The hadronic light-by-light contribution to the muon $g-2$ using staggered fermions

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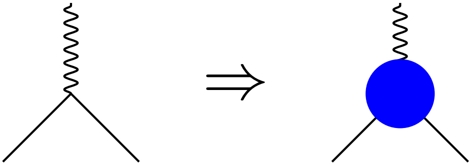
The anomalous magnetic muon moment g_μ

- For Dirac fermions $g_\mu = 2$ at the tree-level.
- Deviation by quantum corrections of the fermion photon vertex:



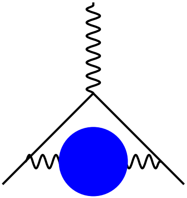
- Corrections quantified by $a_\mu = (g_\mu - 2)/2$.
- The magnetic moment of the muon can be determined precisely in the experiment as well as in theory.
- Sensitive to new physics.
- **Recent experimental value (Fermilab, 2025):** $a_\mu = 1\ 165\ 920\ 705(148) \times 10^{-12}$ (127 ppb) *[Phys.Rev.Lett. 135 (2025) 10, 101802]*

Theoretical determination of a_μ

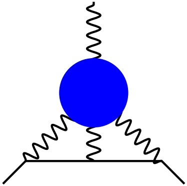
Standard Model contributions and current state results (see *white paper '25* *[arXiv:2505.21476]*):

contrib	$a_\mu \times 10^{11}$
QED	116 584 718.8(2)
Electroweak	154.4(4)
HVP (LO+NLO+NNLO)	7045(61)
HLbL (pheno + latt + NLO)	115.5(9.9)
HLbL (BMW '24)	125.5(11.6)
total SM (WP)	116 592 033(62)

SM uncertainties dominated by two types of hadronic contributions:



LO-HVP
$\mathcal{O}(\alpha^2)$

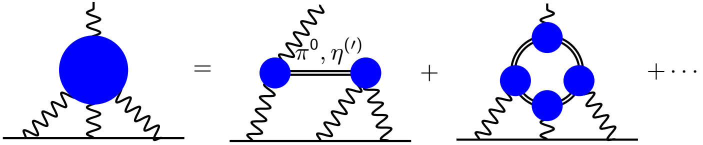


HLbL
$\mathcal{O}(\alpha^3)$

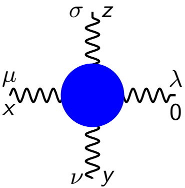
Contribution by hadronic light-by-light scattering

Two approaches:

- **Dispersive Method** *[Colangelo et al '17, Hoferichter et al '18]*:



  - Data driven approach .
  - Input can be provided by lattice calculations *[Mainz '19, ETM '23, BMW '23]*.
- **Evaluate 4pt function on the lattice** **(this work)** *[Phys.Rev.D 111 (2025) 11, 114509]*

$\widetilde{\Pi}_{\mu\nu\lambda\sigma}(x, y, z) =$  $= \langle j_\mu(x) j_\nu(y) j_\lambda(0) j_\sigma(z) \rangle$

  - Need precision of 10% to match the experimental precision.
  - BUT: 4pt functions are in general challenging on the lattice.

Content

Content

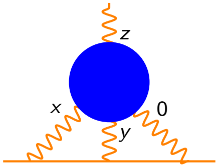
HLbL contributions to a_μ on the lattice

Master formula for hadronic light-by-light contribution to a_μ *[Mainz '21, '22]*:

$$a_\mu^{\mathrm{HLbL}} = \frac{m_\mu e^6}{3} \int_{x,y,z} \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\,(-z_\rho)\,\widetilde{\Pi}_{\mu\nu\sigma\lambda}(x,y,z)$$

- $\widetilde{\Pi}_{\mu\nu\sigma\lambda}(x,y,z) := \langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0)\rangle$: with (conserved) vector currents $j_\mu(x)$; consider light, strange and charm contributions.
- $\mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$: QED-kernel (not unique) *[RBC/UKQCD '17, Mainz '20]*.
- 12-dimensional integral, 8 can be evaluated directly in the simulation.
- Remaining object is rotational invariant (in continuum) $\Rightarrow$ Integrate over $|y|$; Lattice: sample along trajectory in fixed direction, e.g. $y = (n,n,n,n)$, $-L/2 < n < L/2$.

$$a_\mu^{\mathrm{HLbL}} = \frac{m_\mu e^6}{3} 2\pi^2 \int_{|y|} |y|^3 f(|y|) \rightarrow \frac{m_\mu e^6}{3} 2\pi^2 \sum_{y\in\mathrm{trajectory}} |y|^3 f(|y|)$$



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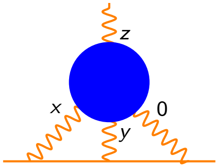

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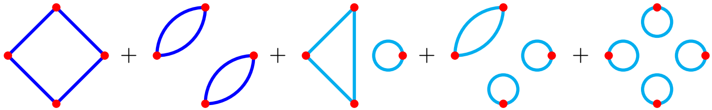
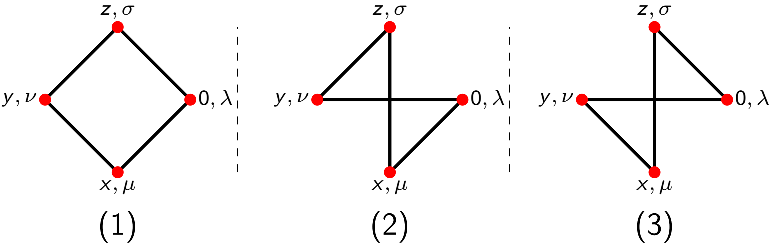
Decomposition of the HLbL tensor $\widetilde{\Pi}$ in terms of Wick contractions:

$\widetilde{\Pi} =$ 

Connected graphs



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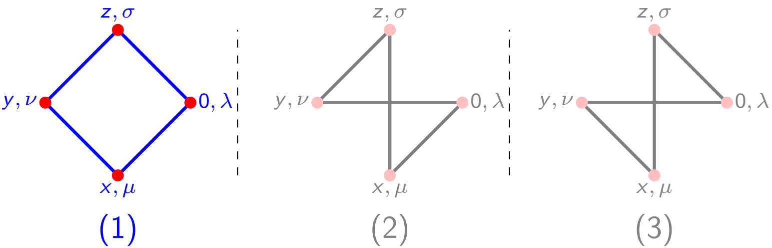
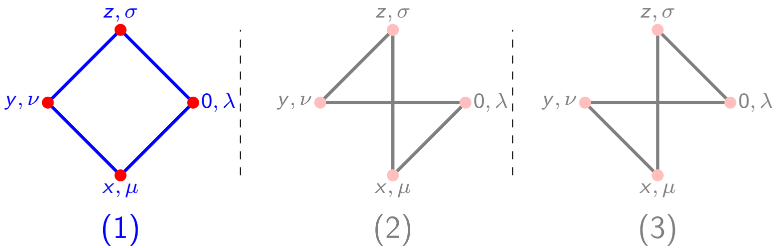


$$a_\mu^{\mathrm{conn}} = \frac{m_\mu e^6}{3} \sum_{x,y,z} \mathcal{L}^{(\mathrm{sym})}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\ (x_\rho - 3z_\rho)\, \mathrm{Re}\left\{ \widetilde{\Pi}^{\mathrm{conn},(1)}_{\mu\nu\sigma\lambda}(x,y,z) \right\}$$

- Re-express contractions (2) and (3) in terms of (1) by exploiting translational invariance *[Mainz '20]*.
- Fully symmetric kernel ($x \leftrightarrow y$, $y \leftrightarrow x - y$) $\mathcal{L}^{(\mathrm{sym})}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$; different choice compared to Mainz and RBC/UKQCD (cannot compare at the integrand level, no window observable).
- Contract directly in the simulation (changing the kernel is non-trivial).

Connected graphs



$$a_\mu^{\mathrm{conn}} = \frac{m_\mu e^6}{3} 2\pi^2 \sum_{|y|} |y|^3 \underbrace{\sum_{x,z} \mathcal{L}^{(\mathrm{sym})}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\ (x_\rho - 3z_\rho)\, \mathrm{Re}\left\{\widetilde{\Pi}^{\mathrm{conn},(1)}_{\mu\nu\sigma\lambda}(x,y,z)\right\}}_{=f(|y|)}$$

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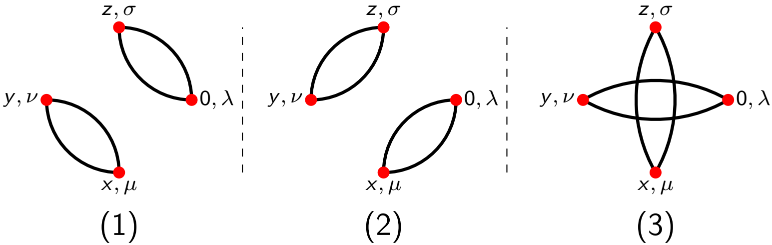
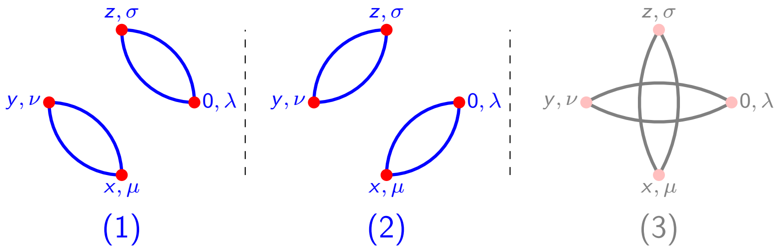
2+2 graphs



z, σ
y, ν
$0, \lambda$
x, μ
(1)
z, σ
y, ν
$0, \lambda$
x, μ
(2)
z, σ
y, ν
$0, \lambda$
x, μ
(3)

2+2 graphs



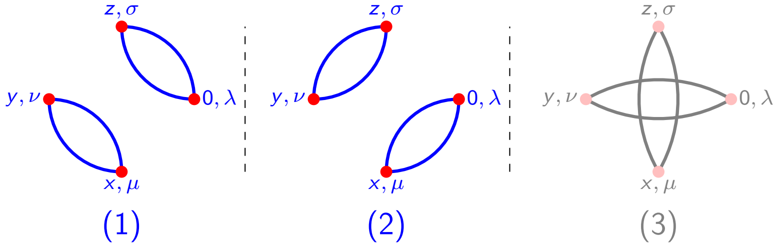
Two versions:

$$a_\mu^{(2+2),1} = \frac{m_\mu e^6}{3} 2\pi^2 \sum_{|y|} |y|^3 \sum_{x,z} \mathcal{L}^{(\mathrm{sym})}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\; (x_\rho + y_\rho - 3z_\rho)\; \widetilde{\Pi}^{(2+2),(1)}_{\mu\nu\sigma\lambda}(x,y,z)$$

$$a_\mu^{(2+2),2} = \frac{m_\mu e^6}{3} 2\pi^2 \sum_{|y|} |y|^3 \sum_{x,z} \mathcal{L}^{(\mathrm{sym})}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\; (y_\rho - 3z_\rho)\; \widetilde{\Pi}^{(2+2),(2)}_{\mu\nu\sigma\lambda}(x,y,z)$$

Fully symmetric kernel: Same as for connected contributions ⇒ re-use, computational costs by the kernel are sub-dominant.

2+2 graphs



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$$a_\mu^{(2+2),2} = \frac{m_\mu e^6}{3} 2\pi^2 \sum_{|y|} |y|^3 \sum_{x,z} \mathcal{L}^{(\mathrm{sym})}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\ (y_\rho - 3z_\rho)\ \widetilde{\Pi}^{(2+2),(2)}_{\mu\nu\sigma\lambda}(x,y,z)$$

Take average as final result (⇒ enhance statistics):

$$a_\mu^{(2+2)} = \frac{1}{2}\left(a_\mu^{(2+2),1} + a_\mu^{(2+2),2}\right)$$

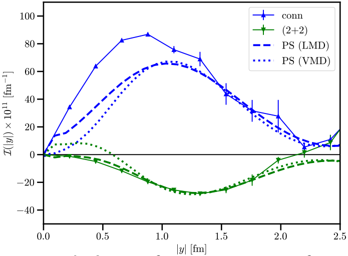
Gauge Ensembles

- $N_f = 2 + 1 + 1$ staggered fermions
- Four steps of stout smearing
- Physical masses
- $L \approx 3, 4, 6$ fm

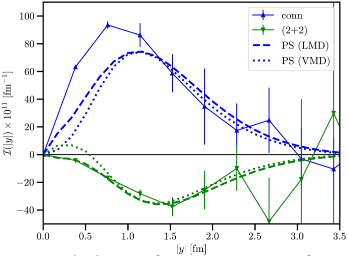
β	$L^3 \times T$	a [fm]	L [fm]	am_l	am_s	#confs (light)	#confs (strange)	#confs (charm)	Id
3.7000	$48^3 \times 64$	0.132	6.3	0.00205349	0.0572911		400		V6-L48
	$32^3 \times 64$		4.2	0.00205349	0.0572911	900	900	900	V4-L32
	$24^3 \times 48$		3.2	0.00205349	0.0572911	700	700	700	V3-L24
3.7553	$56^3 \times 84$	0.112	6.2	0.00171008	0.0476146	500	50*	50*	V6-L56-1
	$56^3 \times 84$		6.2	0.00174428	0.0461862	500		50*	V6-L56-2
	$28^3 \times 56$		3.1	0.00171008	0.0476146	850	850	850	V6-L28
3.8400	$64^3 \times 96$	0.095	6.1	0.001455	0.04075	1150		50*	V6-L64
	$32^3 \times 64$		3.0	0.00151556	0.0431935		50*	50*	V3-L32-1
	$32^3 \times 64$		3.0	0.00143	0.0431935			50*	V3-L32-2
	$32^3 \times 64$		3.0	0.001455	0.04075	1000	1000	1000	V3-L32-3
	$32^3 \times 64$		3.0	0.001455	0.03913			50*	V3-L32-4
3.9200	$40^3 \times 80$	0.079	3.1	0.001207	0.032		200*	200*	V3-L40-1
	$40^3 \times 80$		3.1	0.0012	0.0332856			50*	V3-L40-2
4.0126	$48^3 \times 96$	0.064	3.1	0.00095897	0.0264999		50*	50*	V3-L48-1
	$48^3 \times 96$		3.1	0.001002	0.027318		50*	50*	V3-L48-2
4.1479	$128^3 \times 192$	0.048	6.2	0.000701029	0.0193696			25*	V6-L128

Content

Light quarks: integrand



Integrand, $L = 3$ fm, $a = 0.1097$ fm



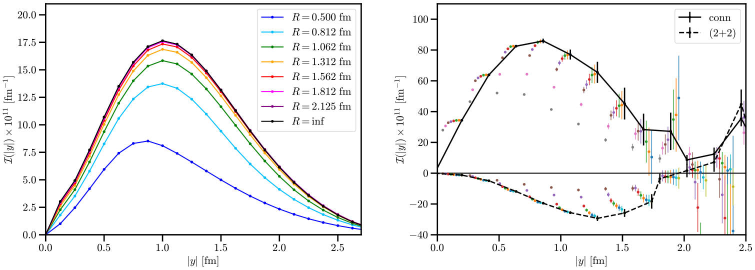
Integrand, $L = 6$ fm, $a = 0.1097$ fm

- Very precise data for small y, more noise for larger y $\rightarrow$ **Noise reduction**.
- Long distance dominated by **light pseudoscalar exchange**.
- Use information from **pseudoscalar transition form factors (TFFs)** to improve results.

Noise reduction

$$a_\mu^{\mathrm{HLbL}} = \frac{m_\mu e^6}{3} 2\pi^2 \int \mathrm{d}|y|\ |y|^3 \int \mathrm{d}^4 x\ \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \int \mathrm{d}^4 z\ (-z_\rho)\ \widetilde{\Pi}_{\mu\nu\sigma\lambda}(x,y,z)$$

- Have only noise at long distances.
- **Cut x-integral if $|x| > R$ and $|x-y| > R$.**
- R depends on $|y|$.



Pseudoscalar pole contribution

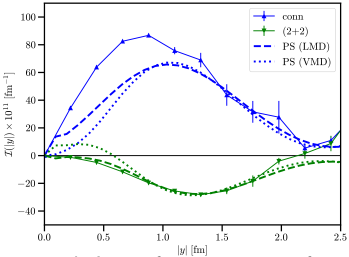
- Use pseudoscalar TFF results *[BMW '23]* in order to supplement the integrand data at large y, where the signal is noise + finite size effect estimate/correction.
- Evaluate contributions for $P = \pi^0, \eta, \eta'$ to the 4pt-function numerically (FFT) using transition form factor (TFF) data *[RBC-UKQCD '23]*:

$$\Pi^P_{\mu\nu\sigma\lambda}(y) = \int_{uv} D_P(u-v) \left(\widetilde{M}^P_{\mu\nu}(u,x,y) \widetilde{M}^P_{\sigma\lambda}(v,z,0) + \cdots \right)$$

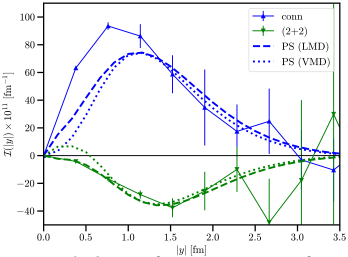
$$\widetilde{M}^P_{\mu\nu}(u,x,y) := i\epsilon_{\mu\nu\rho\sigma} \partial^{(x)}_\rho \partial^{(y)}_\sigma \int_{qk} F_{P\gamma^*\gamma^*}(q^2,k^2)\ e^{iq(x-u)} e^{iq(y-u)}$$

- Perform integration over u, v numerically in finite volume.
- Use LMD (VMD) parameterization to get an upper (lower) bound for the pseudoscalar pole contribution.
- π-pole contribution is enhanced in (separately considered) connected and 2+2 contributions:
  - $\mathcal{I}^{\rm conn} \approx \frac{34}{9} \mathcal{I}^\pi$
  - $\mathcal{I}^{(2+2)} \approx \mathcal{I}^\eta + \mathcal{I}^{\eta'} - \frac{25}{9} \mathcal{I}^\pi$

Light quarks: integrand



Integrand, $L = 3$ fm, $a = 0.1097$ fm

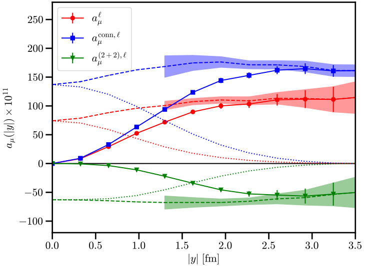


Integrand, $L = 6$ fm, $a = 0.1097$ fm

- Light pseudoscalar pole approximation in finite volume describes data well for $|y| > 1.5$ fm.
- Works for both volumes $\Rightarrow$ Finite size effects are well described by pseudoscalar poles $\Rightarrow$ Can be used for infinite volume extrapolation.
- Only small differences between LMD and VMD parameterization at large $|y|$.
- Notice: Only use $L = 6$ fm for the continuum extrapolation.

Light quarks: strategies

- Replace lattice data by pseudoscalar pole integrand once its contribution is small compared to the statistical error.
- FSE correction: $v^P = a_\mu^{P,\mathrm{cont}} - a_\mu^P(a, V)$



Accumulative sum, $L = 6$ fm, $a = 0.0939$ fm

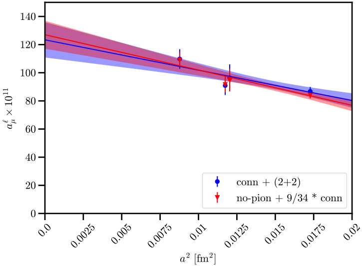
Two approaches:

- Direct calculation of $\mathcal{I}^{\mathrm{cont}} + \mathcal{I}^{(2+2)}$:

$$a_\mu^\ell = \int_0^{y_{\mathrm{cut}}} \mathrm{d}y \left[\mathcal{I}^{\mathrm{cont}}(y) + \mathcal{I}^{(2+2)}(y)\right] + \int_{y_{\mathrm{cut}}}^{\infty} \mathrm{d}y \left[\mathcal{I}^{\pi}(y) + \mathcal{I}^{\eta}(y) + \mathcal{I}^{\eta'}(y)\right] + v^\pi + v^\eta + v^{\eta'}$$

- Calculate $a_\mu^{no-\pi} = \frac{25}{34} a_\mu^{\mathrm{conn}} + a_\mu^{(2+2)}$ (converges at $|y| \sim 2$ fm, smaller stat. error), add $\frac{9}{34} a_\mu^{\mathrm{conn}}$ (smaller stat. error) *[RBC/UKQCD '23]*.

Light quarks: results



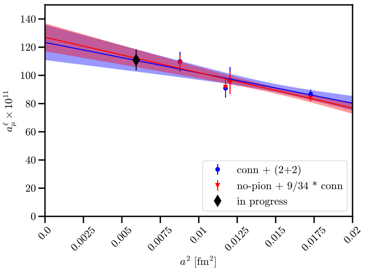
Published in *[Phys.Rev.D 111 (2025) 11, 114509]*:

- 3 lattice spacings based on large volume only ($L = 6$ fm).
- Continuum extrapolation:

$$a_\mu^\ell(a^2) = a_\mu^\ell + \beta_2(\Lambda a)^2$$

- The two approaches agree perfectly.
- Continuum value (conn+(2+2)): $a_\mu^\ell = 122.6(11.6) \times 10^{-11}$
- Compatible within 1σ with *[RBC/UKQCD '23]* and *[Mainz '21]*.

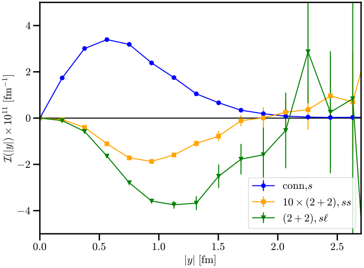
Light quarks: upcoming updates



- Light quark contribution is the largest source of uncertainties:
  - Statistical precision
  - Systematics: currently have three data points only
- Add another ensemble at $a = 0.077$ fm (black data point).
- Increase statistics of the two coarsest ensembles.

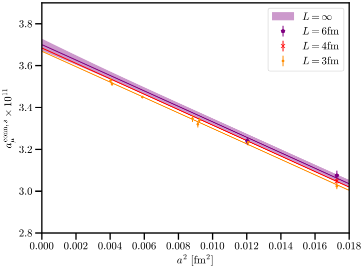
Content

Strange connected: integrand



- Sub-percent precision, systematic errors become relevant.
- Have to take into account the error of the lattice spacing (need muon mass in lattice units for the kernel calculation).
- For the sea quark masses, have $m_s \neq m_s^{\mathrm{phys}} \Rightarrow$ correction necessary.
- Finite volume effects are small but statistically relevant.
- Strategy: 11 ensembles with 3 volumes and 5 lattice spacings.

Strange connected: results

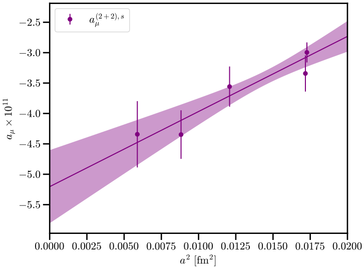


- Close to linear behavior in a^2, also consider quadratic term and scale dependence.
- Consider finite volume effects.
- Mismatch of strange quark mass.
- Ansatz:

$$a_\mu(a^2, M_{ss}) = a_\mu^{\mathrm{conn},s} + \beta_2(\Lambda a)^2 + \delta_2\ (\Lambda a)^2\ \alpha^n(a^{-1}) + \beta_4(\Lambda a)^4 + \gamma\frac{\delta M_{ss}}{M_{ss}^{\mathrm{phys}}} + \lambda\ e^{-Lm_\pi}$$

- Continuum value: $a_\mu^{\mathrm{conn},s} = 3.694(25)_{\mathrm{stat}}(8)_{\mathrm{syst}} \times 10^{-11}$
- RBC/UKQCD: $3.5(0.1)\times 10^{-11}$ *[RBC-UKQCD '23]*

Strange 2+2



- Statistical error larger than for connected diagrams.
- Linear continuum extrapolation:

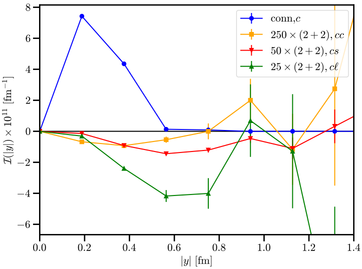
$$a_\mu^{(2+2),s}(a^2) = a_\mu^{(2+2),s} + \beta_2(\Lambda a)^2$$

- Continuum value: $a_\mu^{(2+2),s} = -5.4(0.8)_{\mathrm{stat}}(0.2)_{\mathrm{syst}} \times 10^{-11}$ (large cancelation with connected contribution)
- **Total strange contribution**:
  - Our result: $a_\mu^s = -1.7(0.8)_{\mathrm{stat}}(0.3)_{\mathrm{syst}} \times 10^{-11}$
  - RBC/UKQCD: $-0.0(2.3) \times 10^{-11}$ *[RBC-UKQCD '23]*
  - Mainz: $-0.6(2.0) \times 10^{-11}$ *[Mainz '19]*

Content

Charm connected: integrand and continuum extrapolation

- Statistical errors small compared to systematic uncertainties.
- No visible finite size effects.
- Main difficulty: Discretization artifacts; poor sampling of the integrand
  $\Rightarrow$ Sample in different directions regarding y: Evaluate integrand along different trajectories:
  $y \propto (n, n, n, n), (0, n, n, n)$
- Finer lattice spacing desirable $\Rightarrow$ need to improve the kernel for short distances.



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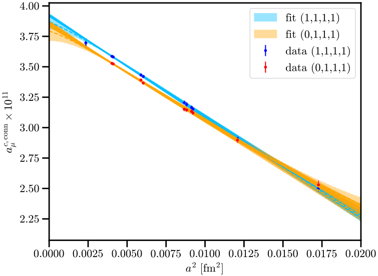


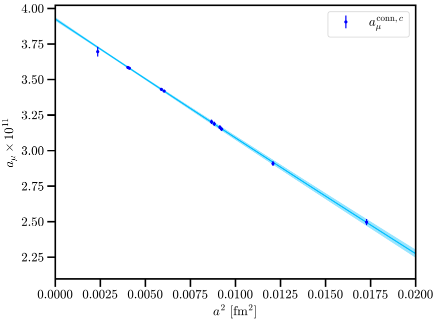
- Tree-level improvement using the free (quark-loop) value for given a and trajectory:

$$a_\mu^{\mathrm{impr}}(a) = a_\mu^{\mathrm{latt}}(a) - a_\mu^{\mathrm{free}}(a) + a_\mu^{\mathrm{cont,free}}(a)$$

- Use same ansatz as for strange connected (no volume term):

$$a_\mu(a, M_{\eta_c}) = a_\mu^{\mathrm{conn},c} + \beta_2(\Lambda a)^2 + \delta_2(\Lambda a)^2\, \alpha_s^n(a^{-1}) + \beta_4(\Lambda a)^4 + \gamma\frac{\delta M_{\eta_c}}{M_{\eta_c}^{\mathrm{phys}}}$$

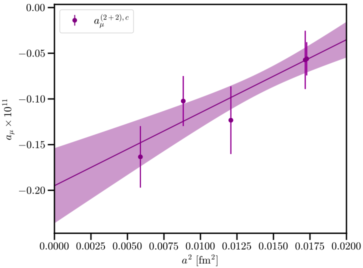
Charm connected: results and comparison



- Continuum value: $a_\mu^{\mathrm{conn},c} = 3.92(1)_{\mathrm{stat}}(26)_{\mathrm{syst}} \times 10^{-11}$
- Significant tension to other lattice calculations:
  - Mainz: $3.1(4) \times 10^{-11}$ *[Eur.Phys.J.C 82 (2022) 8, 664]*
- Recent perturbative calculation: $3.65(25) \times 10^{-11}$ *[Bijnens et al, Phys.Lett.B 873 (2026) 140178]*

  $\Rightarrow$ Agrees with our result within 1σ.

Charm, 2+2



- Linear continuum extrapolation:

$$a_\mu^{(2+2),s/c}(a^2) = a_\mu^{(2+2),s/c} + \beta_2(\Lambda a)^2$$

- Continuum value: $a_\mu^{(2+2),c} = -0.185(51)_{\mathrm{stat}}(04)_{\mathrm{syst}} \times 10^{-11}$
- **Total charm contribution:** $a_\mu^c = 3.73(5)_{\mathrm{stat}}(26)_{\mathrm{syst}} \times 10^{-11}$

Content

Summary

contribution	$a_\mu \times 10^{11}$
Light connected	220.1(13.5)
Light 2+2	$-101.1(12.8)$
Light total	122.6(11.6)
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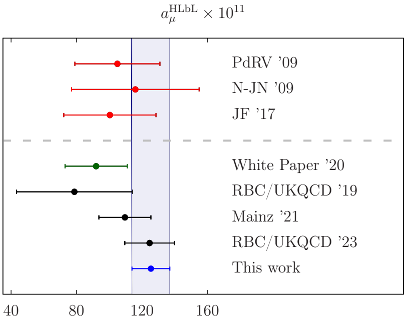

Status[Phys.Rev.D 111 (2025) 11, 114509]:

- Results for light, strange and charm quark contributions.
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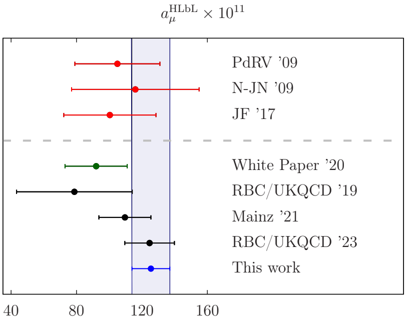

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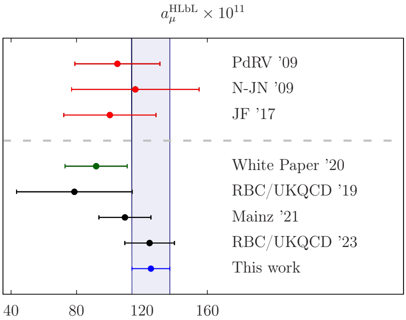

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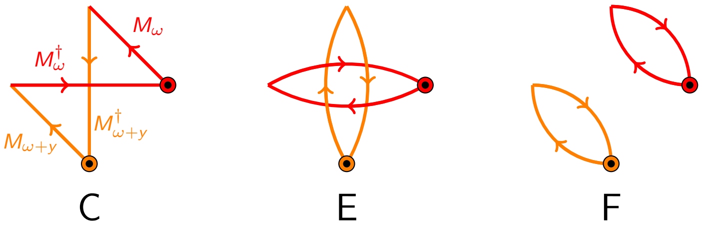
Thanks for your attention!

Backup slides

Technical approach

Conserved vector currents for staggered fermions $\chi(x)$, $\bar{\chi}(x)$:

$$j_\mu(x) = -\frac{1}{2}\eta_\mu(x)\left[\bar{\chi}(x+\hat{\mu})\ U_\mu^\dagger(x)\ \chi(x) + \bar{\chi}(x)\ U_\mu(x)\ \chi(x+\hat{\mu})\right]$$



- For given direction $\hat{n}$, consider all combinations of (y, ω) with $y = |y|\hat{n}$, $\omega = |\omega|\hat{n}$, exploit (anti-)periodicity.
- Repeat calculation for all $y' = y + h$, where $h_\mu = 0, \pm 1$ ($\mathcal{O}(a^2)$-smearing)
- Use local stochastic sources in color space $\Rightarrow$ Reduce number of matrix-matrix multiplications

Dealing with staggered tastes

$$a_\mu^{\mathrm{HLbL}} \propto \sum_{|y|} \mathcal{I}(|y|)$$

- Have to sample $\mathcal{I}(|y|)$ for a given set of points $|y|$
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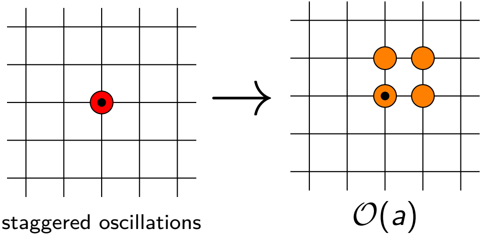
Project by applying "smearing":

$$\mathcal{I}(y) \to \mathcal{I}^{(1)}(y) = \prod_\mu \mathcal{S}_\mu^{(1)} \mathcal{I}(y) \qquad \mathcal{S}_\mu^{(1)} f(y) := \frac{f(y) + f(y+\hat{\mu})}{2}$$



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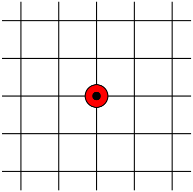
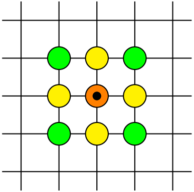
Project by applying "smearing":

$$\mathcal{I}(y) \to \mathcal{I}^{(2)}(y) = \prod_\mu \mathcal{S}_\mu^{(2)} \mathcal{I}(y) \qquad \mathcal{S}_\mu^{(2)} f(y) := \frac{f(y-\hat{\mu}) + 2f(y) + f(y+\hat{\mu})}{4}$$



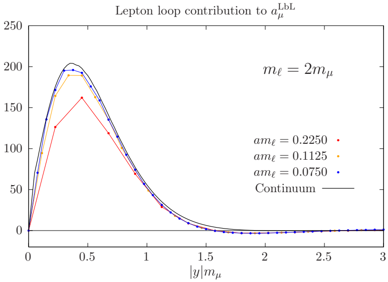
staggered oscillations

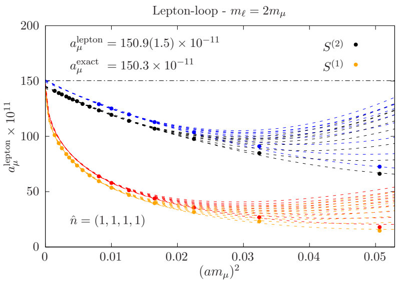
$$\longrightarrow$$



$\mathcal{O}(a^2)$

Test: Lepton loop on the lattice





- No staggered oscillations visible at the integrand level
- Ansatz: $a_\mu(a) = c_0 + c_1 a + c_2 a^2 + c_4 a^4$
- $a_\mu^{\mathrm{lepton}} = 150.9(1.5) \times 10^{-11}$ (lattice) vs $a_\mu^{\mathrm{lepton}} = 150.3 \times 10^{-11}$ (exact)
- **Values consistent**