The hadronic contribution to the running of α and the electroweak mixing angle

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AC, Djukanovic, von Hippel, Kuberski, Meyer, Miura, Ottnad, Risch, Wittig

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JOHANNES GUTENBERG UNIVERSITÄT MAINZ

HIM Helmholtz-Institut Mainz

GSI

A key hadronic quantity: $\Delta\alpha_{\rm had}$

- The electromagnetic gauge coupling is a fundamental parameter in the Standard Model
  $\rightarrow$ $\alpha(M_Z^2)$ relevant input parameters for high-energy colliders

- One introduces an effective coupling at any time-like momentum transfer q^2

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}, \qquad \Delta\alpha(q^2) = \Delta\alpha_{\rm lep}(q^2) + \Delta\alpha^{(5)}_{\rm had}(q^2) + \Delta\alpha_{\rm top}(q^2)$$

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- $\Delta\alpha_{\rm lep}$ and $\Delta\alpha_{\rm top}$ computed reliably in PT
- $\Delta\alpha^{(5)}_{\rm had}$ dominated by non-PT QCD effects

- Traditional methods relies on data-drive dispersive approach (strong tension in the $\pi^+\pi^-$ channel)

The success of LQCD in clarifying $g-2$ strongly motivates a lattice determination of $\Delta\alpha_{\rm had}$

Preliminaries

- Electroweak couplings as a function of the momentum transfer q^2

$$\alpha(q^2) = \alpha/(1 - \Delta\alpha(q^2)), \qquad \sin^2\theta_W(q^2) = \sin^2\theta_W(1 + \Delta\sin^2\theta_W(q^2))$$

Leading hadronic contribution

$$\Delta\alpha_{\mathrm{had}}(q^2) = 4\pi\alpha\bar{\Pi}^{\gamma\gamma}(q^2), \qquad (\Delta\sin^2\theta_W)_{\mathrm{had}}(q^2) = -4\pi\alpha/\sin^2\theta_W\bar{\Pi}^{Z\gamma}(q^2)$$

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- Time Momentum Representation (TMR) $\quad \bar{\Pi}(q^2) = \Pi(q^2) - \Pi(0) = \int_0^\infty \mathrm{d}t G(t) K(t, q^2)$

  [Bernecker, Meyer 2011; Francis *et al.* 2013]

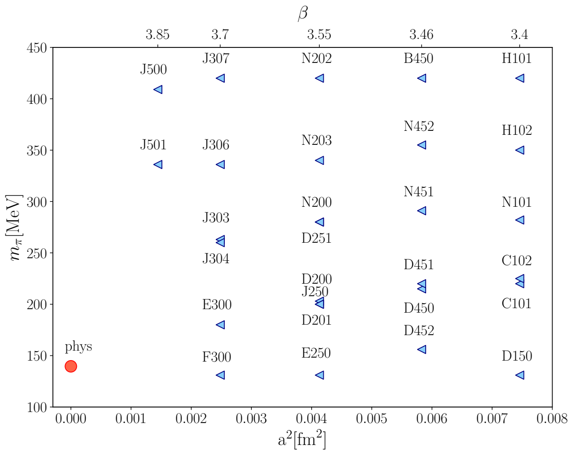
- In the $SU(3)$-flavour basis $j_k^a = \bar{q}\gamma_k(\lambda_a/2)q$, $a = 3, 8, 0$

$$G_{\mu\nu}^{\gamma\gamma}(x) = G_{\mu\nu}^{33}(x) + \frac{1}{3}G_{\mu\nu}^{88}(x) + \frac{4}{9}G_{\mu\nu}^{cc}(x)$$

$$G_{\mu\nu}^{Z\gamma}(x) = \left(\frac{1}{2} - \sin^2\theta_W\right)G_{\mu\nu}^{\gamma\gamma}(x) - \frac{1}{6\sqrt{3}}G_{\mu\nu}^{08}(x) - \frac{1}{18}G_{\mu\nu}^{cc}(x)$$

Lattice setup - CLS ensembles

- $N_f = 2+1$ non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions [Lüscher and Schaefer, JHEP 1107 036 - JHEP 1502 043 - 1712.04884 - 2003.13359]
- Five values of $a \in [0.039, 0.087]$ fm
- Full range of pion masses $m_\pi \in [130, 420]$ MeV
- Two different chiral trajectories:
  - $\rightarrow$ $\Phi_4 \propto \mathrm{Tr}(M_q) = \Phi_4^{\text{phys}}$
  - $\rightarrow$ some ensembles on $m_s \approx m_s^{\mathrm{phys}}$



Computational Strategy

- We propose the following decomposition for the subtracted HVP

$$\begin{aligned}\bar{\Pi}(Q^2) &= [\Pi(Q^2) - \Pi(Q^2/4)] + [\Pi(Q^2/4) - \Pi(Q^2/16)] + [\Pi(Q^2/16) - \Pi(0)] \\ &= \widehat{\Pi}(Q^2) + \widehat{\Pi}(Q^2/4) + \bar{\Pi}(Q^2/16)\end{aligned}$$

- Clear separation of the different euclidean regions gives access to $0.25\ \mathrm{GeV}^2 \leq Q^2 \leq 12.0\ \mathrm{GeV}^2$

HV: $4.0\ \mathrm{GeV}^2 \leq Q^2 \leq 12.0\ \mathrm{GeV}^2$

MV: $1.0\ \mathrm{GeV}^2 \leq Q^2/4 \leq 3.0\ \mathrm{GeV}^2$

LV: $0.25\ \mathrm{GeV}^2 \leq Q^2/16 \leq 0.75\ \mathrm{GeV}^2$

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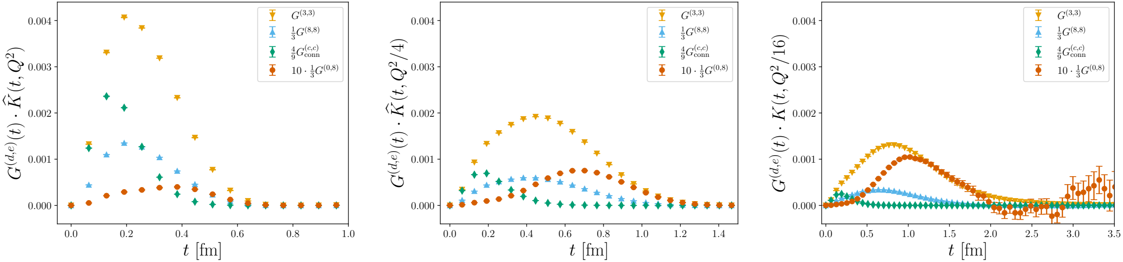

Computational Strategy

- Two discretisations of the vector current, the local (L) and point-split conserved (C)

Set 1: Improvement coefficients from large-volume [2510.06869]

Set 2: Improvement coefficient from Schrödinger Functional setup [1805.07401, 2010.09539]

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- Isovector contribution [S. Kuberski *et al.* 2024]
  - → removal of $O(a^2 \log(a))$ lattice artefacts through pQCD in the high-energy spacelike region
  - → We modify the $K(t, Q)$ QED kernel via

$$K_{\mathrm{sub}}(t, Q, Q_m) = K(t, Q) - \frac{4}{3}\frac{Q^2}{Q_m^4}\sin^4\left(\frac{Q_m x_0}{2}\right),$$

  - → Subtracted contribution restored with pQCD through the Adler function

$$b(Q^2, Q_m^2) = \frac{Q^2}{3Q_m^2}\left[\Pi(4Q_m^2) - \Pi(Q^2)\right]$$

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- Isovector contribution [S. Kuberski *et al.* 2024]
  - → removal of $O(a^2 \log(a))$ lattice artefacts through pQCD in the high-energy spacelike region
- Isoscalar contribution: computational strategy depending on the Euclidean region
  - → HV region:

$$\widehat{\Pi}^{(8,8)} = \widehat{\Pi}^{(3,3)} + \widehat{\Delta}_{\rm ls}(Q^2), \qquad \widehat{\Delta}_{\rm ls}(Q^2) = \int_0^\infty \mathrm{d}x_0 \left[G^{(8,8)}(x_0) - G^{(3,3)}(x_0)\right] \hat{K}(x_0, Q^2)$$

  - → MV and LV region: obtained through a straightforward computation by employing the non-subtracted kernel

Computational Strategy

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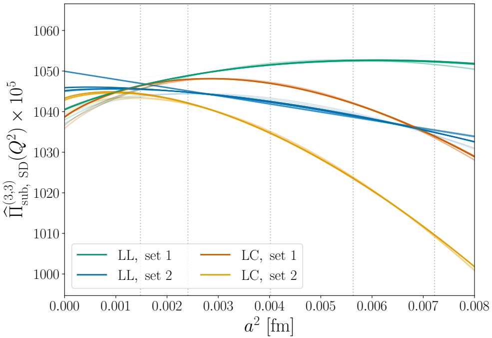
- Isoscalar contribution: computational strategy depending on the Euclidean region

- Charm-connected contribution $\bar{\Pi}^{(c,c)}(Q^2)$ computed analogously to the isovector HVP

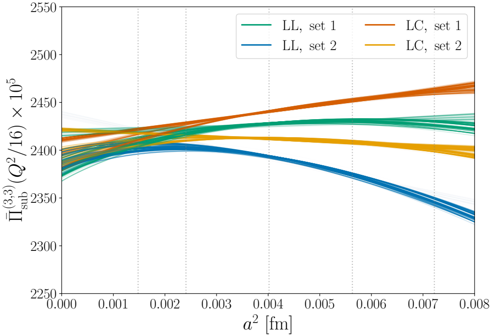
- The $Z\gamma$ mixed isoscalar $\bar{\Pi}^{(0,8)}$ vanishes linearly in $m_s - m_l$ and no subtracted kernel is required

The isovector channel: cutoff dependence

HV: $Q^2 = 9.0\ \mathrm{GeV}^2$



LV: $Q^2/16 = 0.56\ \mathrm{GeV}^2$



- Dependence of $\bar{\Pi}^{(3,3)}$ on a^2 at physical quark masses for HV and LV virtuality regions
- Each line represents a fit in the model average: four sets of data differ by $O(a^2)$
- $\sqrt{t_0}$ from a combination of f_π and f_K [2510.20450]

The isovector channel: chiral dependence

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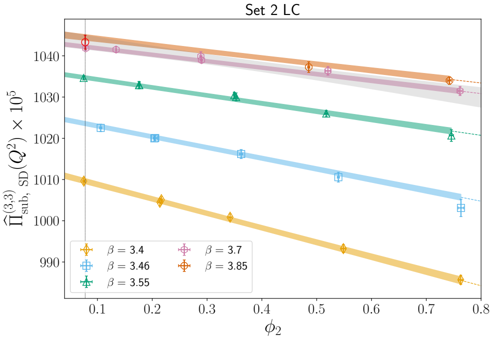
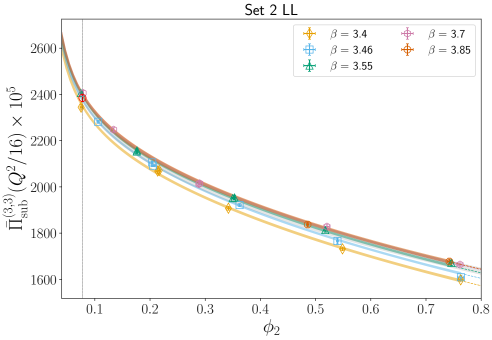


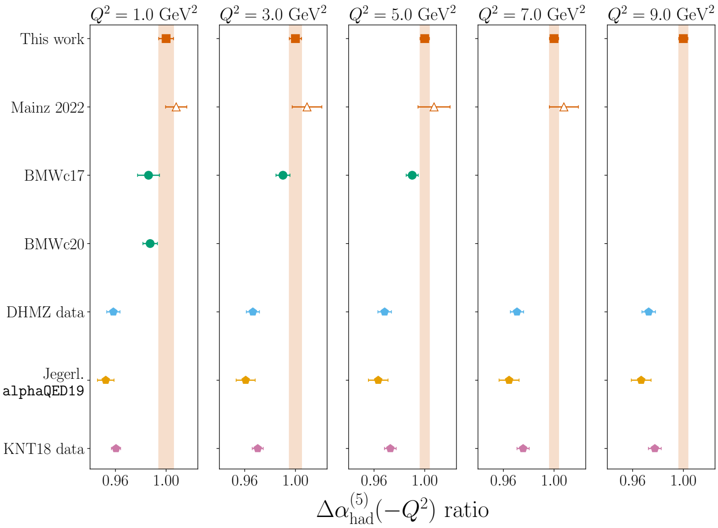


- Dependence of $\bar{\Pi}^{(3,3)}$ on $\phi_2 = 8t_0 m_\pi^2$ for HV and LV virtuality regions
- Well constrained chiral dependence across the range of pion masses
- Strong chiral dependence observed in the LV region

Comparison with other determinations at space-like momenta

Ratio of $\Delta\alpha_{\mathrm{had}}$ divided by our results for different Q^2

- Agreement with Mainz 2022 [M. Cè *et al.*, 2021]
- 1-2 σ disagreement between this work and [Borsanyi *et al.*, 2018; Borsanyi *et al.*, 2021]
- 3-7 σ tension between this work and [Keshavarzi, Nomura, and Teubner 2020; Davier *et al.* 2020; Jegerlehner 2020]



Hadronic running at the Z-pole

- Convert lattice results for $\Delta\alpha_{\mathrm{had}}^{(5)}(-Q_0^2)$ to an estimate of $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$
- Euclidean Split Technique: the Adler function approach [Jegerlehner, hep-ph/9901386, 0807.4206]

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_z^2) = \Delta\alpha_{\mathrm{had}}^{(5)}(-Q_0^2) + \left[\Delta\alpha_{\mathrm{had}}^{(5)}(-M_z^2) - \Delta\alpha_{\mathrm{had}}^{(5)}(-Q_0^2)\right] + \left[\Delta\alpha_{\mathrm{had}}^{(5)}(M_z^2) - \Delta\alpha_{\mathrm{had}}^{(5)}(-M_z^2)\right]$$

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- $\Delta\alpha^{(5)}_{\text{had}}(-Q_0^2)$: input from Lattice QCD

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- $\Delta\alpha^{(5)}_{\text{had}}(-Q_0^2)$: input from Lattice QCD
- From the Adler function $D(Q^2)$, known in massive pQCD at three-loops

$$\left[\Delta\alpha^{(5)}_{\text{had}}(-M_z^2) - \Delta\alpha^{(5)}_{\text{had}}(-Q_0^2)\right]_{\text{pQCD/Adler}} = \frac{\alpha}{3\pi}\int_{Q_0^2}^{M_Z^2} \frac{\mathrm{d}Q^2}{Q^2} D(Q^2)$$

Determination of $D(Q^2)$

- **AdlerPy** [R. F. Hernández, 2311.04849; J. Erler *et al.*, 2308.05740]
- Jegerlehner's software package **pQCDAdler**

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- Perturbation theory: $\left[\Delta\alpha^{(5)}_{\rm had}(M_z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_z^2)\right] = 0.000\ 045(2)$ [Jegerlehner, CERN Yellow Report, 2020]

Perturbative $D(Q^2)$: three different setups

AdlerPy

- $\overline{\text{MS}}$ coupling and quark masses
- Massive PT: exact to α_s; expansion + spline at α_s^2; leading α_s^3 tail terms; double bubbles included
- FLAG24 input values

pQCD-bfmom

- Pole masses, BF-MOM coupling
- Massive PT: exact to α_s; expansion + Padé at α_s^2; no α_s^3 terms ; no double bubbles
- Legacy input values

pQCD-updated

- Purpose: update input parameters to match AdlerPy
- Coupling tables regenerated: the α_s RG running was rebuilt
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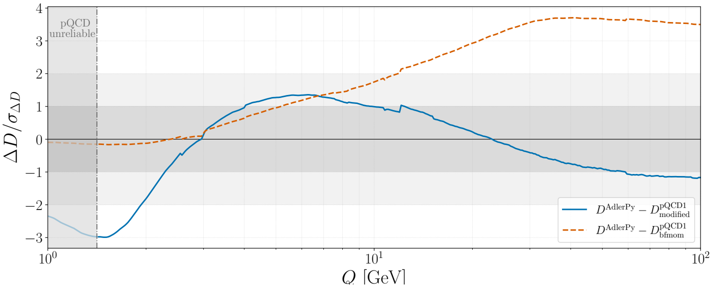
At the Z pole, quark-mass effects are negligible, and the residual tension reflects the coupling scheme – $\overline{\mathrm{MS}}$ vs BF-MOM. At lower Q, sensitivity to the mass scheme increases, exposing the limitations of pole masses



AdlerPy as the reference implementation

Cleaner scheme choices across scales

- **High Q (Z-pole)**: $\overline{\mathrm{MS}}$ coupling, without finite BF-MOM matching residuals
- **Intermediate Q**: short-distance $\overline{\mathrm{MS}}$ masses avoid the pole-mass ambiguity and reduce threshold sensitivity

More complete massive sector

- Leading $O(\alpha_s^3)$ mass corrections, heavy-quark double bubbles, and QED $O(\alpha)$ (the massless series is *identical* through α_s^4)

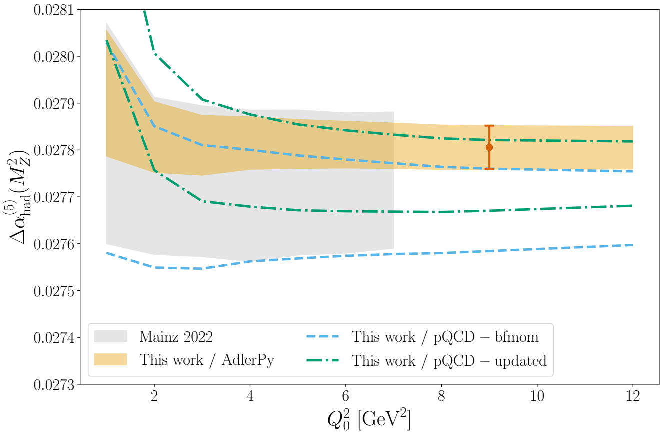
Transparent implementation and error budget

- RunDec running and decoupling; open and reproducible code
- Bootstrap over independent inputs, explicit truncation error
- Smaller error in the running-dominant $2 - 20\ \mathrm{GeV}$ window

Evaluation of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$

- **Lattice input:**
  $\Delta\alpha^{(5)}_{\rm had}(-Q_0^2)$ for $3~\mathrm{GeV}^2 \leq Q_0^2 \leq 12~\mathrm{GeV}^2$
- **Perturbative running:**
  Evaluation of the Adler function with three different methods
  1. AdlerPy
  2. pQCD-bfmom
  3. pQCD-updated

  Input values from FLAG24 for 1 and 3
- **Threshold energy:** $Q_0^2 = 9~\mathrm{GeV}^2$



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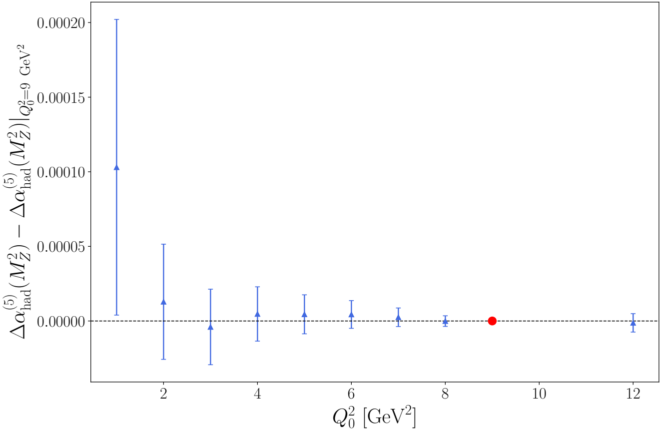
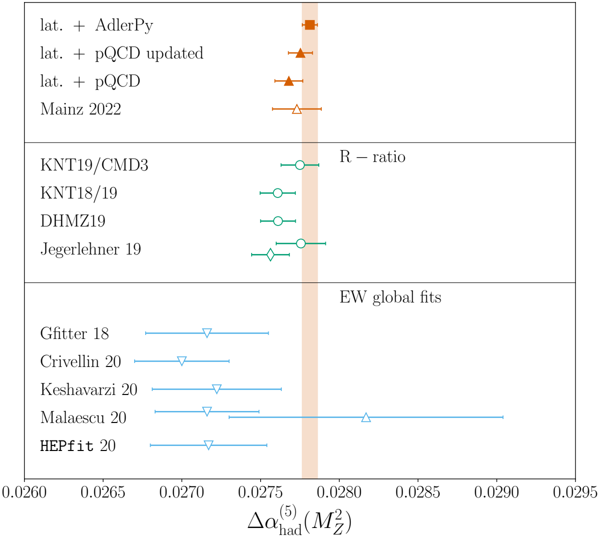


Comparison with phenomenology and electroweak fits

- We obtain at the Z-pole

$$\boxed{\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027\,821(34)_{\mathrm{lat}}(35)_{\mathrm{pQCD}}}$$

- Large tension observed in space-like region is substantially reduced at the Z-pole
- Tension at the level of $1-2\sigma$ with dispersive evaluations
- Global electroweak fits compatible within 2σ

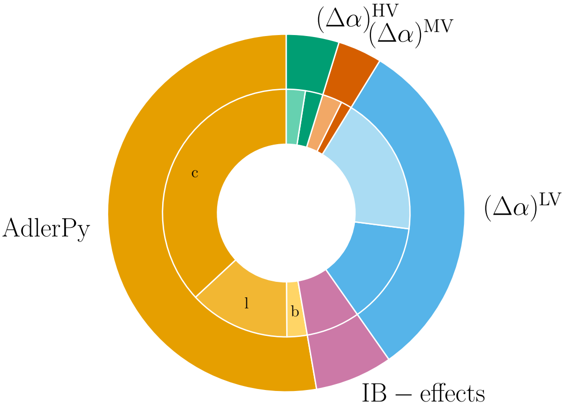


Main contribution to the variance of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$

- **Threshold energy:** $Q_0^2 = 9\ \mathrm{GeV}^2$
- Good balance between lattice and pQCD

Error budget dominated by:

- **AdlerPy**: charm contribution to $D(Q^2)$
- **Lattice**: $(\Delta\alpha)^{\mathrm{LV}}$ contribution
- **IB-effects:** by far subleading



Future prospects and FCC-ee targets

Are we on track?

Mainz 2022:

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = 0.027\,73(15)\ [0.54\%]$$

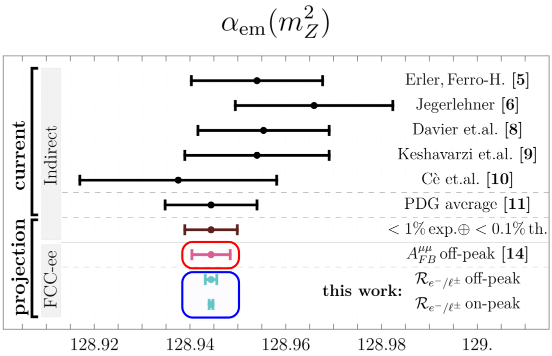
Mainz 2025:

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = 0.027\,821(49)\ [0.17\%]$$

Future prospects and FCC-ee targets

- The electroweak precision program at FCC-ee requires $\delta\Delta\alpha^{(5)}_{\text{had}} \sim 3 \times 10^{-5}$, based on FB asymmetry muon production measurements
  [P. Janot, JHEP 02 (2016) 053]

- A novel method based on Z-pole measurements could reach a statistical sensitivity of $\delta\Delta\alpha^{(5)}_{\text{had}} \sim 1 \times 10^{-5}$
  [M. Riembau, 2501.05508]



[M. Riembau, 2501.05508]

How do we get there, given the current precision of $\delta\Delta\alpha_{\text{had}} = 4.9 \times 10^{-5}$?

Parametric error modelling

- Hadronic running on the lattice up to Q_0^2, then evolution to the Z-pole using pQCD
- Increasing the matching scale Q_0^2
  - $\rightarrow$ decreases the weight of perturbative running, and thus its uncertainty
  - $\rightarrow$ requires higher spacelike momenta on the lattice, where discretization effects become more relevant

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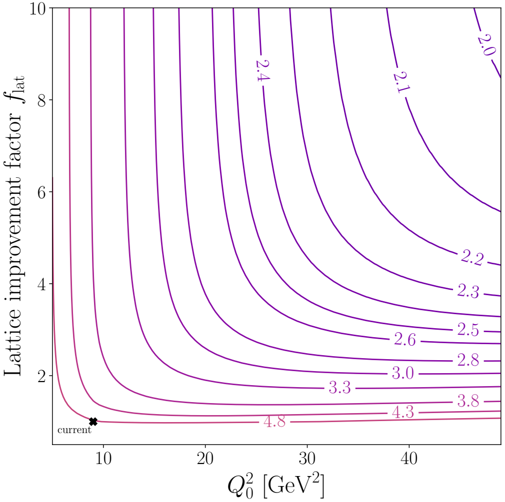
- We construct a predictive model of the total uncertainty as a function of three ingredient:
  - $\rightarrow$ the precision of the non-perturbative inputs entering pQCD
  - $\rightarrow$ the precision of the lattice determination at the matching point
  - $\rightarrow$ the choice of the matching scale itself

Exploring improvement scenarios

- Projected total uncertainty σ_{tot} in the (Q_0^2, f_{lat}) plane. Shown are contour lines of the total error in units of 10^{-5}
- The current uncertainty is denoted by a black cross
- No improvement factor assumed in pQCD input parameters
- Pushing to higher Q_0^2 alone does not meet the desired precision level. Combined with moderate lattice improvements, it delivers substantial boost in precision



Conclusions

Summary

- Updated determination of the HVP contribution to the running of the electroweak gauge couplings
- Significant tension at space-like momenta with data-drive estimates
- Our result at the Z-pole is a factor of two more precise than recent phenomenological evaluations

$$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027\,821(34)_{\mathrm{lat}}(35)_{\mathrm{pQCD}}[49]\ [0.17\ \%]$$

Outlook: path to FCC-ee precision

- The FCC-ee EW program requires $\delta\Delta\alpha_{\mathrm{had}} \sim 3\times10^{-5}$, and possibly $\delta\Delta\alpha_{\mathrm{had}} \sim 1\times10^{-5}$
- Increasing the matching scale Q_0^2 alone is insufficient, but combining this with moderate lattice improvements $(\times 2 - \times 4)$ yields substantial gains
- Significant progress in pQCD inputs needed for the 1×10^{-5} regime, and LQCD can play a crucial role

Thank You!

FVC and noise reduction strategies

- Finite-volume corrections via spacelike [Hansen and Patella, 1904.10010, 2004.03935] and timelike [Meyer, 1105.1892][Lellouch and Lüscher, hep-lat/0003023] pion form factor
  - → Correct to $(m_\pi L)_{\mathrm{ref}} = 4.29$ for $a \neq 0$ [Borsanyi et al., 2002.12347] [Djukanovic et al., 2411.07696]
  - → Correct to $L = \infty$ in the continuum at the physical point

FVC and noise reduction strategies

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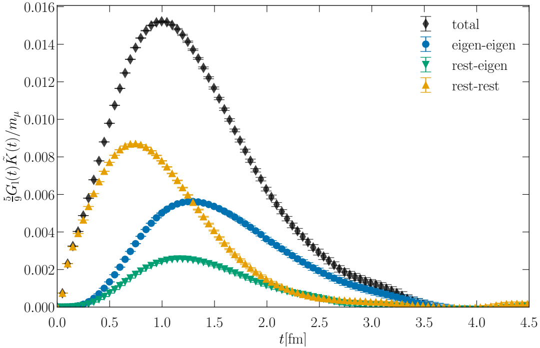
- Low-Mode Averaging (LMA) [L. Giusti *et al.*, hep-lat/0402002, T.A. DeGrand *et al.*, hep-lat/0401011]
  - $\rightarrow$ Improved estimator for the light-connected correlation function
  - $\rightarrow$ LMA is used for all ensembles with $m_\pi < 280$ MeV



Ensemble F300
$m_\pi = 130$ MeV, $\quad a = 0.049$ fm
[Plot by S. Kuberski]

FVC and noise reduction strategies

- Finite-volume corrections via spacelike [Hansen and Patella, 1904.10010, 2004.03935] and timelike [Meyer, 1105.1892][Lellouch and Lüscher, hep-lat/0003023] pion form factor
- Low-Mode Averaging (LMA) [L. Giusti *et al.*, hep-lat/0402002, T.A. DeGrand *et al.*, hep-lat/0401011]
- Bounding Method in isovector and isoscalar channels

$$0 \leq G^{(\alpha,\gamma)}(t_c)e^{-E_{\mathrm{eff}}(t-t_c)} \leq G^{(\alpha,\gamma)}(t) \leq G^{(\alpha,\gamma)}(t_c)e^{-E_0(t-t_c)}$$

  where

  $\rightarrow$ E_0 is the ground-state energy level contributing to the vector correlation function

  $\rightarrow$ E_{eff} is the logarithmic derivative of the vector correlation function

FVC and noise reduction strategies

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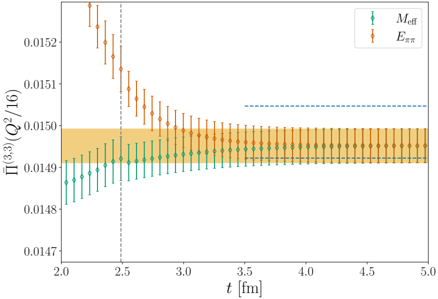
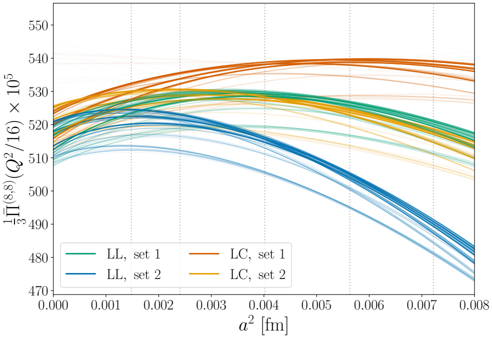
- Spectral reconstruction of the $\pi\pi$ contribution in isovector channel [Djukanovic et al., 2411.07696][Nolan Miller @ Lattice24]

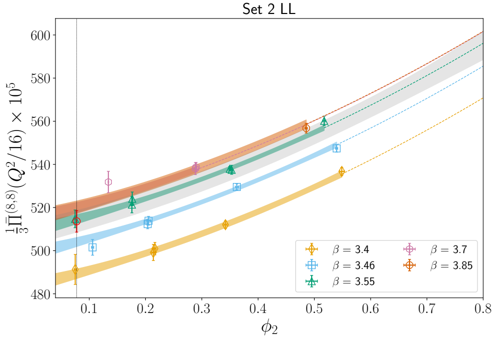
- $N_{\pi\pi} = 4$ states used to reconstruct the correlator
- Solves signal-to-noise problem beyond $t = 2.5$ fm
- Increase the precision by a factor of 2 on physical point ensemble E250: 0.2% for $\bar{\Pi}^{(3,3)}$



The isoscalar channel

- Chiral-continuum dependence of $\bar{\Pi}^{(8,8)}$ in the LV region at $Q^2/16 = 0.56\ \mathrm{GeV}^2$

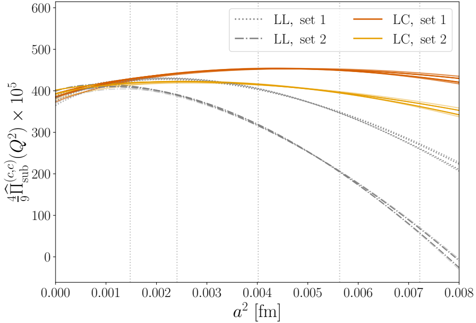


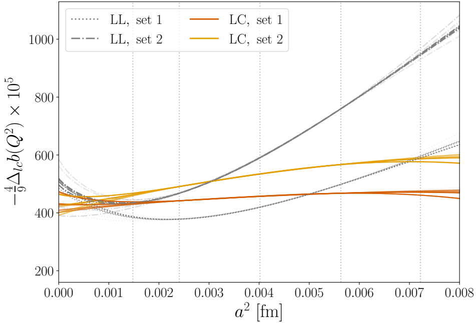


- Divergent behaviour of light-connected and disconnected pieces is cancelled
- Bounding method in the LV region to mitigate noise at large Euclidean distances

Charm connected contribution - HV region

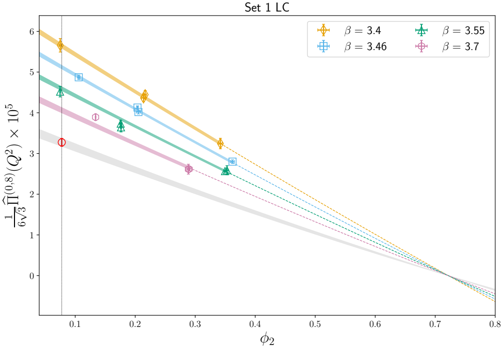
- $\widehat{\Pi}^{(c,c)} = (\widehat{\Pi}^{(c,c)})_{\mathrm{sub}} + b^{(c,c)}$ chiral-continuum extrapolation at $Q^2 = 9\ \mathrm{GeV}^2$
- $b^{(c,c)}(Q^2) = 2b^{(3,3)}(Q^2) + \Delta_{\mathrm{lc}}b$
- Very good agreement despite significantly different cutoff effects

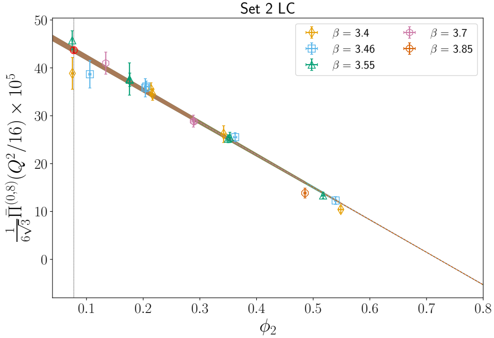




$\bar{\Pi}^{08}$ contribution

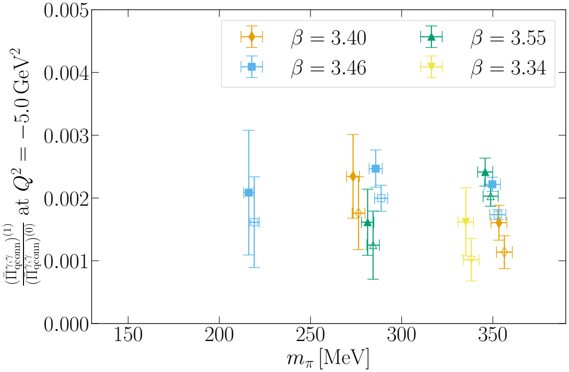
- $\bar{\Pi}^{(0,8)}$ chiral-continuum extrapolation at $Q^2 = 9$ GeV
- Results for MV (left) and LV (right) virtuality regions
- $SU(3)_f$ breaking $\rightarrow$ parametrically suppressed at short distance





Isospin-breaking effects

- **Lattice calculation:**
  - → Down to $m_\pi \sim 200$ MeV
  - → Only connected diagrams



Isospin-breaking effects

- **Lattice calculation:**
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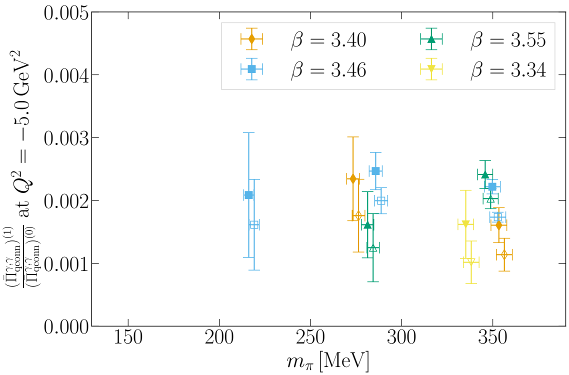
- **Phenomenological model:**
  - a Charged pion loops with VMD form factors [J. Parrino *et al.* 2501.03192]
  - b Pseudoscalar meson exchanges ($\pi^0,\ \eta,\ \eta',\ \eta_c$) [V. Biloshytskyi *et al.* 2209.02149]
  - c Strong IB, based on $\omega - \rho$ mixing [D. Erb *et al.* 2505.24344]



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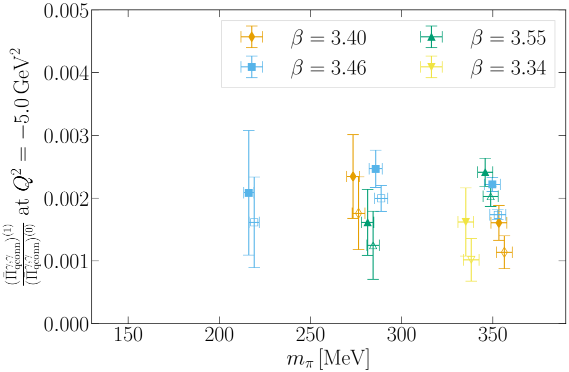
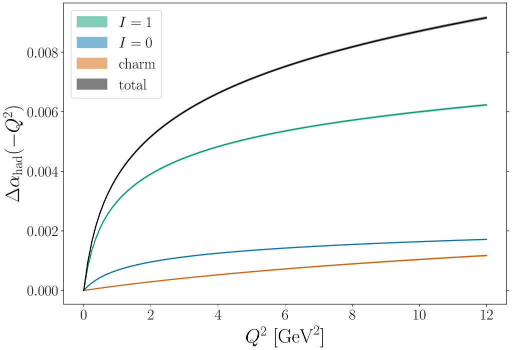
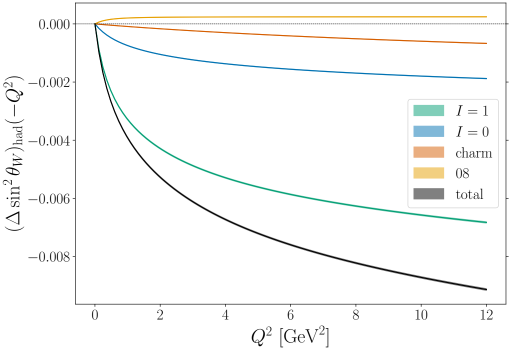


$$\boxed{\Delta_{\text{IB}}\Delta\alpha_{\text{had}}(9\ \text{GeV}^2) = 0.75(1.28)\times 10^{-5}}$$

Small relative size of IB effects

Running of $\Delta\alpha_{\mathrm{had}}$ and $(\sin^2\theta_W)_{\mathrm{had}}$





- Total HVP contribution to $\Delta\alpha_{\mathrm{had}}$ and $(\Delta\sin^2\theta_W)_{\mathrm{had}}$
- Results in the range $0 \leq Q^2 \leq 12\ \mathrm{GeV}^2$
- Rational approximation of the running through a Padé Ansatz [AC *et al.*, 2511.01623]

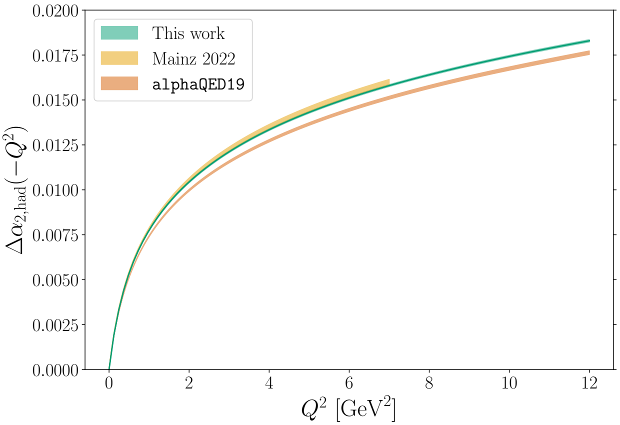
Electroweak mixing angle

- Running of $\alpha_2 = g^2/(4\pi^2)$

$$\alpha_2(q^2) = \frac{\alpha_2}{1 - \Delta\alpha_2(q^2)}$$

- Phenomenological estimates systematically below our determinations, by about 3.5% at $Q^2 = 12\ \mathrm{GeV}^2$

- Factor of two improvement in precision for the running of the electroweak mixing angle compared to [J. Erler *et al.*, 2406.16691]



- New measurements expected from P2 [D. Becker *et al.*, 1802.04759] and MOLLER [MOLLER Experiment, 1411.4088] collaborations

Parametric error modelling

- We quantify how much each perturbative input $\alpha_s,\ m_c(m_c),\ m_b(m_b)$ shifts $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$

$$g_i = \frac{\partial \Delta\alpha^{(5)}_{\rm had}(M_Z^2)}{\partial p_i}, \qquad p_i \in \{\alpha_s,\ m_c(m_c),\ m_b(m_b)\}$$

- This allows to parametrize the total uncertainty

$$\sigma^2_{\rm pQCD}(Q_0^2) = \sum_{i,j} g_i(Q_0^2) C_{ij} g_j(Q_0^2) + \sigma^2_{\rm trunc} \quad \Longrightarrow \quad \sigma^2_{\rm tot}(Q_0^2) = \sigma^2_{\rm pQCD}(Q_0^2) + \sigma^2_{\rm lat}(Q_0^2)$$

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- Introduce improvement factors that rescale the corresponding uncertainties

$$f_i = \frac{\sigma_{p_i}}{\sigma_{p_i}^{\mathrm{curr}}}, \qquad p_i \in \{\alpha_s,\ m_c(m_c),\ m_b(m_b)\}, \qquad f_{\mathrm{lat}} = \frac{\sigma_{\mathrm{lat}}}{\sigma_{\mathrm{lat}}^{\mathrm{curr}}}$$

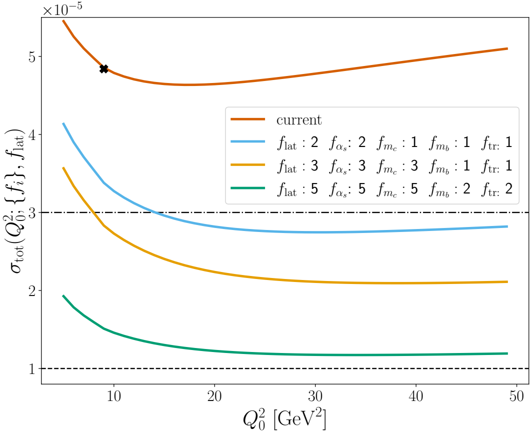
$$\sigma^2_{\mathrm{tot}}(Q_0^2; \{f_i\}, f_{\mathrm{lat}}) = \sigma^2_{\mathrm{pQCD}}(Q_0^2; \{f_i\}) + \sigma^2_{\mathrm{lat}}(Q_0^2; f_{\mathrm{lat}})$$

Pathways to ultimate precision

- Total uncertainty $\sigma_{\rm tot}$ as a function of the matching scale Q_0^2 for different improvement scenarios

- Moderate improvement scenarios are capable of reaching the required accuracy of $\delta\Delta\alpha^{(5)}_{\rm had} \sim 3 \times 10^{-5}$ foreseen by [P. Janot, JHEP 02 (2016) 053]

- Reaching the target precision of $\delta\Delta\alpha^{(5)}_{\rm had} \sim 1 \times 10^{-5}$ anticipated in [M. Riembau, 2501.05508] demands substantial advances in both lattice and pQCD inputs. Further suppression of the truncation uncertainty is required



- Lattice QCD can help improve the pQCD inputs [ALPHA 25, 2501.06633]