

Experimental Inputs for Hadronic Light-by-Light Scattering from the **BESIII** Collaboration

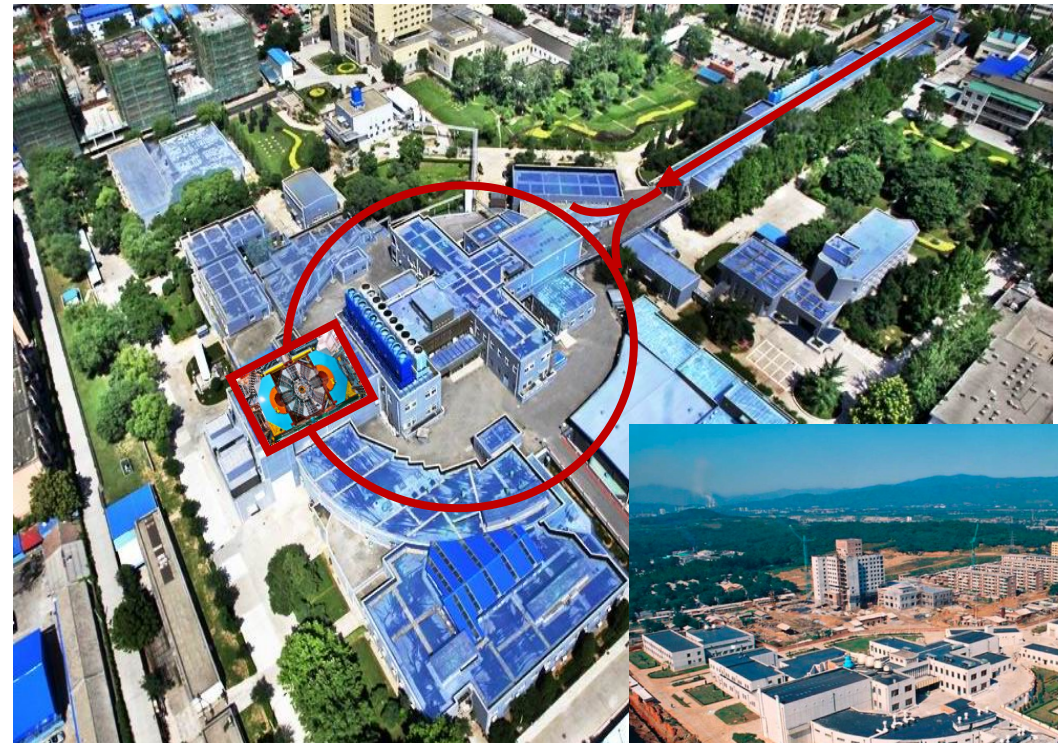
Achim Denig, Yuping Guo, **Max Lellmann**,
Jan Muskalla, Christoph F. Redmer,
Kangshuai Zhu

on behalf of the BESIII collaboration

9th Plenary Workshop of the g-2 Theory Initiative

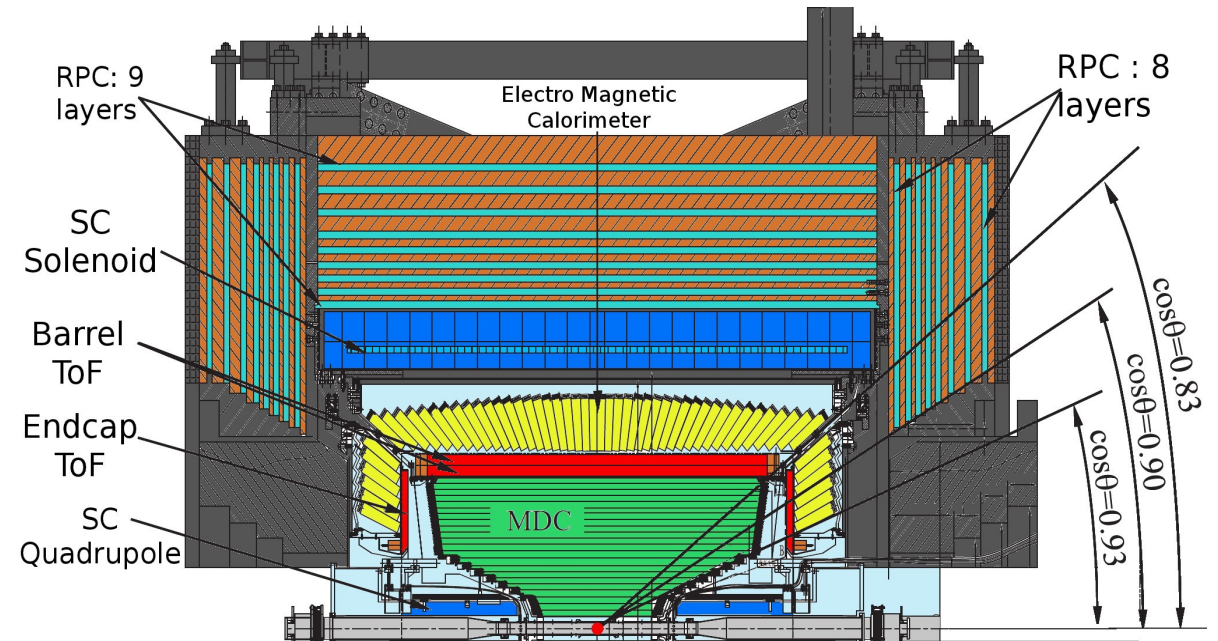
The **BESIII** Experiment

- Electron Positron Collisions in the τ -charm energy range
 - $E_{\text{cm}} = 1.86\text{-}4.95\text{ GeV}$
- Multi purpose detector
 - Tracking, TOF, Calorimetry, Magnet, Muon counters in flux return yoke
- Design luminosity of $1.0\text{ nb}^{-1}/\text{s}$ exceeded
- Largest dataset of 20.3 fb^{-1} at 3.773 GeV
- **Perfectly suited for two-photon physics at small momentum transfers**



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Muon Chambers

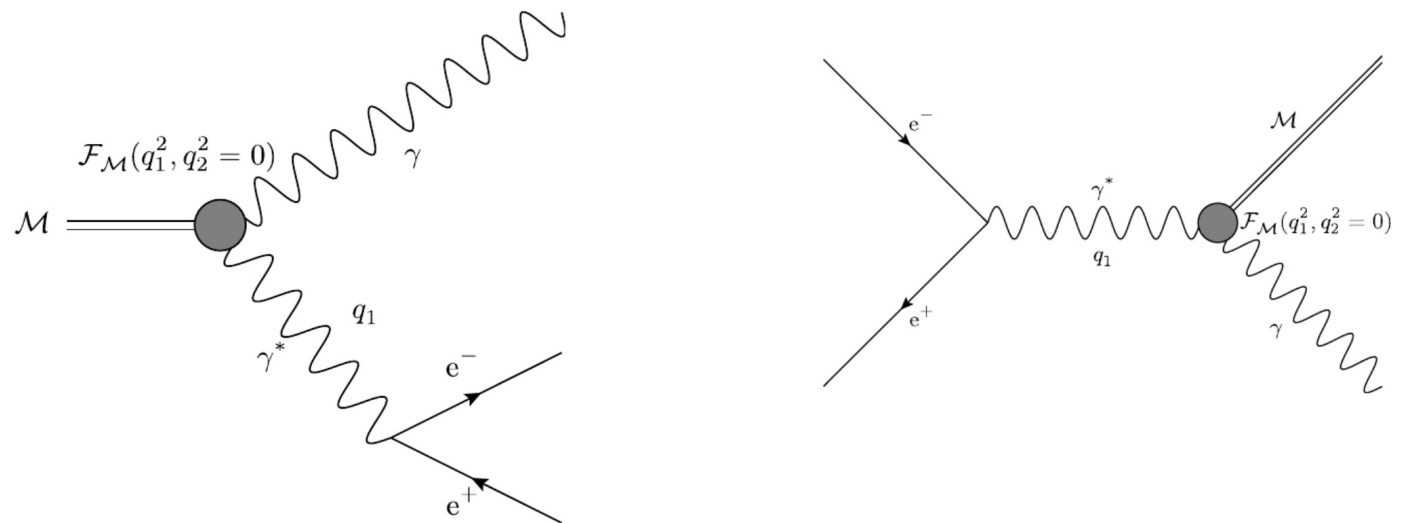
Superconducting Solenoid

Electromagnetic Calorimeter

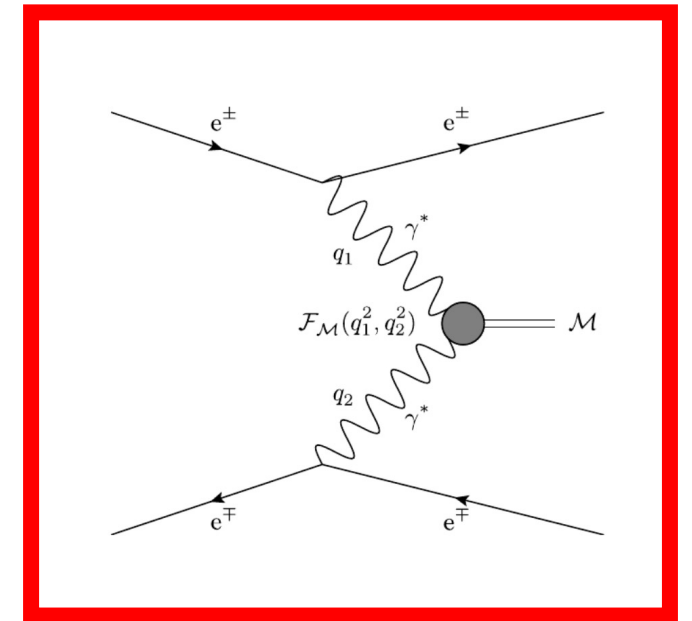
Time-of-Flight System

Drift Chamber

Scope of this talk

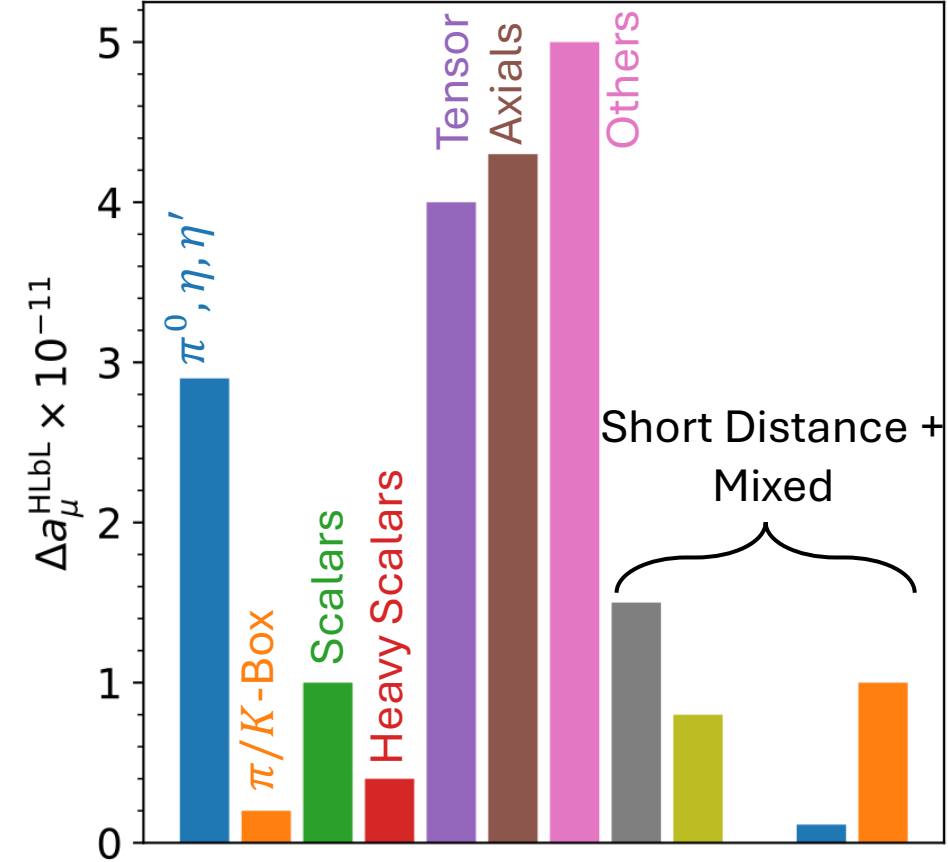
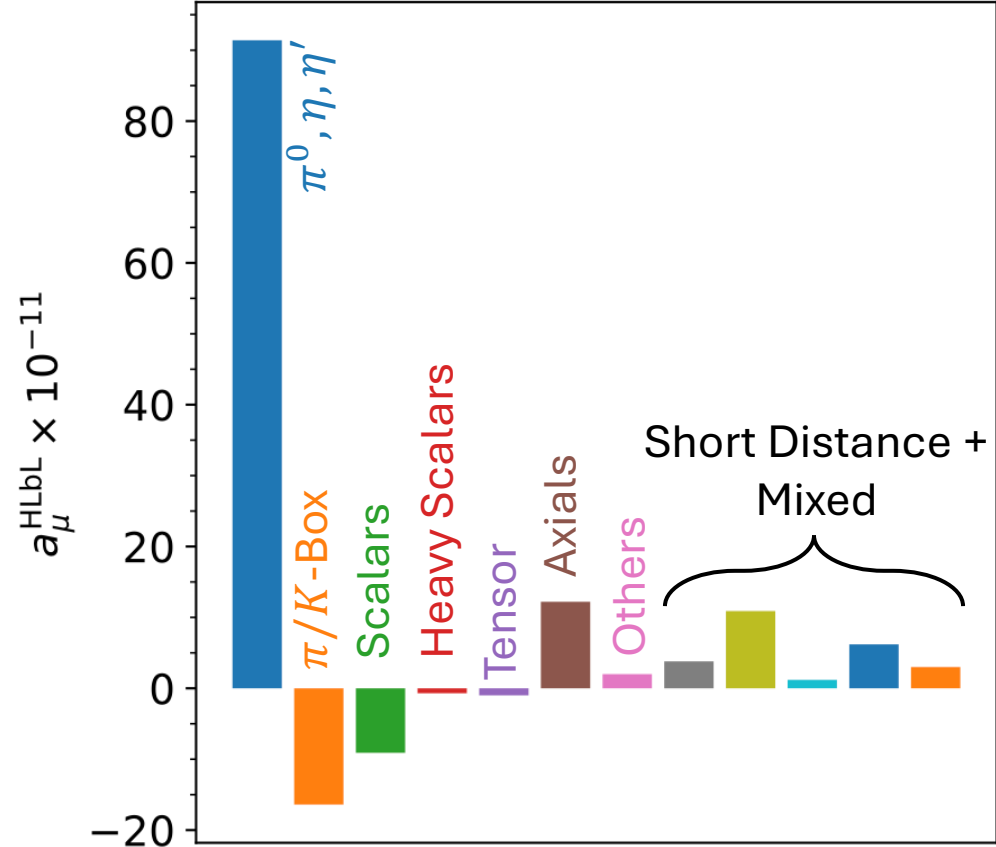


time-like



space-like

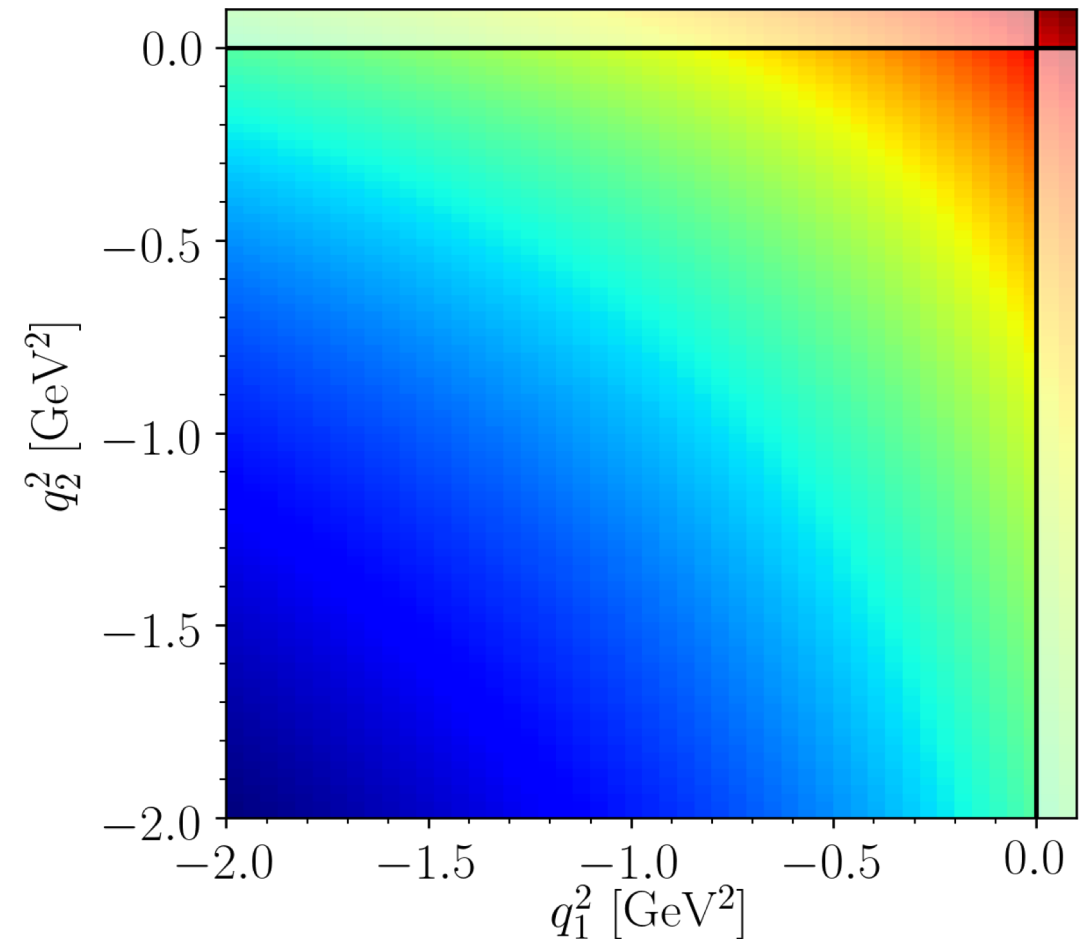
The Hadronic Light-by-Light Contribution



Collider Studies of Transition Form Factors

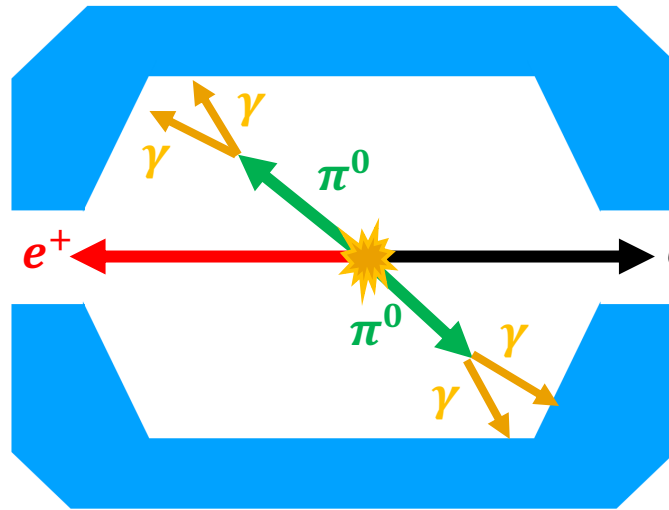
- Strong Q^2 dependence of transition form factor and cross sections
- Momentum transfer coupled to lepton scattering angles $Q^2 = -q^2 \approx 4EE' \sin^2 \frac{\theta}{2}$
 - Detectors do not cover full solid angle!
 - At BESIII: $\theta > 20^\circ$
 - Restriction to Q^2

➤ Different tagging methods

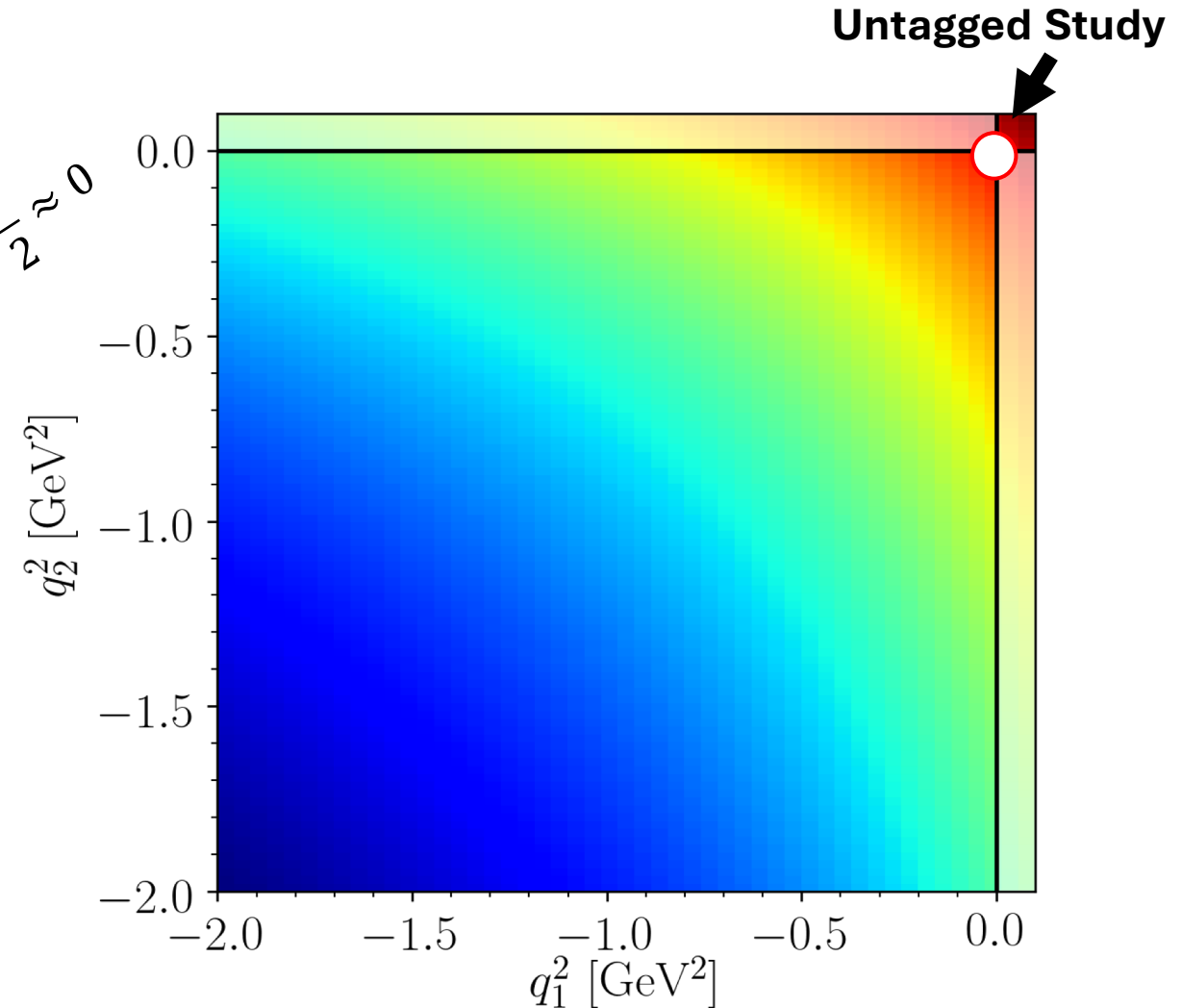


Collider Studies of Transition Form Factors

Untagged Studies

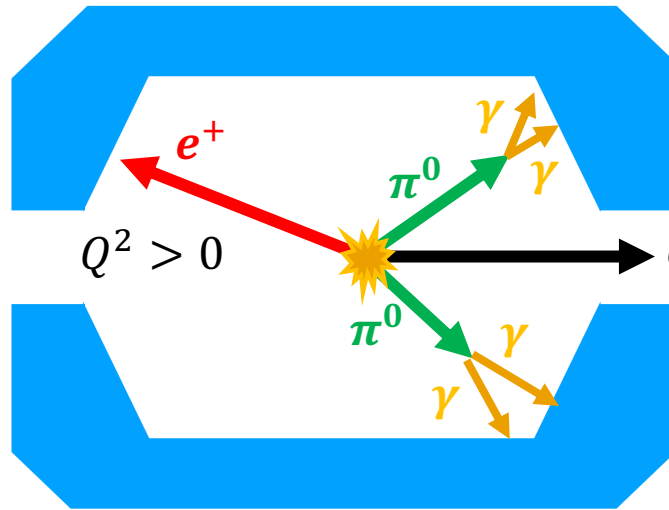


- Scattering of two on-shell photons
- Must be transversally polarized
 - Only helicity 0 and 2 contributions
- Study of radiative widths



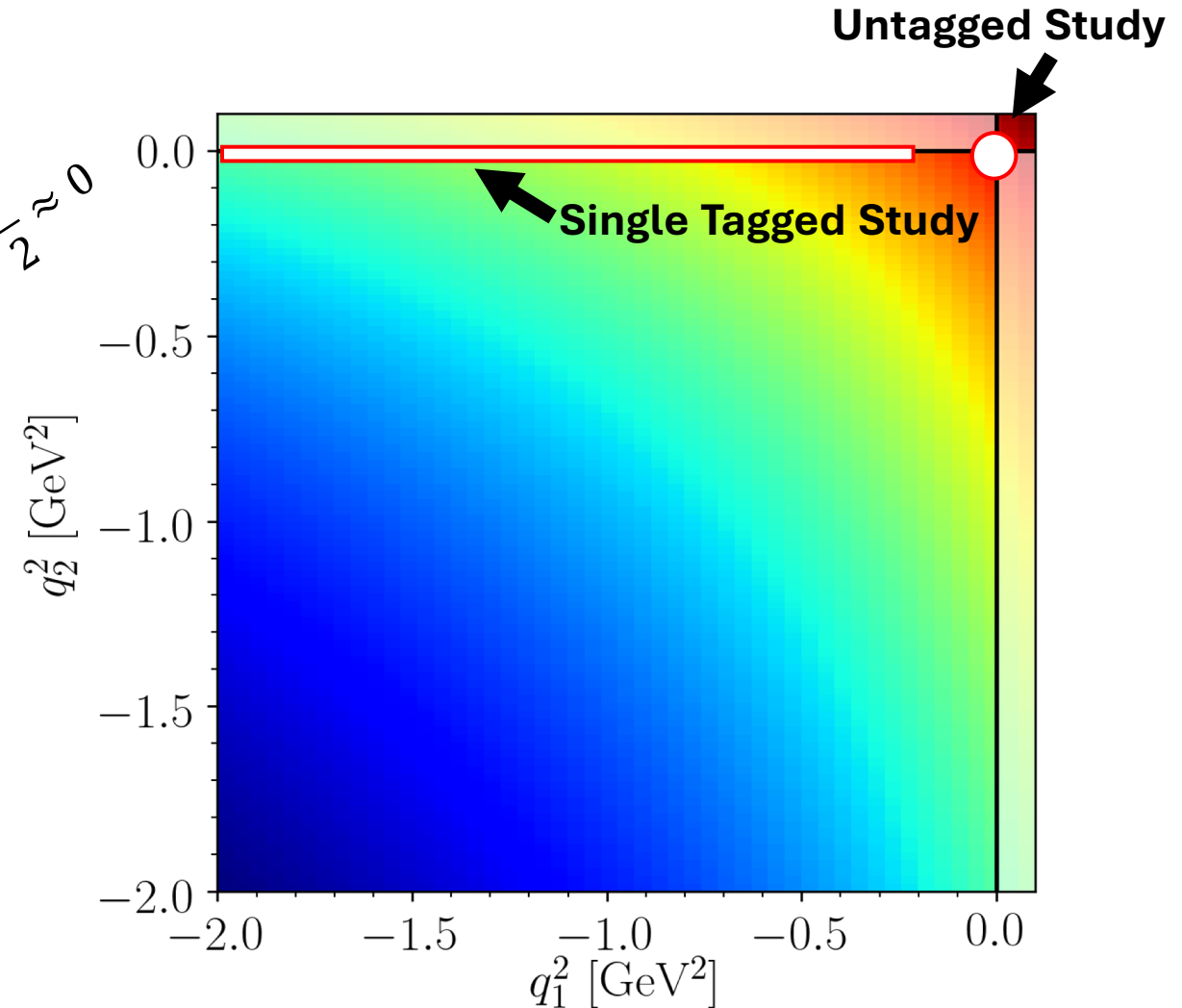
Collider Studies of Transition Form Factors

Single Tagged Studies



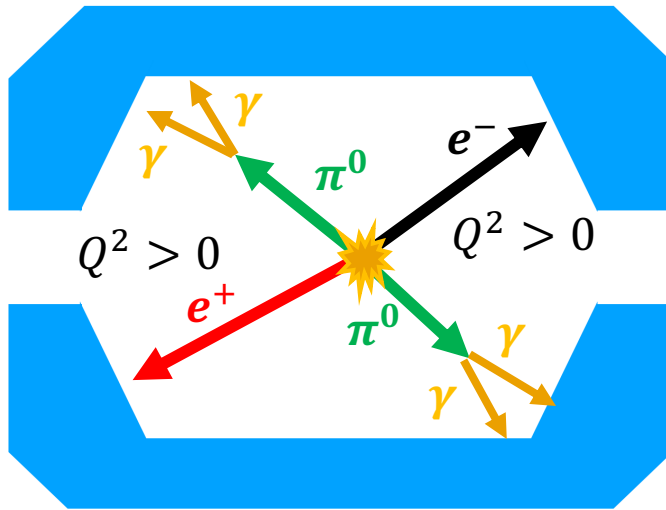
- One photon on-shell, one photon off-shell
 - Much smaller cross section
- TT and LT configurations
 - Helicity 0, 1, and 2
- Study TFFs as function of one Q^2

$$Q^2 \approx 4EE' \sin^2 \frac{\theta}{2} \approx 0$$

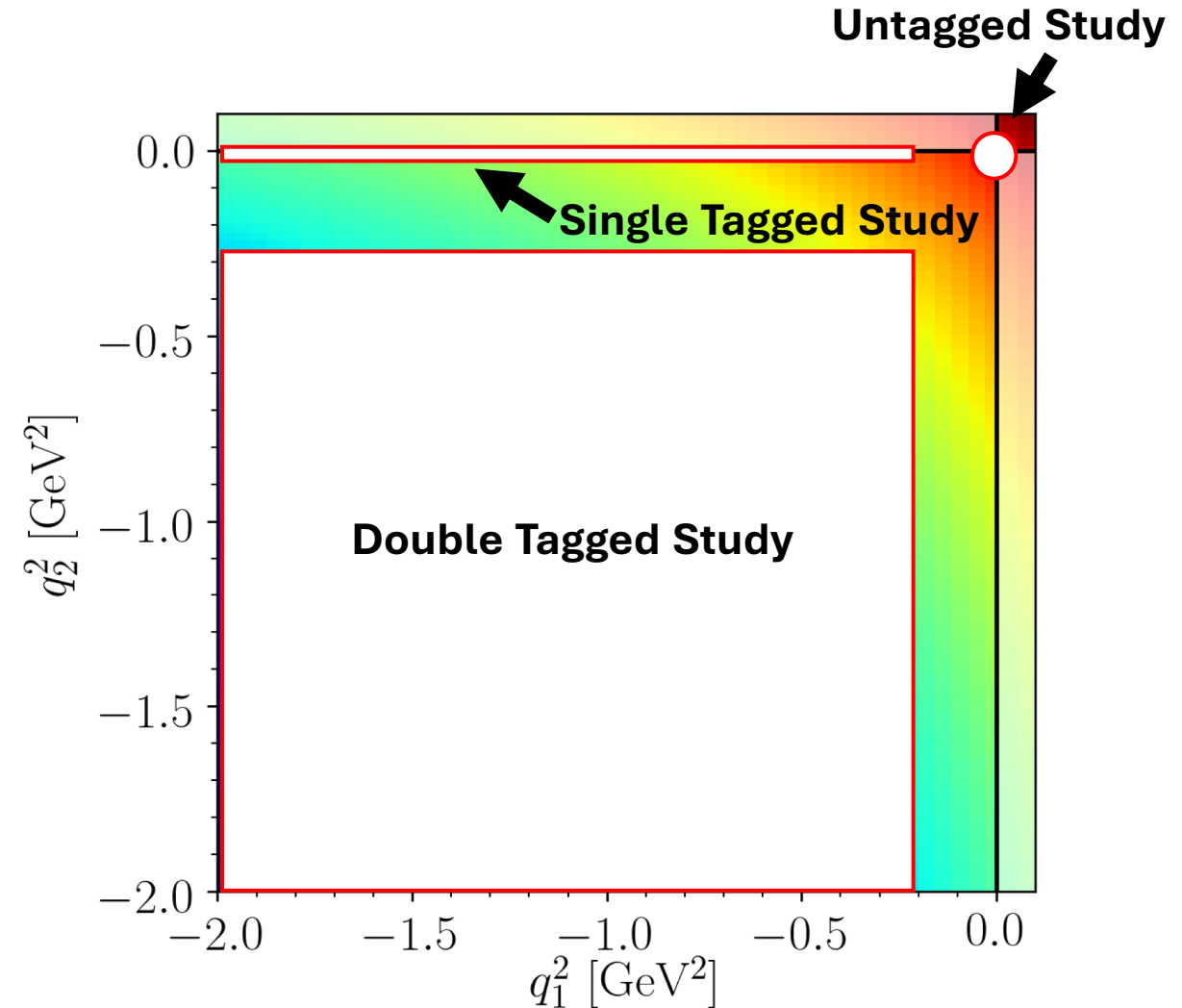


Collider Studies of Transition Form Factors

Double Tagged Studies



- Scattering of two off-shell photons
 - Tiny cross sections
- Study full functional dependence of TFFs on Q^2
- **Only BaBar η' measurement**



Collider Studies of Transition Form Factors

The $e^+e^- \rightarrow e^+e^- M$ cross sections can be related directly to different $\gamma^*\gamma^* \rightarrow M$ helicity cross sections and response functions

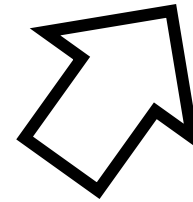
$$\begin{aligned}
 d\sigma = & \frac{\alpha^2}{8\pi^4 Q_1^2 Q_2^2} \frac{\sqrt{X}}{s(1-4m^2/s)^{1/2}} \cdot \frac{d^3\vec{p}'_1}{E'_1} \cdot \frac{d^3\vec{p}'_2}{E'_2} \frac{4}{(1-\varepsilon_1)(1-\varepsilon_2)} \\
 & \times \left\{ \frac{1}{2} (\sigma_0 + \sigma_2) + \left[\varepsilon_1 + \frac{2m^2}{Q_1^2}(1-\varepsilon_1) \right] \left[\varepsilon_2 + \frac{2m^2}{Q_2^2}(1-\varepsilon_2) \right] \sigma_{LL} \right. \\
 & + \left[\varepsilon_2 + \frac{2m^2}{Q_2^2}(1-\varepsilon_2) \right] \sigma_{TL} + \left[\varepsilon_1 + \frac{2m^2}{Q_1^2}(1-\varepsilon_1) \right] \sigma_{LT} + \frac{1}{2} \varepsilon_1 \varepsilon_2 \cos(2\tilde{\phi}) \tau_{TT} \\
 & \left. + \left[\varepsilon_1(1+\varepsilon_1) + \frac{4m^2}{Q_1^2} \varepsilon_1(1-\varepsilon_1) \right]^{1/2} \left[\varepsilon_2(1+\varepsilon_2) + \frac{4m^2}{Q_2^2} \varepsilon_2(1-\varepsilon_2) \right]^{1/2} \cos \tilde{\phi} (\tau_0 + \tau_1) \right\}
 \end{aligned}$$

Collider Studies of Transition Form Factors

The $e^+e^- \rightarrow e^+e^- M$ cross sections can be related directly to different $\gamma^*\gamma^* \rightarrow M$ helicity cross sections and response functions

“Luminosity Functions”

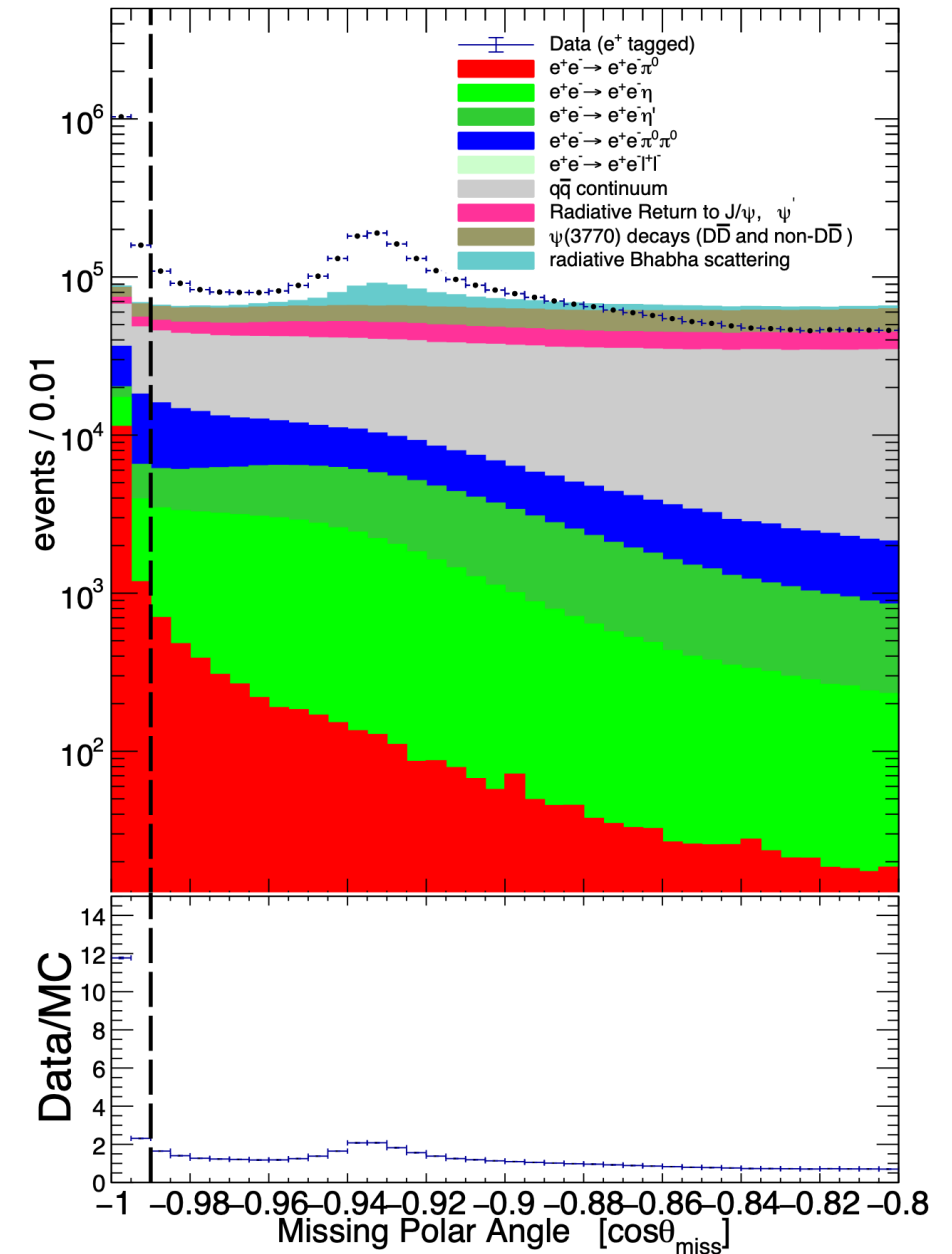
$$\frac{d\sigma}{dQ_1^2 dQ_2^2 dW} = \frac{d^3 L_{TT}}{dQ_1^2 dQ_2^2 dW} (\sigma_0 + \sigma_2) + \frac{d^3 L_{LT}}{dQ_1^2 dQ_2^2 dW} \sigma_{LT} + \frac{d^3 L_{TL}}{dQ_1^2 dQ_2^2 dW} \sigma_{TL} + \frac{d^3 L_{LL}}{dQ_1^2 dQ_2^2 dW} \sigma_{LL} + \dots$$



$$d\sigma = \frac{\alpha^2}{8\pi^4 Q_1^2 Q_2^2} \frac{\sqrt{X}}{s(1-4m^2/s)^{1/2}} \cdot \frac{d^3 \vec{p}'_1}{E'_1} \cdot \frac{d^3 \vec{p}'_2}{E'_2} \frac{4}{(1-\varepsilon_1)(1-\varepsilon_2)} \times \left\{ \frac{1}{2} (\sigma_0 + \sigma_2) + \left[\varepsilon_1 + \frac{2m^2}{Q_1^2} (1-\varepsilon_1) \right] \left[\varepsilon_2 + \frac{2m^2}{Q_2^2} (1-\varepsilon_2) \right] \sigma_{LL} + \left[\varepsilon_2 + \frac{2m^2}{Q_2^2} (1-\varepsilon_2) \right] \sigma_{TL} + \left[\varepsilon_1 + \frac{2m^2}{Q_1^2} (1-\varepsilon_1) \right] \sigma_{LT} + \frac{1}{2} \varepsilon_1 \varepsilon_2 \cos(2\tilde{\phi}) \tau_{TT} + \left[\varepsilon_1 (1+\varepsilon_1) + \frac{4m^2}{Q_1^2} \varepsilon_1 (1-\varepsilon_1) \right]^{1/2} \left[\varepsilon_2 (1+\varepsilon_2) + \frac{4m^2}{Q_2^2} \varepsilon_2 (1-\varepsilon_2) \right]^{1/2} \cos \tilde{\phi} (\tau_0 + \tau_1) \right\}$$

Study of $\gamma\gamma^* \rightarrow \pi^0$

- Single Tag Measurement (2.9 fb^{-1} @ 3.773 GeV)
 - Require quasi-real photon
- Select ...
 - ... one final state lepton
 - ... require small scattering angle of missing momentum $\rightarrow$ small missing Q^2

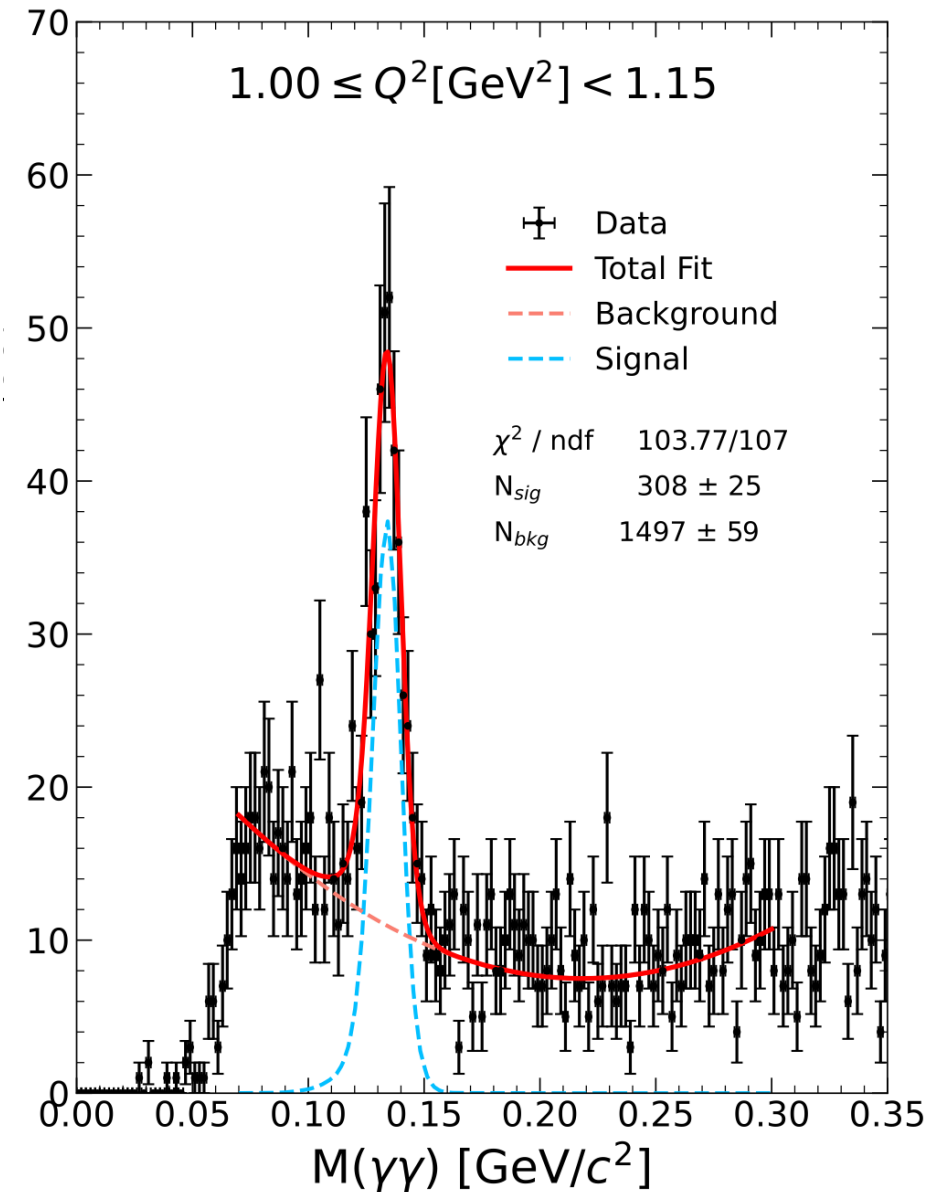


Study of $\gamma\gamma^* \rightarrow \pi^0$

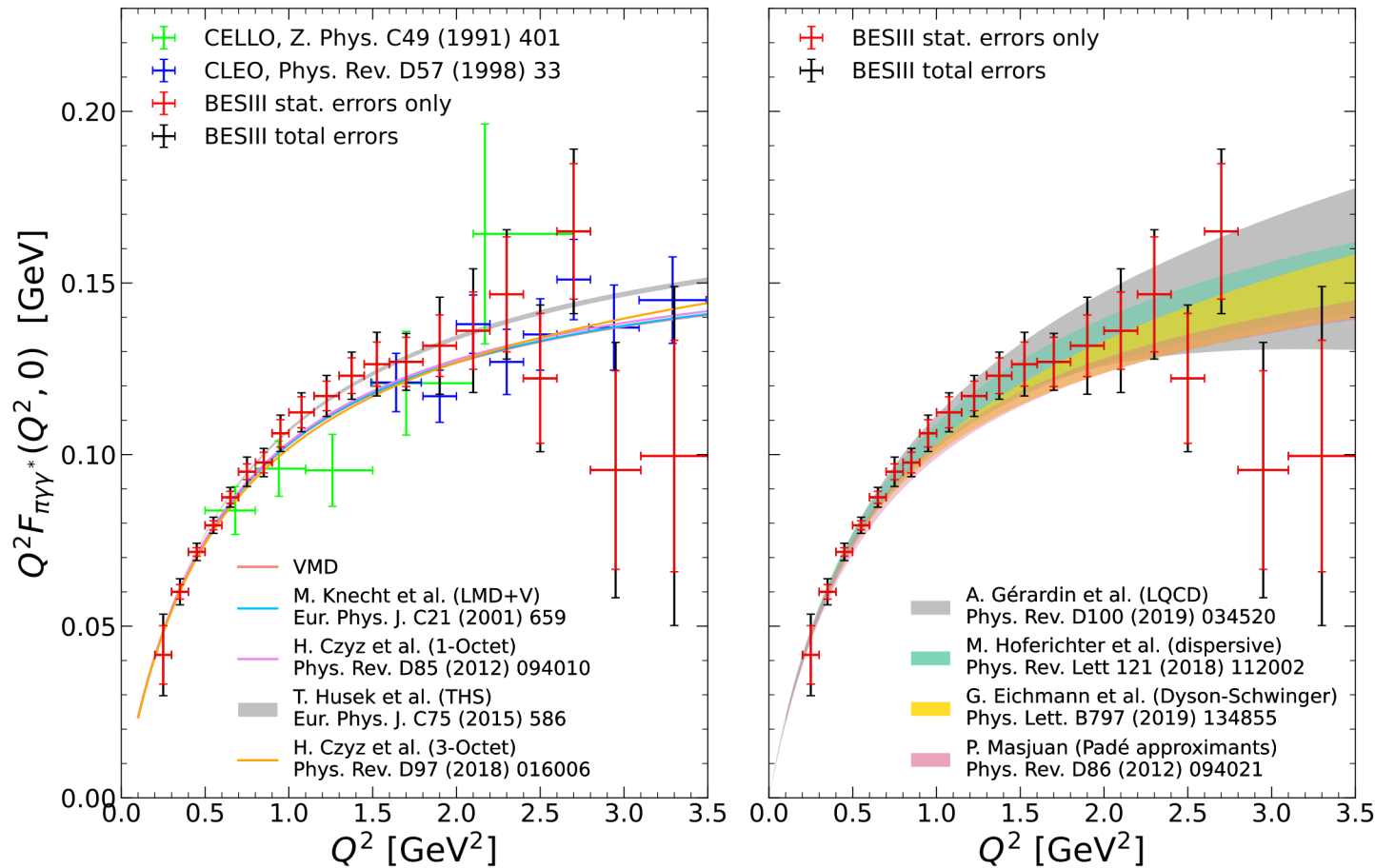
- Single Tag Measurement (2.9 fb^{-1} @ 3.773 GeV)
 - Require quasi-real photon
- Select ...
 - ... one final state lepton
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- Background suppression and subtraction

$$|F(Q^2, 0)|^2 = \frac{N}{\Delta Q^2 \textcolor{red}{L} \textcolor{green}{\varepsilon} (\textcolor{blue}{1} + \delta)} \left(\frac{d\sigma_{WZW}}{dQ^2} \right)^{-1}$$

- Normalize to **luminosity**, **efficiency**, **point-like cross section**, and **corrections for radiative effects and finite 2nd virtuality**



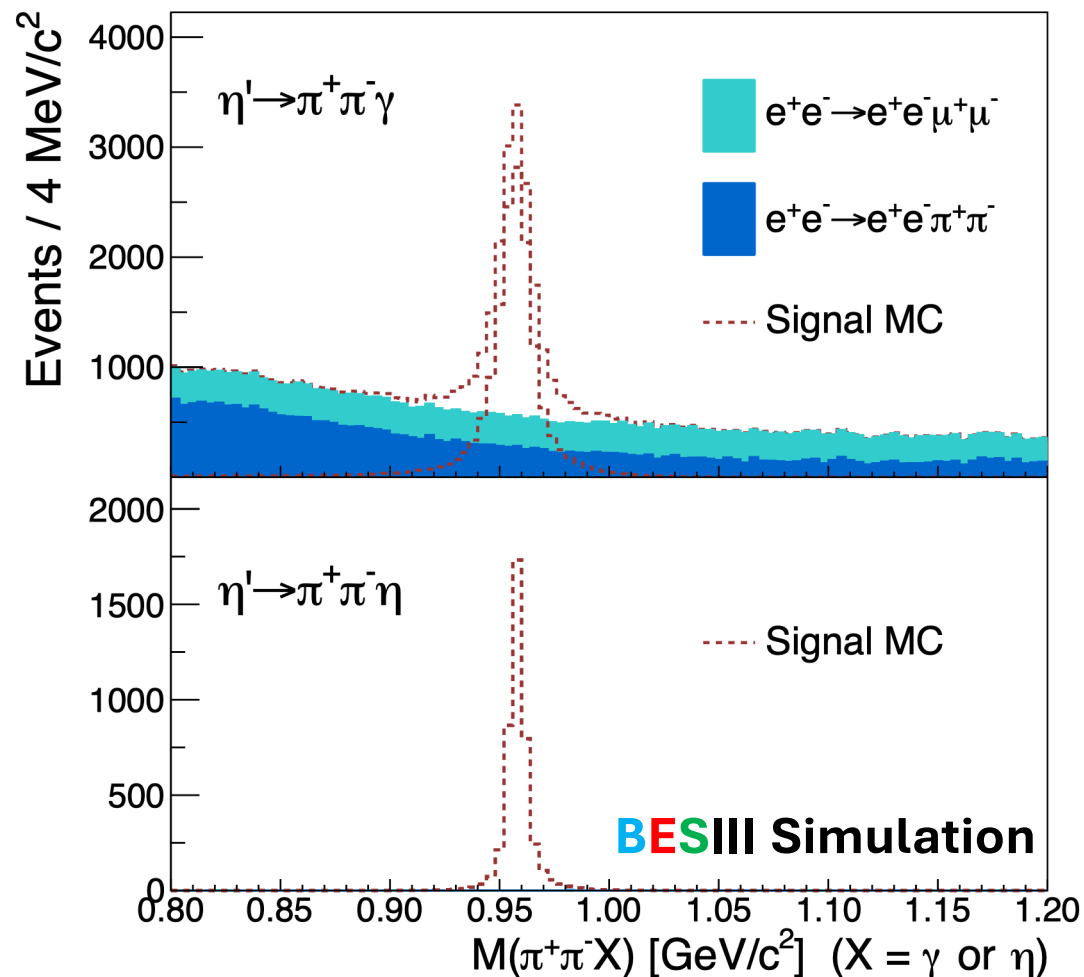
Study of $\gamma\gamma^* \rightarrow \pi^0$



- Good agreement with previous measurements
- Most precise measurement at small Q^2
- Excellent agreement with
 - Lattice QCD
 - Dispersive calculations
 - Dyson-Schwinger
- **To be repeated with 20 fb⁻¹!**
 - Also for η and η'

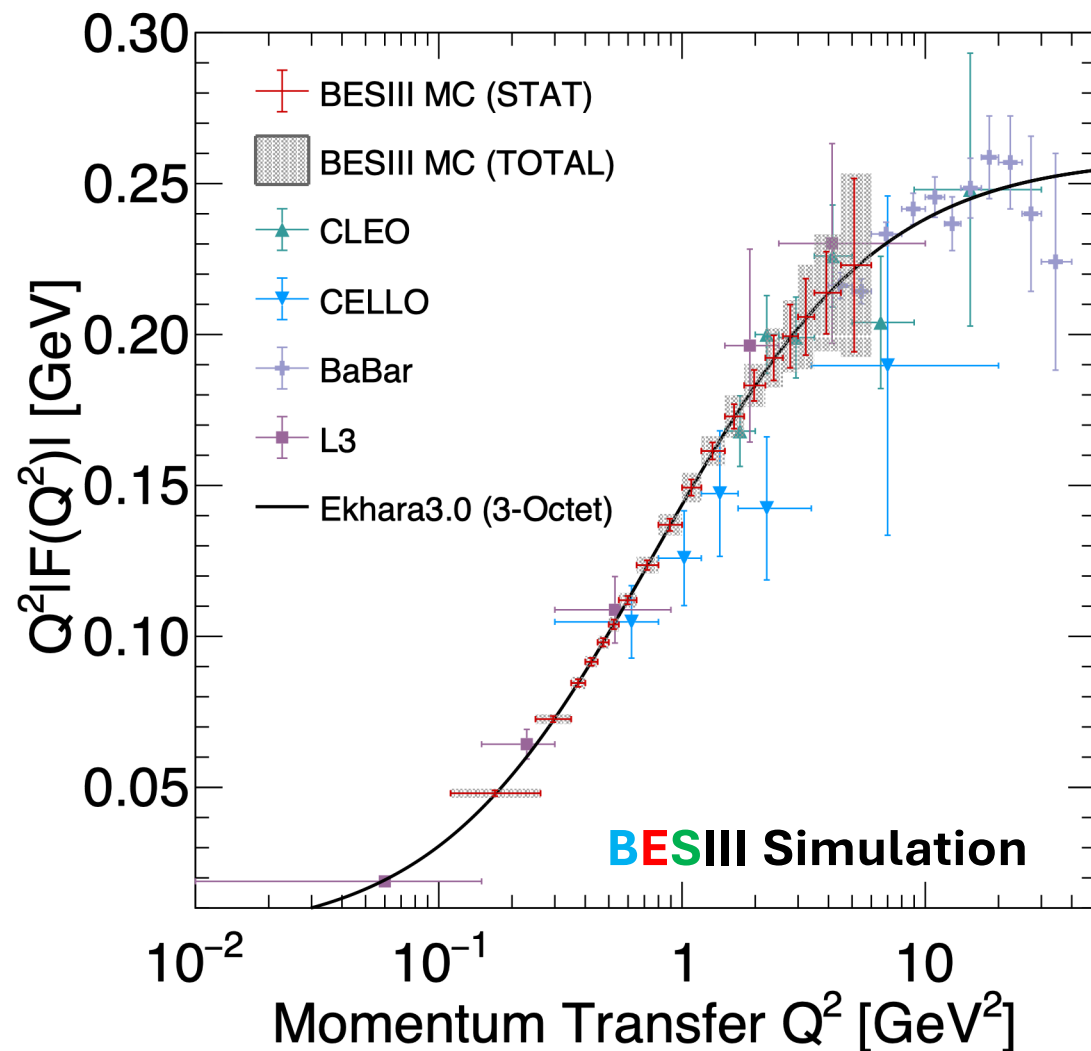
Study of $\gamma\gamma^* \rightarrow \eta'$

- Same principles as $\gamma\gamma^* \rightarrow \pi^0$ measurement
- Based on full 20.3 fb⁻¹ of 3.773 GeV data
 - 7x larger luminosity than π^0 measurement
- Selection of $\eta' \rightarrow \pi^+\pi^-\gamma$ and $\eta' \rightarrow \pi^+\pi^-\eta(\rightarrow \gamma\gamma)$
- **By far most precise measurement below 4 GeV²**
- **Close to publication**
(Spokesperson Review/last step)



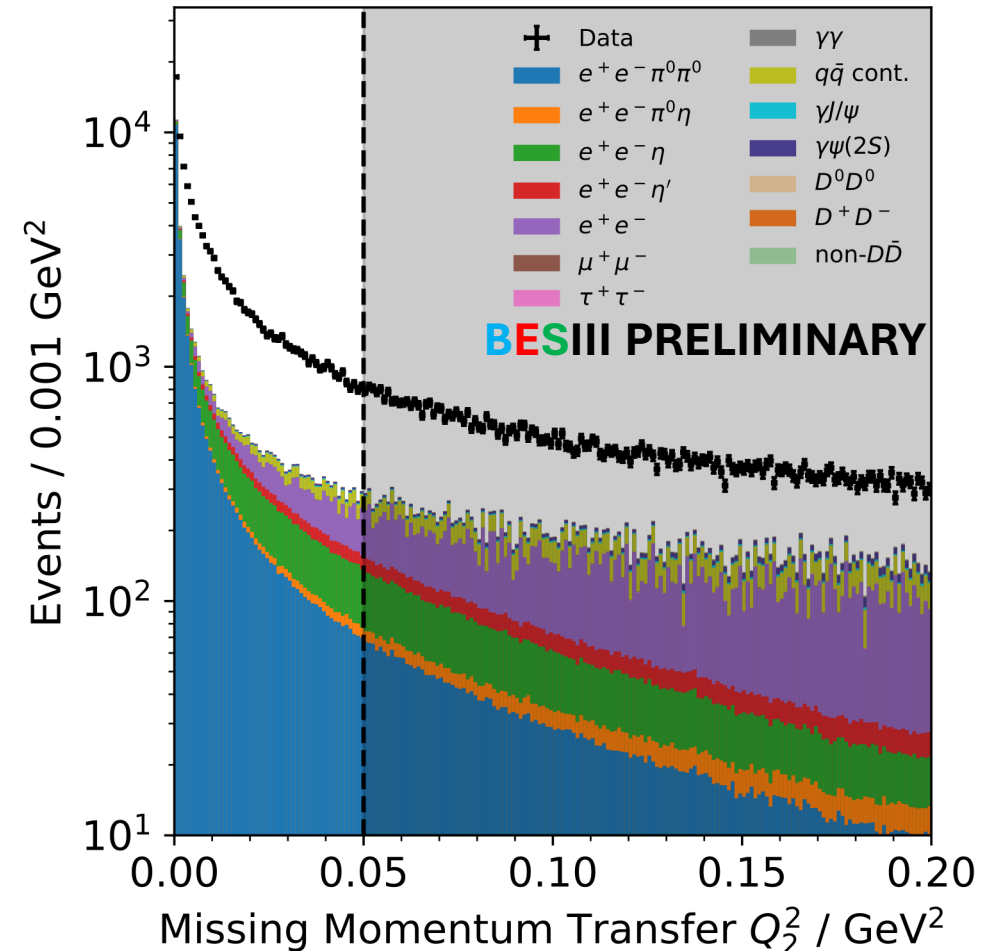
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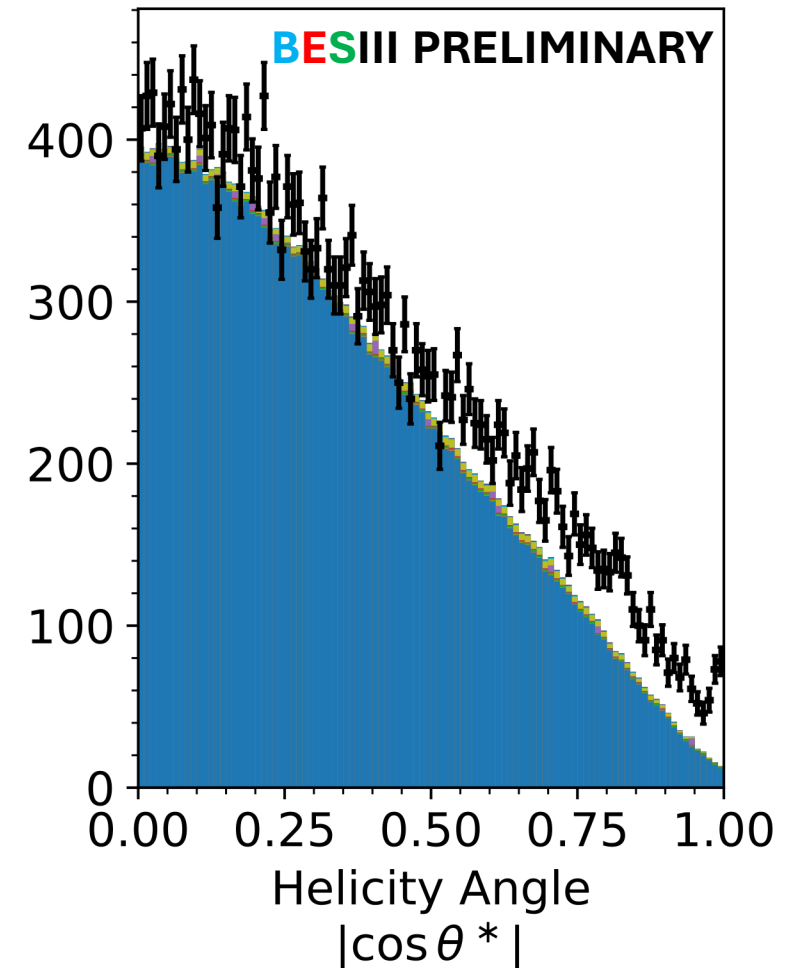
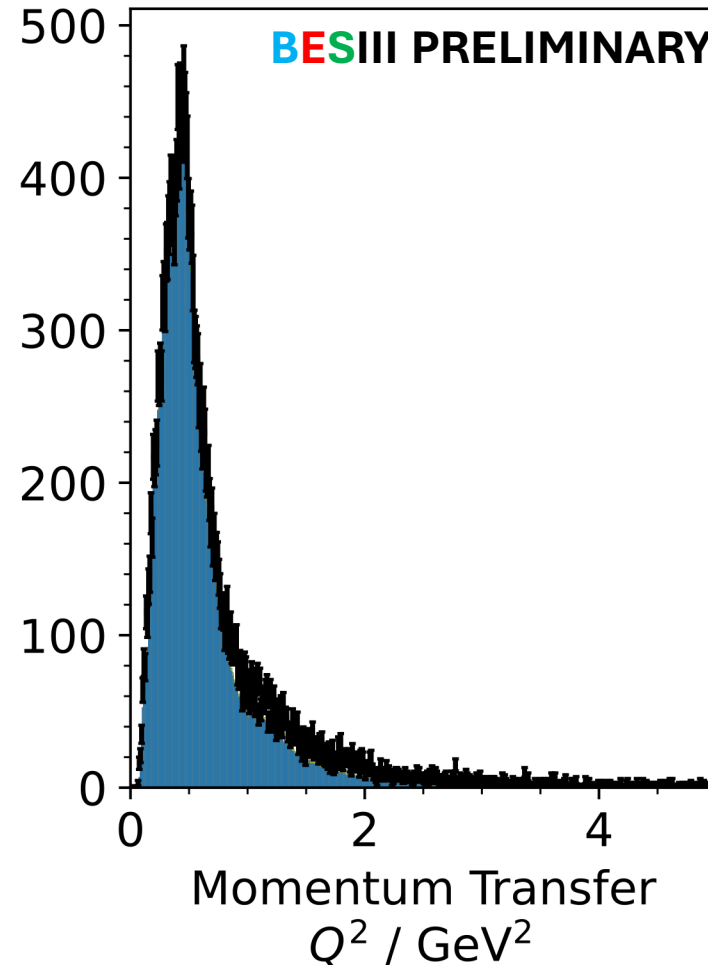
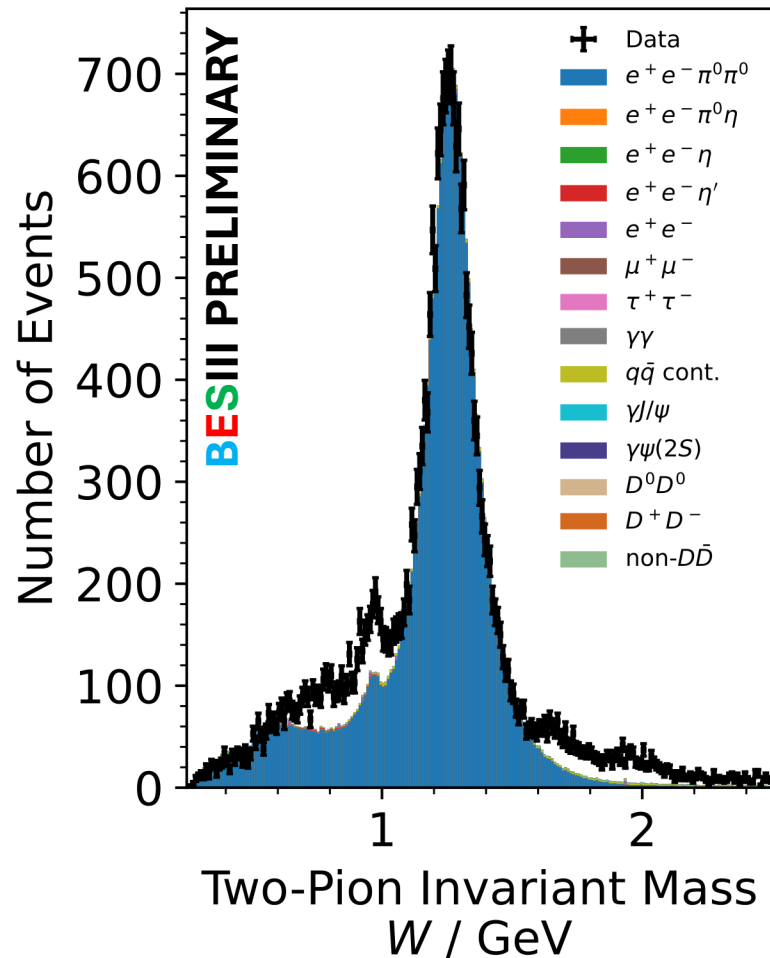
Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$

- Single Tag Measurement (20.3 fb⁻¹ @ 3.773 GeV)
- Select ...
 - ... one final state lepton
 - ... four photons (2x $\pi^0 \rightarrow \gamma\gamma$)
- Require vanishing second virtuality
 - Improved resolution of Q_{miss}^2 using kin. fit
→ Direct constraint of Q_{miss}^2
- Further suppression of background using kinematic constraints



Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$

Simulations according to [ML et. al. , Comp. Phys. Comm. 327, 110153, (2026)]



Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$

- Data driven background subtraction

$$\frac{d(\sigma_{TT} + \varepsilon \sigma_{LT})}{d|\cos \theta^*|} = \frac{N}{\Delta Q^2 \Delta W \Delta|\cos \theta^*| L \epsilon} \left(\frac{d^2 L_{TT}}{dQ^2 dW^2} \right)^{-1}$$

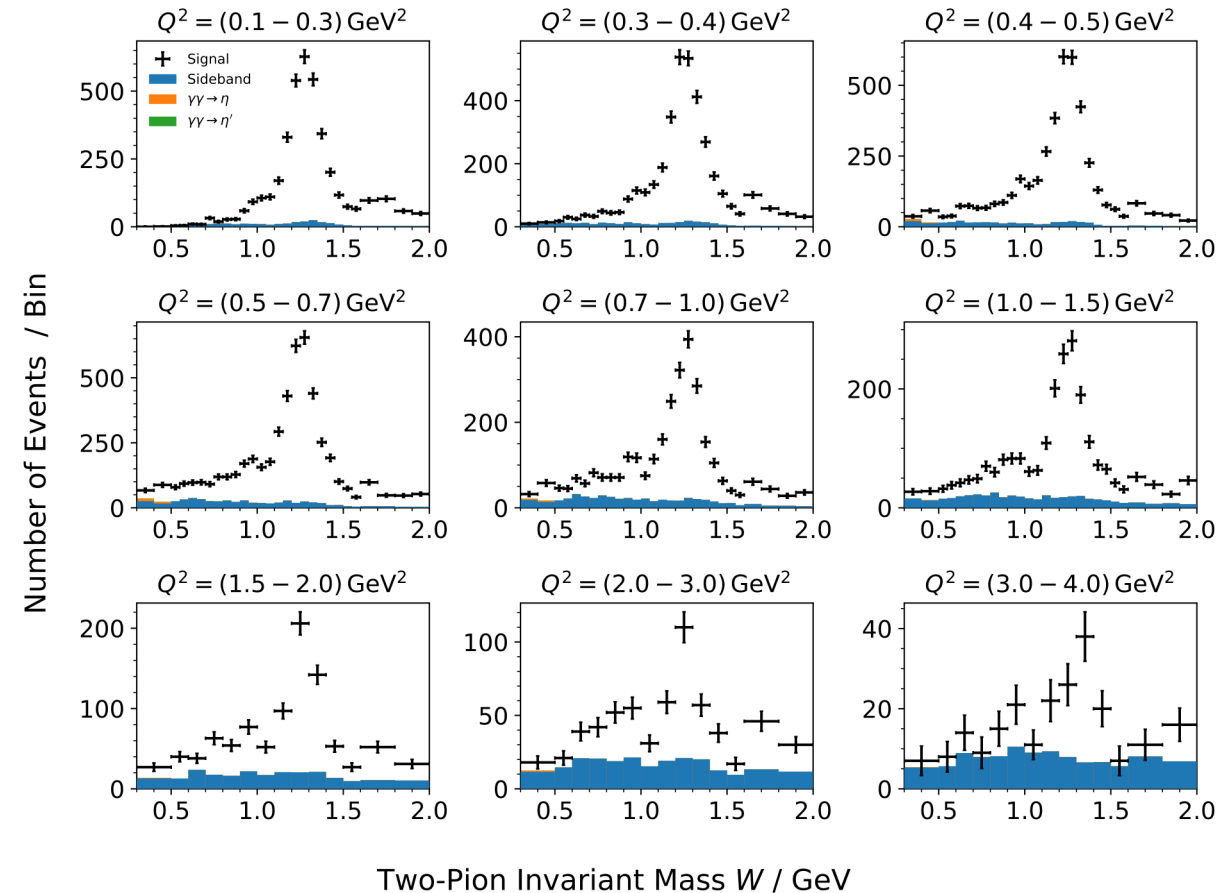
- Normalize event yield to

- Bin widths
- Integrated luminosity
- Reconstruction efficiency
- Luminosity Function

→ TT luminosity function → correct LT term

→ correction for finite second Q^2

BESIII PRELIMINARY



Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$

[Phys. Rev. D, vol. 101, no. 5, p. 054008, 2020,
Phys. Lett. B, vol. 789, pp. 366–372, 2019, Prog. Part. Nucl. Phys.,
vol. 107, pp. 20–68, 2019]

BESIII PRELIMINARY

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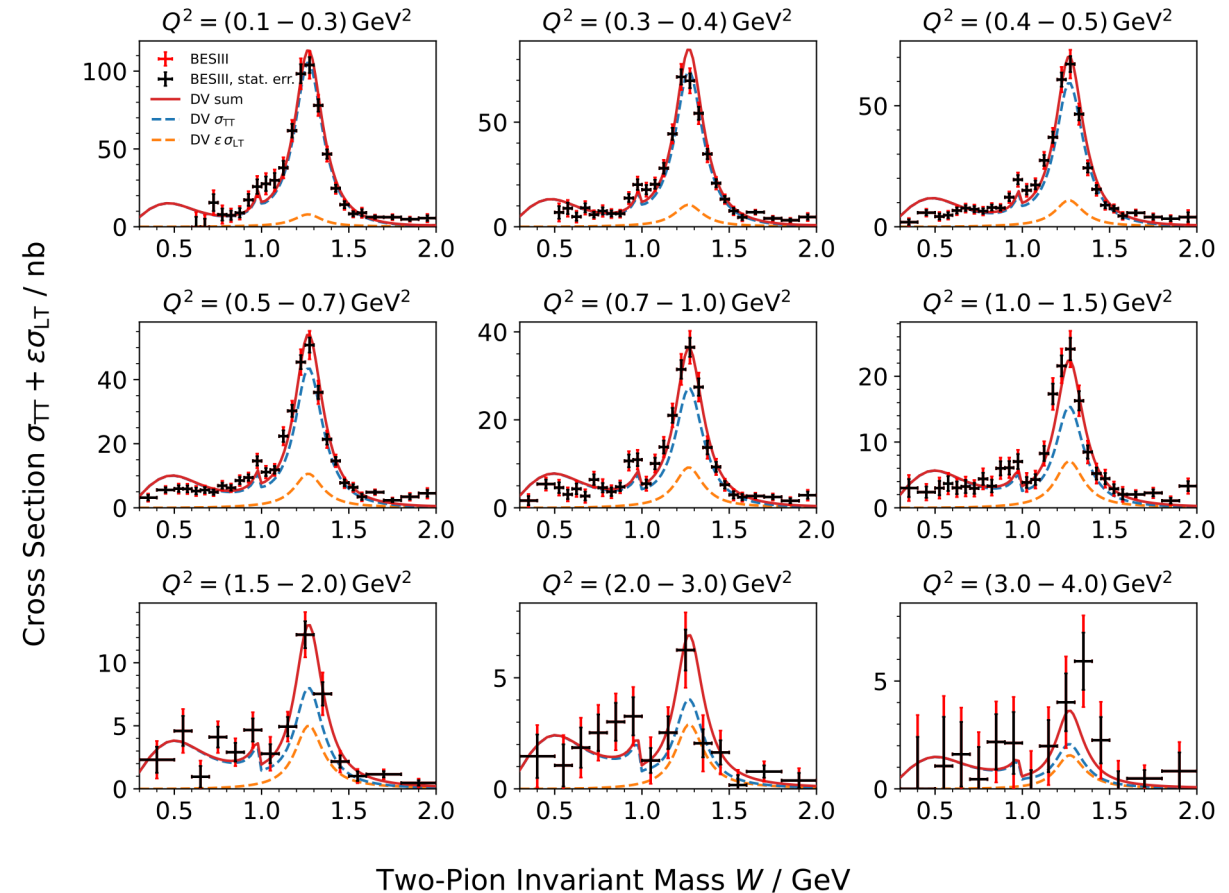
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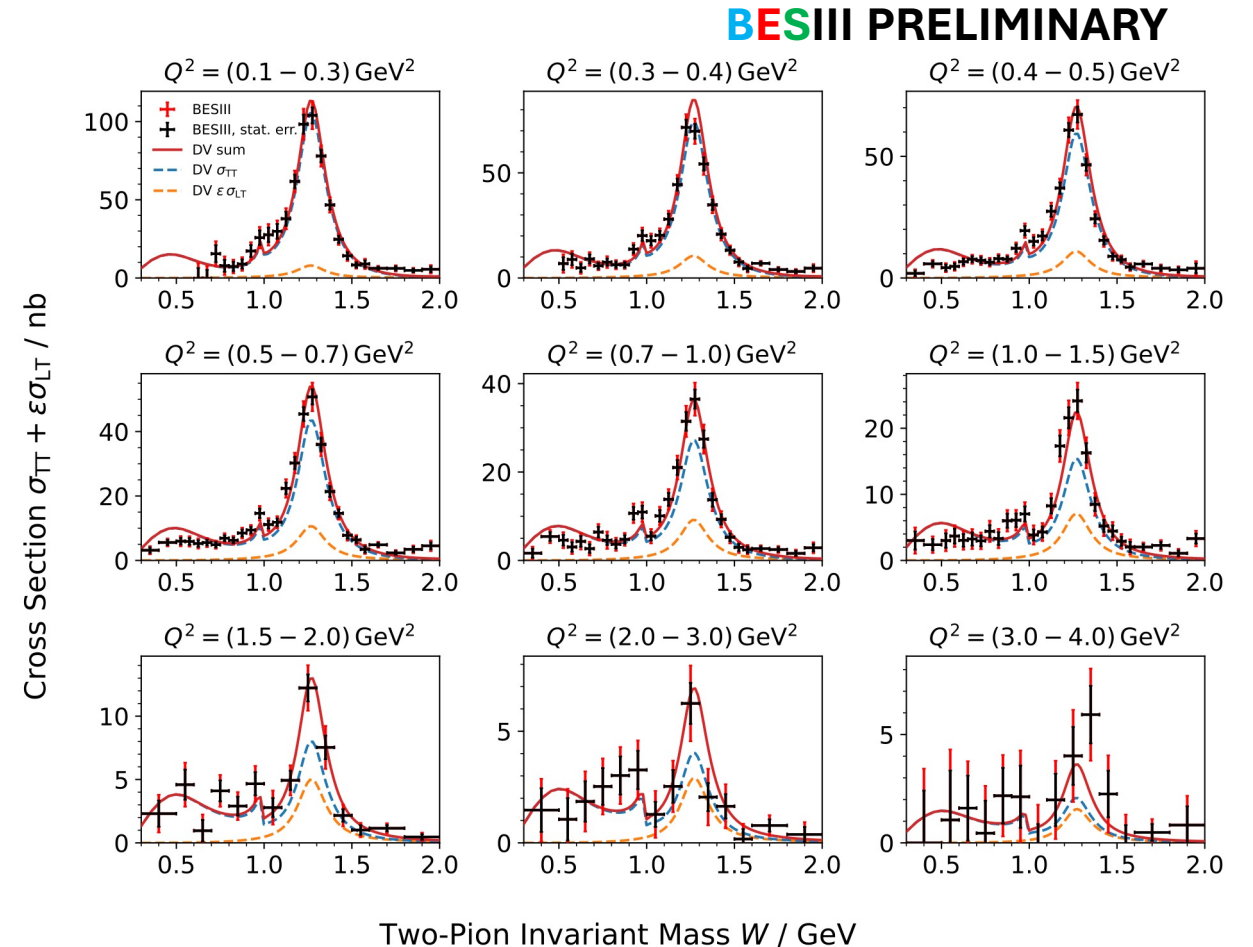
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Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$

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- The cross sections contain information on Transition Form Factors
- Different resonances:
 - Scalars: $f_0(500)$ & $f_0(980)$
 - Tensor: $f_2(1270)$
 - Others statistically irrelevant
- Extraction of TFFs through partial wave analysis

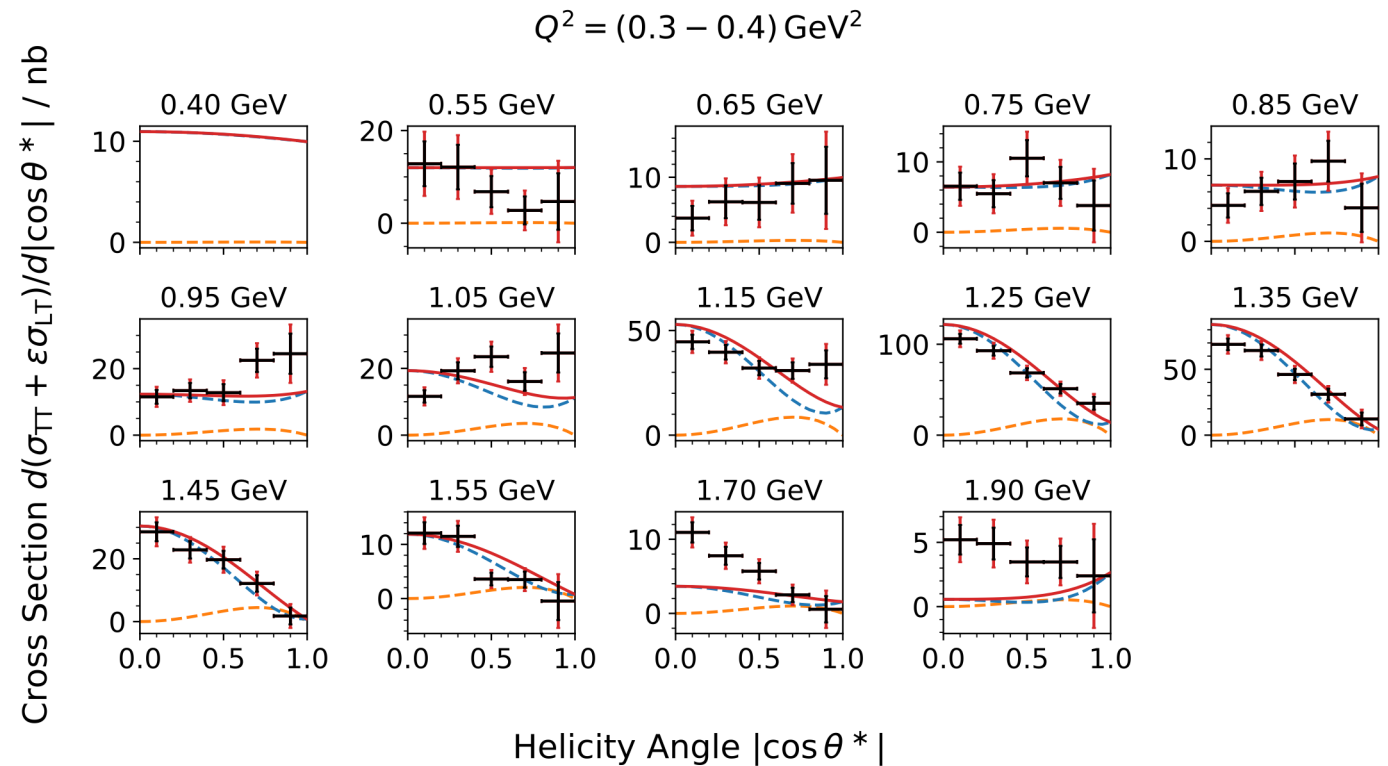


Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$

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Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$

- The cross sections contain information on Transition Form Factors

- Cross section's dependence on partial waves

$$\frac{d\sigma}{d|\cos\theta^*|} = 4\pi \left(1 + \frac{Q^2}{W^2}\right) \left(|S Y_0^0 + D_0 Y_2^0 e^{i\phi}|^2 + 2\varepsilon |D_1 Y_2^1|^2 + |D_2 Y_2^2|^2 \right)$$

- Different resonances:

- Scalars: $f_0(500)$ & $f_0(980)$
- Tensor: $f_2(1270)$
- Others statistically irrelevant

- Parameterization of S wave using Omnès function

- Provided by Danilkin et. al.

- Parameterization of $f_2(1270)$ by relativistic Breit-Wigner

- Extraction of TFFs through partial wave analysis

- TFFs are given by the ratio of amplitudes at Q^2 and $Q^2=0$

- Normalization needs to be determined
- Combined fit with untagged data

Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$

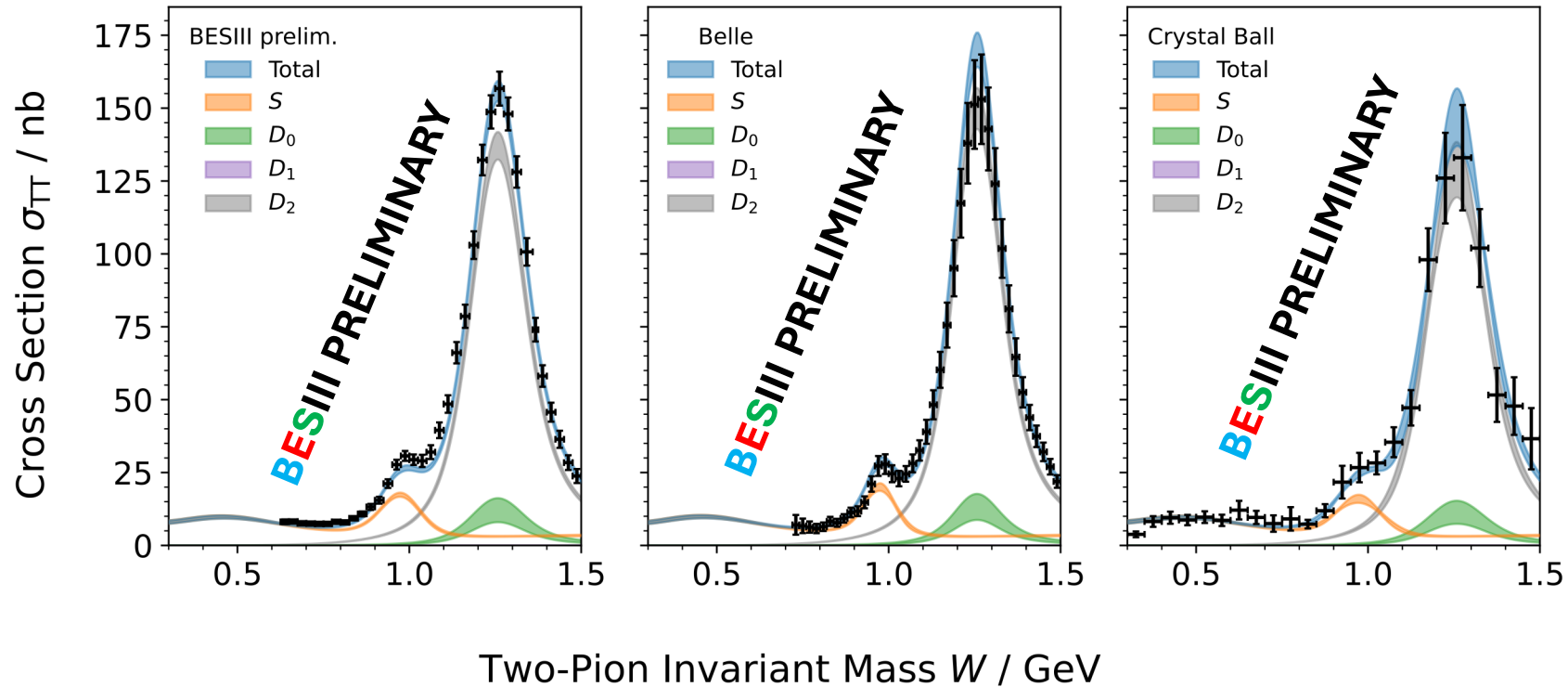
Fix normalization at $Q^2=0$
using untagged data!

$$Q^2 = 0 \quad |\cos \theta^*| < 0.8$$

EPJ Web Conf., vol. 291 (2024), p. 01002

Phys.Rev.D 79 (2009) 052009

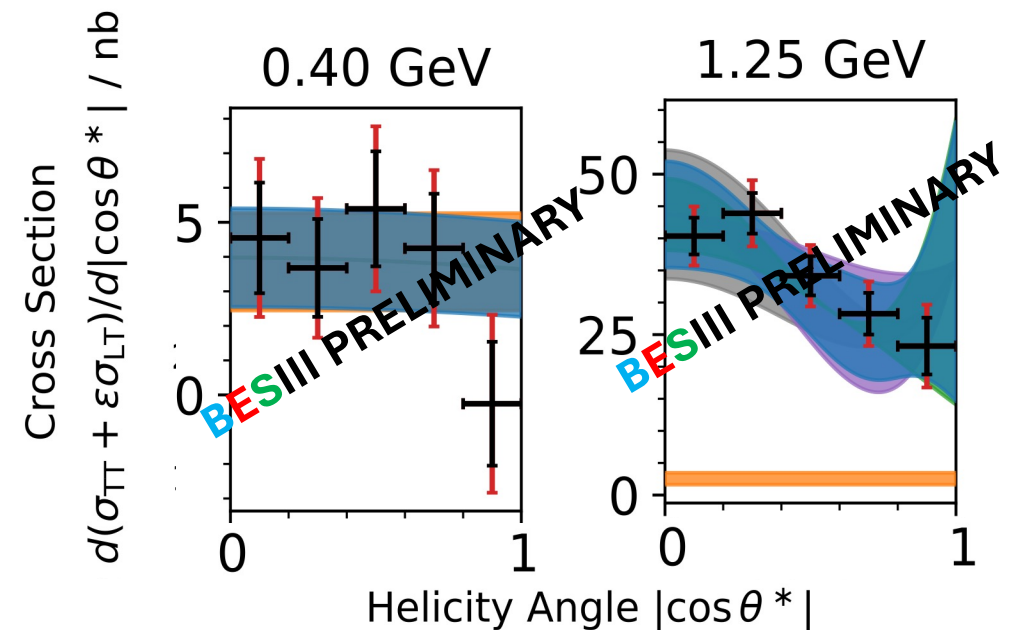
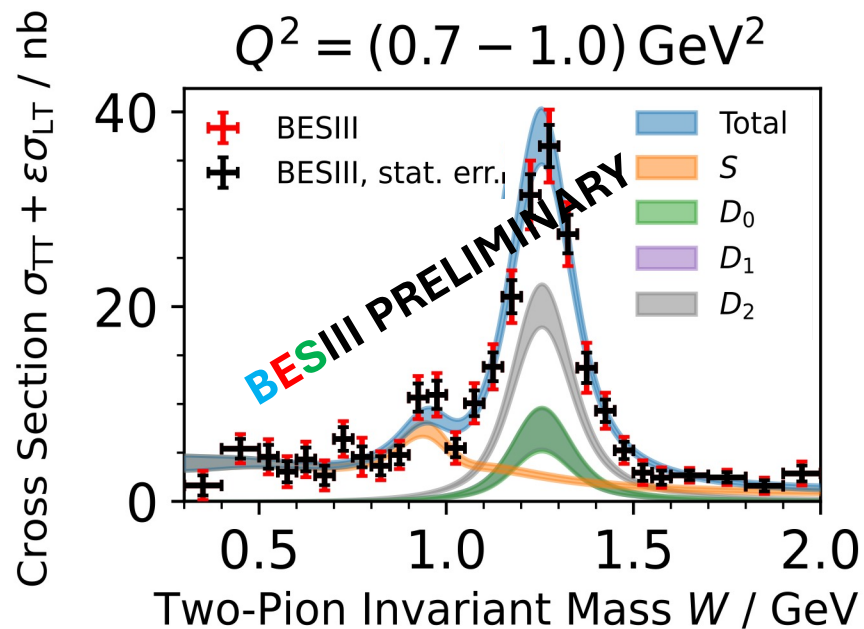
Phys.Rev.D 41 (1990) 3324



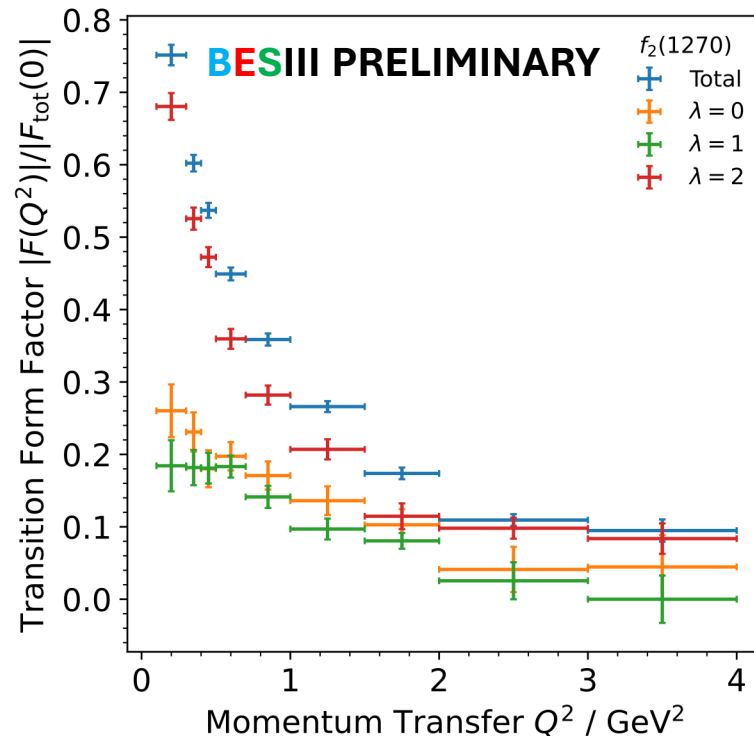
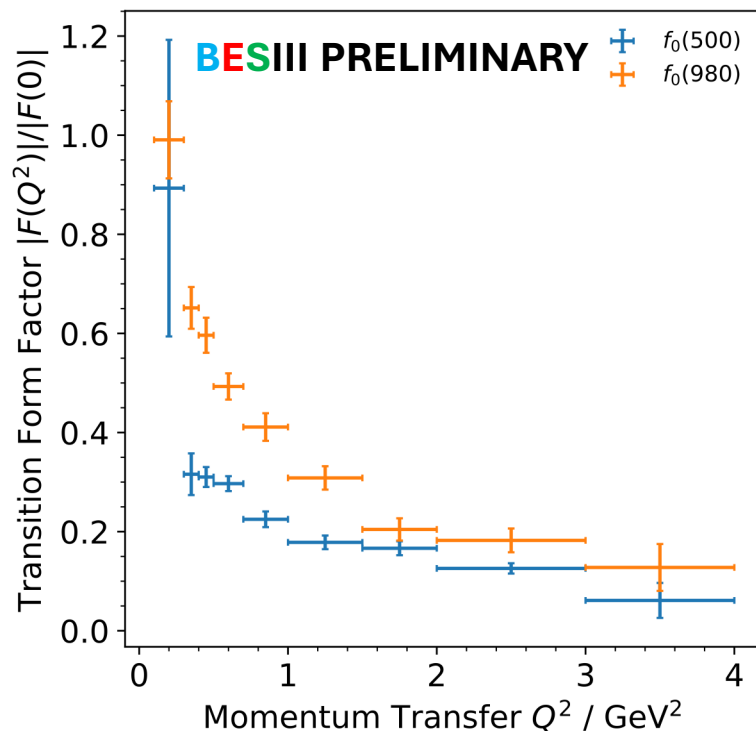
Convolution of the fit function with Gaussian function to
compensate smearing effects

Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$

- Fit to single tagged data to extract Q^2 dependence of TFFs
- Angular modulations used to differentiate resonances
- **Smearing effects taken into account**



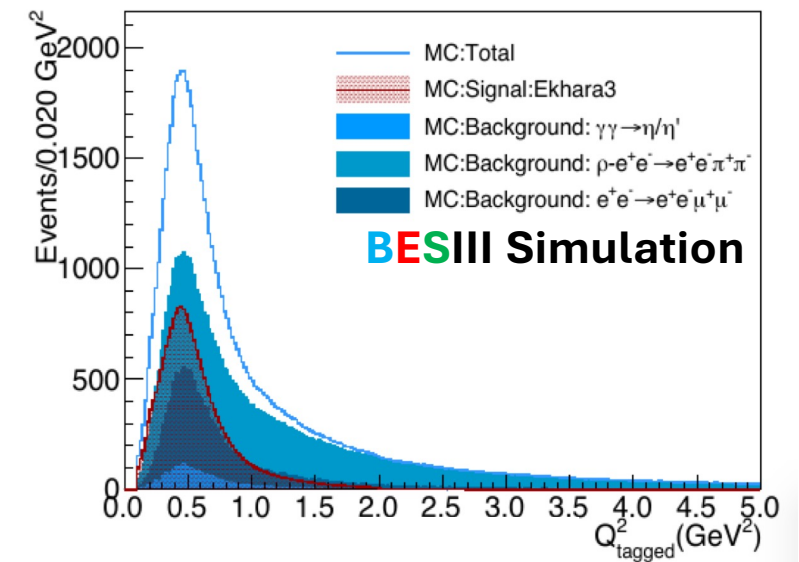
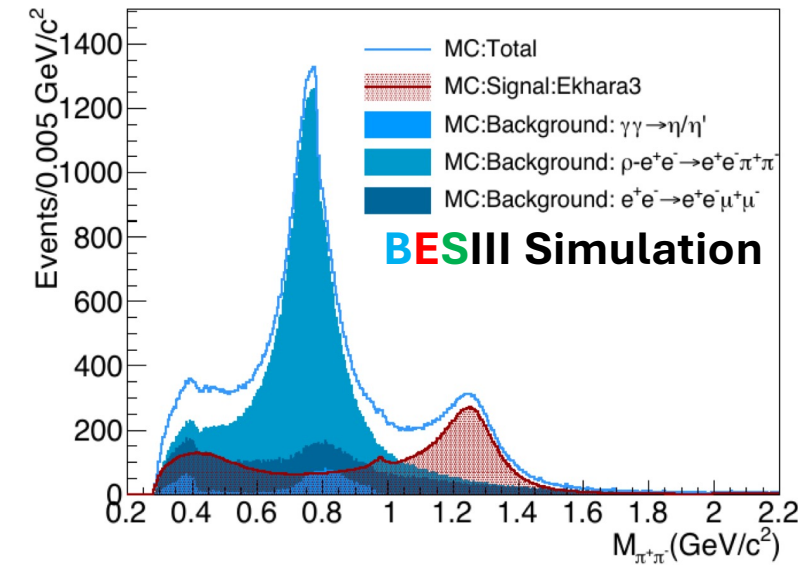
Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^0\pi^0$



- First extraction of TFFs in a_μ relevant Q^2 range
- First measurement of $f_0(500)$ TFF
- Lowest Q^2 and most precise measurement of $f_0(980)$ and $f_2(1270)$ TFFs
 - Compatible with high Q^2 Belle data!

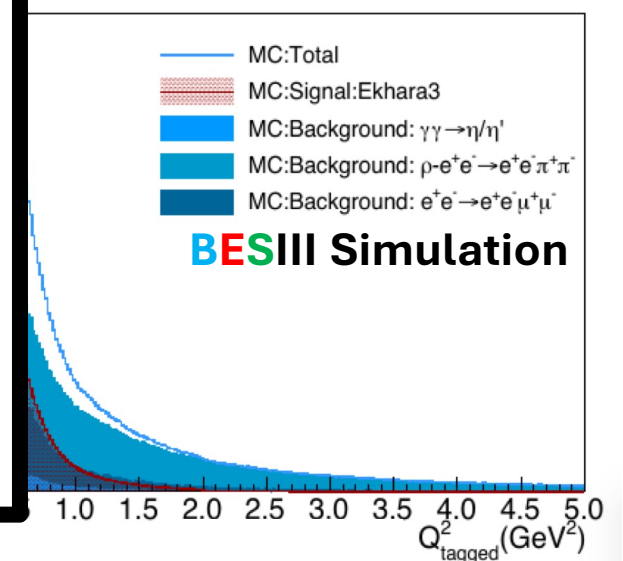
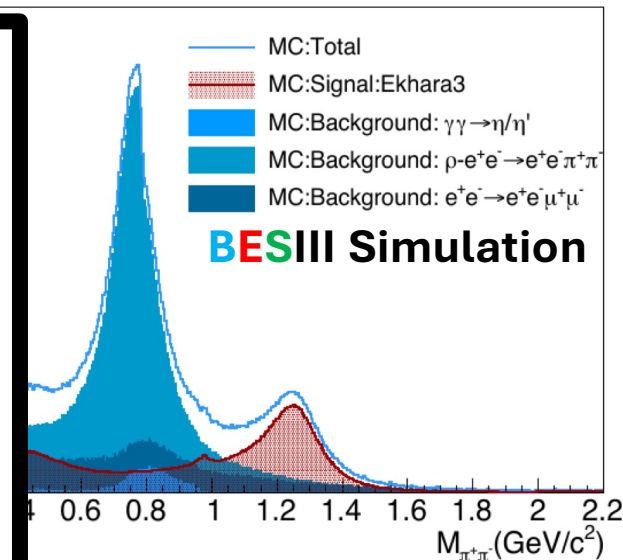
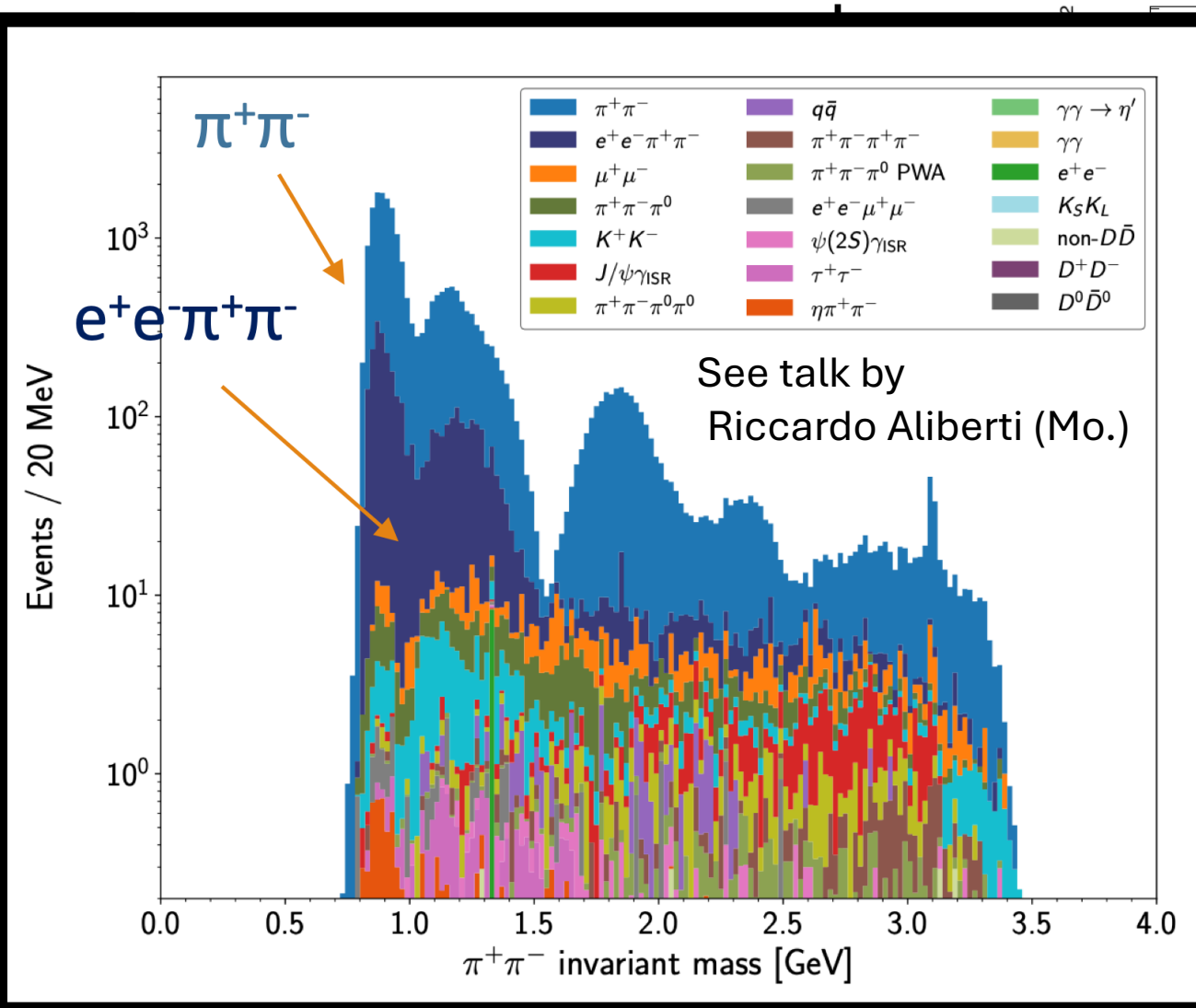
Study of $\gamma\gamma^* \rightarrow f_0/f_2 \rightarrow \pi^+\pi^-$

- The same measurement with charged pions is work in progress
- Significant irreducible background in $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$
 - Dominating source of events
 - Must be subtracted using Monte Carlo
 - **Limited by knowledge of pion-vector form factor**
- Similar phase space coverage as neutral-pion measurement but higher statistics



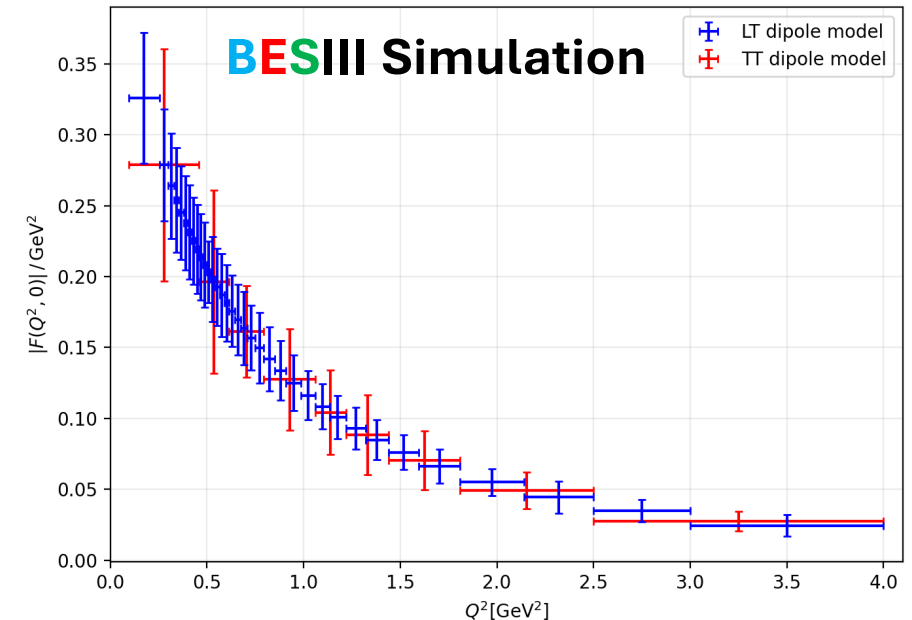
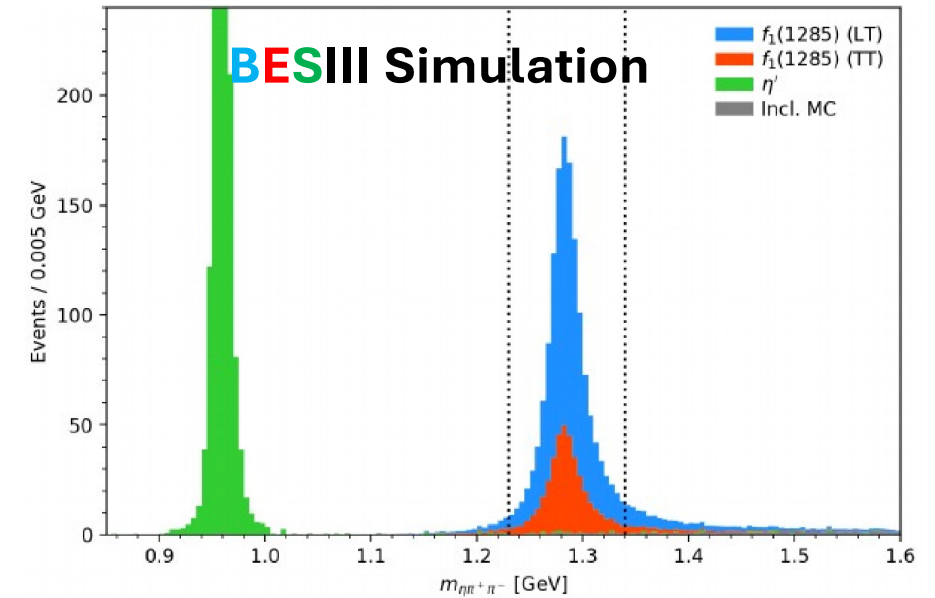
Study of

- The same meas in progress
- Significant irreducible $e^+e^-\pi^+\pi^-$
 - Dominating source
 - Must be subtracted
 - Limited by know
- Similar phase sp measurement b

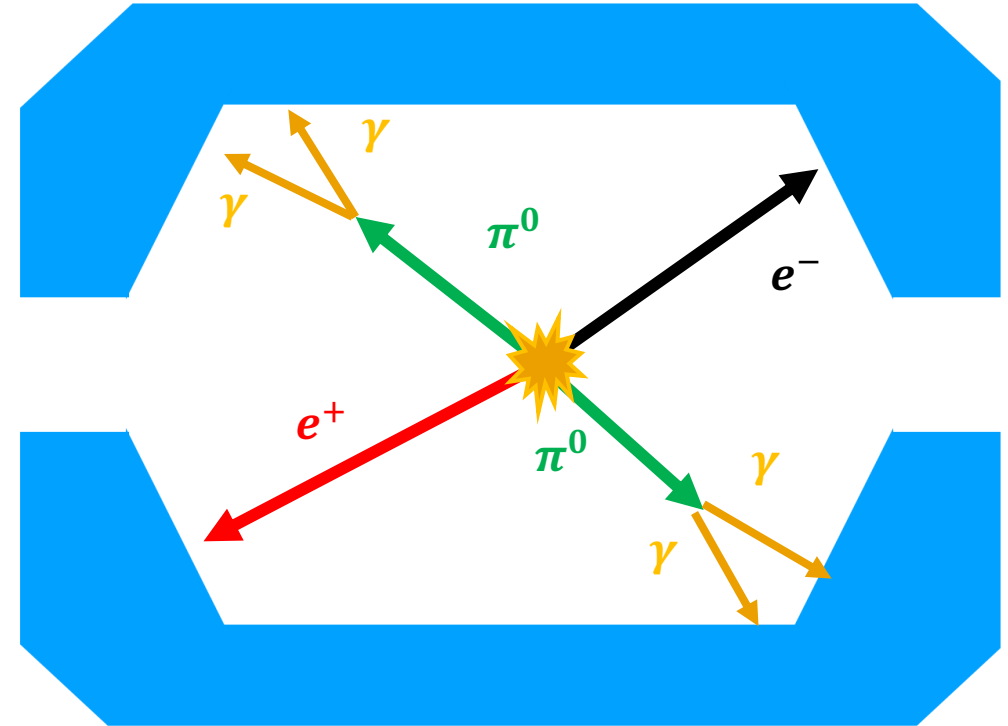


Study of $\gamma\gamma^* \rightarrow f_1(1285)$

- Single tagged study of $f_1(1285) \rightarrow a_0\pi / f_0\eta \rightarrow \pi^+\pi^-\eta$
- Simulation and partial wave analysis based on the phenomenological framework by X.-L. Ren et. al.
[Phys. Rev. D 110 (2024) 094043]
- Separation of TT and LT $f_1(1285)$ TFFs
- **First measurement of the $f_1(1285)$ TFFs**

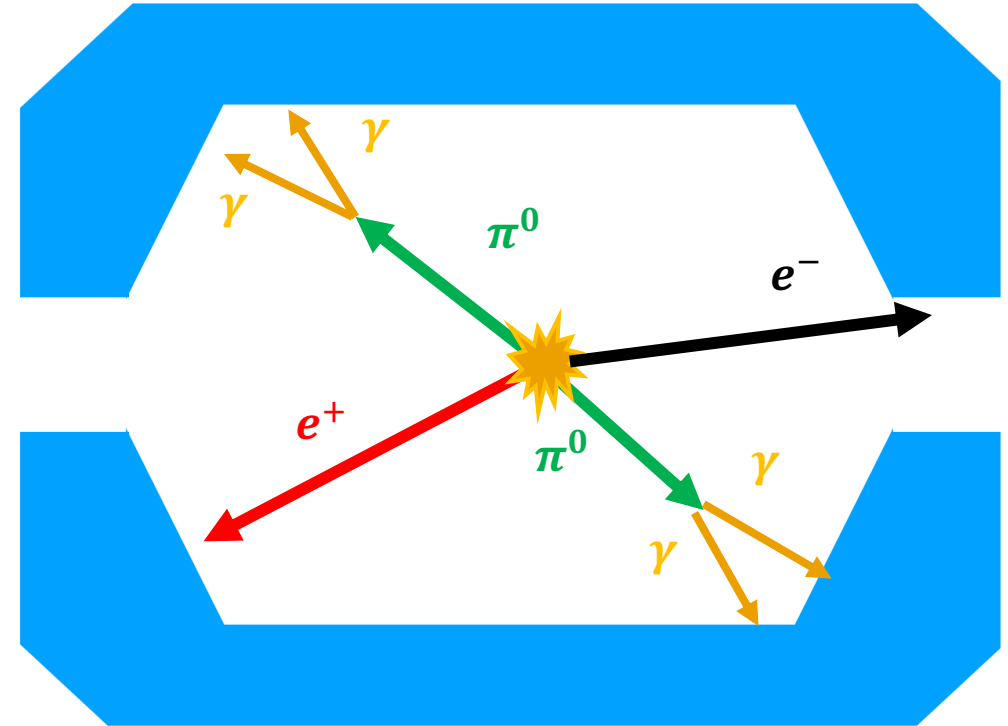


Double-Tagged Measurements



Not-So-Double-Tagged Measurements

New Idea: Use non finite second virtuality from single tagged events for double virtual studies!

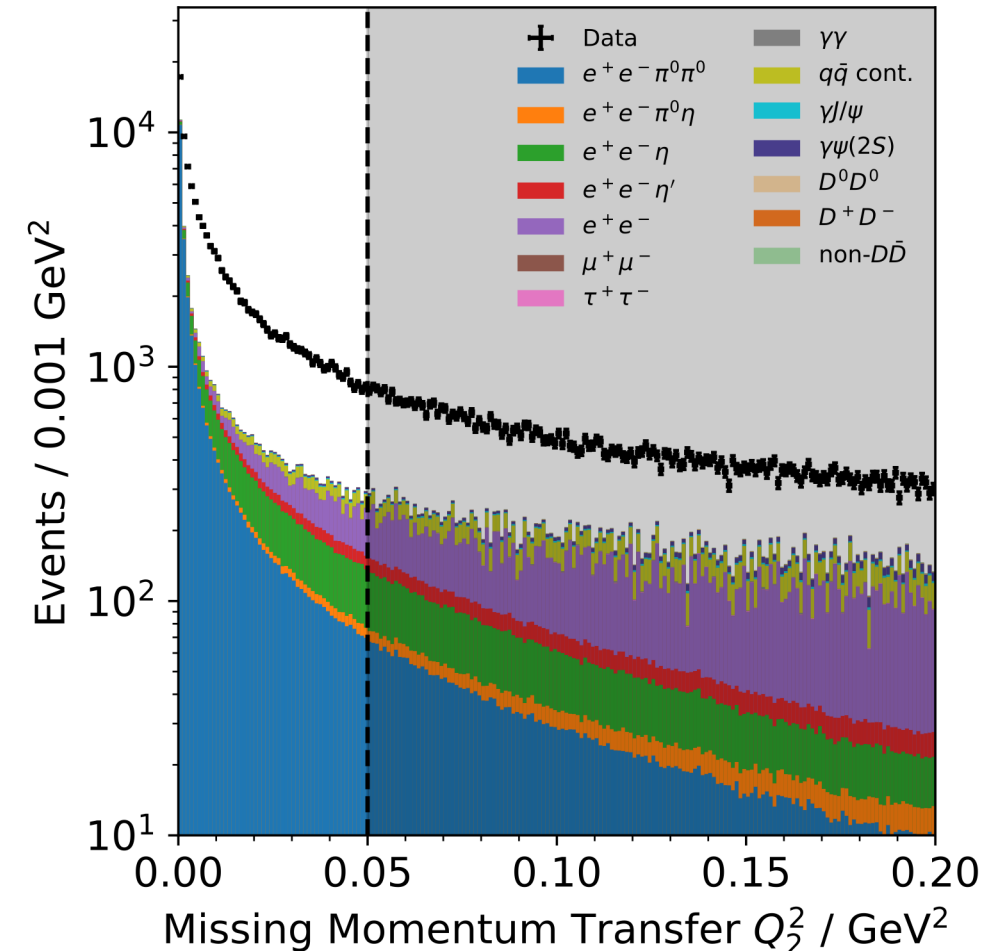


Study of $\gamma^* \gamma^* \rightarrow f_2(1270) \rightarrow \pi^0 \pi^0$

BESIII Simulation

Use the same selection as single tagged $\gamma \gamma^* \rightarrow \pi^0 \pi^0$ study, apply different missing Q^2 constraint!

- More difficult background situation
 - Single virtual events need to be subtracted!
- Large background contamination in mass range of scalar resonances
 - Focus on $f_2(1270)$ mass range

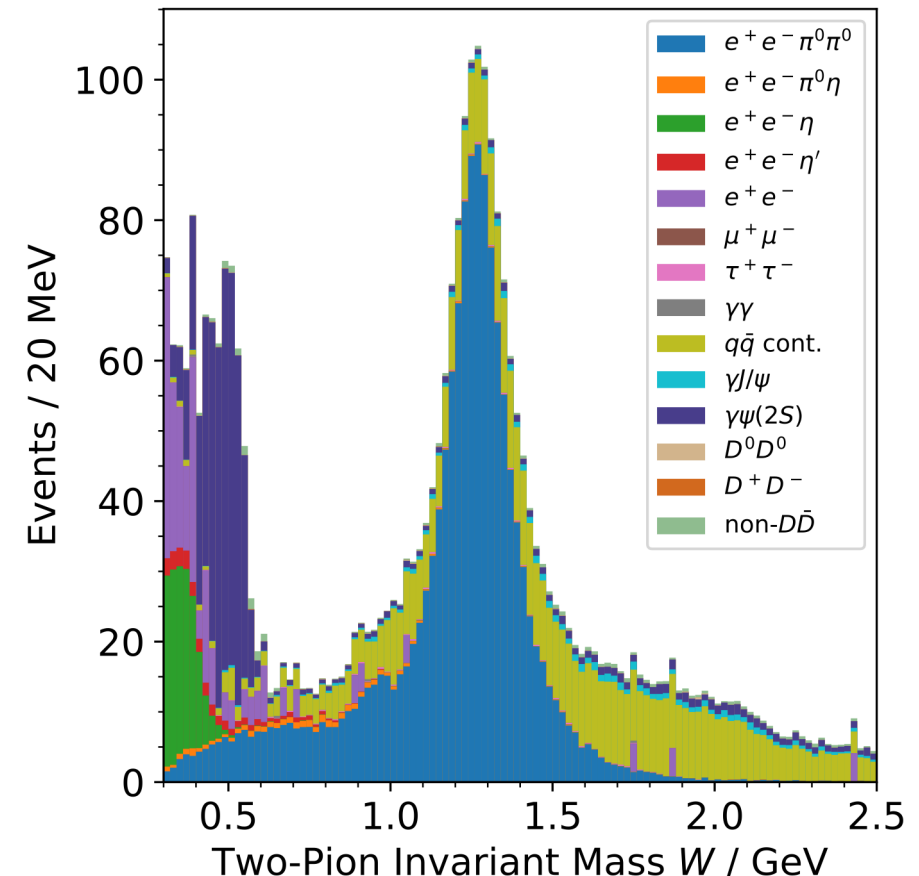


Study of $\gamma^* \gamma^* \rightarrow f_2(1270) \rightarrow \pi^0 \pi^0$

BESIII Simulation

Use the same selection as single tagged $\gamma \gamma^* \rightarrow \pi^0 \pi^0$ study, apply different missing Q^2 constraint!

- More difficult background situation
 - Single virtual events need to be subtracted!
- Large background contamination in mass range of scalar resonances
 - Focus on $f_2(1270)$ mass range

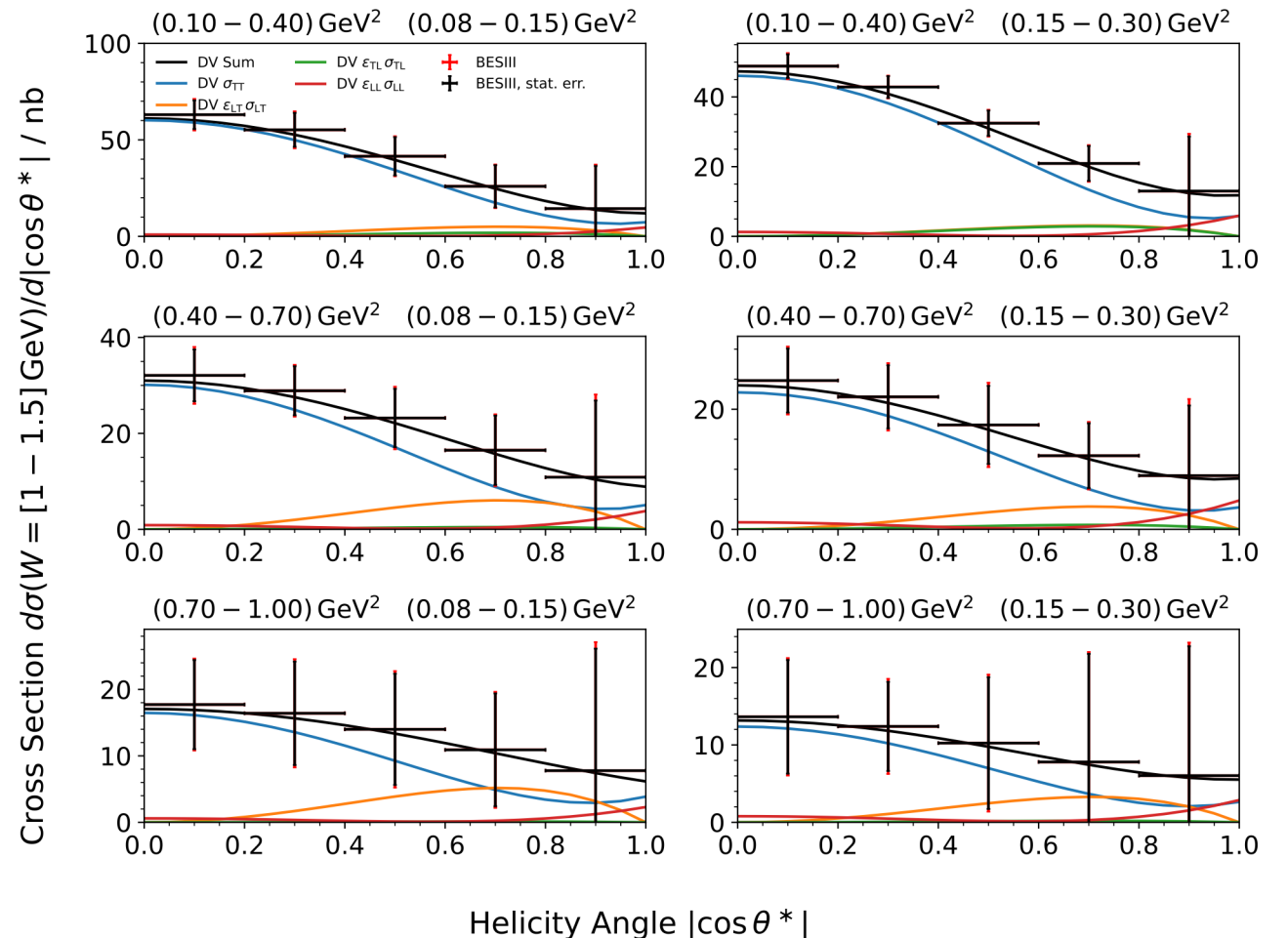


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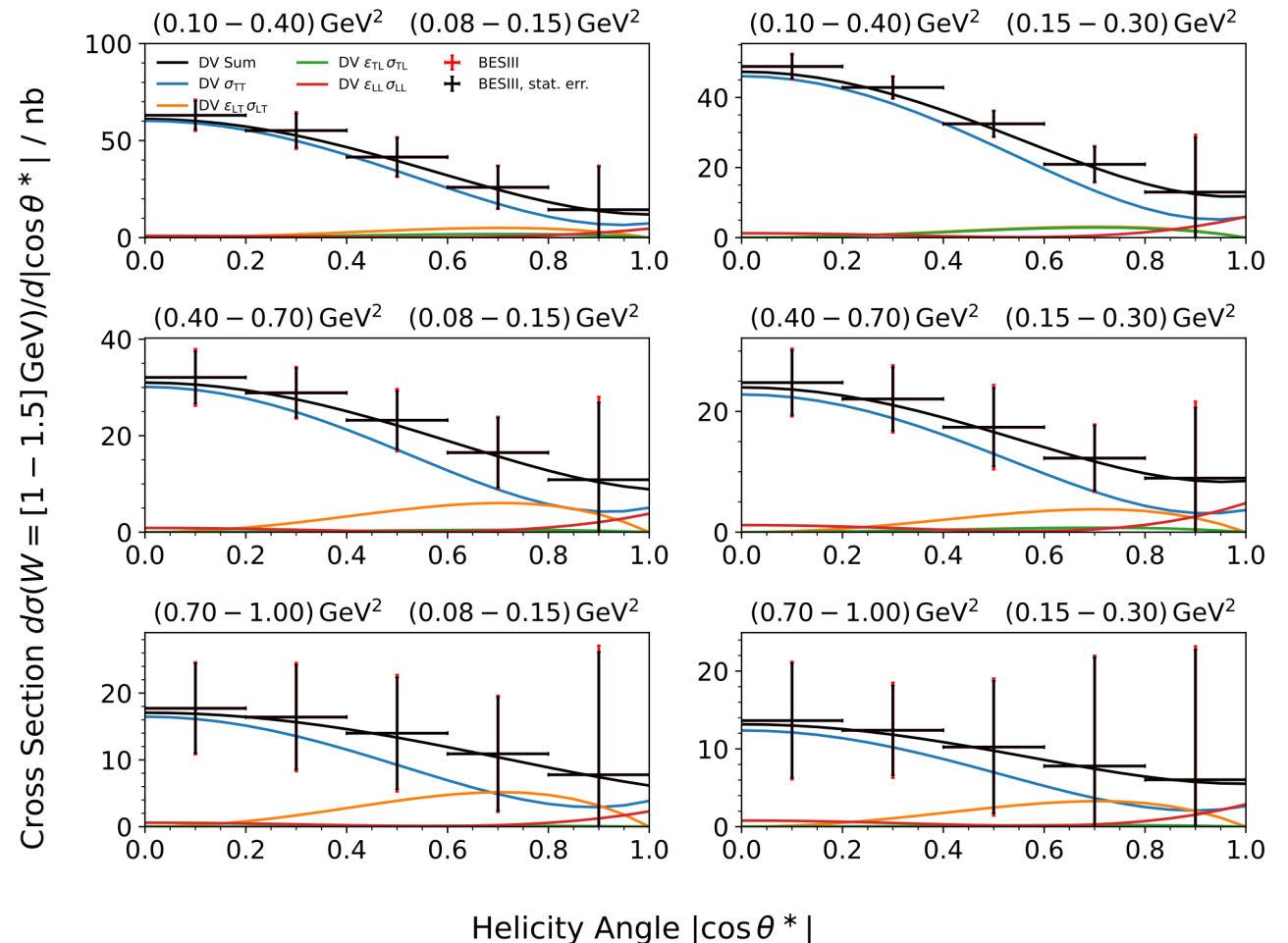
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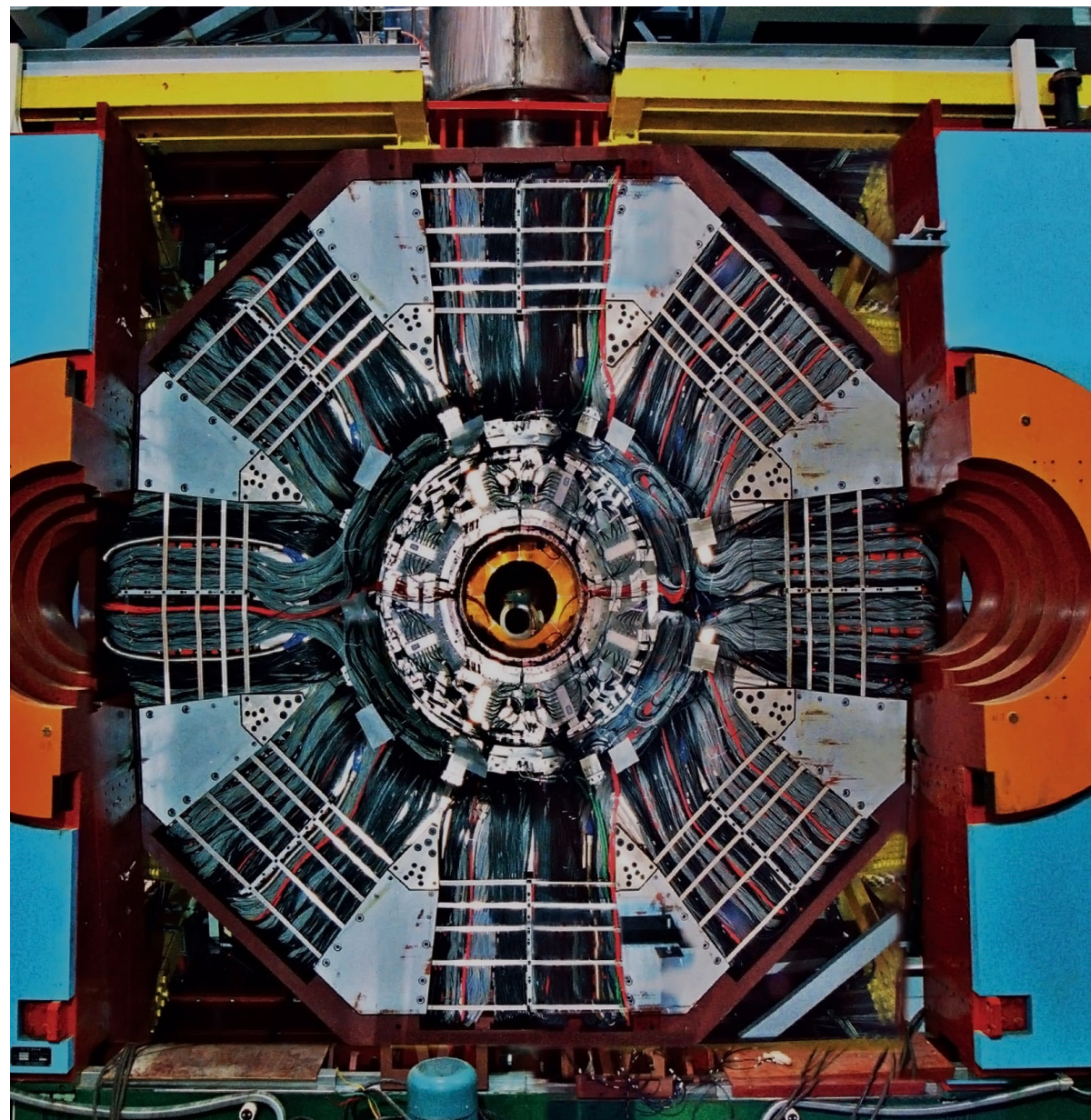
- More difficult background situation
 - Single virtual events need to be subtracted!
- Large background contamination in mass range of scalar resonances
 - Focus on $f_2(1270)$ mass range

Applicable to other final states!



Summary

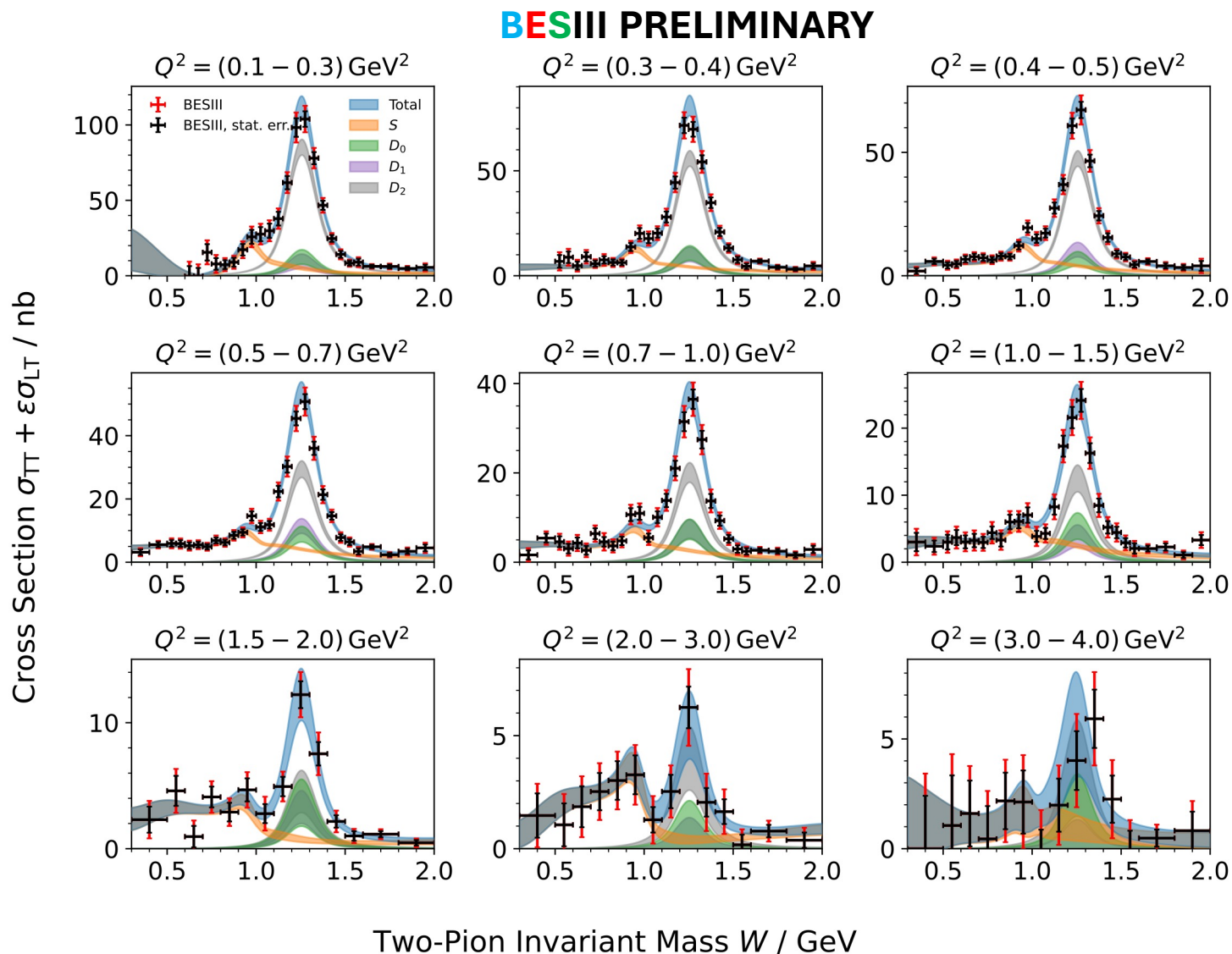
- BESIII is perfectly suited to provide data on space-like TFFs at small to intermediate Q^2
- World best measurement of $\pi^0, f_0(500), f_0(980),$ and $f_2(1270)$ TFFs
- **New technique to access double virtual events**
- Several measurements in preparation/planned:
 - $\gamma\gamma^* \rightarrow \pi^0, \eta, \eta', \pi^+\pi^-, \pi^0\eta \dots$
 - $\gamma\gamma^* \rightarrow f_1(1285) \rightarrow a_0(980)\pi/f_0(500)\eta$
 - $\gamma^*\gamma^* \rightarrow \pi^0, \eta, \eta', \pi^0\pi^0$
 - $\gamma\gamma^* \rightarrow K\bar{K}\pi, \pi^+\pi^-\pi^0$
 - Study of azimuthal angular dependencies
 - ...
- **More data to come: 10 fb⁻¹ @ 4.68 GeV and possible BEPCII-UP upgrade**



Backup

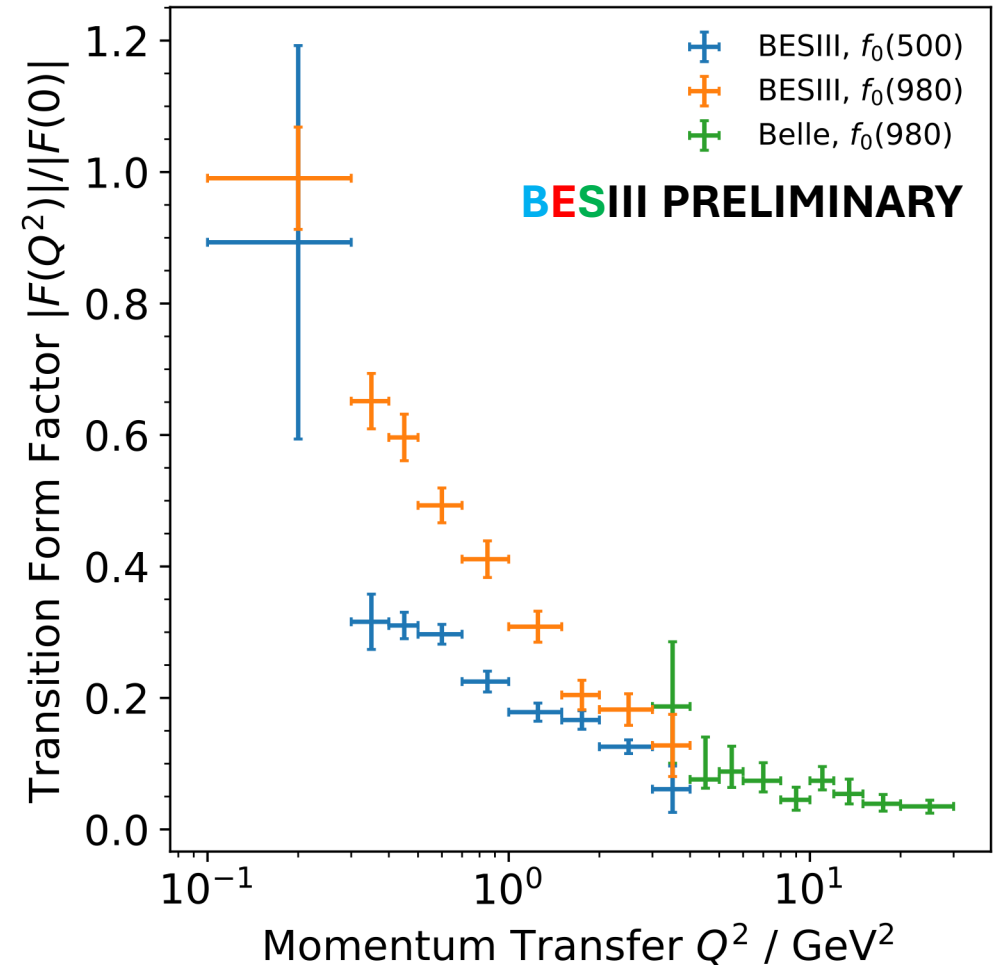
Fit Results

- Fit converges with $\chi^2 / ndf = 736 / 1449$
 - Large systematic uncertainties of untagged Belle measurement
- Good description of all structures
- Normalization mostly fixed by untagged Belle results
- BESIII preliminary and Crystal Ball untagged results necessary to fix smearing and normalization of $f_0(500)$ and $f_0(980)$ TFFs



Comparison To Belle

- First determination of $f_0(500)$ TFF!
- First measurement of $f_0(980)$ TFF below 3 GeV^2
- Good agreement to Belle $f_0(980)$ data
- Large uncertainty of $f_0(500)$ TFF at small momentum transfers
 - Missing constraints at small W and small Q^2



Comparison To Belle

- First measurement below 3 GeV^2
- Helicity-2 component dominant
- Very slight tension to existing helicity-1 Belle data
- Otherwise the data seems to be described by a smooth distribution
- "Plateau" in helicity-1 TFF
 - Drop towards zero required by Landau-Yang?

BESIII PRELIMINARY

