The Muon $g-2$ HVP contribution from the BMW/DMZ collaboration

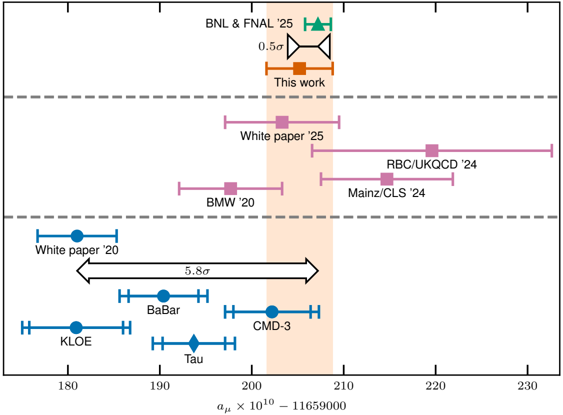
Bálint C. Tóth

University of Wuppertal
Budapest–Marseille–Wuppertal collaboration (BMW)

[BMW/DMZ, Nature 653 (2026) 373, arXiv:2407.10913v2]

A. Boccaletti, Sz. Borsanyi, A. Cotellucci, M. Davier, Z. Fodor, F. Frech, A. Gérardin, D. Giusti, A.Yu. Kotov, L. Lellouch, Th. Lippert, A. Lupo, B. Malaescu, S. Mutzel, A. Portelli, A. Risch, M. Sjö, F. Stokes, K.K. Szabo, B.C. Toth, G. Wang, Z. Zhang

Overview

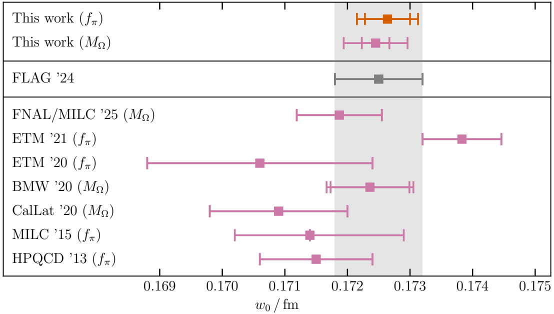


$$a_\mu^{\text{LO-HVP}} = 714.1(2.2)(2.5)[3.3] \; (v1)$$

$$\longrightarrow \quad a_\mu^{\text{LO-HVP}} = 715.1(2.5)(2.3)[3.4]$$

- Remarkable success of Standard Model
- Hybrid approach
  - Long distance contribution from e^+e^- data
    - ($t \geq 2.8\,\text{fm}$) gives $\lesssim 5\,\%$ of total result
    - At low energies e^+e^- data are consistent
  - $\gtrsim 95\,\%$ of total result from lattice
  - Reduced noise, finite volume effects, taste breaking

Scale determination



See [Davide Giusti at 16:15]

- w_0 scale as intermediate scale

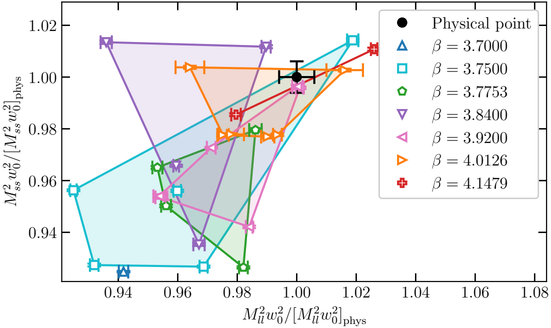
⟶ precise value of w_0 in QCD+QED is needed

- Value of w_0 is now obtained from $\Gamma(\pi \to \mu\bar{\nu}_\mu[\gamma])$:

$$[w_0]_{\text{qcd+qed}} = 0.17264(36)(33)[48]\,\text{fm}$$

- M_Ω determination is kept as a cross-check
- Remeasured w_0 with Zeuthen-flow
- Blinded analysis

Simulations



- Tree-level Symanzik gauge action
- $N_f = 2+1+1$ staggered fermions
- 4 steps of stout smearing, $\varrho = 0.125$
- $L \sim 6\,\text{fm}$, $T \sim 9\,\text{fm}$
- M_π and M_K are around physical point

β	a[fm]	$L \times T$
~~3.7000~~	~~0.1315~~	~~48×64~~
3.7500	0.1191	56×96
3.7753	0.1116	56×84
3.8400	0.0952	64×96
3.9200	0.0787	80×128
4.0126	0.0640	96×144
4.1479	0.0483	128×192

Fit functions

- Anomalous dimensions in the Symanzik effective theory
  $\longrightarrow$ a^2 scaling modified by powers of strong coupling $\alpha_s(a)$ [Husung et al. 2019]

$$a^2 \longrightarrow a^2\alpha_s^\gamma(a)$$

- γ (and prefactors) not yet known for our action and observables
- Fitting several $a^2\alpha_s^\gamma(a)$ terms simultaneously: unstable with available lattice spacings
- Strategy: fit a single $a^2\alpha_s^\gamma(a)$ term at a time, vary γ
  $\longrightarrow$ estimate uncertainty coming from logarithmic corrections
- Allow

$$\gamma \in \{0.0\,,\, 0.5\,,\, 1.0\,,\, 1.5\,,\, 2.0\,,\, 2.5\}$$

- Ansatz to parameterize lattice artifacts:

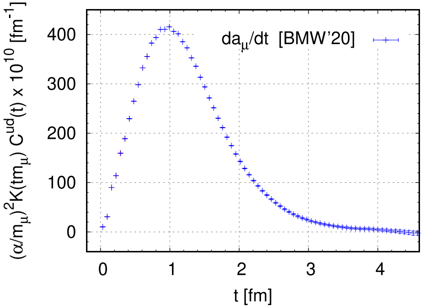
$$A(a) = A_0 + A_2 \cdot (a/w_0)^2\alpha_s^\gamma(a) + A_4 \cdot (a/w_0)^4 + A_6 \cdot (a/w_0)^6$$

$a_\mu^{\text{LO-HVP}}$ from lattice QCD

- Compute electromagnetic current-current correlator

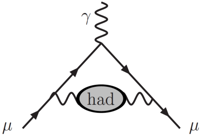
$$C(t) = \langle J_\mu(t) J_\nu(0) \rangle$$

$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt\ K(t)\ C(t)$$



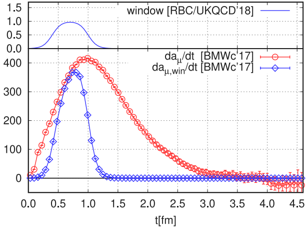
- $K(t)$ describes the leptonic part of the diagram

[Blum'02], [Bernecker,Meyer '11], [HPQCD'14], ...



Window observable

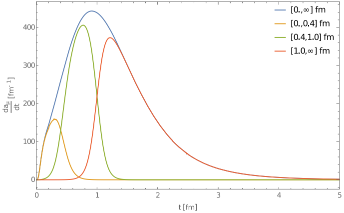
- Restrict correlator to window between $t_1 = 0.4\,\text{fm}$ and $t_2 = 1.0\,\text{fm}$ [RBC/UKQCD’18]



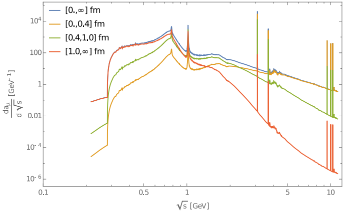
(144×96^3, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

- Less challenging than full a_μ
  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)

- Effect in $\sqrt{s}$



$\longleftrightarrow$

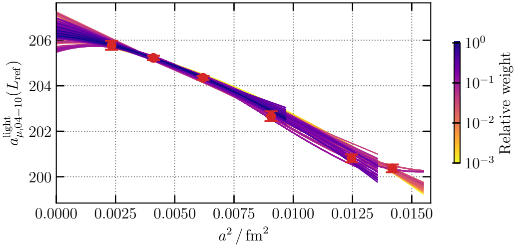


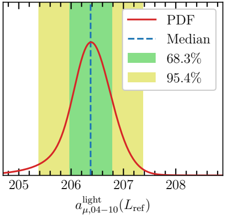
Intermediate-distance window [0.4 fm , 1.0 fm] light-connected

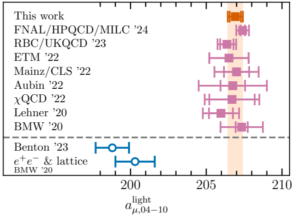
Number of fits	50688	
Fits with $P > 0.1$	80%	
Median	206.37	
Total error	0.50	0.24%
Statistical error	0.37	0.18%
Systematic error	0.33	0.16%
Pseudoscalar fits	0.01	0.00%
M_{ss} value	0.01	0.00%
w_0 value	0.18	0.09%
Lattice spacing cut	0.07	0.04%
Fit polynomial order	0.18	0.09%
Log corrections γ	0.10	0.05%

- With FV-corrections included

$$a^{\text{light}}_{\mu,04-10} = 206.92(37)(34)[50]$$





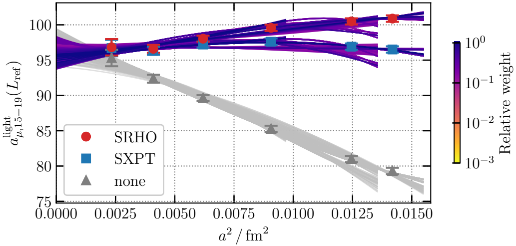


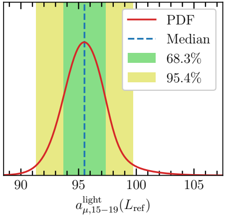
Long-distance window [1.5 fm , 1.9 fm] light-connected

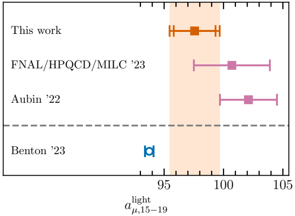
Number of fits	50688	
Fits with $P > 0.1$	100%	
Median	95.51	
Total error	2.10	2.20%
Statistical error	1.75	1.84%
Systematic error	1.16	1.22%
Pseudoscalar fits	0.02	0.02%
M_{ss} value	0.02	0.02%
w_0 value	0.62	0.65%
Taste correction	0.28	0.30%
Lattice spacing cut	0.22	0.23%
Fit polynomial order	0.62	0.64%
Log corrections γ	0.20	0.21%

- With FV-corrections included

$$a_{\mu,15-19}^{\text{light}} = 97.57(1.76)(1.17)[2.11]$$



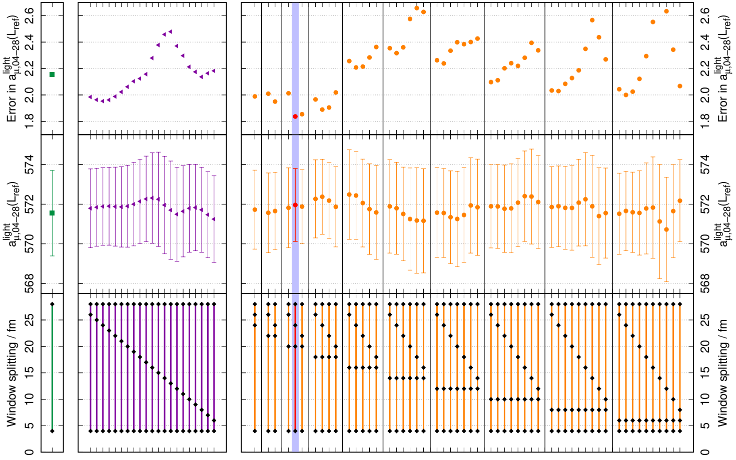




Multi-window fits

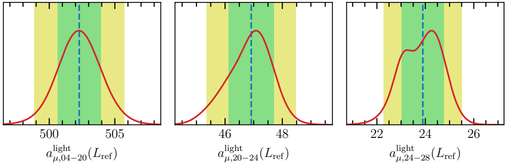
- Try different splittings of [0.4 fm , 2.8 fm] into one, two, or three sub-windows
- Fit sub-windows together in a correlated manner
- Optimize for minimal final variance
- With new fit functions, optimal splitting:

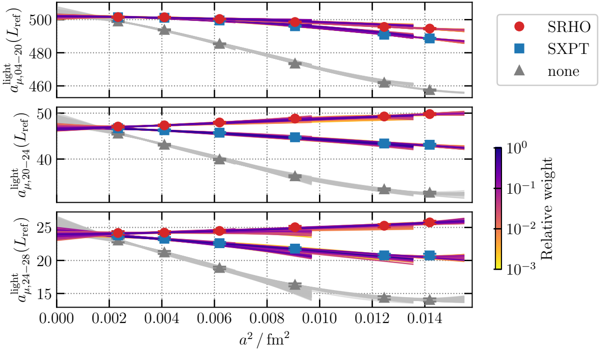
  0.4 fm – 2.0 fm – 2.4 fm – 2.8 fm



[0.4 fm , 2.8 fm] light-connected

Number of fits	248396544							
Fits with $P > 0.1$	80%							
Median	572.99		502.28		46.91		23.90	
Total error	2.75	0.48%	1.72	0.34%	0.79	1.68%	0.88	3.67%
Statistical error	1.84	0.32%	1.24	0.25%	0.41	0.88%	0.41	1.70%
Systematic error	2.04	0.36%	1.19	0.24%	0.67	1.43%	0.78	3.25%
Pseudoscalar fits	0.05	0.01%	0.03	0.01%	0.02	0.03%	0.01	0.04%
M_{ss} value	0.03	0.01%	0.02	0.00%	0.01	0.02%	0.00	0.02%
w_0 value	1.72	0.30%	1.20	0.24%	0.31	0.66%	0.21	0.86%
Taste correction	0.86	0.15%	0.31	0.06%	0.32	0.69%	0.45	1.89%
Lattice spacing cut	0.40	0.07%	0.16	0.03%	0.20	0.43%	0.15	0.62%
Fit polynomial order	0.40	0.07%	0.23	0.05%	0.20	0.42%	0.27	1.11%
Log corrections γ	0.23	0.04%	0.06	0.01%	0.08	0.17%	0.10	0.42%
	04 – 28		04 – 20		20 – 24		24 – 28	





Long-distance window [1.0 fm , ∞] light-connected

- Fit using 3 sub-windows

$$a^{\text{light}}_{\mu,10-\infty} = a^{\text{light}}_{\mu,10-32} + a^{\text{light}}_{\mu,32-36} + a^{\text{light}}_{\mu,36-\infty}$$

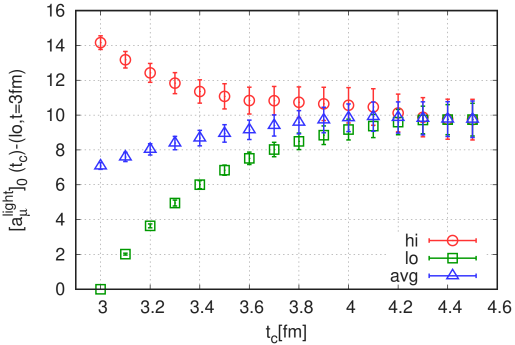
- Bounding method [Lehner 2016] [Borsanyi *et.al.* 2017]

$$a^{\text{light}}_{\mu,36-\infty}\Big|_{\text{lo}} = a^{\text{light}}_{\mu,36-40}$$

$$a^{\text{light}}_{\mu,36-\infty}\Big|_{\text{up}} = a^{\text{light}}_{\mu,36-40} + \left(\frac{a^{\text{light}}_{\mu,39-40}}{a^{\text{2-pion}}_{\mu,39-40}}\right) a^{\text{2-pion}}_{\mu,40-\infty}$$

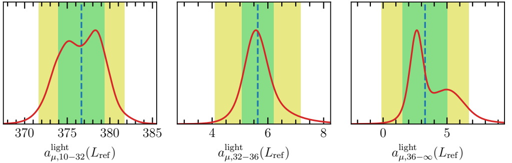
- $a^{\text{2-pion}}_{\mu,39-40}$, $a^{\text{2-pion}}_{\mu,40-\infty}$ are computed using

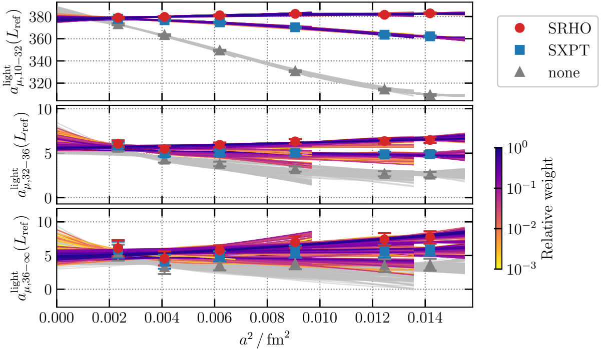
$$\varphi(t) = \frac{\cosh[E_{2\pi}(t-T/2)]+1}{\cosh(E_{2\pi}T/2)-1}$$



Long-distance window [1.0 fm , ∞] light-connected

	10 – ∞		10 – 32		32 – 36		36 – ∞	
Number of fits	496793088							
Fits with $P > 0.1$	100%							
Median	386.06		376.66		5.64		3.28	
Total error	3.64	0.94%	2.70	0.72%	0.77	13.66%	1.75	53.22%
Statistical error	2.68	0.70%	1.47	0.39%	0.63	11.16%	0.97	29.51%
Systematic error	2.46	0.64%	2.27	0.60%	0.44	7.89%	1.45	44.29%
Pseudoscalar fits	0.06	0.02%	0.05	0.01%	0.02	0.28%	0.02	0.55%
Taste correction	1.49	0.39%	1.40	0.37%	0.13	2.33%	0.04	1.34%
Bounding	1.22	0.32%	0.00	0.00%	0.01	0.20%	1.21	36.93%
Lattice spacing cut	0.54	0.14%	0.53	0.14%	0.07	1.25%	0.11	3.38%
Fit polynomial order	0.98	0.25%	0.57	0.15%	0.31	5.54%	0.30	9.24%
Log corrections γ	0.64	0.16%	0.62	0.16%	0.03	0.61%	0.06	1.70%





Long-distance window [1.0 fm , ∞] light-connected

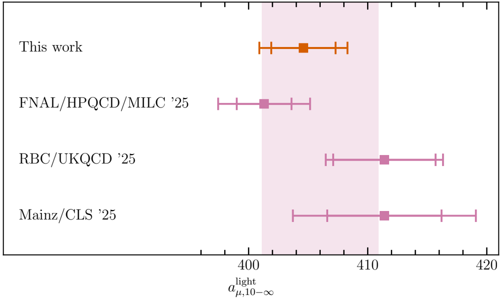
- Isospin symmetric point:
  At current level of precision, WP25 ≈ BMW

- FV corrections obtained from pheno + NNLO SXPT

$$\Delta a^{I=1}_{\mu,10-\infty}(L_{\text{ref}}, \infty \to \infty, \infty) = 17.16(51)$$
$$\Delta a^{I=1}_{\mu,10-\infty}(L_{\text{ref}}, T_{\text{ref}} \to L_{\text{ref}}, \infty) = -0.47$$

- Result to compare:

$$a^{\text{light}}_{\mu,10-\infty} = 404.6(2.7)(2.5)[3.7]$$



Precision gain compared to [BMW '20]

- Effect of finest ensemble $a = 0.0483\,\text{fm}$

$$\text{with:} \quad a_{\mu,10-\infty}^{\text{light}}(L_{\text{ref}}) = 386.1(2.7)(2.5)[3.6]$$
$$\text{without:} \quad a_{\mu,10-\infty}^{\text{light}}(L_{\text{ref}}) = 383.2(4.0)(4.1)[5.7]$$

$\longrightarrow$ Finest ensemble reduces the error by $1 - 3.6/5.7 = 37\%$

- Effect of data-driven tail $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$

$$a_{\mu,10-28}^{\text{light}}(L_{\text{ref}}) = 366.4(1.8)(2.2)[2.8]$$
$$a_{\mu,10-\infty}^{\text{light}}(L_{\text{ref}}) = 386.1(2.7)(2.5)[3.6]$$

$\longrightarrow$ Data-driven tail reduces the error by $1 - 2.8/3.6 = 22\%$

Result compared to [BMW '20]

- Add up three windows, $a^{\text{light}}_{\mu,00\text{-}04} + a^{\text{light}}_{\mu,04\text{-}28} + a^{\text{light}}_{\mu,28\text{-}\infty}$

$$[\text{BMW/DMZ '24}] \qquad a_\mu^{\text{light}}(L_{\text{ref}}) = 640.1(2.8)(2.5)[3.7]$$

$$[\text{BMW '20}] \qquad a_\mu^{\text{light}}(L_{\text{ref}}) = 633.7(2.1)(4.2)[4.7]$$

$\longrightarrow$ Shift in central value: 6.4

  - New lattice spacing: 2.8
  - Type-I (M_Ω) vs Type-II (w_0): 4.0
  - Rest: differences in the analysis procedures

- Significance of difference
  - Statistical correlation: $r = 0.41$
  - Taste correction: $3.8 \to 1.5$, fully correlated
  - Bounding: no error assigned in [BMW '20] vs 1.2 in [BMW/DMZ '24] $\longrightarrow$ uncorrelated
  - Rest: $1.8 \to 1.5$. Take both extremes: (a) fully uncorrelated, (b) fully correlated

$$\text{(a):} \qquad [\text{BMW/DMZ '24}] - [\text{BMW '20}] = 6.4(4.5), \quad 1.4\sigma$$

$$\text{(b):} \qquad [\text{BMW/DMZ '24}] - [\text{BMW '20}] = 6.4(3.8), \quad 1.7\sigma$$

All contributions to $a_\mu^{\text{LO-HVP}}$

00 – 28	light and disconnected	619.6(2.3)(2.1)[3.1]	this work
00 – 28	strange	53.12(14)(11)[18]	this work
00 – 28	charm	14.68(24)(29)[38]	this work
00 – 28	light qed	-1.86(53)(27)	this work
00 – 28	light sib	7.06(64)(35)	this work
00 – 28	disconnected qed	-1.00(46)(28)	this work
00 – 28	disconnected sib	-4.39(58)(63)	this work
00 – 28	strange qed	-0.0136(85)(77)	this work
00 – ∞	disconnected charm	0.0(1)	BMW'17, Section 4 in Supp. Mat.
00 – ∞	charm qed	0.0182(36)	ETM'19
00 – ∞	bottom	0.271(37)	HPQCD'14
28 – ∞	tail from data-driven	27.59(26)(45)[52]	this work
00 – ∞	total	715.1(2.5)(2.3)[3.4]	

[BMW/DMZ '24] vs [BMW '20]

$$\text{[BMW/DMZ '24]} \qquad a_\mu^{\text{LO-HVP}} = 715.1(2.5)(2.3)[3.4]$$
$$\text{[BMW '20]} \qquad a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$$

$\longrightarrow$ Shift in central value: 7.6

- Statistical correlation: $r = 0.16$
  Reduced due to new ensemble and $a_{\mu,00\text{-}\infty}$ vs $a_{\mu,00\text{-}28}$
- Taste correction: $3.8 \to 0.9$, fully correlated
- Finite volume corrections: 1.9, fully correlated
- Scale setting: M_Ω vs w_0 $\longrightarrow$ uncorrelated
- Rest: Take both extremes: (a) fully uncorrelated, (b) fully correlated

$$\text{(a):} \qquad \text{[BMW/DMZ '24]} - \text{[BMW '20]} = 7.6(5.2), \quad 1.5\sigma$$
$$\text{(b):} \qquad \text{[BMW/DMZ '24]} - \text{[BMW '20]} = 7.6(4.5), \quad 1.7\sigma$$

Conclusions

- Lattice results suggest that no new physics is needed at current level of precision
- Lattice groups agree in different windows
- Disagreement between lattice and data-driven
  $\longrightarrow$ further investigation needed
- Upcoming experiments
  - MUonE at CERN for spacelike HVP
  - New BaBar e^+e^- analysis
  - New KLOE analysis
  - New BES III, BELLE-II, CMD-3, SND-2 data and analysis
  - J-PARC, entirely new method for a_μ measurement

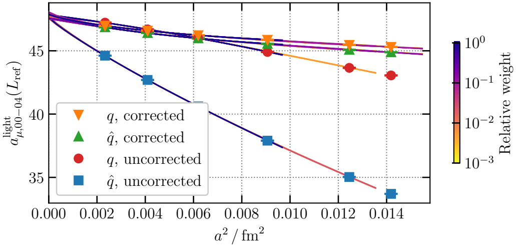
Backup

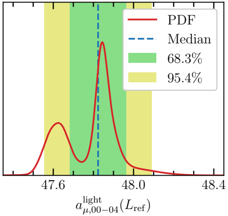
Short-distance window [0.0 fm , 0.4 fm] light-connected

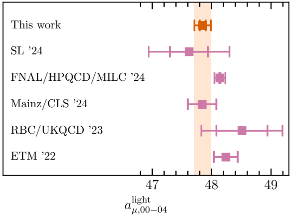
Number of fits	8192	
Fits with $P > 0.1$	54%	
Median	47.82	
Total error	0.14	0.29%
Statistical error	0.05	0.10%
Systematic error	0.13	0.28%
Pseudoscalar fits	0.00	0.00%
M_{ss} value	0.00	0.00%
w_0 value	0.01	0.02%
Tree correction and/or $\hat{q}$	0.09	0.19%
Lattice spacing cut	0.05	0.11%
Fit polynomial order	0.06	0.12%

- With FV-corrections included

$$a^{\text{light}}_{\mu,00-04} = 47.85(5)(13)[14]$$





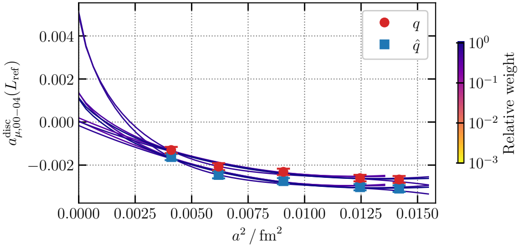


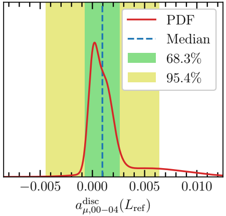
Short-distance window [0.0 fm , 0.4 fm] disconnected

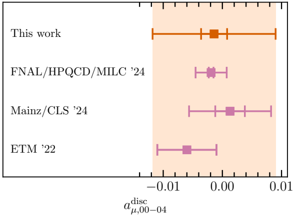
Number of fits	2560	
Fits with $P > 0.1$	100%	
Median	0.0010	
Total error	0.0027	284%
Statistical error	0.0022	226%
Systematic error	0.0016	172%
Pseudoscalar fits	0.0000	1%
M_{ss} value	0.0000	1%
w_0 value	0.0002	16%
Tree correction and/or $\hat{q}$	0.0001	6%
Lattice spacing cut	0.0005	47%
Fit polynomial order	0.0018	192%

- With FV-corrections included

$$a^{\text{disc}}_{\mu,00-04} = -0.0014(22)(101)[104]$$





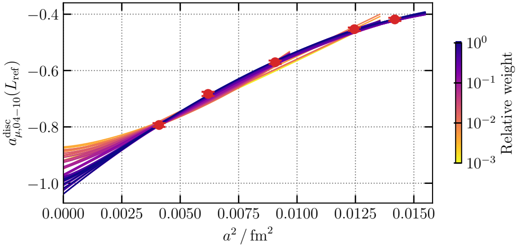


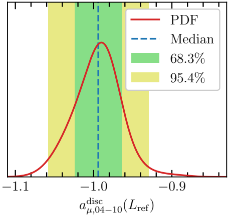
Intermediate-distance window [0.4 fm , 1.0 fm] disconnected

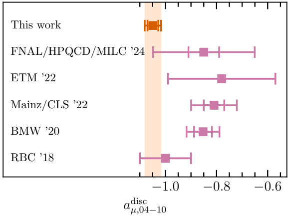
Number of fits	30720	
Fits with $P > 0.1$	48%	
Median	-0.994	
Total error	0.032	-3.2%
Statistical error	0.021	-2.1%
Systematic error	0.024	-2.4%
Pseudoscalar fits	0.000	-0.0%
M_{ss} value	0.000	-0.0%
w_0 value	0.008	-0.8%
Lattice spacing cut	0.013	-1.3%
Fit polynomial order	0.012	-1.2%
Log corrections γ	0.015	-1.5%

- With FV-corrections included

$$a^{\rm disc}_{\mu,04-10} = -1.049(21)(24)[32]$$

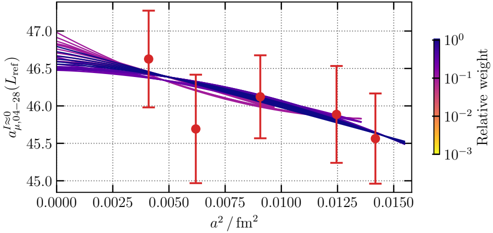


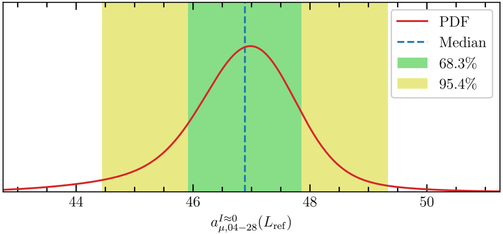




$[0.4\,\mathrm{fm}\,,\,2.8\,\mathrm{fm}]\quad I=0$

Number of fits	7680	
Fits with $P > 0.1$	100%	
Median	46.89	
Total error	1.22	2.61%
Statistical error	1.19	2.54%
Systematic error	0.28	0.59%
Pseudoscalar fits	0.01	0.02%
M_{ss} value	0.01	0.01%
w_0 value	0.17	0.36%
Lattice spacing cut	0.04	0.08%
Fit polynomial order	0.25	0.54%
Log corrections γ	0.01	0.02%



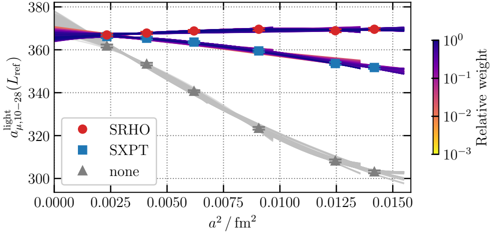


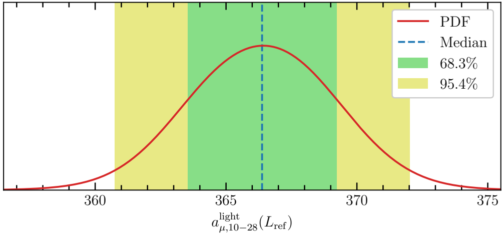
$$a_{\mu,00-28}^{\text{light+disc}} = \frac{9}{10}\,a_{\mu,04-28}^{\text{light}} + a_{\mu,04-28}^{I\approx0} + a_{\mu,00-04}^{\text{light}} + a_{\mu,00-04}^{\text{disc}}$$

$$a_{\mu,00-28}^{\text{light+disc}} = 619.6(2.3)(2.1)[3.1]$$

Long-distance window [1.0 fm , 2.8 fm] light-connected

Number of fits	50688	
Fits with $P > 0.1$	99%	
Median	366.38	
Total error	2.84	0.78%
Statistical error	1.76	0.48%
Systematic error	2.23	0.61%
Pseudoscalar fits	0.04	0.01%
M_{ss} value	0.03	0.01%
w_0 value	1.64	0.45%
Taste correction	0.90	0.25%
Lattice spacing cut	0.40	0.11%
Fit polynomial order	0.33	0.09%
Log corrections γ	0.36	0.10%





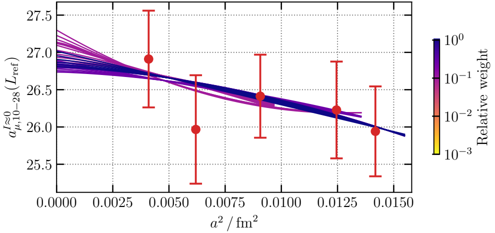
Long-distance window [1.0 fm, 2.8 fm] $I = 0$

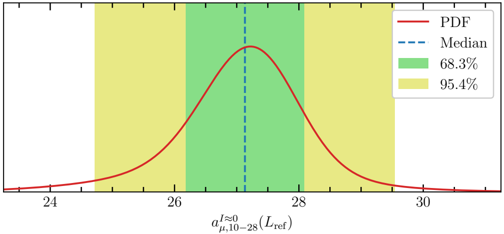
Number of fits	7680	
Fits with $P > 0.1$	100%	
Median	27.13	
Total error	1.21	4.45%
Statistical error	1.18	4.36%
Systematic error	0.24	0.88%
Pseudoscalar fits	0.01	0.03%
M_{ss} value	0.01	0.02%
w_0 value	0.14	0.53%
Lattice spacing cut	0.03	0.12%
Fit polynomial order	0.22	0.83%
Log corrections γ	0.00	0.01%

$$a_{\mu,10-28}^{I\approx 0} \equiv \frac{1}{10} a_{\mu,10-28}^{\text{light}} + a_{\mu,10-28}^{\text{disc}}$$

$$a_{\mu,10-28}^{\text{light+disc}} = \frac{9}{10} a_{\mu,10-28}^{\text{light}} + a_{\mu,10-28}^{I\approx 0}$$

$$a_{\mu,10-28}^{\text{light+disc}} = 365.6(2.4)(2.0)[3.1]$$

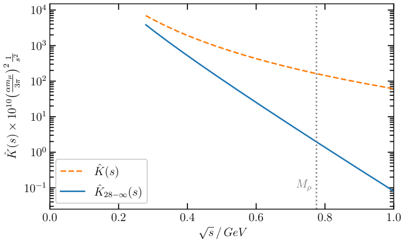


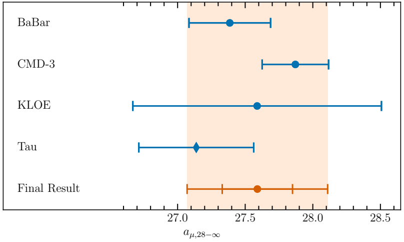


Tail contribution: $t \geq 2.8\,\mathrm{fm}$

- Tail $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ contributes $\lesssim 5\%$ to final result for a_μ
- Tail dominated by cross section below ρ peak: $\sim 75\,\%$ for $\sqrt{s} \leq 0.63\,\mathrm{GeV}$
  $\longrightarrow$ Tension between experiments significantly reduced
- For small $\sqrt{s}$ radiative-correction issues are less pronounced [DHLMZ '23]
- Independently integrate each experiment that has over 95 % coverage
- Weighted average taken w/ and w/out τ: $\chi^2/\mathrm{dof} = 1.0$ and 0.8

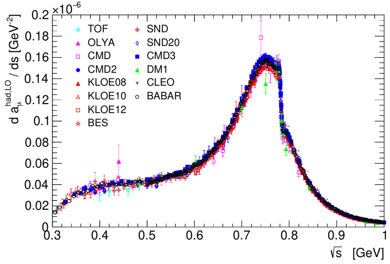
$$a_{\mu,28\text{-}\infty}^{\text{LO-HVP}} = 27.59(26)(45)[52]$$



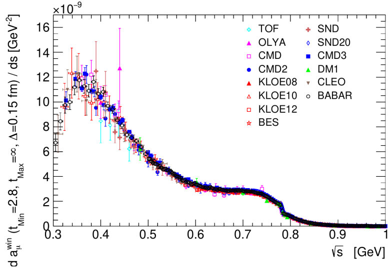


Tail contribution: $t \geq 2.8\,\mathrm{fm}$

$a_\mu^{\text{LO-HVP}}$



$a_{\mu,28-\infty} \equiv a_\mu^{\text{LO-HVP}}|_{\text{tail}}$



Include additional systematic variations

- Averaging before integrating vs. integrating before averaging: 0.12
- Effect of experiments with less coverage: 0.16
- Removing/including CMD3: 0.31
- Removing/including BaBar/KLOE: 0.26

Still negligible effect on final $a_\mu^{\text{LO-HVP}}$