# Hadronic light-by-light contribution to muon $g-2$

## RBC/UKQCD update 2026

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# Outline

# Results from previous work

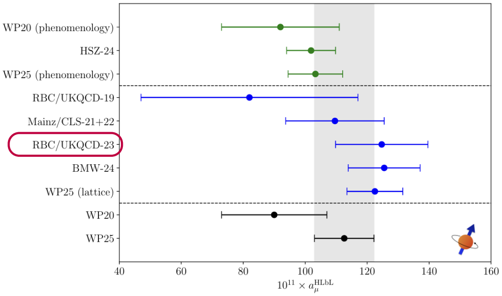
The HLbL contribution calculated with the 48I unitary mass ensemble in [Blum et al., 2023]:

$$a_\mu^{\text{HLbL}} \times 10^{10} = 12.47\,(1.15)_{\text{stat}}(0.95)_{\text{syst}}\,[1.49].$$

Significant sources of systematic uncertainty come from $a^2$-correction and $\pi^0$-exchange contribution [Blum et al., 2023].

## Previous work and improvements

| | |
|---|---|
| 48I light $\pi^0$-exchange | $2.00\ (0.11)_{\text{stat}}(0.28)_{\text{syst}}\ [0.30]$ |
| 48I light FV-correction | $-0.47\ (0.11)_{\text{syst}}$ |
| 48I light $m_\pi$-correction | $0.35\ (0.07)_{\text{stat}}(0.17)_{\text{syst}}\ [0.19]$ |
| This talk: | Improved long distance formula |
| 48I light $a^2$-correction | $0.00\ (0.83)_{\text{syst}}$ |
| This talk: | New calculation on 64I |
| 48I light no-pion | $0.31\ (0.22)_{\text{stat}}(0.31)_{\text{syst}}\ [0.38]$ |




Summary of HLbL calculations.

# HLbL master formula

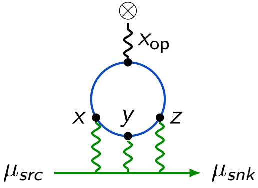
The master formula for the HLbL contribution to muon $g-2$ in position space is given by [Blum et al., 2017]:

$$a_\mu^{\mathrm{HLbL}} = \frac{2me^2}{3}\frac{1}{VT}\sum_{x_{\mathrm{op}}}\sum_{x,y,z}\frac{1}{2}\,\epsilon_{ijk}\left(x_{\mathrm{op}} - x_{\mathrm{ref}}(x,y,z)\right)_j (6e^4)\,\mathcal{H}_{k\rho\sigma\lambda}(x_{\mathrm{op}},x,y,z)\,\mathcal{M}_{i\rho\sigma\lambda}(x,y,z)$$

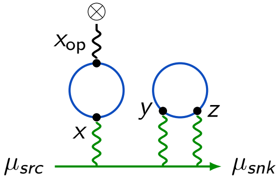
where $\mathcal{H}$ is the hadronic four-point function, $\mathcal{G}$ is the continuum QED weighting function evaluated in infinite volume, and $x_{\mathrm{ref}}(x,z,y)$ is the moment-method reference position.

$$\mathcal{M}_{i\rho\sigma\lambda}(x,y,z) = \frac{1}{2}\,\mathrm{Tr}\Big[\frac{1}{6}\,i^3\,\mathcal{G}_{\rho\sigma\lambda}(x,y,z)\,\Sigma_i\Big]$$

$$6e^4\,\mathcal{H}_{k\rho\sigma\lambda}(x_{\mathrm{op}},x,y,z) = \langle 0|\,T\{J_k(x_{\mathrm{op}})J_\rho(x)J_\sigma(y)J_\lambda(z)\}\,|0\rangle$$



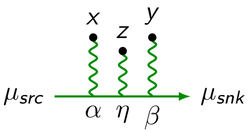

Connected diagram




Leading order disconnected diagram

# QED weighting function

$$\mathcal{M}_{i\rho\sigma\lambda}(x,y,z) = \frac{1}{2}\,\mathrm{Tr}\Big[\frac{1}{6}\, i^3\, \mathcal{G}_{\rho\sigma\lambda}(x,y,z)\,\Sigma_i\Big]$$




The QED weighting function $\mathcal{G}$ can be expressed in terms of muon and photon propagators:

$$\mathfrak{G}_{\sigma\kappa\rho}(y,z,x) = \lim_{\substack{t_{\mathrm{src}}\to-\infty \\ t_{\mathrm{snk}}\to\infty}} e^{m_\mu(t_{\mathrm{snk}}-t_{\mathrm{src}})} \int_{\alpha,\beta,\eta,x_{\mathrm{snk}},x_{\mathrm{src}}} G(x,\alpha)\,G(y,\beta)\,G(z,\eta)\,S_\mu(x_{\mathrm{snk}},\beta)\,i\gamma_\sigma\,S_\mu(\beta,\eta)\,i\gamma_\kappa\,S_\mu(\eta,\alpha)\,i\gamma_\rho\,S_\mu(\alpha,x_{\mathrm{src}})$$

**Remove the infrared divergence:**

$$\mathfrak{G}^{(1)}_{\sigma\kappa\rho}(y,z,x) = \frac{1}{2}\,\mathfrak{G}_{\sigma\kappa\rho}(y,z,x) + \frac{1}{2}\left[\mathfrak{G}_{\rho\kappa\sigma}(x,z,y)\right]^\dagger$$

**Subtraction scheme:**

$$\mathfrak{G}^{(2)}_{\sigma\kappa\rho}(y,z,x) = \mathfrak{G}^{(1)}_{\sigma\kappa\rho}(y,z,x) - \mathfrak{G}^{(1)}_{\sigma\kappa\rho}(z,z,x) - \mathfrak{G}^{(1)}_{\sigma\kappa\rho}(y,z,z)$$

$$i^3\,\mathcal{G}_{\rho\sigma\kappa}(x,y,z) = \mathfrak{G}^{(2)}_{\rho\sigma\kappa}(x,y,z) + [\text{all perms}]$$

# Comparison with the Mainz subtraction scheme

Due to the current conservation in $\mathcal{H}$ we can allow several forms of subtraction schemes to reduce the discretization error:

**RBC/UKQCD subtraction [Blum et al., 2017]**

$$\mathfrak{G}^{(2)}_{\sigma\kappa\rho}(y,z,x) = \mathfrak{G}^{(1)}_{\sigma\kappa\rho}(y,z,x) - \mathfrak{G}^{(1)}_{\sigma\kappa\rho}(z,z,x) - \mathfrak{G}^{(1)}_{\sigma\kappa\rho}(y,z,z)$$

**Mainz subtraction [Asmussen et al., 2023; Chao et al., 2021]**

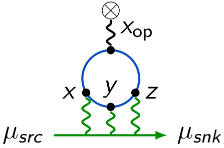
$$\begin{aligned}\mathfrak{G}^{(\mathrm{Mainz})}_{\sigma\kappa\rho}(y,z,x) = \mathfrak{G}^{(1)}_{\sigma\kappa\rho}(y,z,x) & \\ & - \partial^y_\sigma\big((y-x)_\alpha\, e^{-\Lambda m_\mu^2 (y-x)^2/2}\big)\, \mathfrak{G}^{(1)}_{\alpha\kappa\rho}(x,z,x) \\ & - \partial^z_\kappa\big((z-x)_\alpha\, e^{-\Lambda m_\mu^2 (z-x)^2/2}\big)\, \mathfrak{G}^{(1)}_{\sigma\alpha\rho}(y,x,x)\end{aligned}$$

note that while $a_\mu^{HLbL}$ is independent of the subtraction scheme, the integrands and partial sums are *not directly comparable*.

# Long distance + short distance calculations

We define a cutoff function $R_{\text{max}}$:

$$R_{\text{max}} = \max\left(|x-y|,\ |x-z|,\ |y-z|\right).$$




The total can be written as the sum of all contributions from two regions:

$$a_\mu^{\text{HLbL}} = a_\mu\big(R_{\text{max}} < R_{\text{max}}^{\text{cut}}\big) + a_\mu\big(R_{\text{max}} > R_{\text{max}}^{\text{cut}}\big)$$

- The first term is the partial sum of the $a_\mu^{\text{HLbL}}$ summand over vertex coordinates within the range $0 \leq R_{\text{max}} < R_{\text{max}}^{\text{cut}}$; the second term is the reverse partial sum with respect to the same cutoff.
- $R_{\text{max}}^{\text{cut}}$ needs to be large enough so that $a_\mu\left(R_{\text{max}} > R_{\text{max}}^{\text{cut}}\right) \approx a_\mu^{\pi^0\text{-exchange}}$. Our previous work [Blum et al., 2023] used $R_{\text{max}}^{\text{cut}} = 4.0$ fm in the reported final result.

# Long distance + short distance calculations

$$a_\mu^{\text{HLbL}} = a_\mu\left(R_{\text{max}} < R_{\text{max}}^{\text{cut}}\right) + a_\mu\left(R_{\text{max}} > R_{\text{max}}^{\text{cut}}\right)$$

## Direct and $\pi^0$-exchange (indirect) calculations

| $a_\mu(R_{\text{max}} < R_{\text{max}}^{\text{cut}})$ | $a_\mu(R_{\text{max}} > R_{\text{max}}^{\text{cut}})$ |
|---|---|
| • Direct calculation using two point source propagators ($\sim$ 2000 propagators per configuration).<br>• Field-sparsening is employed to reduce contraction costs. | • Long distance region is dominated by the $\pi^0$-exchange contribution.<br>• Indirect calculation using $\pi^0$ transition form factors (TFF). |

# Long-distance $\pi^0$ exchange contribution

In the long-distance region where $|z-y| \ll \min(|x-z|, |x-y|)$, the largest contribution to $\mathcal{H}_{k\rho\sigma\lambda}$ comes from the $\pi^0$ intermediate state:

$$\mathcal{H}_{k\rho\sigma\lambda}(x_{\text{op}}, x, y, z) \approx \mathcal{H}^{\pi 0}_{k\rho\sigma\lambda}(x_{\text{op}}, x, y, z),$$

$$(6e^4)\, \mathcal{H}^{\pi 0}_{k\rho\sigma\lambda}(x_{\text{op}}, x, y, z) = \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2E_{\pi,\vec{q}}} \langle 0| T\{J_k(x_{\text{op}}) J_\rho(x)\} |\pi^0_{\vec{q}}\rangle \langle \pi^0_{\vec{q}}| T\{J_\sigma(y) J_\lambda(z)\} |0\rangle$$

The $\pi^0$ transition form factor is given by:

$$\widetilde{\mathcal{F}}_{k\rho}(\tilde{x}, q) = \langle 0| T\{J_k(\tfrac{\tilde{x}}{2}) J_\rho(-\tfrac{\tilde{x}}{2})\} |\pi^0_{\vec{q}}\rangle = \epsilon_{k\rho\alpha\beta}\, \tilde{x}_\alpha q_\beta\, \tilde{\mathcal{F}}_{\pi^0\gamma\gamma}\left(\tilde{x}^2, q \cdot \tilde{x}\right)$$

# Long-distance $\pi^0$ exchange contribution

The $\pi^0$ transition form factor is given by:

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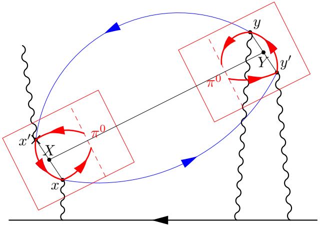
$\mathcal{H}^{\pi_0}$ can then be expressed in terms of products of form factors and scalar pion propagator:

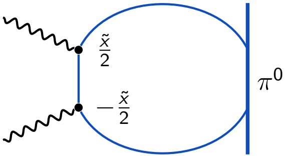
$$(6e^4)\, \mathcal{H}^{\pi_0}_{k\rho\sigma\lambda}(x', x, y', y) = \widetilde{\mathcal{F}}_{k\rho}(\tilde{x}, -i\tfrac{\partial}{\partial X})\, \widetilde{\mathcal{F}}_{\sigma\lambda}(\tilde{y}, -i\tfrac{\partial}{\partial Y})\, D_{\pi_0}(|R_\pi|)$$

$$\boxed{\begin{aligned} X &= \tfrac{x'+x}{2}, \quad Y = \tfrac{y'+y}{2} \\ R_\pi &= X - Y \\ \tilde{x} &= x' - x, \quad \tilde{y} = y' - y \end{aligned}}$$

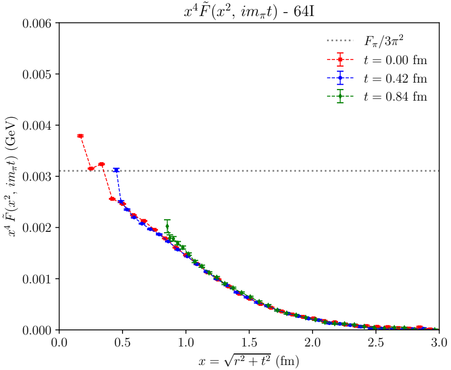


# Long-distance $\pi^0$ exchange contribution

$$\widetilde{\mathcal{F}}_{k\rho}(x,q) = \langle 0|T\{J_k(\tfrac{x}{2})J_\rho(-\tfrac{x}{2})\}|\pi^0_{\vec{q}}\rangle = \epsilon_{k\rho\alpha\beta}x_\alpha q_\beta\, \tilde{\mathcal{F}}_{\pi^0\gamma\gamma}(x^2, q\cdot x)$$




$\pi^0 \to \gamma\gamma$ transition form factor



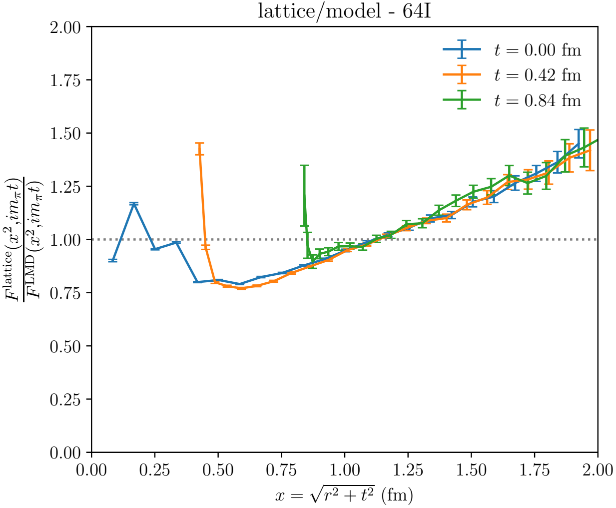

$\tilde{\mathcal{F}}_{\pi^0\gamma\gamma}(x^2, q\cdot x)$ has a weak dependence on $q\cdot x$.

- Ratio of the lattice scalar form factor to the LMD model form factor.

$$F^{\mathrm{VMD}} = \frac{m_V^2}{q_1^2+m_V^2}\,\frac{m_V^2}{q_2^2+m_V^2}$$

$$F^{\mathrm{TE}} = \frac{1}{2}\left(\frac{m_V^2}{q_1^2+m_V^2} + \frac{m_V^2}{q_2^2+m_V^2}\right)$$

$$F^{\mathrm{LMD}} = F^{\mathrm{VMD}} + \frac{8\pi^2 F_\pi^2}{3m_V^2}\left(F^{\mathrm{TE}} - F^{\mathrm{VMD}}\right)$$



# Long-distance $\pi^0$ exchange contribution

$$(6e^4)\,\mathcal{H}^{\pi_0}_{\kappa\rho\sigma\lambda}(x',x,y',y) = \widetilde{\mathcal{F}}_{\kappa\rho}\big(\tilde{x}, -i\tfrac{\partial}{\partial X}\big)\,\widetilde{\mathcal{F}}_{\sigma\lambda}\big(\tilde{y}, -i\tfrac{\partial}{\partial Y}\big)\,D_{\pi_0}(|R_\pi|)$$

For leading order in $1/|R_\pi|$, the derivative with respect to the propagator can be written as:

$$-i\frac{\partial}{\partial X^\beta} D_\pi(|X-Y|) \approx \left(im_\pi \frac{(X-Y)_\beta}{|X-Y|}\right) D_{\pi_0}(|X-Y|).$$

## Two formulations of $\mathcal{H}^{\pi^0}$

**Previous work (2023): derivative approximation on the entire $\mathcal{H}^{\pi_0}$**

$$(6e^4)\,\mathcal{H}^{\pi_0(RBC-UKQCD23)}_{\kappa\rho\sigma\lambda}(x',x,y',y) = \widetilde{\mathcal{F}}_{\kappa\rho}\big(\tilde{x}, im_\pi\hat{R}_\pi\big)\,\widetilde{\mathcal{F}}_{\sigma\lambda}\big(\tilde{y}, -im_\pi\hat{R}_\pi)\big)\,D_{\pi_0}(|R_\pi|)$$

**Improved (this work): derivative approximation only on $\widetilde{\mathcal{F}}_{\pi^0\gamma\gamma}(\tilde{x}^2, -i\frac{\partial}{\partial X})$**

$$(6e^4)\,\mathcal{H}^{\pi_0(\mathrm{new})}_{\kappa\rho\sigma\lambda}(x',x,y',y) \approx \epsilon_{\kappa\rho\alpha\beta}\epsilon_{\sigma\lambda\gamma\zeta}\,\tilde{x}_\alpha\tilde{y}_\gamma\,\widetilde{\mathcal{F}}^E_{\pi^0\gamma\gamma}(\tilde{x}^2, im_\pi\hat{R}_\pi\cdot\tilde{x})\,\widetilde{\mathcal{F}}^E_{\pi^0\gamma\gamma}(\tilde{y}^2, -im_\pi\hat{R}_\pi\cdot\tilde{y})$$
$$\times\left(-i\frac{\partial}{\partial X^\beta}\right)\left(-i\frac{\partial}{\partial Y^\zeta}\right) D_{\pi^0}(|R_\pi|)$$

# Long-distance $\pi^0$ exchange contribution

$$(6e^4)\,\mathcal{H}^{\pi_0}_{\kappa\rho\sigma\lambda}(x',x,y',y) = \widetilde{\mathcal{F}}_{\kappa\rho}\big(\tilde{x}, -i\tfrac{\partial}{\partial X}\big)\,\widetilde{\mathcal{F}}_{\sigma\lambda}\big(\tilde{y}, -i\tfrac{\partial}{\partial Y}\big)\,D_{\pi_0}(|R_\pi|)$$

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$$(6e^4)\,\mathcal{H}^{\pi_0(\text{new})}_{\kappa\rho\sigma\lambda}(x',x,y',y) \approx \epsilon_{\kappa\rho\alpha\beta}\epsilon_{\sigma\lambda\gamma\zeta}\,\tilde{x}_\alpha\tilde{y}_\gamma\,\widetilde{\mathcal{F}}^E_{\pi^0\gamma\gamma}(\tilde{x}^2, im_\pi\hat{R}_\pi\cdot\tilde{x})\,\widetilde{\mathcal{F}}^E_{\pi^0\gamma\gamma}(\tilde{y}^2, -im_\pi\hat{R}_\pi\cdot\tilde{y})$$
$$\times\left(-i\tfrac{\partial}{\partial X^\beta}\right)\left(-i\tfrac{\partial}{\partial Y^\zeta}\right) D_{\pi^0}(|R_\pi|)$$

# Calculation details

Using approximations for $\mathcal{H}^{\pi_0}$ described in the previous slide, we perform Monte Carlo importance sampling to evaluate:

$$a_\mu^{\pi_0\text{-exch}} = \frac{1}{VT} \sum_{x',x,y',y} I^{\pi^0\text{-exchange}}(x', x, y', y)$$

using the following **weighting function**:

$$w(x', x, y', y) = \frac{1}{|x' - x|^4 + \tilde{a}^2} \frac{1}{|y' - y|^4 + \tilde{b}^2} \frac{135\,\text{MeV}\ K_1(135\,\text{MeV}\ R_\pi)}{4\pi^2 R_\pi}$$

**$\pi^0$ propagation distance:** $R_\pi = \left| \frac{x'+x}{2} - \frac{y'+y}{2} \right|, \quad (\tilde{a} = \tilde{b} = 30.0)$

| | 48I | 64I |
|---|---|---|
| $m_\pi$ (MeV) | 139 | 139 |
| $m_K$ (MeV) | 499 | 508 |
| $a^{-1}$ (GeV) | 1.730 | 2.359 |
| $a$ (fm) | 0.114 | 0.084 |
| $L$ (fm) | 5.47 | 5.35 |
| # configs | 113 | 119 |

Lattice parameters for 48I and 64I ensembles

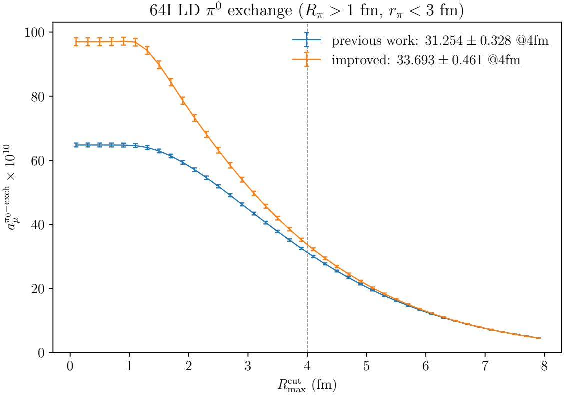
# Long-distance $\pi^0$-exchange contribution - $\mathcal{H}^{\pi^0}(2023)$ vs $\mathcal{H}^{\pi^0}(\text{improved})$

BLINDED, PRELIMINARY results

- Cut functions for the reverse partial sum

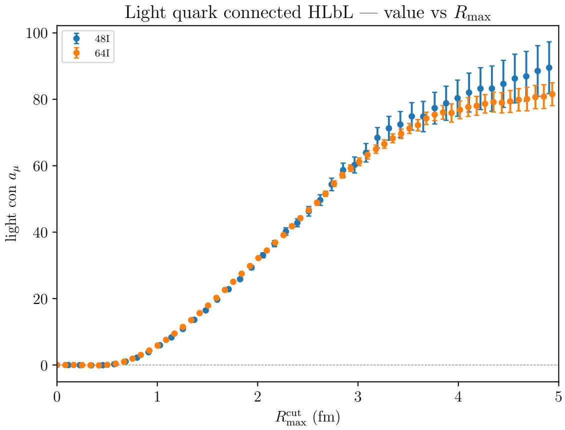
$$R_{\max} = \max\left(|x-y|, |x-z|, |y-z|\right)$$
$$r_\pi = \max\left(|x_{op}-x|, |y-z|\right)$$
$$R_\pi = \left|\frac{x_{op}+x}{2} - \frac{y+z}{2}\right|$$

- The relative uncertainty between previous work and this work at $R_{\max}^{\text{cut}} = 4.0$ fm is $\sim 7.8\%$, which is within the 14% estimated systematic error of the $\pi^0$-exchange calculation in [Blum et al., 2023].



BLINDED, PRELIMINARY results

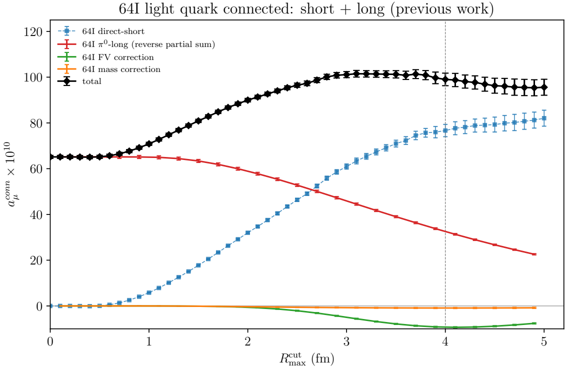
- Light quark contributions are evaluated with unitary $\pi^0 = 139$ MeV mass ensembles.
- The $R_{\text{max}}^{cut}$ in the direct, partial sum calculation is the same cutoff variable as in the reverse partial sum calculation.



# 64I total light quark connected - $\mathcal{H}^{\pi^0}$ (2023)

BLINDED, PRELIMINARY results

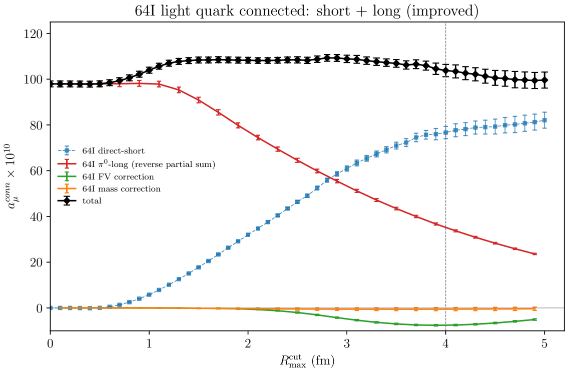
- Light quark contributions are evaluated with unitary $\pi^0 = 139$ MeV mass ensembles.
- Displayed curves are partial sums with respect to $R_{\text{max}}^{\text{cut}}$, except for 64I long distance, which is a reverse partial sum.
- In this plot, the long distance calculation is performed using RBC/UKQCD-23 formula for $\mathcal{H}^{\pi^0}$.



# 64I total light quark connected - $\mathcal{H}^{\pi^0}$(improved)

BLINDED, PRELIMINARY results

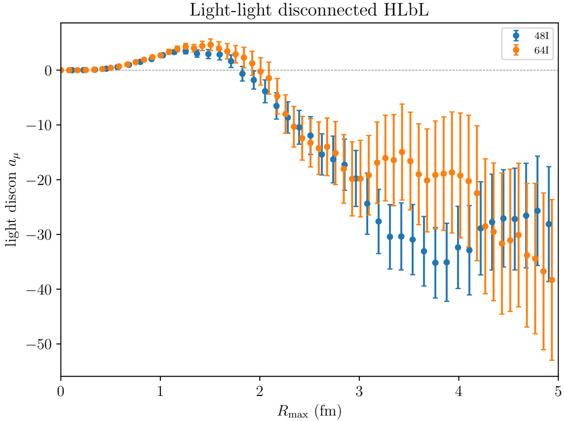
- Compared to $\mathcal{H}^{\pi^0}$(2023), the total plateaus at earlier at $R_{\text{max}}^{\text{cut}} \sim$ 1.5 fm.
- $\mathcal{H}^{\pi^0}$(improved) has less systematic uncertainty compared to $\mathcal{H}^{\pi^0}$(2023), due to the former using fewer derivative approximations.
- Earlier plateau allows us to consider lower $R_{max}^{cut}$ cutoffs to reduce statistical uncertainty



# Light quark disconnected (unitary $\pi^0 = 139$ MeV mass ensembles)

BLINDED, PRELIMINARY results

- Large statistical error in the non-negligible long-distance tail of the light quark disconnected diagram.
- Instead of adding the noisy disconnected contribution at long-distance $R_{\max}^{\text{cut}}$, we use the hybrid approach.



# Results: hybrid approach

BLINDED, PRELIMINARY results

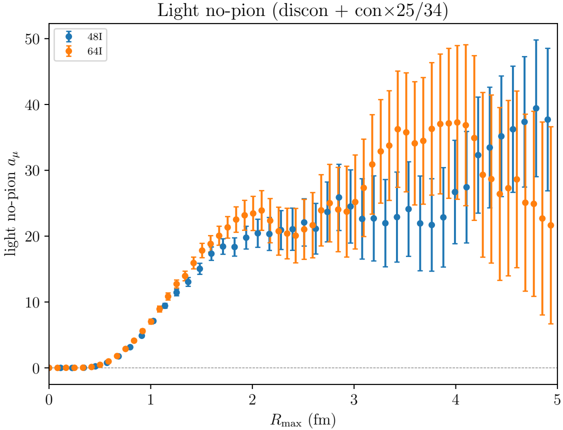
- Long-distance discon/con ratio [Bijnens and Relefors, 2016]:

$$\lim_{R\to\infty} \frac{a_\mu^{\text{discon}}(R_{\max} > R)}{a_\mu^{\text{con}}(R_{\max} > R)} = -\frac{1}{2} \cdot \frac{(e_u^2 + e_d^2)^2}{e_u^4 + e_d^4} = -\frac{25}{34}$$

- Hybrid approach:

$$a_\mu^{\text{no-pion}} = a_\mu^{\text{discon}} + \frac{25}{34}\, a_\mu^{\text{con}},$$
$$a_\mu^{\text{total}} = a_\mu^{\text{no-pion}} + \frac{9}{34}\, a_\mu^{\text{con}}.$$

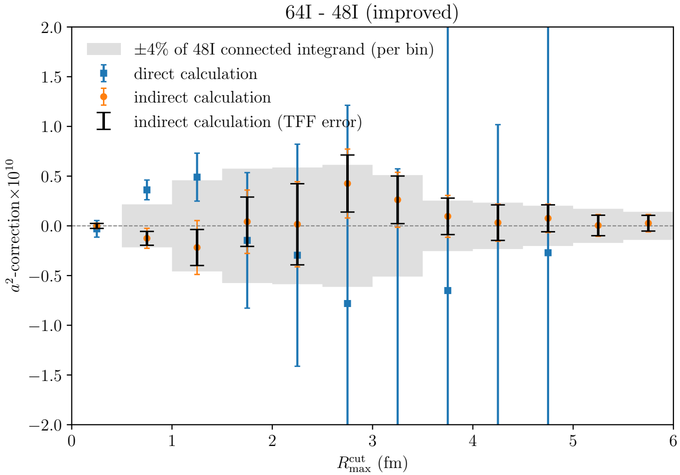
- In our previous work we truncated $a_\mu^{\text{no-pion}}$ at an earlier plateau value, $\sim$ 2.5 fm, and add it to less noisy connected contributions at larger $R_{\max}^{\text{cut}}$



# Light quark connected: lattice-spacing effects

BLINDED, PRELIMINARY results

- The plot compares binned $a^2$-correction between direct and $\pi^0$-long distance calculations.
- Recall that we estimated $\pm$ 8% systematic error in our previous work. [Blum et al., 2023].
- 8% correction to 48I corresponds to 4% difference between 48I and 64I
- Shaded error-band is $\pm$ 4% of the 48I connected integrant.

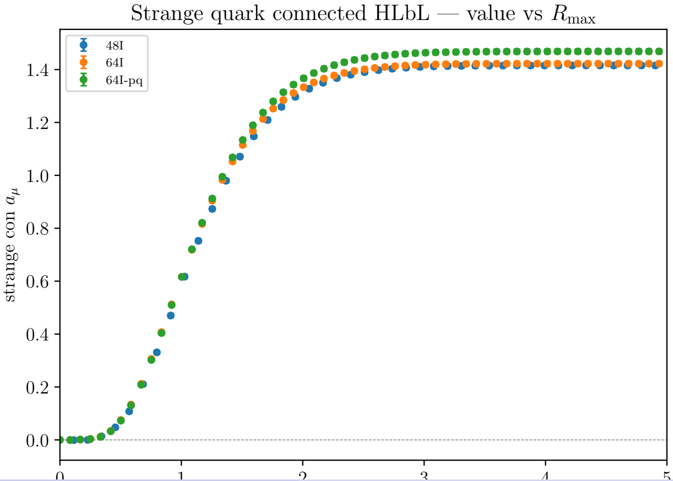


# Strange quark connected

BLINDED, PRELIMINARY results

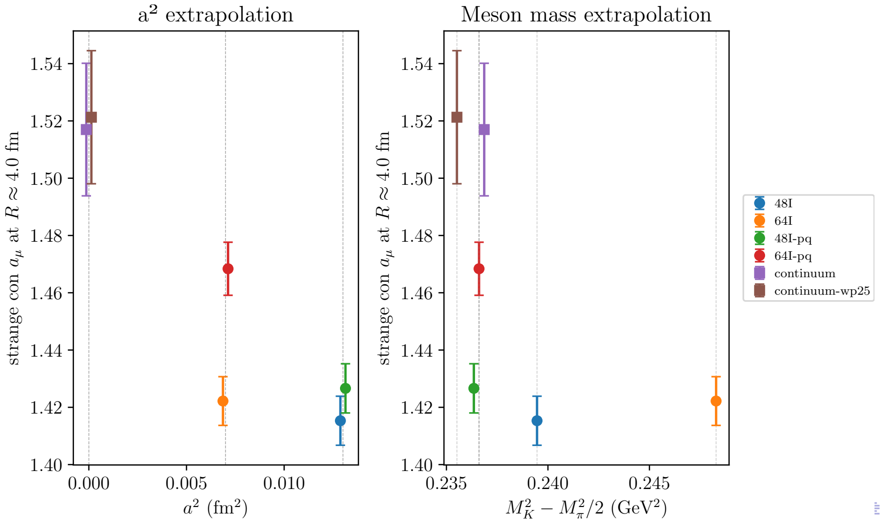
- The direct calculation of the strange quark connected contribution for the 48I, 64I and 64I-pq ensembles.
- Pion and kaon masses of the plotted ensembles:

| | $m_{\pi^0}$ (MeV) | $m_K$ (MeV) |
|---|---|---|
| 48I | 139 | 499 |
| 64I | 139 | 508 |
| 64I-pq | 135 | 496 |



# Strange quark connected: continuum limit

BLINDED, PRELIMINARY results



# Summary

Significant sources of systematic uncertainty come from $a^2$-correction and $\pi^0$-exchange contribution [Blum et al., 2023]

| | |
|---|---|
| 48I light $\pi^0$-exchange ($R_{\mathrm{max}} > 4\,\mathrm{fm}$) | $2.00\ (0.11)_{\mathrm{stat}}(0.28)_{\mathrm{syst}}\ [0.30]$ |
| this talk: | Improved long distance formula ✓ |
| 48I light $a^2$-correction | $0.00\ (0.83)_{\mathrm{syst}}$ |
| this talk: | New calculation on 64I ✓ |

Future directions:

- Evaluate the charged pion contribution to HLbL to help improve the long distance calculation.
- Perform continuum limit extrapolation for the direct calculation; finalize estimates for $\pi^0$ mass and FV corrections; unblind the results.

# Thank you!

# References I


- Asmussen, Nils et al. [2023]. “Hadronic light-by-light scattering contribution to the muon g − 2 from lattice QCD: semi-analytical calculation of the QED kernel”. In: *JHEP* 04, p. 040. DOI: 10.1007/JHEP04(2023)040. arXiv: 2210.12263 [hep-lat].
- Bijnens, Johan and Johan Relefors [2016]. “Pion light-by-light contributions to the muon $g-2$”. In: *JHEP* 09, p. 113. DOI: 10.1007/JHEP09(2016)113. arXiv: 1608.01454 [hep-ph].
- Blum, Thomas et al. [Aug. 2017]. “Using infinite-volume, continuum QED and lattice QCD for the hadronic light-by-light contribution to the muon anomalous magnetic moment”. In: *Phys. Rev. D* 96 [3], p. 034515. DOI: 10.1103/PhysRevD.96.034515. URL: https://link.aps.org/doi/10.1103/PhysRevD.96.034515.
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# References II

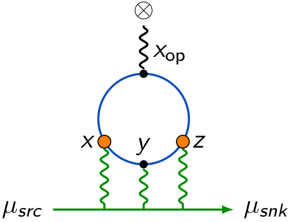

Chao, En-Hung et al. [2021]. "Hadronic light-by-light contribution to $(g-2)_\mu$ from lattice QCD: a complete calculation". In: *Eur. Phys. J. C* 81.7, p. 651. DOI: 10.1140/epjc/s10052-021-09455-4. arXiv: 2104.02632 [hep-lat].

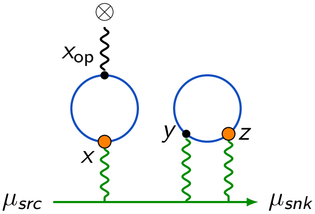
# Short-distance calculations

$$(6e^4)\,\mathcal{H}^{\text{con-no-perm}}_{k\rho\sigma\lambda}(x_{\text{op}},x,y,z) = -6\Big\langle \mathrm{Re}\sum_{q=u,d,s} e_q^4\,\mathrm{Tr}\big[\gamma_k S_q(x_{\text{op}},x)\gamma_\rho S_q(x,z)\gamma_\lambda S_q(z,y)\gamma_\sigma S_q(y,x_{\text{op}})\big]\Big\rangle$$

$$(6e^4)\,\mathcal{H}^{\text{discon-no-perm}}_{k\rho\sigma\lambda}(x_{\text{op}},x,y,z) = 3\Big\langle \Pi_{k\rho}(x_{\text{op}},x)\big(\Pi_{\lambda\sigma}(z,y)-\langle\Pi_{\lambda\sigma}(z,y)\rangle\big)\Big\rangle$$

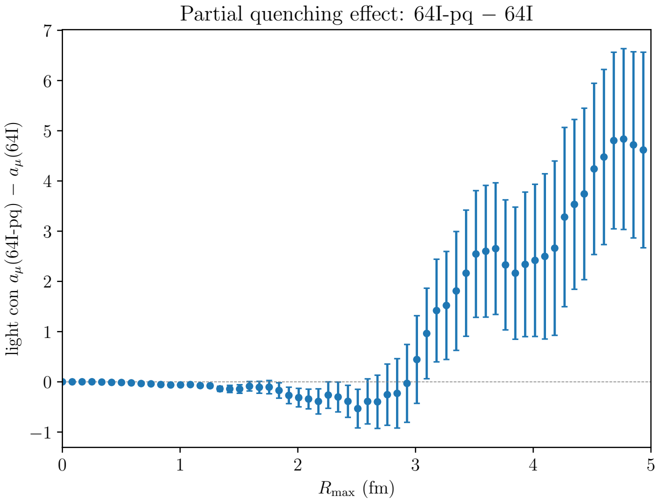
$$\Pi_{\rho\sigma}(x,y) \equiv \sum_q e_q^2\,\mathrm{Tr}\big[\gamma_\rho S_q(x,y)\gamma_\sigma S_q(y,x)\big]$$



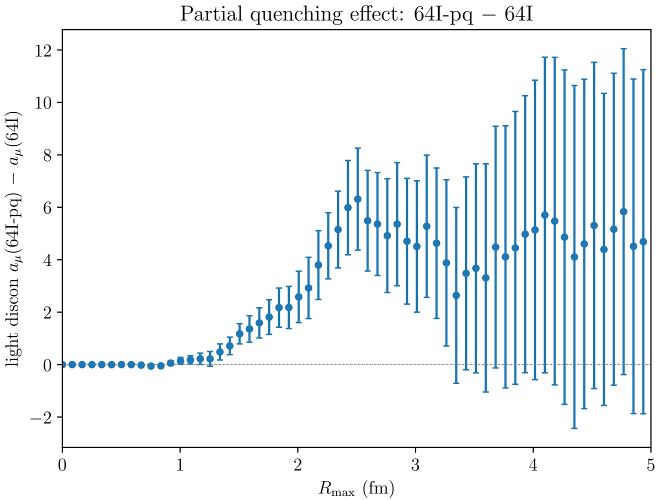
# Light quark connected: $\pi^0$ mass correction

BLINDED, PRELIMINARY results



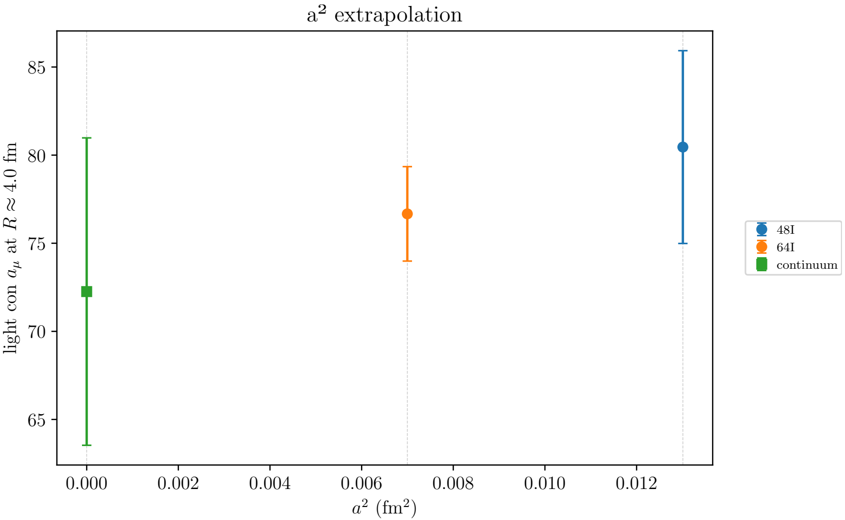
# Light quark disconnected: $\pi^0$ mass correction

BLINDED, PRELIMINARY results



# Light quark connected: continuum limit

BLINDED, PRELIMINARY results



# Light quark disconnected: continuum limit

BLINDED, PRELIMINARY results