = K_\mu(s) - \frac{m_\mu^2}{m_e^2}K_e(s) \simeq -\frac{m_\mu^4}{s^2}\int_0^1 \mathrm{d}x \frac{x^4}{1-x+x^2(m_\mu^2/s)}$$

◇ LQCD: time-momentum representation[1]
** See A. Beltran's talk*

1. Bernecker and Meyer, EPJA 47, 148 (2011), 1107.4388 [hep-lat]

HVP Leading Channels



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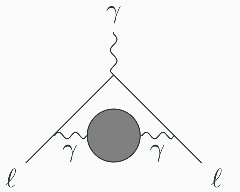

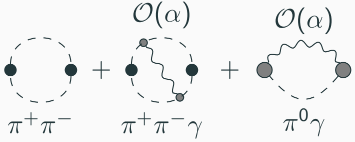

- $\pi^+\pi^-$: vector-meson-dominance (VMD) model

$$\mathrm{Im}\,\Pi^{(\pi\pi)}(s) = \frac{\alpha}{12}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi(s)|^2\theta(s - 4m_\pi^2)$$

$a_\ell^{\mathrm{HVP,LO}}$	μ $(\times 10^{-10})$	e $(\times 10^{-14})$	$\mu - e$ $(\times 10^{-10})$
VMD model[1]	508.6	140.0	−90.0
KNT eval.[2]	503.46(1.91)	138.59(54)	−89.1(4)

1. V. Biloshytskyi, D. Erb, H. B. Meyer, J. Parrino, and V. Pascalutsa, Eur. Phys. J. C 86, 497 (2026), arXiv:2509.08115 [hep-ph].
2. A. Keshavarzi, D. Nomura, and T. Teubner, Phys. Rev. D 101, 014029 (2020), arXiv:1911.00367 [hep-ph].

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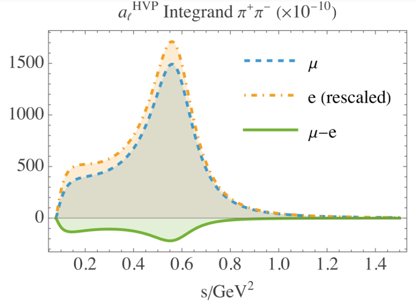

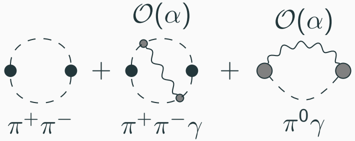

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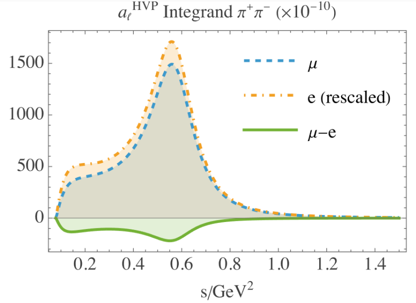

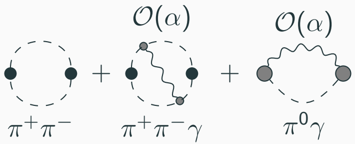

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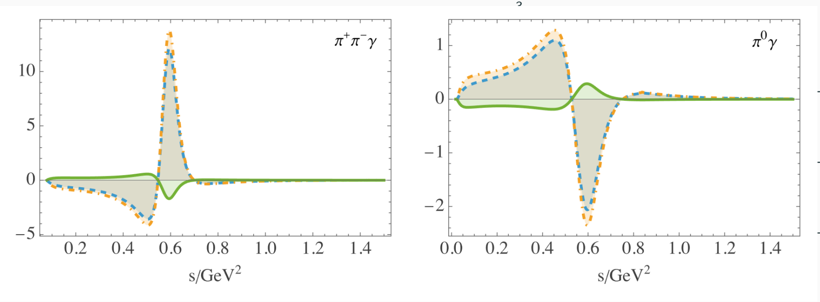
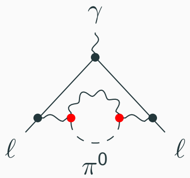


Pseudoscalar with χPT



$\pi\gamma\gamma$ vertex: $\mathcal{L}_{\text{WZW}} = -\frac{\alpha}{4\pi f_\pi}\pi^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$

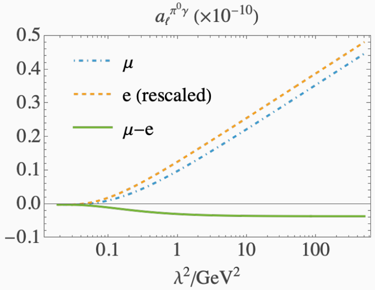
$\Rightarrow$ UV divergence

- ◇ UV divergence $\Rightarrow$ Transition Form Factor (TFF) $\Rightarrow$ empirical

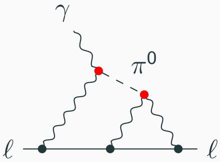
$$a_{\mu-e}^{\pi^0\gamma}(\text{w/ TFF}) \simeq -0.45\times 10^{-11}$$

- ◇ In $a_{\mu-e}$ divergences canceled
  $\Rightarrow$ can be independent of TFF
  $\Rightarrow$ pure χPT result

$$a_{\mu-e}^{\pi^0\gamma}(\text{w/o TFF}) = -0.34(5)\times 10^{-11}$$



Potential Extension to HLbL

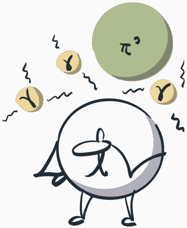


UV divergence from the same vertices...

The leading behavior*

$$a_\ell^{\pi^0,\,\mathrm{LbL}} = \left(\frac{\alpha}{\pi}\right)^3 \frac{3m_\ell^2}{16\pi^2 f_\pi^2}\,\ln^2\frac{\Lambda_{\mathrm{UV}}}{m_{\pi^0}} + \mathcal{O}\left(\ln\frac{\Lambda_{\mathrm{UV}}}{m_{\pi^0}}\right) + \text{finite terms},$$

Stay tuned. . .



*.

1. M. Knecht et al., Phys. Rev. Lett. 88, 071802 (2002), arXiv:hep-ph/0111059 [hep-ph].
2. M. J. Ramsey-Musolf et al., Phys. Rev. Lett. 89, 041601 (2002), arXiv:hep- ph/0201297.
3. W. J. Marciano et al., Phys. Rev. D 94, 115033 (2016), arXiv:1607.01022 [hep-ph].

Sensitivity to New Physics

- $(g-2)_\ell$ has been treated as a potential probe to BSM effects.
- $a_{\mu-e}$ provides a deeper reach into new physics parameter space than a_ℓ.
- Flavour universal new physics: $a_\ell^{\mathrm{BSM}} \propto m_\ell^2 \Lambda^{-2}$, ($\Lambda$ being mass scale of the heavy new particle).
- In supersymmetric models, new physics corrections to a_ℓ

$$a_{\mu-e}^{\mathrm{BSM}} = m_\mu^2 \left(\frac{1}{\Lambda_\mu^2} - \frac{1}{\Lambda_e^2} \right).$$

Sensitivity to New Physics (Dark Photon coupling ε)

- Dark photon (A'): at low-energy limit, correction to a_ℓ at one-loop

$$a_\ell^{A'} = \frac{\alpha}{2\pi} \times \varepsilon^2 \int_0^1 \mathrm{d}x \frac{2m_\ell^2 x^2 (1-x)}{m_\ell^2 x^2 + m_{A'}^2 (1-x)}.$$

With conditions:

1. exp. improvement to $\alpha\,(\sim 10^{-14})$ and $a_e(\sim 10^{-15})$
2. theo. improvement of QED and EW
3. $\delta a_{\mu-e}^{\mathrm{HVP}} \simeq 0.15\,\delta a_\mu^{\mathrm{HVP}}$

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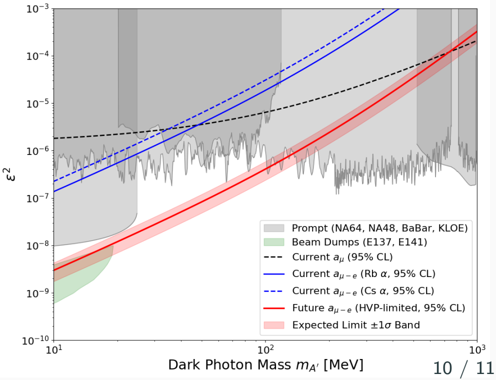
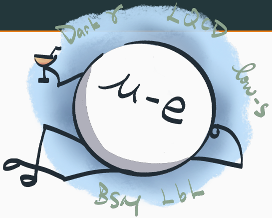


Conclusions



- Introduction of a sub-GeV window quantity of

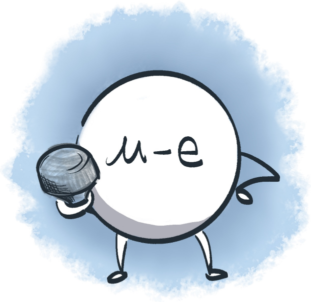
$$a_{\mu-e} = a_\mu - (m_\mu^2/m_e^2)\, a_e$$

  that utilize the strong correlation of $(g-2)_\ell$ HVP contribution at high-energies.
- Formulated as a modification of the kernel function in the dispersion relation.
- Largely reduces sensitivity at HVP energy regime where lattice-QCD and data-driven approach presents persistant tension.
- A powerful probe to low-energy hadronic effects.
- Advantages in the field of: χPT, LbL, BSM etc.

arXiV: 2606.08329 [hep-ph]
"Lepton $g-2$ non-universality of hadronic contributions and a sub-GeV window to New Physics"

Thank you!

Questions?



Backup Slide: Kernel function approximation

AMM dispersion relation and a_e approximation:

$$a_\ell^{\mathrm{HVP}} = \frac{\alpha}{\pi^2} \int_{s_0}^{\infty} \mathrm{d}s \, \frac{\operatorname{Im}\Pi(s)}{s} K_\ell(s) \, , \; K_\ell(s) = \frac{m_\ell^2}{s} \int_0^1 \mathrm{d}x \, \frac{x^2(1-x)}{1 - x + x^2(m_\ell^2/s)}$$

$$a_e^{\mathrm{HVP}} \cong \frac{\alpha m_e^2}{3\pi} \Pi'(0) \equiv \frac{\alpha}{3\pi^2} m_e^2 \int_{s_0}^{\infty} \mathrm{d}s \, \frac{\operatorname{Im}\Pi(s)}{s^2} .$$

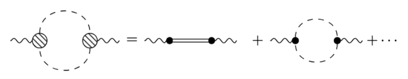
Kernel function expression comes from:

$$\begin{aligned} K_{\mu-e}(s) &= K_\mu(s) - \frac{m_\mu^2}{m_e^2} K_e(s) \\ &\cong -\frac{m_\mu^4}{s^2} \int_0^1 \mathrm{d}x \, \frac{x^4}{1 - x + x^2(m_\mu^2/s)} \\ &= K_\mu(s) - \frac{m_\mu^2}{3s} . \end{aligned}$$

Backup Slide: $\pi^+\pi^-$ channel spectral function

$$\mathrm{Im}\,\Pi^{(\pi\pi)}(s) = \frac{\alpha}{12}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi(s)|^2\theta(s - 4m_\pi^2)$$

The generic spectral function: the one-loop scalar-QED expression (cf., Peskin) augmented with the pion electromagnetic form factor, $F_\pi(q^2)$. In the VMD picture, the latter is essentially described by the ρ-meson exchange.



Two-pion contribution to HVP. The blobs represent the VMD form factor of the pion, and dashed lines are pion propagators. The double line represents ρ-meson propagator.

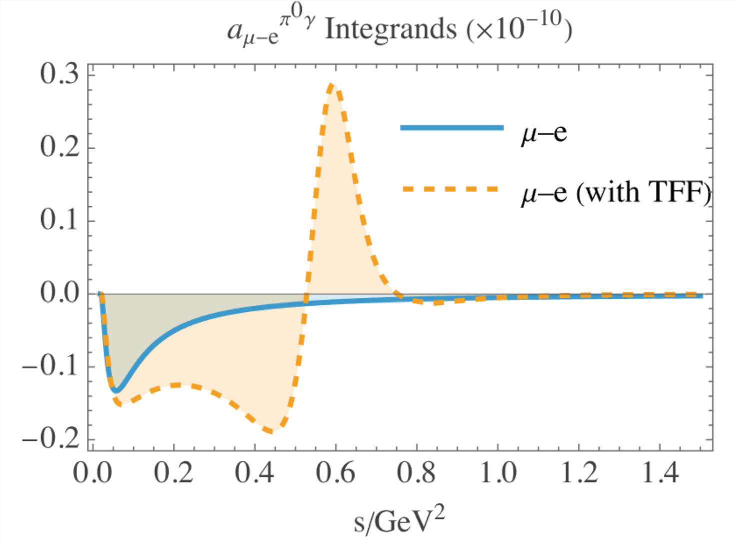
Backup Slide: Sub-leading channel results

Channel	$a_\mu^{\mathrm{HVP,LO}}$ $(\times 10^{-10})$	$a_e^{\mathrm{HVP,LO}}$ $(\times 10^{-14})$	$a_{\mu-e}^{\mathrm{HVP,LO}}$ $(\times 10^{-10})$
$\pi^+\pi^-\gamma$	0.711	0.237	-0.304
$\pi^0\gamma$	0.103	0.034	-0.044

Table 1: LO HVP contributions to the muon and electron anomalous magnetic moment and $(\mu - e)$ from sub-leading $\pi^+\pi^-\gamma$ and $\pi^0\gamma$ channels (with TFF).

$$
\begin{aligned}
a_{\mu-e}^{\pi^0\gamma}&(\text{without TFF}) \\
&= \frac{\alpha^3 m_{\pi^0}^2}{1728\,\pi^5 f_\pi^2\, r^4}\Bigg[r^2\left(4r^4+81r^2+66\right) - \frac{6}{\sqrt{4r^2-1}}\left(64r^4+28r^2-11\right)\arccos\frac{1}{2r} \\
&\qquad - 6(2r^6+9r^4-11)\log r + 18(6r^2+1)\left(\log^2 r + \arccos^2\frac{1}{2r}\right)\Bigg] \\
&= -\,0.34(5)\times 10^{-11}\,.
\end{aligned}
$$

Backup Slide: $\pi^0\gamma$ HVP integrand plot w/o TFF



Backup Slide: Sensitivity to the fine structure constant α

Validity condition for $a_{\mu-e}$:

$$\delta\left(m_\mu^2/m_e^2 \times a_e^{\mathrm{had,exp}}\right) \leq \delta a_\mu^{\mathrm{had,theo}}$$

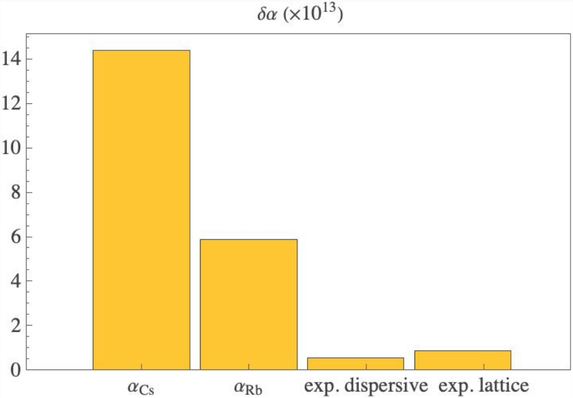
To satisfy the validity condition, the precision of the fine-structure constant, which is calculated based on theoretical uncertainties from data-driven and lattice method respectively, must be improved to

$$\delta\alpha \leq \begin{cases} 5.39 \times 10^{-14} & \text{dispersive} \\ 8.65 \times 10^{-14} & \text{lattice} \end{cases}$$

where the lepton mass ratio [PDG], experimental measurement of a_e [Phys. Rev. Lett. 130, 071801 (2023)], α measurement using Cs atoms [Science 360, 191 (2018)] and Rb atoms [Nature 588, 61 (2020)], the weak contribution [Rev. Mod. Phys. 97, 025002 (2025)] and QED contributions of a_e^{QED} [WP25, Atoms 7 28 (2019), Phys. Rev. D 97, 036001 (2018)] (where α is introduced).

Current α measurement uncertainty $\delta\alpha(\text{Cs, Rb}) \sim 10^{-12}$.

Backup Slide: Sensitivity to the fine structure constant α



Backup Slide: Sensitivity to the fine structure constant α

Complete validity condition

$$\delta\left(m_\mu^2/m_e^2 \times a_e^{\mathrm{had,exp}}\right) \leq \delta a_\mu^{\mathrm{had,theo}}$$

requires

$$\delta a_e^{\mathrm{exp}} \leq 8.58 \times 10^{-15}$$

$$\delta\alpha \leq 5.39 \times 10^{-14}\sqrt{1 - 1.36 \times 10^{28}\delta\left(a_e^{\mathrm{exp}}\right)^2}$$

where largest $\delta\alpha$ at $\sim 10^{-14}$.

Backup Slide: New Physics

flavor-universal new physics, parametrized by a contact dimension-5 operator,

$$\mathcal{L}_{\text{universal}} = \frac{1}{\Lambda^2} \sum_{\ell=e,\mu} m_\ell \bar{\psi}_\ell \sigma_{\mu\nu} F_{\mu\nu} \psi_\ell.$$

In supersymmetric models, flavour universality does not have to hold exactly, giving two different effective values of Λ for the muon and electron:

$$a_{\mu-e}^{\text{BSM}} = m_\mu^2 \left(\frac{1}{\Lambda_\mu^2} - \frac{1}{\Lambda_e^2} \right).$$

The ultimate sensitivity to $\Lambda_\mu^{-2} - \Lambda_e^{-2}$ can be better than to Λ_μ^{-2} and Λ_e^{-2} separately.

Dark photon model at low-energy limit:

$$\mathcal{L}_{d.\,ph.} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(F'_{\mu\nu})^2 + \frac{1}{2} m_{A'}^2 A'^2_\mu + \sum_\ell \bar{\psi}_\ell \gamma_\mu (i\partial_\mu - eA_\mu - e\varepsilon A'_\mu)\psi_\ell.$$

Corrections to the leptonic $a_{\mu(e)}$ can be written as [Phys. Rev. D 80, 095002 (2009)]

$$a_\ell^{A'} = \frac{\alpha}{2\pi} \times \varepsilon^2 \int_0^1 \mathrm{d}x \frac{2m_\ell^2 x^2 (1-x)}{m_\ell^2 x^2 + m_{A'}^2 (1-x)} \Rightarrow \operatorname{Im} \Pi(s) \propto \varepsilon^2 \delta(s - m_{A'}^2).$$