Isospin breaking corrections to the hadronic vacuum polarization contribution to the muon g-2

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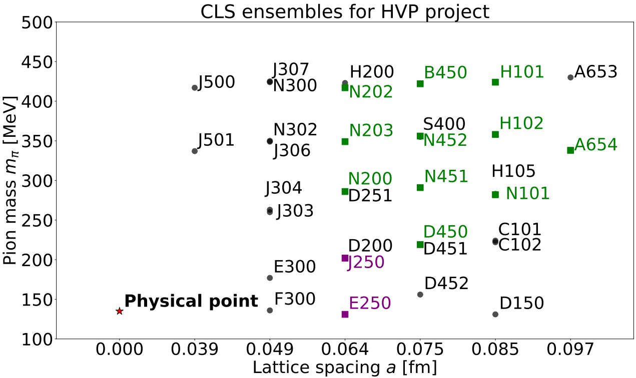
CLS ensembles

- isoQCD ensembles of the CLS collaboration[1]
- six different lattice spacings and large range of pion masses down to the physical point used in isoQCD calculation[2]
- green ones are finished or in production, purple ones planned for this project



[1] arXiv: 1411.3982 (hep-lat).
[2] arXiv: 2312.13753.

Theoretical framework

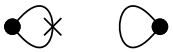
- time momentum representation[1] and euclidean windows[2]
- isoQCD ensembles ($m_u = m_d = m_{ud}$ and no QED effects)
- treat these additional effects as small perturbation and expand in Δm_d, Δm_u, Δm_s and in e [3]

$\Rightarrow$ many more new diagrams ($\mathcal{O}(20)$ diagrams in total)

- using QED_L to put photon on the lattice
- complementary approach to $CCS + QED_\infty$

[1] arXiv: 1107.4388 (hep-lat).
[2] arXiv: 1801.07224 (hep-lat).
[3] arXiv: 1303.4896 (hep-lat); arXiv: 1110.6294 (hep-lat).

Set of local diagrams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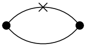
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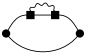
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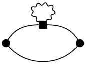
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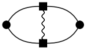
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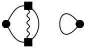
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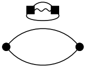
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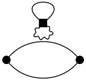
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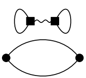
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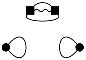
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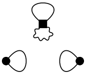
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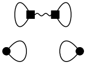
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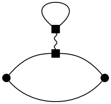
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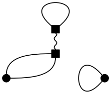
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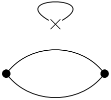
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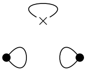
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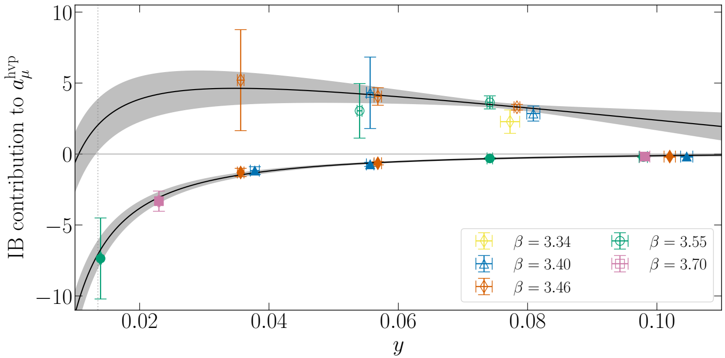
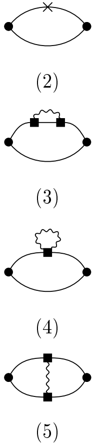
Current status of isospin breaking computations



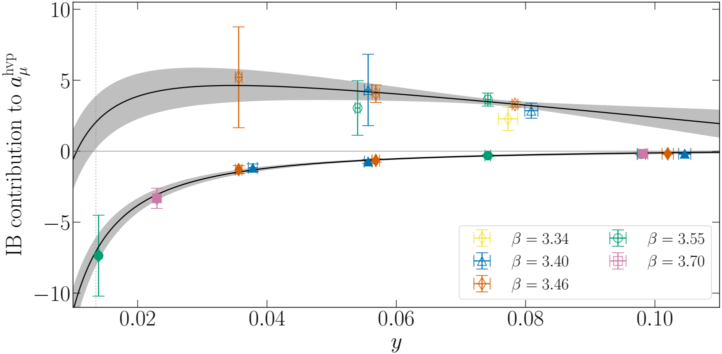


Chiral extrapolation of the isospin breaking corrections[1]

- connected part not well constrained, in particular in the physical pion mass region

[1] arXiv: 2411.07969 (hep-lat).

Current status of isospin breaking computations



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Chiral extrapolation of the isospin breaking corrections[1]

- missing disconnected mass insertion diagram and self-energy
- methods for conn. and disc. QED diagrams differ substantially (QED_L vs. QED_∞)

[1] arXiv: 2411.07969 (hep-lat).

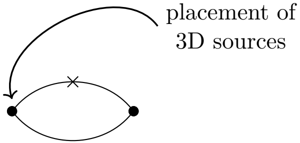
Computational setup

- current setup produces large errors at small pion mass
- idea: employ different noise estimator to improve errors
- current setup of computation: placing a three dimensional stochastic source at the source time of the diagram

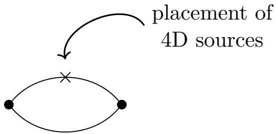
$\Rightarrow$ $C(x_0, y_0) = \sum_{\vec{x},\vec{y},z} \langle \gamma_\mu \eta(\vec{x'}, x_0) \eta^\dagger(\vec{x}, x_0) D^{-1}(x; z') D^{-1}(z; y) \gamma_\mu D^{-1}(y; x) \rangle_\eta$

- new setup: 4D volume sources at the mass insertion vertex

$\Rightarrow$ $C(x_0, y_0) = \sum_{\vec{x},\vec{y},z} \langle \gamma_\mu D^{-1}(x; z') \eta(z') \eta^\dagger(z) D^{-1}(z; y) \gamma_\mu D^{-1}(y; x) \rangle_\eta$



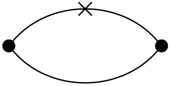
1: Connected mass insertion diagram for the valence quark



1: Connected mass insertion diagram for the valence quark

Computational setup

1: Disconnected mass insertion



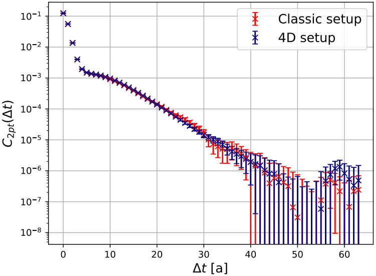
2: Connected mass insertion

- allows for computation of disconnected diagrams (split even estimator)[1]
- small cost for disconnected diagram (lower number of sources and shared solves)

[1] arXiv: 1903.10447 (hep-lat).

Improvement mass insertion correlation function

- reduction of statistical noise at equal computational cost ⇒ reduced errors
- Test on N451 ($a = 0.076\text{fm}$, $m_\pi = 291\text{MeV}$)



Two-point connected correlation function for N451 (1000 configurations)

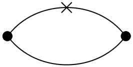


Diagram 1: Connected mass insertion diagram for the valence quark

Improvement mass insertion correlation function

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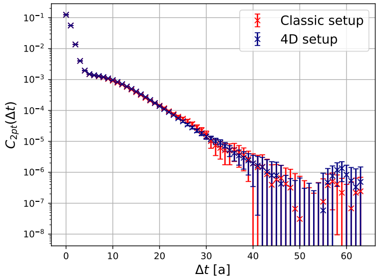

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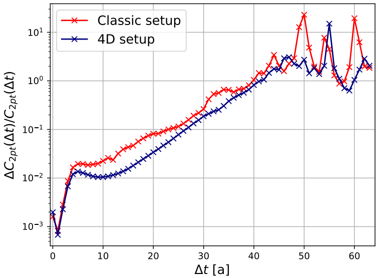
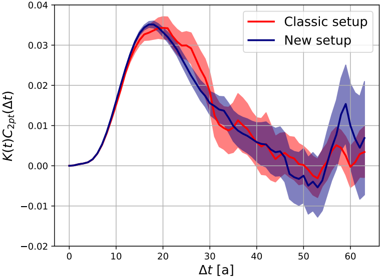


Rel. error of the two-point connected correlation function for N451 (1000 configurations)

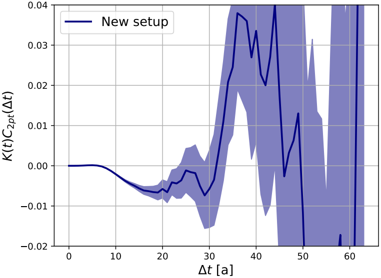
Comparison Connected and disconnected mass insertion at $m_\pi \approx 280 MeV$

- connected diagram more important
- disconnected diagram dominates uncertainty

$\Rightarrow$ improvement for strong isospin-breaking contributions due to new setup around 30%



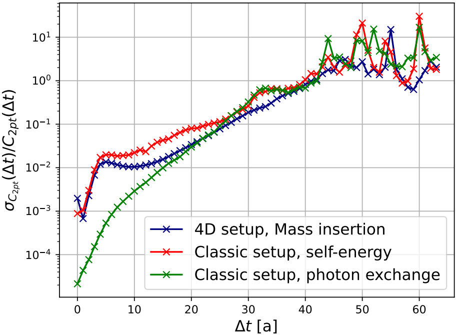
Integrand of connected mass insertion for N451



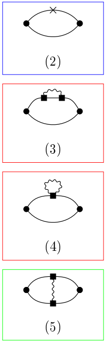
Integrand of disconnected mass insertion for N451

Photon connected contributions

- no direct improvement on connected diagrams including QED
- mass insertion and self-energy with largest contributions
- photon self-energy has highest relative error



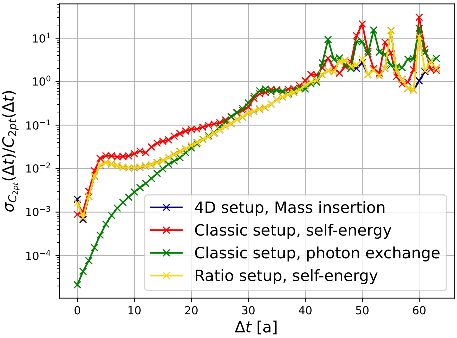
Relative error of connected diagrams

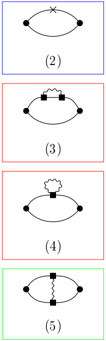


Photon connected contributions

- perform ratios to reduce the error on the self-energy:

$$C_{se}^{(new)} = \frac{C_{se}^{(classic)}}{C_{mi}^{(classic)}} C_{mi}^{(new)}$$

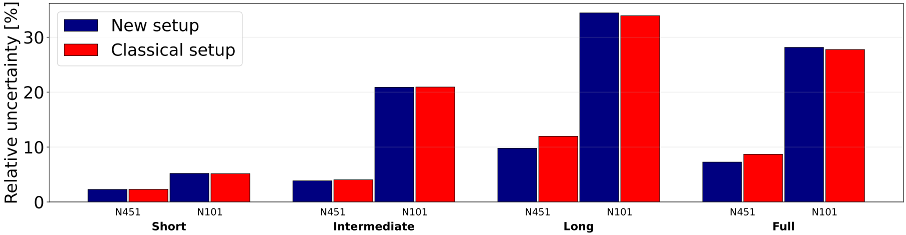




Relative error of connected diagrams

Improvement on connected diagrams?

- comparison for two ensembles at equal pion mass (N451: $a = 0.076\mathrm{fm}$, $m_\pi = 291\mathrm{MeV}$ and N101: $a = 0.085\mathrm{fm}$, $m_\pi = 282\mathrm{MeV}$)
- new setup does not improve the overall uncertainty for the connected part alone



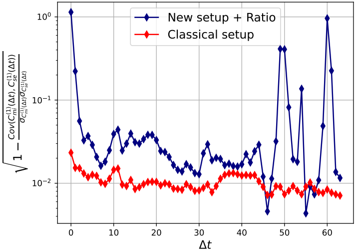
Relative uncertainties for the connected part of δa_μ

Improvement on connected diagrams?

- reason: reduced correlation ⇒ smaller cancellation of errors
- error of mass insertion and self-energy of same size

⇒ if combining strong isospin-breaking with QED effects, new setup is not beneficial anymore

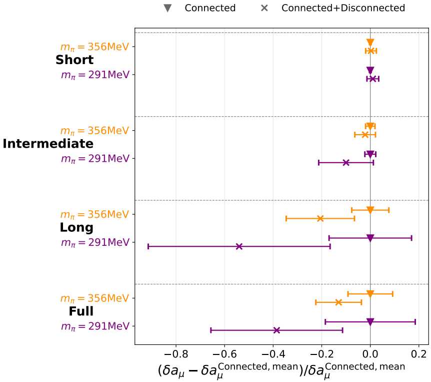
$$\Rightarrow \sigma_{mi-se} \approx \sigma_{mi} \cdot \sqrt{1 - \frac{Cov(C_{mi}^{(1)}(\Delta t), C_{se}^{(1)}(\Delta t))}{\sigma_{C_{mi}^{(1)}(\Delta t)} \sigma_{C_{se}^{(1)}(\Delta t)}}}$$



Correlation between connected mass insertion and self-energy

Inclusion of disconnected diagrams

- QCD+QED valence sector at $a = 0.076\mathrm{fm}$: connected diagrams and disconnected mass insertion as well as disconnected self-energy
- higher statistics at $m_\pi = 356\mathrm{MeV}$
- negative shift, increase with decreasing pion mass
- shift mainly from long distance regime



Effect of the disconnected mass insertion and self-energy

Strong isospin breaking only - R_{38K}

- separation of QED effects and strong isospin-breaking effects introduced by Erb et al.[1]:

$$a_\mu^{HVP,38} = a_{\mu,em}^{HVP,38} + \left(\Delta M_K^{phys} - \Delta M_K^{lat,EM}\right) \cdot R_{38K}$$

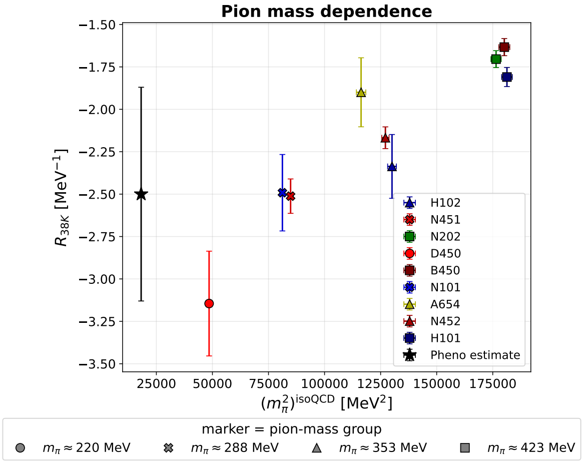
$$R_{38K} = \frac{\frac{\partial a_\mu^{(3,8)}}{\partial(m_u - m_d)}}{\frac{\partial M_K}{\partial(m_u - m_d)}}$$

- $a_\mu^{HVP,38}$ zero at leading order $\Rightarrow$ "isospin-violating part of the HVP"

[1] arXiv: 2505.24344 (hep-lat).

Pion mass dependency of R_{38K}

- R_{38K} shows strong pion mass dependency
- mild dependency on the lattice spacing



Pion mass dependency of R_{38K}

Summary and Outlook

Summary:

1. isospin breaking corrections are essential for precision calculations
2. disconnected contributions produce negative shift in the case of HVP
3. new noise estimator reduces error for strong isospin-breaking, however correlations matter

Outlook:

1. investigate correlation further
2. extend calculations to the full set of QCD+QED diagrams
3. improve statistics on existing ensembles and extend ensemble space