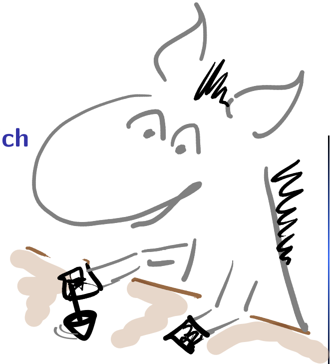
9th Plenary Workshop of the Muon $g-2$ Theory Initiative

McMule update

Sophie Kollatzsch

Paul Scherrer Institute / University of Zurich



McMule :: fixed-order (resummation planned) Monte Carlo framework (integrator, generator WIP)
for lepton processes at NNLO in QED & beyond

. . . of interest today compared to status during phase I of RadioMonteCarLow2

- $ee \to \mu\mu\gamma$ :: NLO → NLO + dNNLO ISC
- $ee \to \pi\pi\gamma$ :: NLO ISC → NLO ISC + physics-justified FsQED

f.l.t.r.: **M.Rocco** (Turin), Tarzan (mule), M.Ronchi (Mainz), Mula (mule), **A.Signer** (Zurich & PSI), Franco (mule), Foscolo (mule), **Y.Fang** (Zurich & PSI), **S.Gündogdu** (Zurich & PSI), Chrigel (mule), J.Wilson (Liverpool), **Y.Ulrich** (Liverpool), Orfeo (mule), F.Hagelstein (Mainz), **S.Kollatzsch** (Zurich & PSI)

not shown: P.Banerjee (NISER Bhubaneswar), A.Coutinho (IFIC), V.Sharkovska (Zurich & PSI), D.Radic (Zurich & PSI), G.Billis (PSI), T.Oruç (ETH Zurich)

***direct contributors to topics presented**

challenges to overcome

$$\sigma = \int \mathrm{d}\Phi_3 \, \Bigg|$$  $$\Bigg|^2$$

$$+ \int \mathrm{d}\Phi_4 \, \Bigg| \qquad \Bigg|^2$$

$$+ \int \mathrm{d}\Phi_5 \, \Bigg| \qquad \Bigg|^2$$

$$+ \ldots$$

challenges to overcome

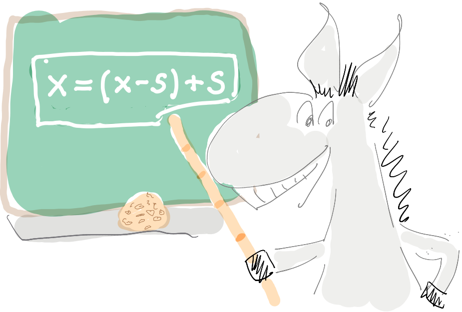
$$\sigma = \int \mathrm{d}\Phi_3 \left| \quad + \quad \right|^2 + \int \mathrm{d}\Phi_4 \left| \quad \right|^2$$

- divergent phase-space integration
  $\Longrightarrow$ FKS$^\ell$ subtraction

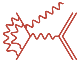
$$\underbrace{\int \mathrm{d}\Phi_3 \quad}_{\text{divergent}} + \underbrace{\int \mathrm{d}\Phi_4 \quad}_{\text{divergent}}$$

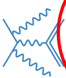
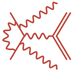
$$= \underbrace{\int \mathrm{d}\Phi_3 \Big(\quad + \int \mathrm{d}\Phi_\gamma \, \mathsf{CT} \Big)}_{\text{finite}}$$

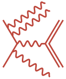
$$+ \underbrace{\int \mathrm{d}\Phi_3 \int \mathrm{d}\Phi_\gamma \Big(\quad - \mathsf{CT} \Big)}_{\text{finite}}$$



challenges to overcome

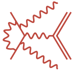
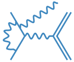
$\sigma = \int \mathrm{d}\Phi_3 \Big|$  $+$  $+$  $+ \dots \Big|^2$

$+ \int \mathrm{d}\Phi_4 \Big|$  $+$  $+ \dots \Big|^2$

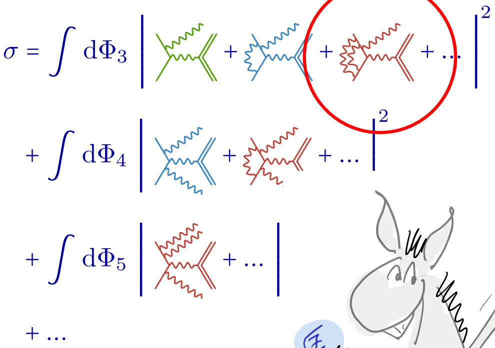
$+ \int \mathrm{d}\Phi_5 \Big|$  $+ \dots \Big|^2$

$+ \dots$

- divergent phase-space integration
  $\Longrightarrow$ FKS$^\ell$ subtraction
- numerical instabilities in phase-space integration
  $\Longrightarrow$ next-to-soft stabilisation
  $+$ OpenLoops

 $\xrightarrow{E_\gamma \to 0} \dfrac{1}{E_\gamma^2}\, \mathcal{E}$ 

$+ \dfrac{1}{E_\gamma} (\mathcal{D} + \mathcal{S})$  $+ \mathcal{O}(E_\gamma^0)$

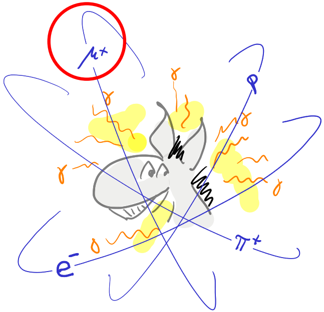


challenges to overcome

- divergent phase-space integration
  $\Longrightarrow$ FKS$^\ell$ subtraction
- numerical instabilities in phase-space integration
  $\Longrightarrow$ next-to-soft stabilisation
  $+$ OpenLoops
- two-loop amplitudes with $m \neq 0$
  $\Longrightarrow$ massification

$$\mathcal{M}(m) = \mathcal{M}(0) \times \left(\sqrt{Z(m)}\right)^{\#\text{ legs}} + \mathcal{O}\left(\frac{m}{\text{scales}}\right)$$

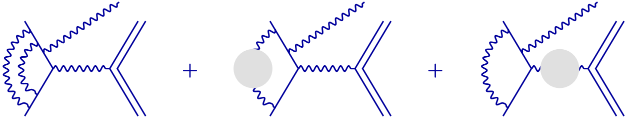
iff $m^2 \ll$ all other scales



part one :: $ee \to \mu\mu\gamma$

$\mu\mu\gamma$ key messages

- McMule can provide the **complete NNLO initial-state corrections** (ISC)

dNNLO ISC $\supset$ 

...including VP effects with resummation

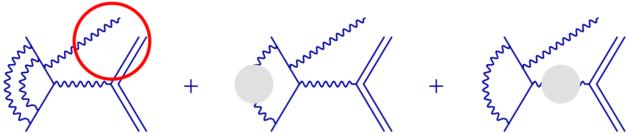
- they can reach $1\,\%$ compared to NLO for $\frac{\mathrm{d}\sigma}{\mathrm{d}x}$
  - VP effects (also with VP in the loop) sizeable part of dNNLO
  - NNLO vs. BabaYaga's NLO+PS differs at most $\mathcal{O}(0.1\,\%)$ for $\frac{\mathrm{d}\sigma}{\mathrm{d}x}$

    if restricted to ISC, without VP

- bottleneck: collinear photon emission $\Longrightarrow$ massification breaks down

  ...KLOE-like small-angle scenario not possible

- McMule can provide the **complete NNLO initial-state corrections** (ISC)

dNNLO ISC $\supset$ 

...including VP effects with resummation

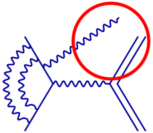
- they can reach $1\,\%$ compared to NLO for $\frac{\mathrm{d}\sigma}{\mathrm{d}x}$
  - VP effects (also with VP in the loop) sizeable part of dNNLO
  - NNLO vs. BabaYaga's NLO+PS differs at most $\mathcal{O}(0.1\,\%)$ for $\frac{\mathrm{d}\sigma}{\mathrm{d}x}$

    if restricted to ISC, without VP

- bottleneck: collinear photon emission $\Longrightarrow$ massification breaks down

  ...KLOE-like small-angle scenario not possible

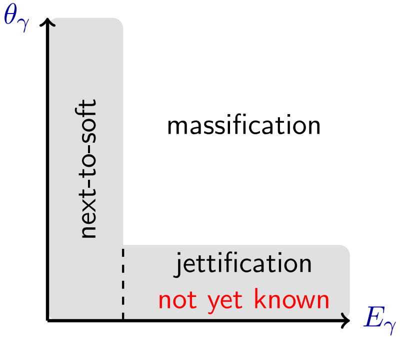
- McMule can provide the **complete N**

dNNLO ISC $\supset$ 

…including VP effects with resumma

- they can reach $1\,\%$ compared to NLO
  - VP effects (also with VP in the loop) sizeable
  - NNLO vs. BabaYaga's NLO+PS di

    if restricted to ISC, without VP



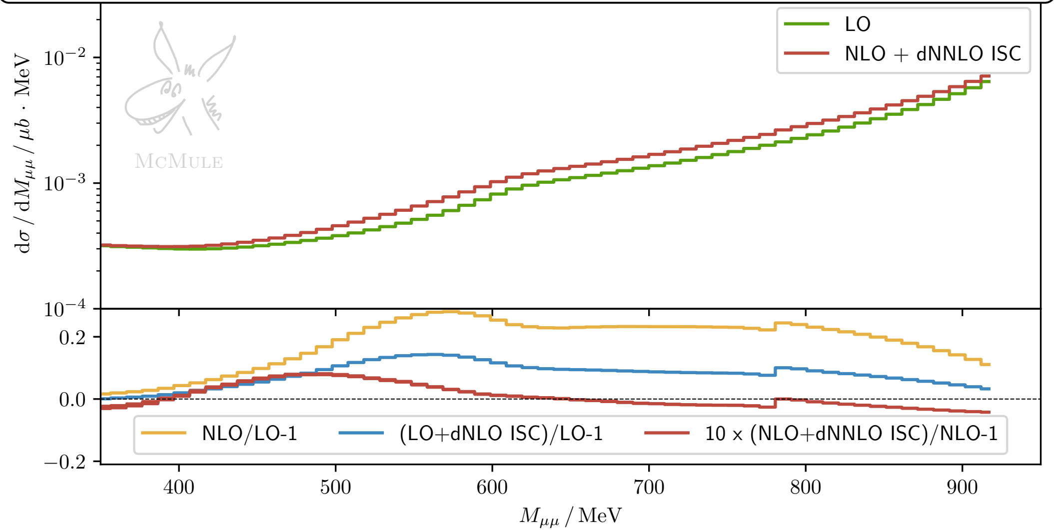
($m^2 \ll$ all other scales)
not valid everywhere

- bottleneck: collinear photon emission $\Longrightarrow$ massification breaks down

  …KLOE-like small-angle scenario not possible

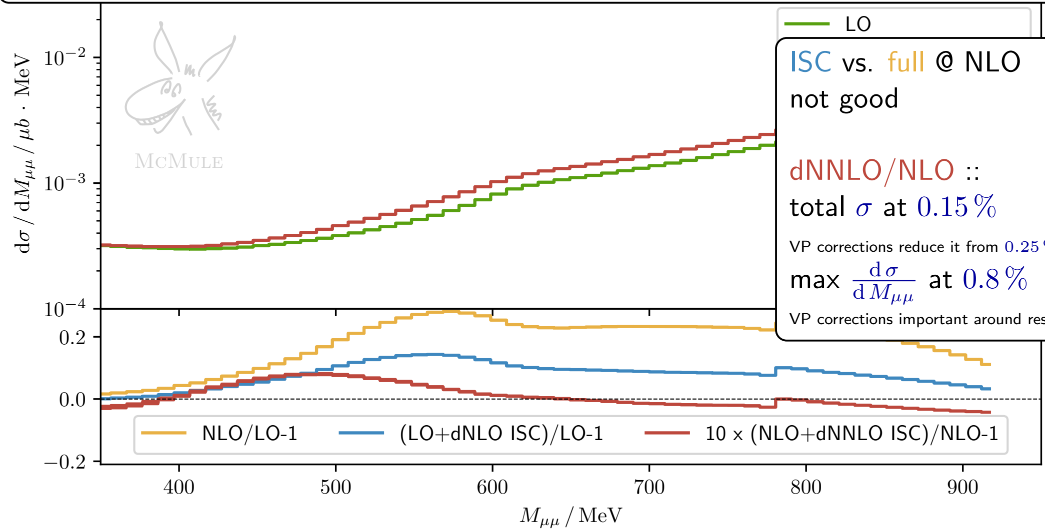
KLOE-like large-angle :: $M_{\mu\mu}$

$\sqrt{s} = 1\,\mathrm{GeV}$, $E_\gamma > 20\,\mathrm{MeV}$, $50^\circ \leq \theta_\gamma \leq 130^\circ$, $0.1\,\mathrm{GeV}^2 \leq M^2_{\mu\mu} \leq 0.85\,\mathrm{GeV}^2$, $50^\circ \leq \theta^\pm \leq 130^\circ$, $|p^z_\pm| > 90\,\mathrm{MeV}$ or $p^\perp_\pm > 160\,\mathrm{MeV}$



KLOE-like large-angle :: $M_{\mu\mu}$

$\sqrt{s} = 1\,\mathrm{GeV},\ E_\gamma > 20\,\mathrm{MeV},\ 50^\circ \le \theta_\gamma \le 130^\circ,\ 0.1\,\mathrm{GeV}^2 \le M^2_{\mu\mu} \le 0.85\,\mathrm{GeV}^2,\ 50^\circ \le \theta^\pm \le 130^\circ,\ |p^z_\pm| > 90\,\mathrm{MeV}$ or $p^\perp_\pm > 160\,\mathrm{MeV}$



ISC vs. full @ NLO not good

dNNLO/NLO ::

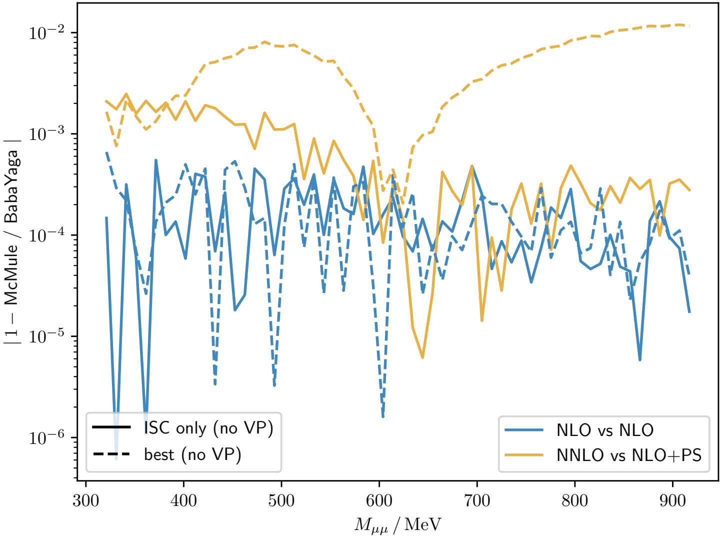
total σ at $0.15\,\%$

VP corrections reduce it from $0.25\,\%$

max $\frac{\mathrm{d}\sigma}{\mathrm{d}M_{\mu\mu}}$ at $0.8\,\%$

VP corrections important around resonance

KLOE-like large-angle :: comparison to BabaYaga :: no VP



techni. validation @ NLO

NNLO vs. NLO+PS

…can be $\mathcal{O}(0.1\%)$

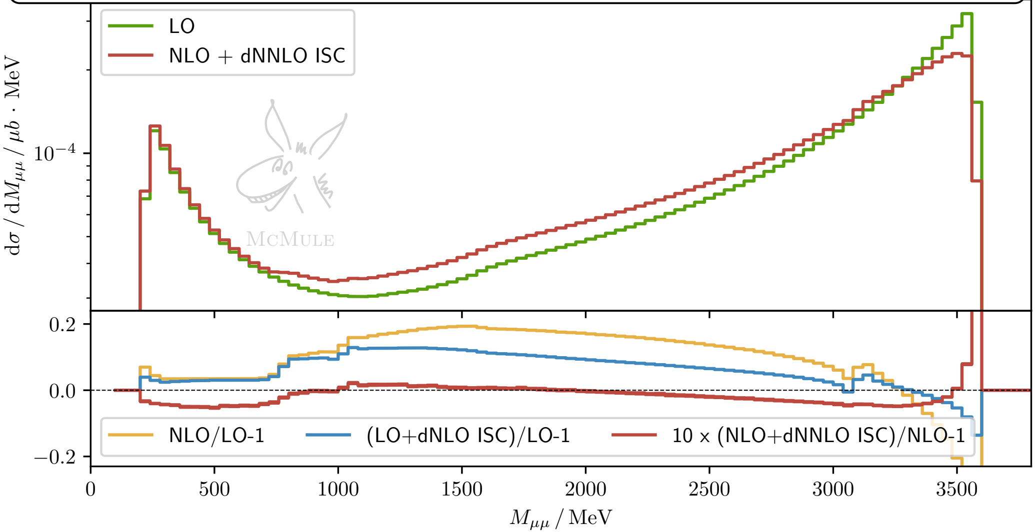
solid line, ISC only

…missing beyond-ISC @ NNLO (McMule) can be $\mathcal{O}(1\%)$

dashed line, using BabaYaga's full NLO+PS

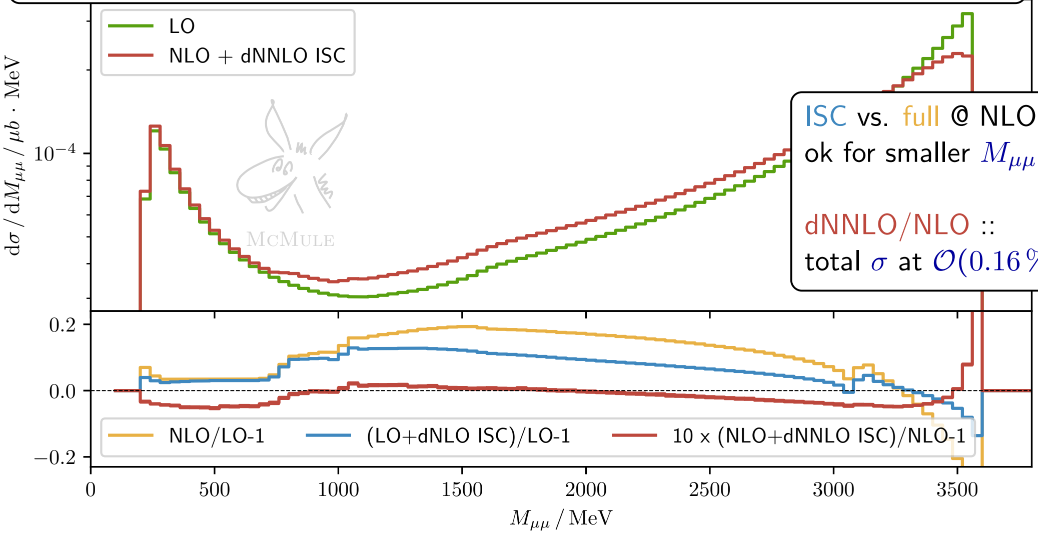
BES-like :: $M_{\mu\mu}$

$\sqrt{s} = 4\,\mathrm{GeV}$, $|\cos\theta^\pm| < 0.93$ $p_\pm^\perp > 300\,\mathrm{MeV}$, $(|\cos\theta_\gamma| < 0.8$ and $E_\gamma > 25\,\mathrm{MeV})$ or $(0.86 < |\cos\theta_\gamma| < 0.92$ and $E_\gamma > 50\,\mathrm{MeV})$



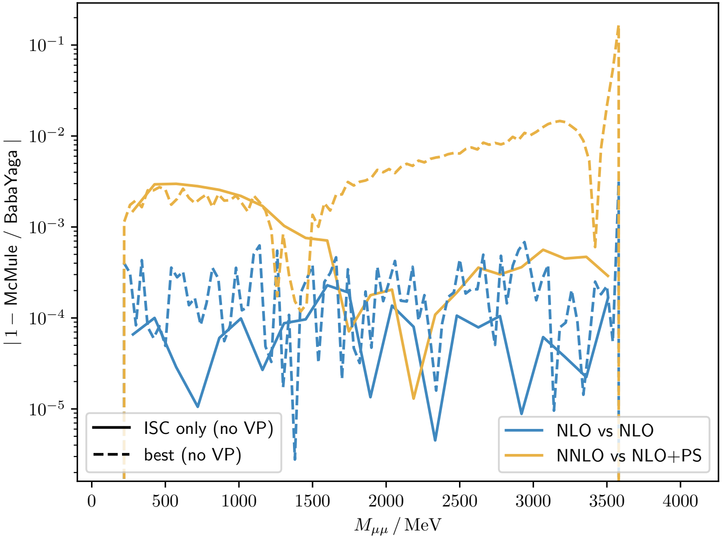
BES-like :: $M_{\mu\mu}$

$\sqrt{s} = 4\,\mathrm{GeV}$, $|\cos\theta^{\pm}| < 0.93$ $p_{\pm}^{\perp} > 300\,\mathrm{MeV}$, $\big(|\cos\theta_\gamma| < 0.8$ and $E_\gamma > 25\,\mathrm{MeV}\big)$ or $\big(0.86 < |\cos\theta_\gamma| < 0.92$ and $E_\gamma > 50\,\mathrm{MeV}\big)$



ISC vs. full @ NLO
ok for smaller $M_{\mu\mu}$

dNNLO/NLO ::
total σ at $\mathcal{O}(0.16\,\%)$



techni. validation @ NLO

NNLO vs. NLO+PS

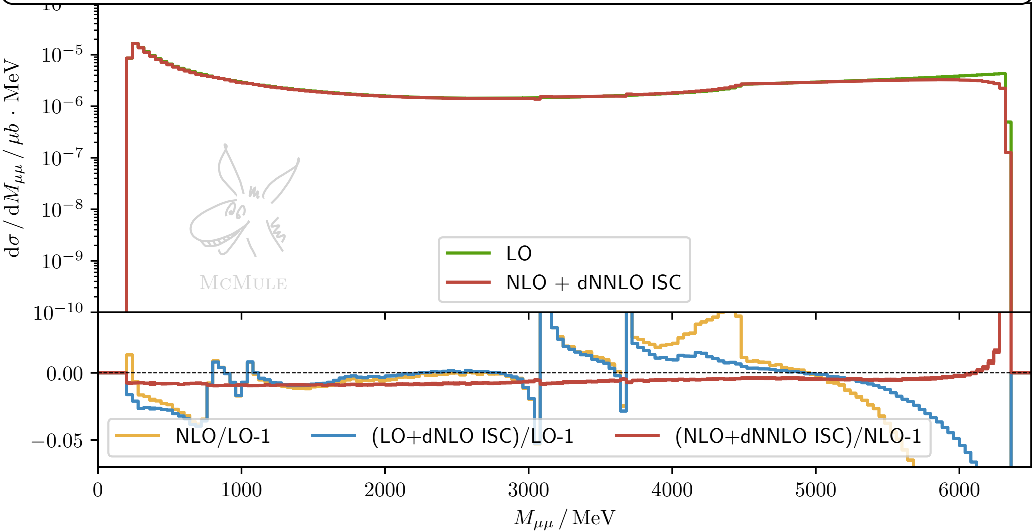
…can be $\mathcal{O}(0.1\%)$

solid line, ISC only

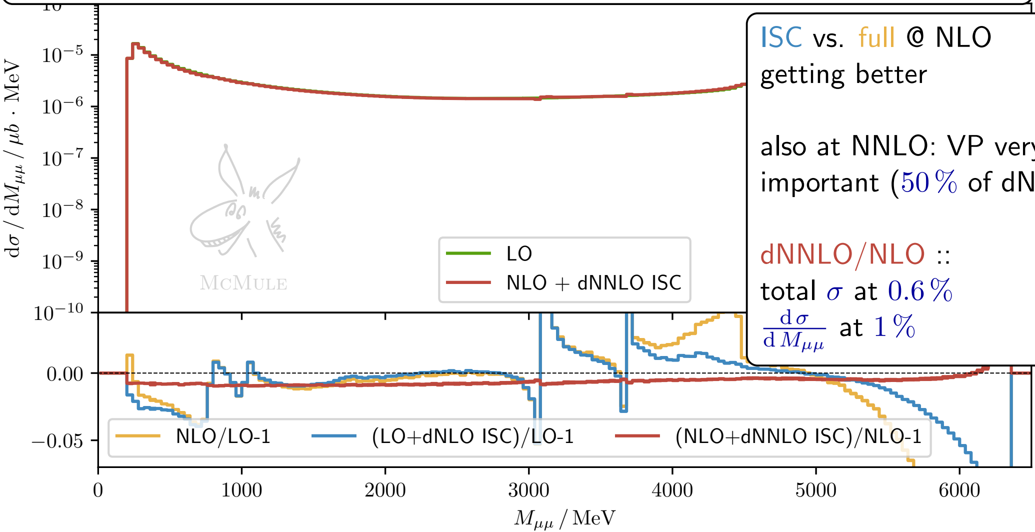
…missing beyond-ISC @ NNLO (McMule) can be $\mathcal{O}(1\%)$

dashed line, using BabaYaga's full NLO+PS

$\sqrt{s}$ = 10 GeV, 0.65 rad ≤ $\theta^{\pm}$ ≤ 2.75 rad, $p_{\pm}$ > 1 GeV, 0.6 rad ≤ θ_{γ} ≤ 2.7 rad, E_{γ} > 3 GeV, $\theta_{\gamma(h)\tilde{\gamma}}$ < 0.3 rad, $M_{XX\gamma}$ > 8 GeV



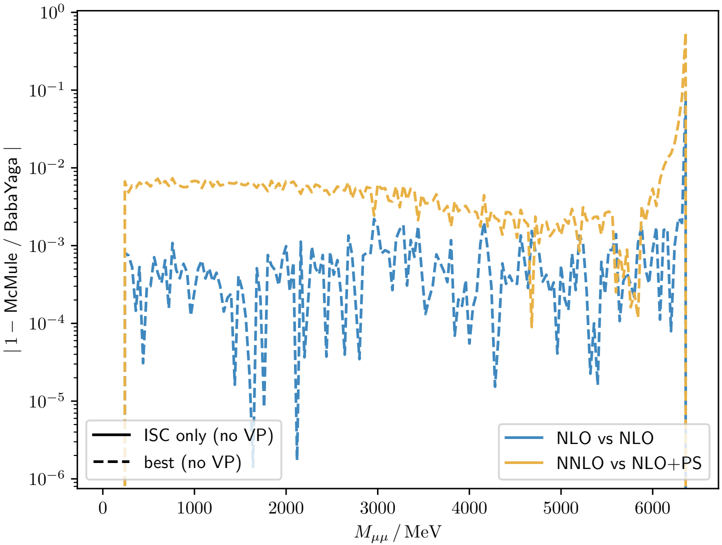
$\sqrt{s} = 10\,\mathrm{GeV}$, $0.65\,\mathrm{rad} \le \theta^\pm \le 2.75\,\mathrm{rad}$, $p_\pm > 1\,\mathrm{GeV}$, $0.6\,\mathrm{rad} \le \theta_\gamma \le 2.7\,\mathrm{rad}$, $E_\gamma > 3\,\mathrm{GeV}$, $\theta_{\gamma(h)\tilde{\gamma}} < 0.3\,\mathrm{rad}$, $M_{XX\gamma} > 8\,\mathrm{GeV}$



ISC vs. full @ NLO getting better

also at NNLO: VP very important (50 % of dNNLO)

dNNLO/NLO ::
total σ at 0.6 %
$\frac{d\sigma}{d M_{\mu\mu}}$ at 1 %

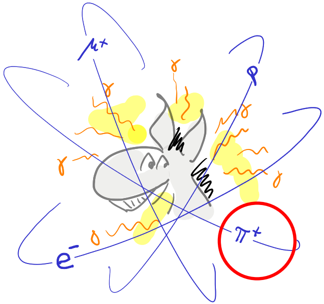


techni. validation @ NLO

NNLO vs. NLO+PS

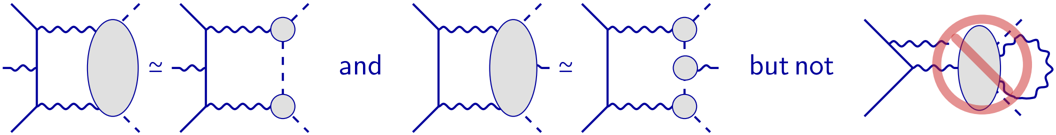
. . . missing beyond-ISC @ NNLO (McMule) below $\mathcal{O}(1\%)$

dashed line, using BabaYaga's full NLO+PS

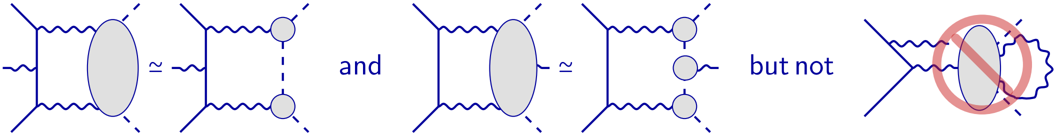


part two :: $ee \to \pi\pi\gamma$

- McMule can provide NNLO ISC soon + mixed corrections using **physics-motivated form factor scalar QED** (FsQED)



- McMule can provide NNLO ISC soon + mixed corrections using **physics-motivated form factor scalar QED** (FsQED)



$\Longrightarrow$ split into well-defined subset C-odd only $\oplus$ estimate of effects beyond that

	KLOE-like large-angle	KLOE-like small-angle
$\frac{\text{dNLO mixed}}{\text{LO} + \text{NLO ISC}}$	$15\,\% \oplus 0.1\,\%$	$1\,\% \oplus 0.001\,\%$

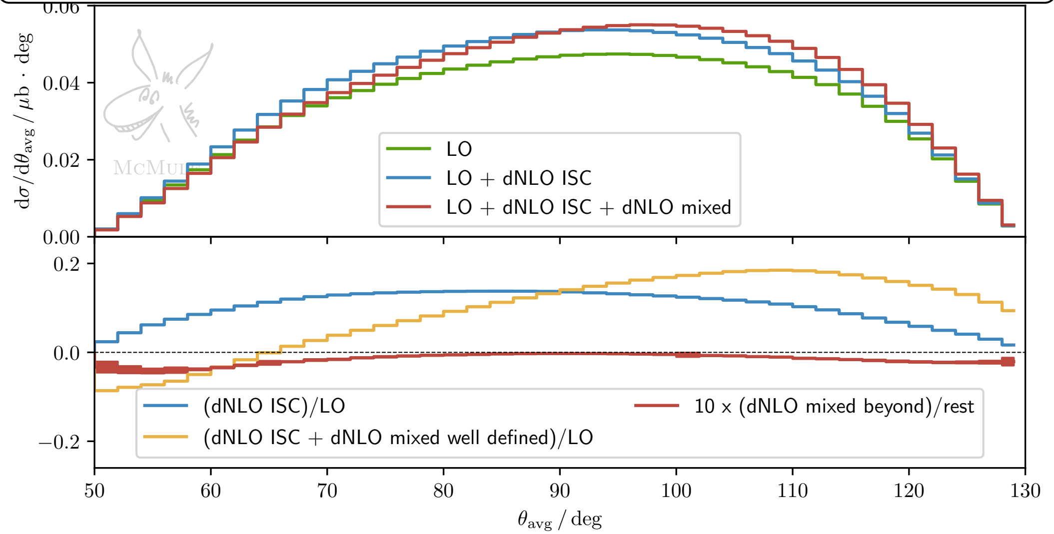
. . . much smaller for BES-like and B scenario

- full mixed vs. physics-motivated mixed

  $\Longrightarrow$ **lower estimate of theory uncertainty**

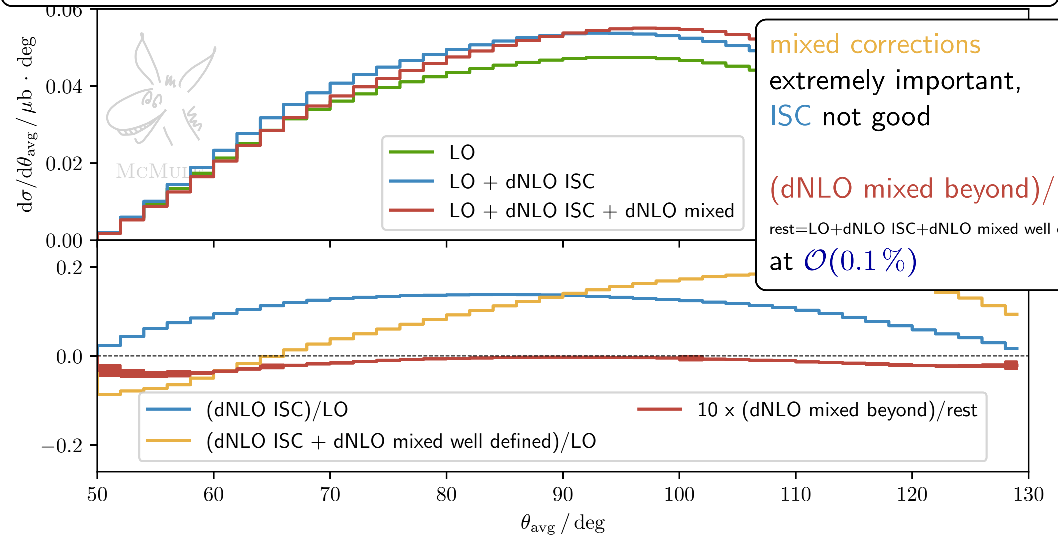
KLOE-like large-angle :: $\theta_{\rm avg} = \frac{1}{2}(\theta^- - \theta^+ + \pi)$

$\sqrt{s} = 1\,\mathrm{GeV}$, $E_\gamma > 20\,\mathrm{MeV}$, $50^\circ \le \theta_\gamma \le 130^\circ$, $0.1\,\mathrm{GeV}^2 \le M^2_{\mu\mu} \le 0.85\,\mathrm{GeV}^2$, $50^\circ \le \theta^\pm \le 130^\circ$, $|p^z_\pm| > 90\,\mathrm{MeV}$ or $p^\perp_\pm > 160\,\mathrm{MeV}$



KLOE-like large-angle :: $\theta_{avg} = \frac{1}{2}(\theta^- - \theta^+ + \pi)$

$\sqrt{s} = 1\,\mathrm{GeV}$, $E_\gamma > 20\,\mathrm{MeV}$, $50^\circ \le \theta_\gamma \le 130^\circ$, $0.1\,\mathrm{GeV}^2 \le M^2_{\mu\mu} \le 0.85\,\mathrm{GeV}^2$, $50^\circ \le \theta^\pm \le 130^\circ$, $|p^z_\pm| > 90\,\mathrm{MeV}$ or $p^\perp_\pm > 160\,\mathrm{MeV}$



mixed corrections extremely important, ISC not good

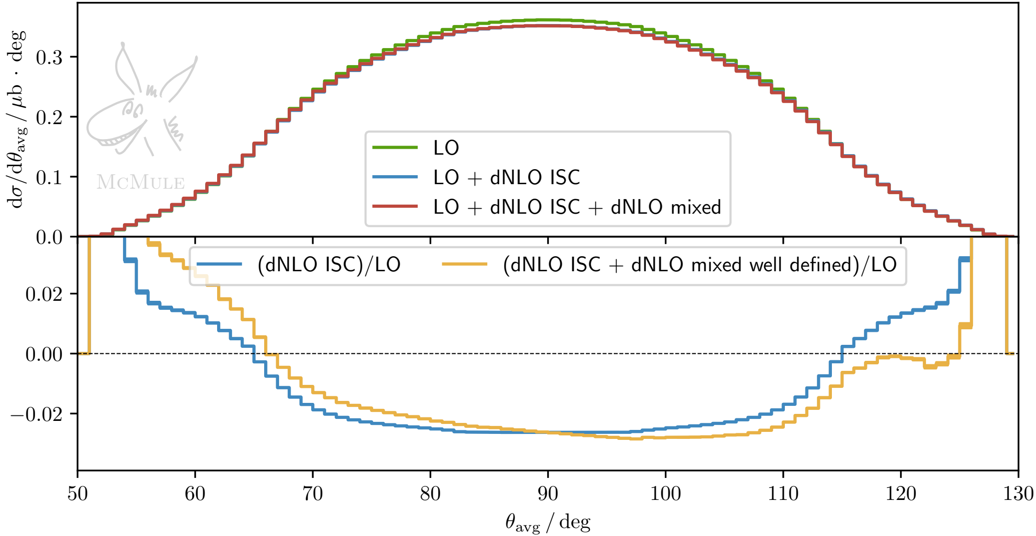
(dNLO mixed beyond)/rest

rest=LO+dNLO ISC+dNLO mixed well defined

at $\mathcal{O}(0.1\,\%)$

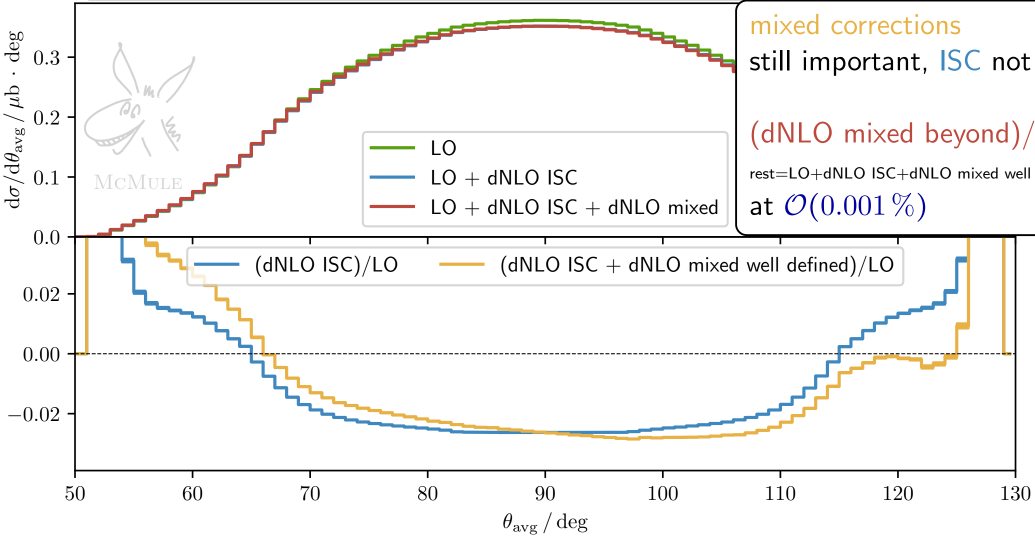
KLOE-like small-angle :: $\theta_{avg} = \frac{1}{2}(\theta^- - \theta^+ + \pi)$

$\theta_{\tilde{\gamma}} \le 15^\circ$ or $\theta_{\tilde{\gamma}} > 165^\circ$, $0.35\,\mathrm{GeV}^2 \le M^2_{\mu\mu} \le 0.95\,\mathrm{GeV}^2$, $50^\circ \le \theta^\pm \le 130^\circ$, $|p^z_\pm| > 90\,\mathrm{MeV}$ or $p^\perp_\pm > 160\,\mathrm{MeV}$



KLOE-like small-angle :: $\theta_{\text{avg}} = \frac{1}{2}(\theta^- - \theta^+ + \pi)$

$\theta_{\tilde\gamma} \leq 15^\circ$ or $\theta_{\tilde\gamma} > 165^\circ$, $0.35\,\mathrm{GeV}^2 \leq M^2_{\mu\mu} \leq 0.95\,\mathrm{GeV}^2$, $50^\circ \leq \theta^\pm \leq 130^\circ$, $|p^z_\pm| > 90\,\mathrm{MeV}$ or $p^\perp_\pm > 160\,\mathrm{MeV}$



mixed corrections

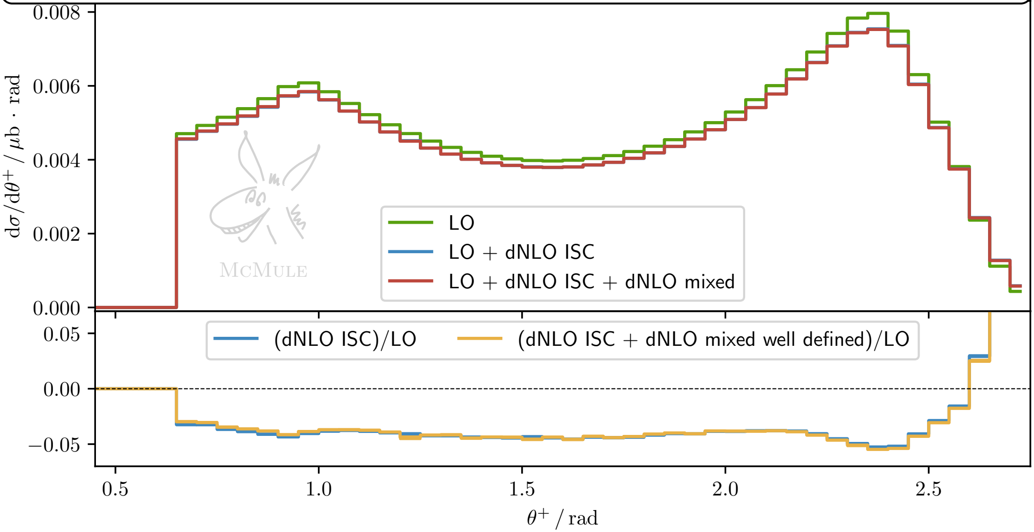
still important, ISC not good

(dNLO mixed beyond)/rest

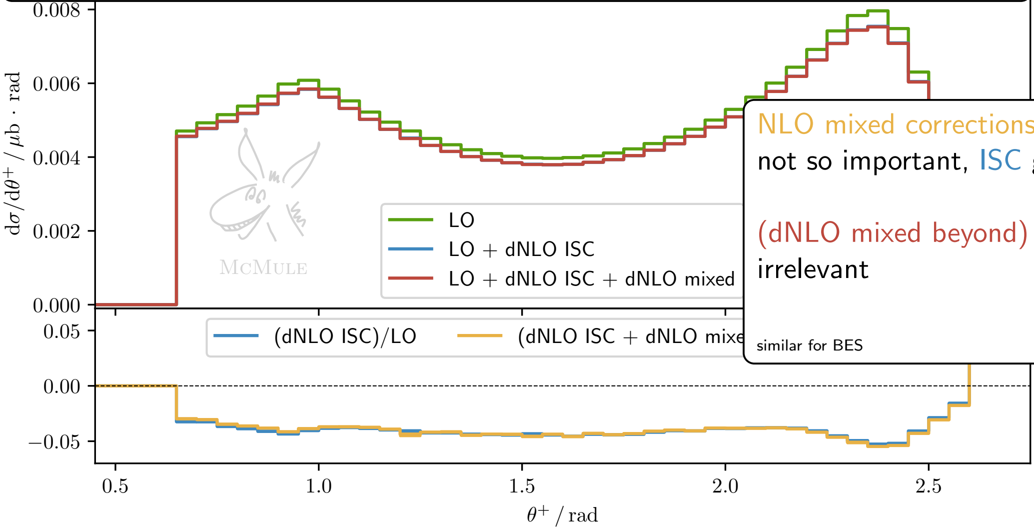
rest=LO+dNLO ISC+dNLO mixed well defined

at $\mathcal{O}(0.001\,\%)$

$\sqrt{s} = 10\,\mathrm{GeV}, 0.65\,\mathrm{rad} \le \theta^\pm \le 2.75\,\mathrm{rad},\ p_\pm > 1\,\mathrm{GeV},\ 0.6\,\mathrm{rad} \le \theta_\gamma \le 2.7\,\mathrm{rad},\ E_\gamma > 3\,\mathrm{GeV},\ \theta_{\gamma(h)\tilde{\gamma}} < 0.3\,\mathrm{rad},\ M_{XX\gamma} > 8\,\mathrm{GeV}$



$\sqrt{s} = 10\,\mathrm{GeV}, 0.65\,\mathrm{rad} \le \theta^\pm \le 2.75\,\mathrm{rad}, p_\pm > 1\,\mathrm{GeV}, 0.6\,\mathrm{rad} \le \theta_\gamma \le 2.7\,\mathrm{rad}, E_\gamma > 3\,\mathrm{GeV}, \theta_{\gamma(h)\tilde{\gamma}} < 0.3\,\mathrm{rad}, M_{XX\gamma} > 8\,\mathrm{GeV}$



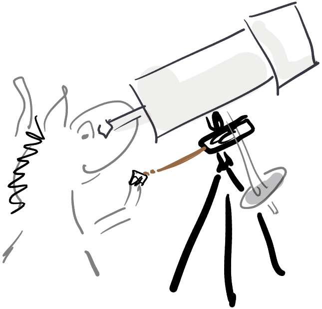
NLO mixed corrections

not so important, ISC good

(dNLO mixed beyond)

irrelevant

similar for BES



outlook

$ee \to \mu\mu\gamma$ :: complete NNLO

- ISC $\to$ FSC
- full NNLO with massification for both m_e & m_μ

  all pieces known, implementation WIP
- resolve massification challenge !?

large-scale pheno comparison within RadioMonteCarLow2

NNLO vs. NLO+PS, VP effects, limitations of FsQED

$ee \to \mu\mu$ :: N^{3}LO ISC in reach

- $ee \to \gamma^*\gamma$ @ NNLO + massification

  $\Longrightarrow$ $ee \to \gamma^* \to \mu\mu$ @ N^{3}LO maybe except (h)LbL
- ...afterwards: extend to N^{3}LO full

$ee \to \pi\pi\gamma$ :: pheno studies

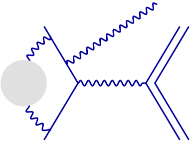
- add NNLO ISC
- estimate rescattering effects via $\rho(770)$?
- apply same implementation to $ee \to K^+K^-(\gamma)$

f.l.t.r.: A.Signer (Zurich & PSI), F.Hagelstein (Mainz), M.Ronchi (Mainz), M.Rocco (Turin)
S.Kollatzsch (Zurich & PSI), Y.Ulrich (Liverpool), S.Gündogdu (Zurich & PSI),
D.Radic (Zurich & PSI), J.Wilson (Liverpool), S.Zanoli (Oxford), Y.Fang (Zurich & PSI)
not shown: P.Banerjee (NISER Bhubaneswar), A.Coutinho (IFIC) V.Sharkovska (Zurich & PSI),
G.Billis (PSI), T.Oruç (ETHZ)

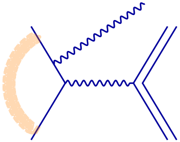
McMule

mule-tools.gitlab.io

question: how to calculate loop diagrams with data input, in a Monte Carlo code?

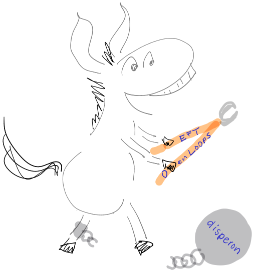


$$\Longrightarrow \quad \frac{\Pi(k^2)}{k^2} = -\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}S\, \frac{\mathrm{Im}\,\Pi(S)}{S(k^2-S)} \quad \Longrightarrow \quad \int_{4m_\pi^2}^{\infty} \mathrm{d}S$$



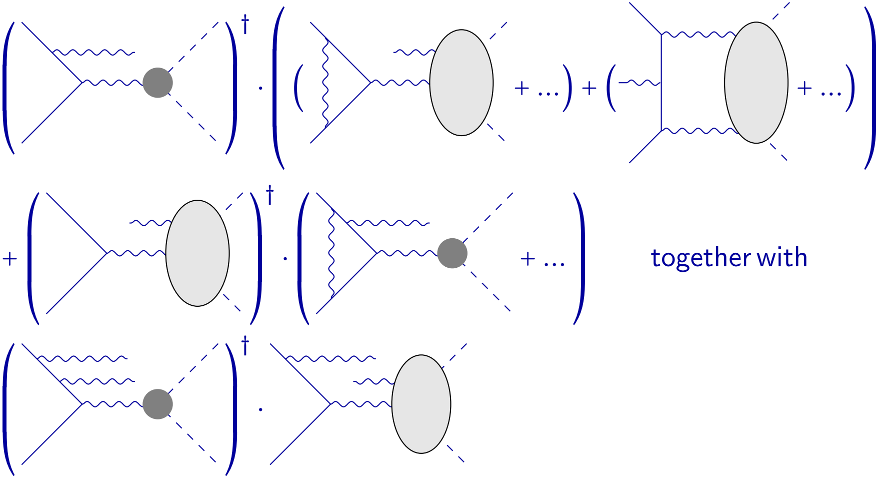
answer: disperon QED

- amplitude generations with disperons using OpenLoops
- universal threshold subtraction
- stability & efficiency at $\int^\infty \mathrm{d}S$ with effective theory method



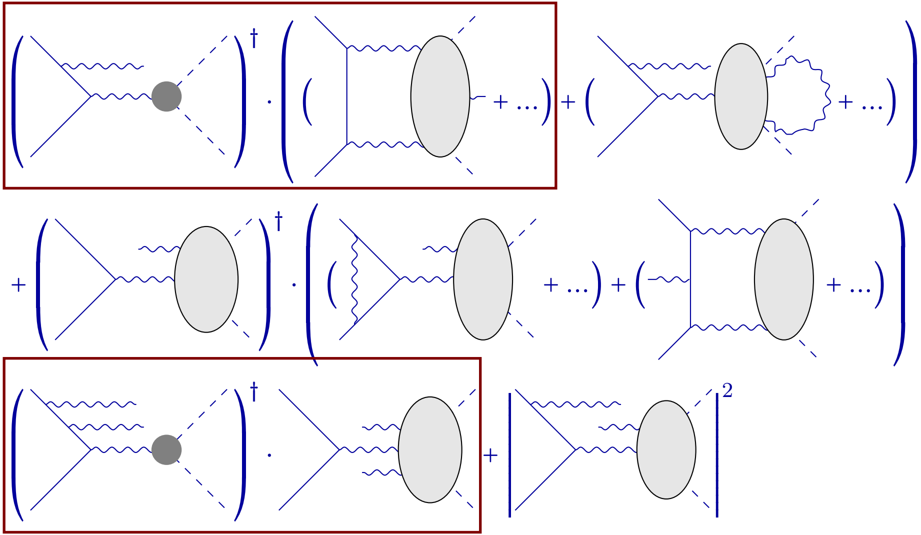
$\Longrightarrow$ **general method**, essential for $ee \to \pi\pi(\gamma)$

the full $q_e^5 q_\pi^3$



together with

subset of $q_e^4 q_\pi^4$



together with