# Hadronic vacuum polarization: $\tau$ data versus lattice QCD

Maarten Golterman

with Noah Allen, Diogo Boito, Kim Maltman, Lucas Mansur, Santi Peris

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# Overview:

- Goal: compare isospin-1 (light-quark connected) lattice results for HVP with estimates based on hadronic $\tau$-decay data
- Motivation: similar comparisons between lattice and $e^+e^-$ data show large discrepancies

- Available:

  $\tau$-based data: combined ALEPH/OPAL/Belle $2\pi$ spectral distribution (Boito *et al.* '20, Allen *et al.* '25)

  combined ALEPH/OPAL $4\pi$ spectral distribution

  combined inclusive vector isovector spectral distribution (many experiments)

  Lattice: $\Pi^{I=1}(Q^2)$ Mainz 2025 and BMW 2020 at various values of $Q^2$

  $a_\mu^{HVP,lqc}$ and window quantities (many collaborations, WP25)

  all in isoQCD (WP25 scheme)

# Construct Euclidean correlators from $\tau$ data

Momentum space: $\bar{\Pi}^{I=1}(Q^2) = Q^2 \int_{s_{\rm th}}^{\infty} ds\, \frac{\rho_V^{I=1}(s)}{s(s+Q^2)} = \bar{\Pi}^{I=1}_{\rm data}(Q^2) + \bar{\Pi}^{I=1}_{\rm PT}(Q^2) + \bar{\Pi}^{I=1}_{\rm DV}(Q^2)$

Coordinate space: $C^{I=1}(t) = \frac{1}{2}\int_{s_{\rm th}}^{\infty} ds\, \sqrt{s}\, e^{-\sqrt{s}t}\, \rho_V^{I=1}(s) = C^{I=1}_{\rm data}(t) + C^{I=1}_{\rm PT}(t) + C^{I=1}_{\rm DV}(t)$

- Break up into two parts:

  $s < m_\tau^2$ use inclusive spectral function from $\tau$ data
  $s > m_\tau^2$ use Perturbation Theory to 4th order (estimate for 5th order) (Baikov, Chetyrkin & Kühn, 2008)
  plus model for Duality Violations (resonances visible in data) with 100% error

- Correct for isospin breaking (present in data, but compare with isoQCD lattice results)
  Easier than for $e^+e^-$ !

# Isospin breaking: strategy

- Assume dominated by $2\pi$ channel, ignore IB effects in other channels

  Work to 1st order in IB parameters $\alpha_{\mathrm{EM}}$ and $m_{\mathrm{d}} - m_{\mathrm{u}}$ (to 1st order IB in $I = 1$ HVP is purely EM)

- For EM current correlator, IB effects have been computed on the lattice – but can't be repurposed for use in the $\tau$ case (see talk by Mattia Bruno)

- isoQCD: $m_\pi = 135$ MeV, $m_K = 494.6$ MeV (WP25 scheme, for our purposes same as FLAG scheme)

- Use data-based approach where possible, use conservative error estimate

# Isospin breaking

Master formula: $\rho_{]\tau^- \to \pi^- \pi^0 \nu_\tau (\gamma)]}(s) = S_{\mathrm{EW}}^{\pi\pi} \beta^3_{\pi^- \pi^0}(s) |f_+(s)|^2 G_{\mathrm{EM}}(s)$

# Isospin breaking

Master formula: $\rho_{]\tau^-\to\pi^-\pi^0\nu_\tau(\gamma)]}(s) = S_{\rm EW}^{\pi\pi}\beta^3_{\pi^-\pi^0}(s)|f_+(s)|^2 G_{\rm EM}(s)$

phase space

short-distance EW correction, $S_{\rm EW}^{\pi\pi} = 1.0233$

radiative corrections in decay
use dispersive determination of
Colangelo, Cottini, Hoferichter & Holz '25/26

form factor: need a model for IB

Correction factor:

$$F_{\rm IB}(s) = \frac{S_{\rm EW}}{S_{\rm EW}^{\pi\pi}}\,\frac{1}{G_{\rm EM}(s)}\,\frac{\beta^3_{\pi^0\pi^0}(s)}{\beta^3_{\pi^-\pi^0}(s)}\,\frac{|f_+^{\rm model}(s)[m_{\pi^-}\to m_{\pi^0};\ m_{K^-,K^0}\to m_{K^{\rm iso}}]|^2}{|f_+^{\rm model}(s)[\text{real world}]|^2}$$

# Isospin breaking: model for form factor

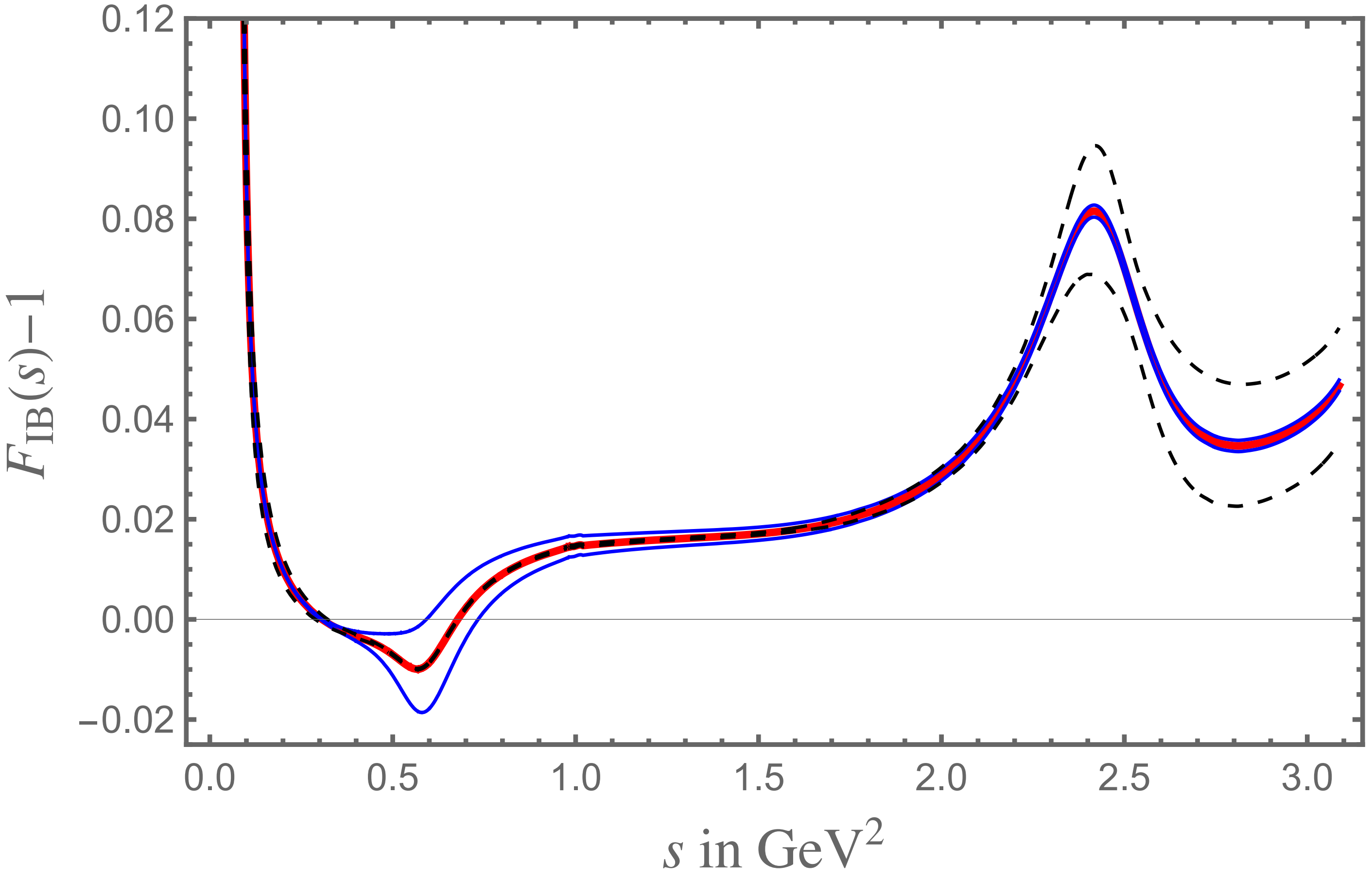
We use a model of Cirigliano, Ecker & Neufeld ‘01 (based on Guerrero & Pich ‘97); schematically:

$$f_+^{\rm model}(s) = \frac{m_\rho^2}{m_\rho^2 - s - i m_\rho \Gamma_\rho(s)}\,\exp[2\tilde{H}_{\pi^-\pi^0}(s) + \tilde{H}_{K^-K^0}(s)]$$

This models the $\rho$ pole (with an $s$-dependent width); the $\widetilde{H}$ represents a chiral loop.

Error associated with IB:
- propagate errors on $G_{\rm EM}$ (from Colangelo *et al.*)
- estimate unknown EM effects on $g_{\rho\pi\pi}$ with factor $(1\pm2\alpha_{\rm EM}/\pi)$ (WP25 recommendation)
- add additional 50% error of IB correction to any quantity

# Isospin breaking – final correction factor

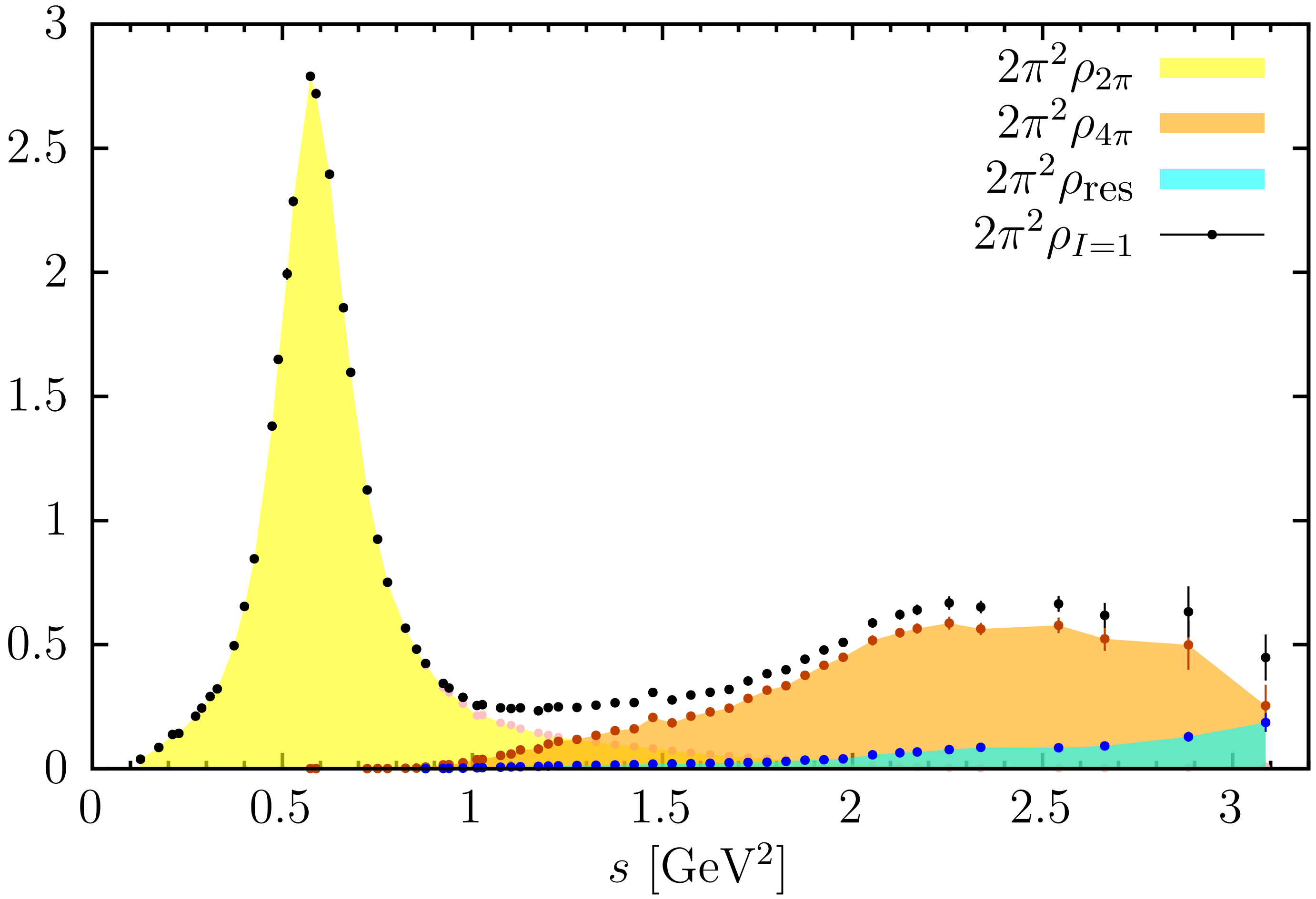



black dashed:
error band on $G_{\mathrm{EM}}$

blue solid:
error band from $(1\pm 2\alpha_{\mathrm{EM}}/\pi)$
factor on $\Gamma_\rho$

# Data: spectral function from hadronic $\tau$ decays

(Boito, Eiben, MG, Maltman, Mansur & Peris '20,
Allen, Boito, MG, Maltman, Mansur & Peris '25)




$2\pi$: ALEPH, OPAL, Belle

$4\pi$: ALEPH, OPAL

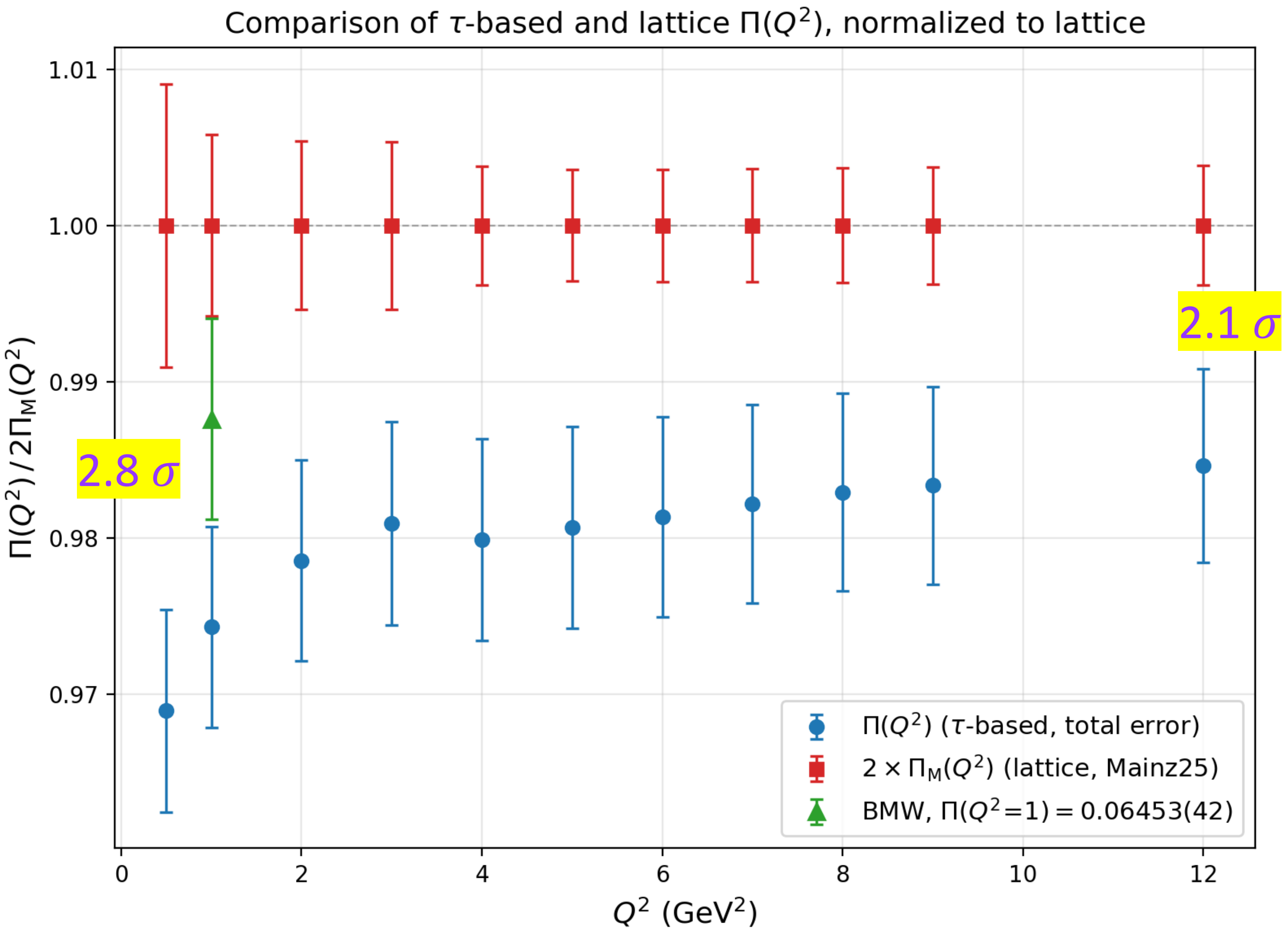
residual channels:

$K\bar{K}$: BaBar

other modes from $e^+e^-$ and CVC

(CLEO $2\pi$: no systematic correlations;
small impact on $2\pi$ combination)

# Comparison of the HVP with Mainz 2025 (arXiv:2511.01623)




large range of $Q^2$ values
($a_\mu^{HVP}$ dominated by $Q^2 < 0.2$ GeV$^2$)

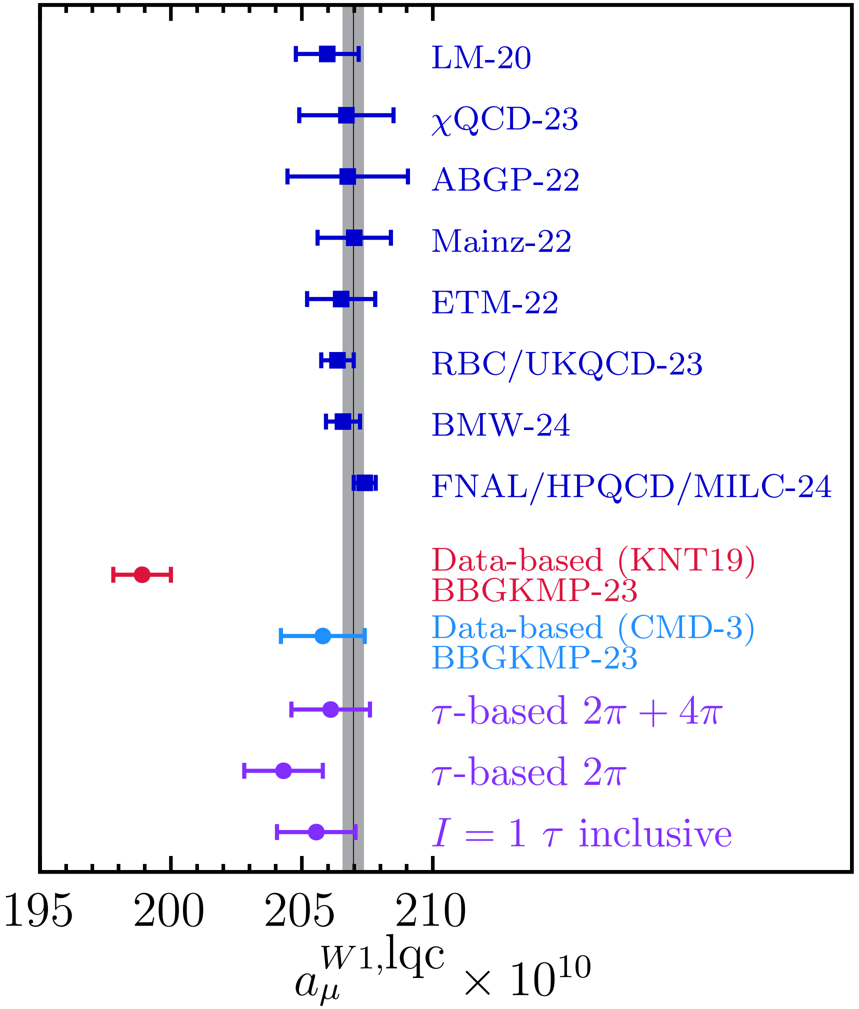
2-3 $\sigma$ discrepancies
errors comparable between
$\tau$-based and Mainz

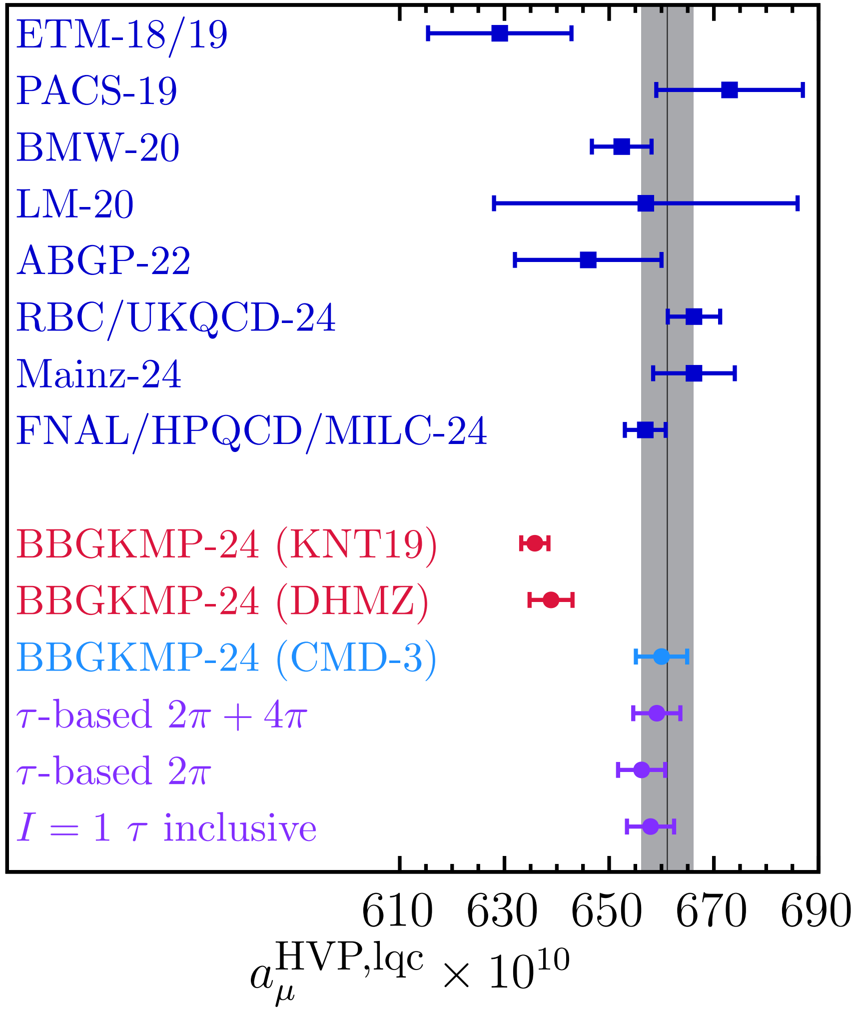
BMW '20 has value at 1 GeV$^2$
1.5 $\sigma$ larger than $\tau$-based value;

also diff. between $Q^2 = 10$ and 1 GeV$^2$
$\lesssim 0.6\ \sigma$ different from $\tau$-based value

# Comparison of RBC/UKQCD window quantities (and $a_\mu^{HVP,lqc}$)

intermediate window



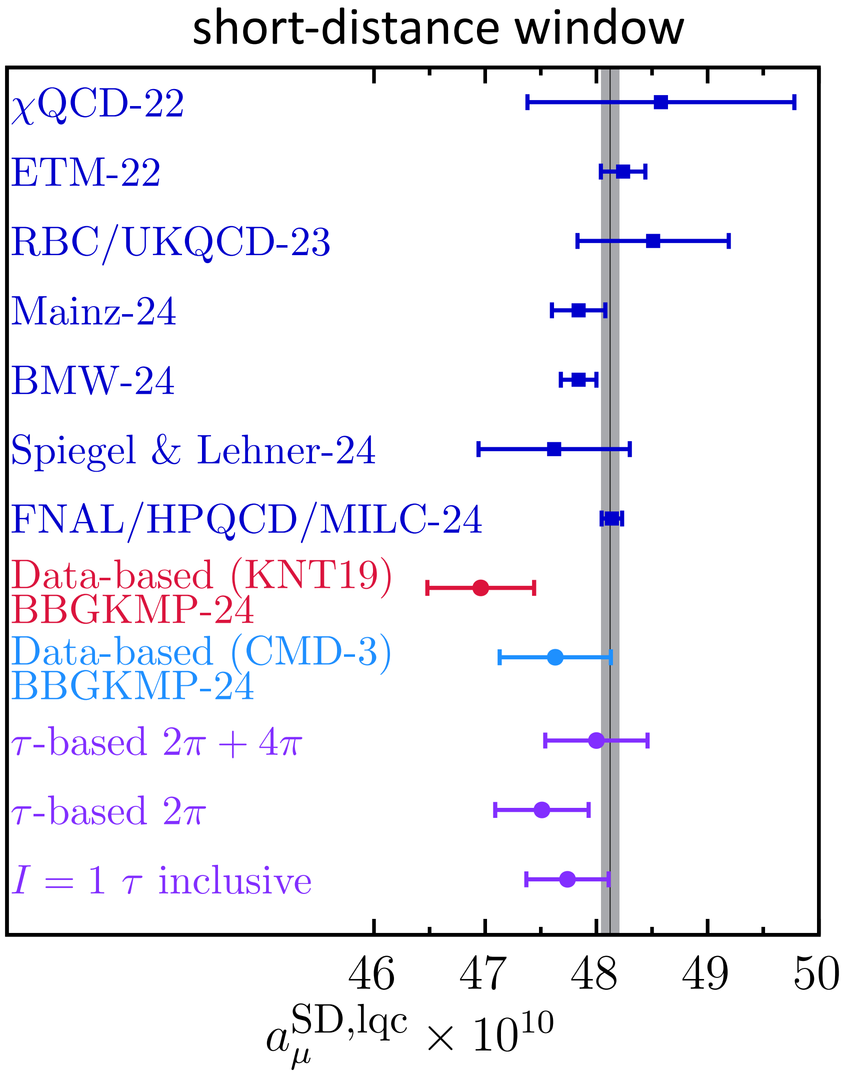



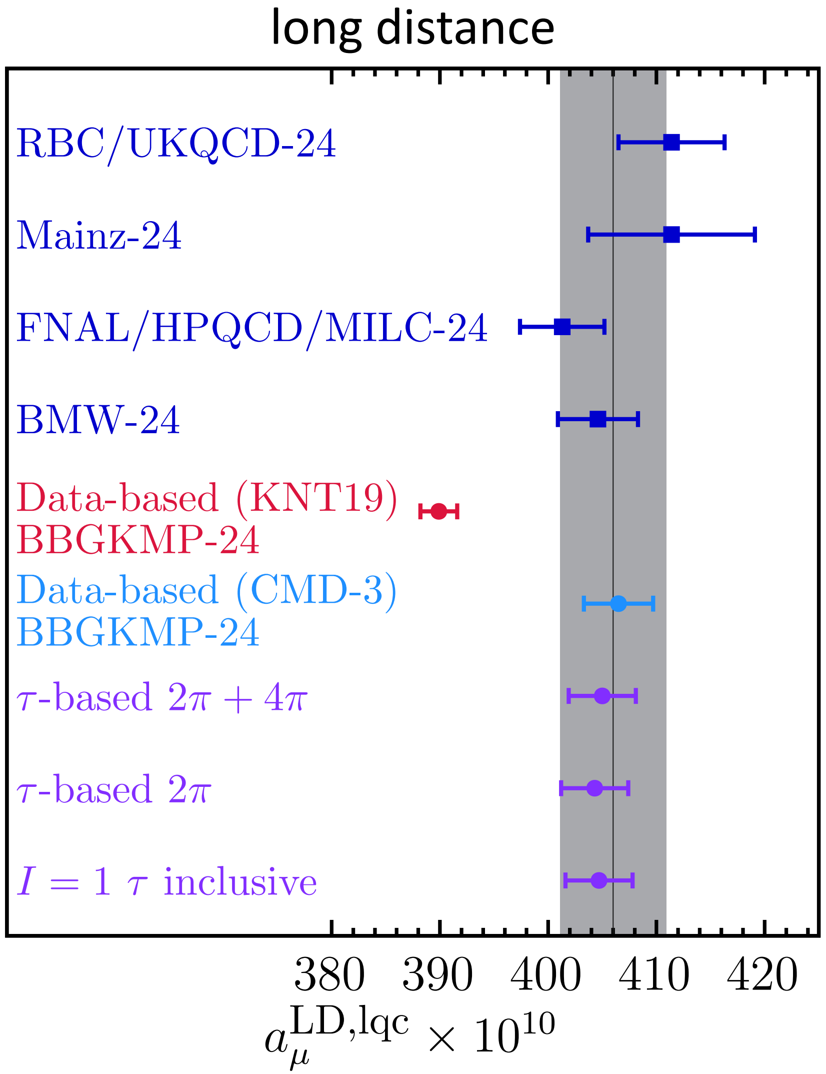

$\tau$-based $2\pi + 4\pi$:
rest KNT19 $e^+e^-$

$\tau$-based $2\pi$:
rest KNT19 $e^+e^-$

$I = 1$ $\tau$ inclusive:
pert.th. @ $s > m_\tau^2$

## Comparison of RBC/UKQCD window quantities (and $a_\mu^{HVP,lqc}$)




$\tau$-based $2\pi + 4\pi$: rest KNT19 $e^+e^-$

$\tau$-based $2\pi$: rest KNT19 $e^+e^-$

$I = 1$ $\tau$ inclusive: pert.th. @ $s > m_\tau^2$

Note small difference between $\tau$-based $2\pi + 4\pi$ and $\tau$-based $2\pi$

# $4\pi$ spectral distributions — comparison

Pais relations: (Pais '60)

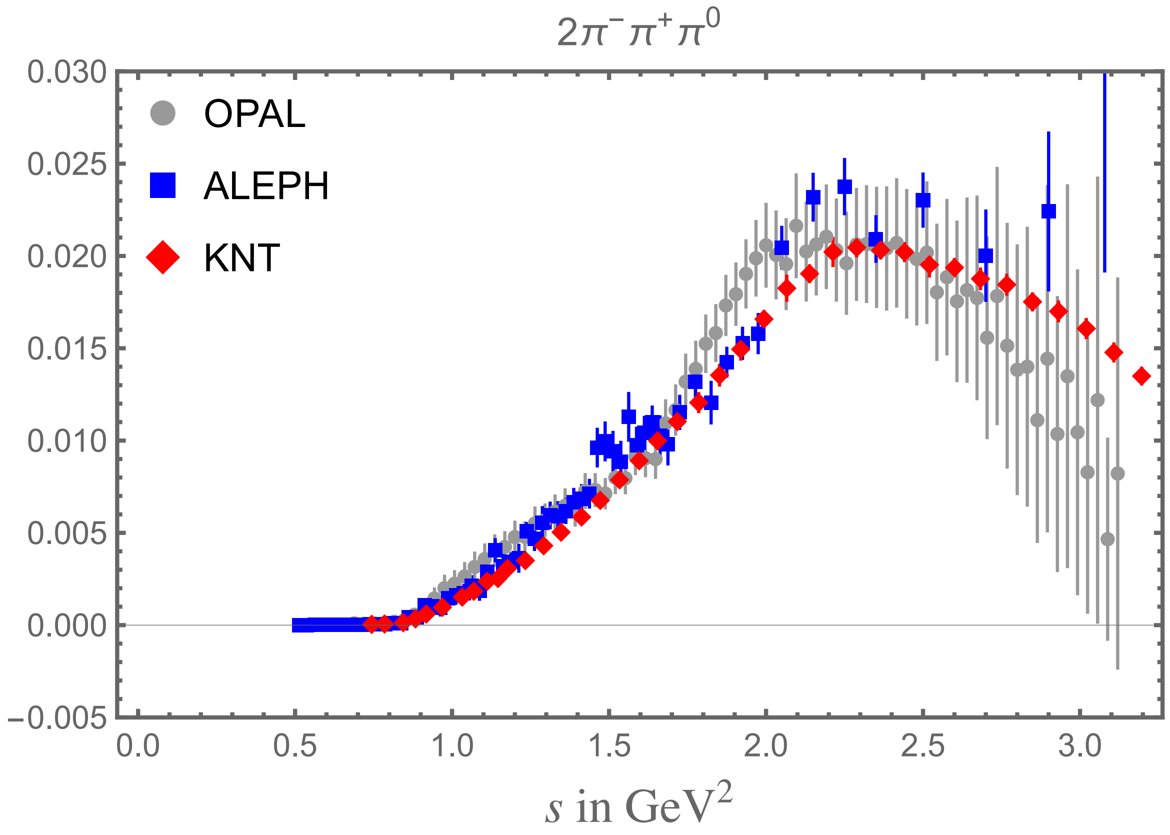
$$\rho_V[\tau, 2\pi^-\pi^+\pi^0](s) = \tfrac{1}{2}\,\rho_V[\mathrm{EM}, 2\pi^-2\pi^+](s) + \rho_V[\mathrm{EM}, \pi^-\pi^+2\pi^0](s)$$

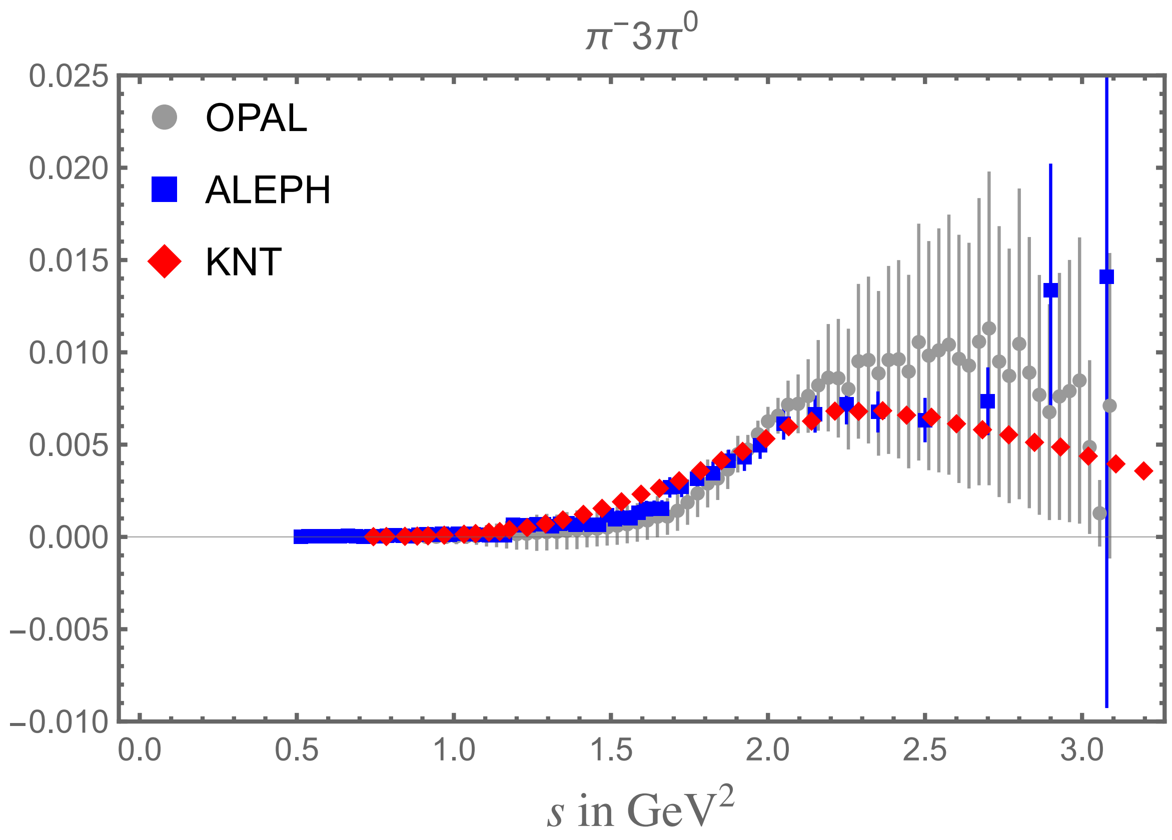
$$\rho_V[\tau, \pi^-3\pi^0](s) = \tfrac{1}{2}\,\rho_V[\mathrm{EM}, 2\pi^-2\pi^+](s)$$

This assumes isospin symmetry, but expect $4\pi$ IB contributions to windows to be very small

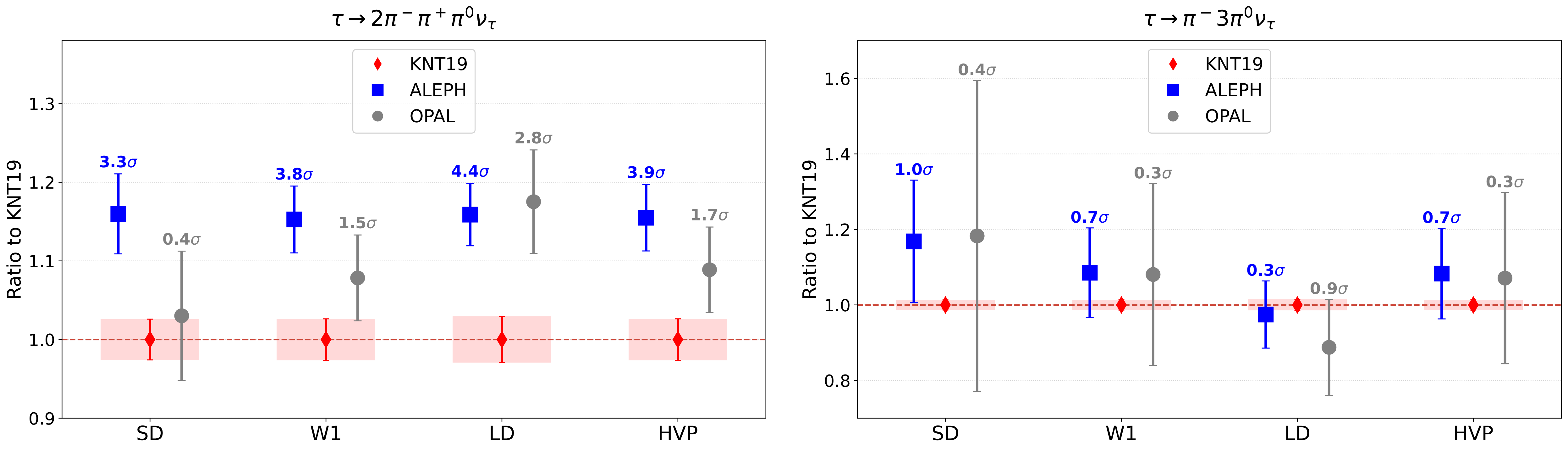
$\Rightarrow$ Compare ALEPH, OPAL $\tau$-based with KNT19 $e^+e^-$ $4\pi$ spectral distributions

# $4\pi$ spectral distributions — comparison



# $4\pi$ spectral distributions — contribution to window quantities




sizeable discrepancies between ALEPH and KNT19 results for $2\pi^-\pi^+\pi^0$ mode

## Conclusions

- Comparison of $\tau$-based HVP-related quantities and lattice results in isoQCD: good agreement

- Compared $I$ = 1 HVP for Euclidean $Q^2$ between 0.5 and 12 GeV$^2$ (Mainz) and window quantities
  some tension with Mainz results, excellent agreement with WP25 window results

- $\tau$-based were corrected for isospin breaking to match lattice; can be improved in the future

- Discrepancies between $\tau$-based and $e^+e^-$ based $4\pi$ spectral distributions?
  Will Belle, BESIII or other experiments produce new $4\pi$ spectral distributions?
  (The ”easy” $2\pi^-\pi^+\pi^0$ mode would already be very useful)