High-precision Monte Carlo predictions for radiative return with BabaYaga@NLO

9th Plenary Workshop of the Muon g-2 Theory Initiative

Marco Ghilardi
On behalf of the BabaYaga@NLO team

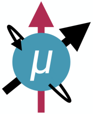
August 3, 2026

INFN National Institute for Nuclear Physics
UNIVERSITÀ DI PAVIA

The Muon $g-2$

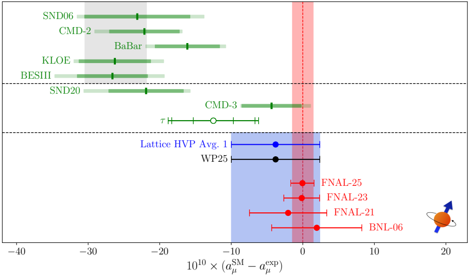
The Muon anomaly

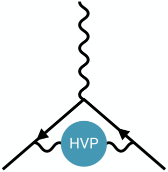
$$\vec{\mu} = -g\mu_B\vec{S} \qquad a_\mu = \frac{(g-2)_\mu}{2}$$



Muon g-2

The current status of the theory–experiment comparison is the following:





https://arxiv.org/abs/2505.21476

$e^+e^- \to \pi^+\pi^-$ channel

$$a_\mu^{\text{HVP-LO}} = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{m_{\pi_0}^2}^{\infty} ds \frac{1}{s^2} \mathcal{R}_0^{\text{had}}(s) \hat{\mathcal{K}}(s)$$

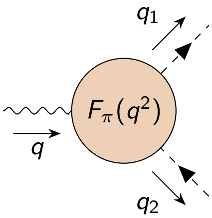
https://arxiv.org/pdf/2312.02053

$e^+e^- \to \pi^+\pi^-$ channel

UNIVERSITÀ DI PAVIA

$$a_\mu^{\text{HVP-LO}} = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{m^2_{\pi_0}}^{\infty} ds\, \frac{1}{s^2} \mathcal{R}_0^{\text{had}}(s) \hat{\mathcal{K}}(s)$$

$$\boxed{\langle \pi^+(q_2)\pi^-(q_1)|J^\mu_\pi(0)|0\rangle = e(q_1 - q_2)^\mu F_\pi\left(q^2\right)}$$

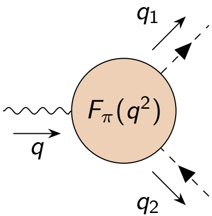


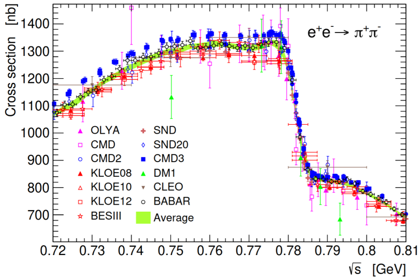
https://arxiv.org/pdf/2312.02053

$e^+e^- \to \pi^+\pi^-$ channel

$$a_\mu^{\text{HVP-LO}} = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{m_{\pi_0}^2}^{\infty} ds \frac{1}{s^2} \mathcal{R}_0^{\text{had}}(s)\hat{\mathcal{K}}(s)$$

$$\boxed{\langle \pi^+(q_2)\pi^-(q_1)|J_\pi^\mu(0)|0\rangle = e(q_1 - q_2)^\mu F_\pi\,(q^2)}$$





https://arxiv.org/pdf/2312.02053

The radiative return

At fixed collider energy $\sqrt{s}$, lower-energy two pion pairs can be studied via initial-state radiation:

$$e^+e^- \to \pi^+\pi^-\gamma, \quad M^2_{\pi\pi} = (p_{\pi^+} + p_{\pi^-})^2 < s$$

The radiative return

At fixed collider energy $\sqrt{s}$, lower-energy two pion pairs can be studied via initial-state radiation:

$$e^+e^- \to \pi^+\pi^-\gamma, \quad M^2_{\pi\pi} = (p_{\pi^+} + p_{\pi^-})^2 < s$$

The measured differential cross section relates to the pion production cross section:

$$\frac{\mathrm{d}\sigma_{\pi^+\pi^-\gamma}}{dM_{\pi\pi}} = H(s, M^2_{\pi\pi})\, \sigma_{\pi^+\pi^-}(M^2_{\pi\pi}),$$

where $H(s, M^2_{\pi\pi})$ is the radiator function.

The radiative return

At fixed collider energy $\sqrt{s}$, lower-energy two pion pairs can be studied via initial-state radiation:

$$e^+e^- \to \pi^+\pi^-\gamma, \quad M^2_{\pi\pi} = (p_{\pi^+} + p_{\pi^-})^2 < s$$

The measured differential cross section relates to the pion production cross section:

$$\frac{\mathrm{d}\sigma_{\pi^+\pi^-\gamma}}{dM_{\pi\pi}} = H(s, M^2_{\pi\pi})\,\sigma_{\pi^+\pi^-}(M^2_{\pi\pi}),$$

where $H(s, M^2_{\pi\pi})$ is the radiator function.

A precise theoretical treatment of radiative corrections is required to extract the pion form factor at the sub-percent level.

One Year Later

UNIVERSITÀ DI PAVIA

2025

- **Tuned NLO comparisons**
  Completed for both $\mu^+\mu^-\gamma$ and $\pi^+\pi^-\gamma$ channels.

- **Preliminary** look at NLOPS matching accuracy.

F. Piccinini, TI 2025

One Year Later

INFN UNIVERSITÀ DI PAVIA

2025

- **Tuned NLO comparisons**
  Completed for both $\mu^+\mu^-\gamma$ and $\pi^+\pi^-\gamma$ channels.
- **Preliminary** look at NLOPS matching accuracy.

F. Piccinini, TI 2025

1 Year $\longrightarrow$

Today

- **Full NLOPS**
  Achieved for $\mu^+\mu^-\gamma$ and $\pi^+\pi^-\gamma$.

  GitHub
- **Beyond FxsQED for pions**
  - GVMD
  - FsQED

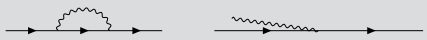
The perturbative accuracy

LL resummation

Approximate calculation of soft-virtual corrections to any order.

1

$\mathcal{C}_\alpha^{\mathrm{LL}}$

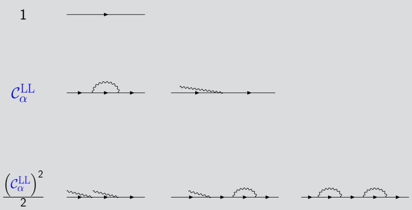


$\frac{\left(\mathcal{C}_\alpha^{\mathrm{LL}}\right)^2}{2}$

The perturbative accuracy

LL resummation

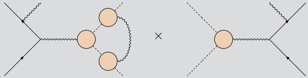
Approximate calculation of soft-virtual corrections to any order.



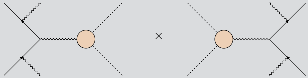
NLO

Exact calculation up to $\mathcal{O}(\alpha)$ w.r.t. the born scattering amplitude.

- Virtual corrections



- Real corrections



The perturbative accuracy

LL resummation

Approximate calculation of soft-virtual corrections to any order.

α^0		
αL	α	
$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
$\sum_{n=3}^{\infty}\frac{\alpha^n}{n!}L^n$	$\sum_{n=3}^{\infty}\frac{\alpha^n}{n!}L^{n-1}$	$\cdots$

NLO

Exact calculation up to $\mathcal{O}(\alpha)$ w.r.t. the born scattering amplitude.

α^0		
αL	α	
$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
$\sum_{n=3}^{\infty}\frac{\alpha^n}{n!}L^n$	$\sum_{n=3}^{\infty}\frac{\alpha^n}{n!}L^{n-1}$	$\cdots$

The perturbative accuracy

NLO+LL (NLOPS)

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\ldots$

The perturbative accuracy

NLO+LL (NLOPS)

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\dots$

NLOPS master formula

$$d\sigma_{\mathrm{NLOPS}} = \sum_{n=1}^{\infty} \exp\{-\mathcal{C}_\alpha^{\mathrm{LL}}\}\, \mathrm{F_{SV}}\, \frac{1}{n!} \left|\mathcal{M}_n^{\mathrm{X}}\right|^2 d\Phi_n(\{p\},\{k\}) \, .$$

The perturbative accuracy

NLOPS master formula

$$d\sigma_{\mathrm{NLOPS}} = \sum_{n=1}^{\infty} \exp\left\{-\mathcal{C}_{\alpha}^{\mathrm{LL}}\right\} \mathrm{F_{SV}} \, \frac{1}{n!} \left|\mathcal{M}_{n}^{\mathrm{X}}\right|^{2} d\Phi_{n}(\{p\},\{k\}) \,.$$

- $\exp\left\{-\mathcal{C}_{\alpha}^{\mathrm{LL}}\right\}$ is the Sudakov Form factor that accounts for non-emission probability.

The perturbative accuracy

NLOPS master formula

$$d\sigma_{\mathrm{NLOPS}} = \sum_{n=1}^{\infty} \exp\{-\mathcal{C}_{\alpha}^{\mathrm{LL}}\}\, \mathrm{F}_{\mathrm{SV}}\, \frac{1}{n!} \left|\mathcal{M}_n^{\mathrm{X}}\right|^2 d\Phi_n(\{p\},\{k\})\,.$$

- $\exp\{-\mathcal{C}_{\alpha}^{\mathrm{LL}}\}$ is the Sudakov Form factor that accounts for non-emission probability.
- $\mathcal{M}_n^X$ stands for $\mathcal{M}^{\mathrm{ex}}$ for $n = 1, 2$ and $\mathcal{M}^{\mathrm{LL}}$ for $n \geq 3$.

The perturbative accuracy

NLOPS master formula

$$\mathrm{d}\sigma_{\mathrm{NLOPS}} = \sum_{\mathrm{n}=1}^{\infty} \exp\{-\mathcal{C}_{\alpha}^{\mathrm{LL}}\}\, \mathrm{F}_{\mathrm{SV}}\, \frac{1}{\mathrm{n}!} \left|\mathcal{M}_{\mathrm{n}}^{\mathrm{X}}\right|^2 \mathrm{d}\Phi_{\mathrm{n}}(\{\mathrm{p}\},\{\mathrm{k}\}) \,.$$

- $\exp\{-\mathcal{C}_{\alpha}^{\mathrm{LL}}\}$ is the Sudakov Form factor that accounts for non-emission probability.

- $\mathcal{M}_n^X$ stands for $\mathcal{M}^{\mathrm{ex}}$ for $n = 1, 2$ and $\mathcal{M}^{\mathrm{LL}}$ for $n \geq 3$.

$$\left|\mathcal{M}_n^{\mathrm{LL}}\right|^2 = \prod_{i=1}^{n-2} \left[-\frac{\alpha}{2\pi} P(z_i)\, \mathcal{I}_{\mathrm{SF}}(k_i, z_i)\, \frac{4\pi^2\,(1-z_i)}{z_i \omega_i^2}\right] \underbrace{\left|\mathcal{M}_2^{\mathrm{ex}}(\{\tilde{p}\},\{\tilde{k}\})\right|^2}_{e^+e^- \to XX\gamma\gamma}$$

The perturbative accuracy

If we define

$$\mathcal{I}(k) = \sum_{i,j=1}^{4} \mathcal{I}_{ij}(k) = \sum_{i,j=1}^{4} \eta_i \eta_j \frac{(p_i \cdot p_j)}{(p_i \cdot k)\,(p_j \cdot k)} k_0^2 \,,$$

then

$$\mathcal{I}_{\mathrm{SF}}(k,z) = -\sum_{ij} \mathcal{I}_{ij}(k) \cdot F_{ij}(k,z)$$

where

$$\begin{cases} F_{ij}(k,z) = 1 - \delta_{ij} \dfrac{(1-z)^2}{1+z^2} & i = \text{fermion} \\ F_{ij}(k,z) = 1 & i = \text{scalar}\,. \end{cases}$$

The perturbative accuracy

If we define

$$\mathcal{I}(k) = \sum_{i,j=1}^{4} \mathcal{I}_{ij}(k) = \sum_{i,j=1}^{4} \eta_i \eta_j \frac{(p_i \cdot p_j)}{(p_i \cdot k)\,(p_j \cdot k)} k_0^2 \, ,$$

then

$$\mathcal{I}_{\mathrm{SF}}(k,z) = -\sum_{ij} \mathcal{I}_{ij}(k) \cdot F_{ij}(k,z)$$

where

$$\begin{cases} F_{ij}(k,z) = 1 - \delta_{ij} \dfrac{(1-z)^2}{1+z^2} & i = \text{fermion} \\ F_{ij}(k,z) = 1 & i = \text{scalar}\, . \end{cases}$$

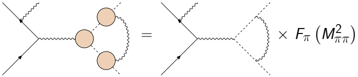
REMARK: In the limit $k \cdot p \to 0$ we correctly reproduce the massive unpolarized splitting function of *S. Dittmaier, hep-ph/9904440*

The Pion internal structure

F×sQED

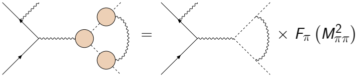
2601.19530

The standard way which respects IR cancellation is $\mathrm{F} \times \mathrm{sQED}$:



The Pion internal structure

The standard way which respects IR cancellation is $\mathrm{F} \times \mathrm{sQED}$:



To include the pion form factor into loop integration:

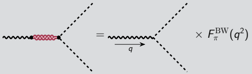
F×sQED

2601.19530

GVMD

$$F^{\mathrm{BW}}_\pi(q^2) = \sum_{v=1}^{n_r} F^{\mathrm{BW}}_{\pi,v}(q^2)$$

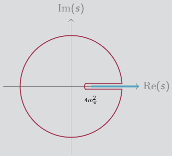
$$F^{\mathrm{BW}}_{\pi,v}(q^2) = \frac{c_v}{c_t}\frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$



FsQED

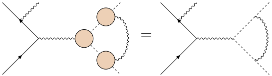
$$F_\pi(q^2) = 1 - \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\frac{\mathrm{Im}F_\pi(s')}{q^2 - s'}$$

$$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\mathrm{Im}F_\pi(s') = 1$$



The Pion internal structure

The standard way which respects IR cancellation is $\mathrm{F} \times \mathrm{sQED}$:

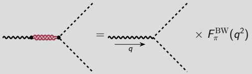


To include the pion form factor into loop integration:

GVMD

$$F_\pi^{\mathrm{BW}}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{\mathrm{BW}}(q^2)$$

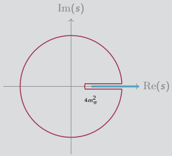
$$F_{\pi,v}^{\mathrm{BW}}(q^2) = \frac{c_v}{c_t}\frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$



FsQED

$$F_\pi(q^2) = 1 - \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\frac{\mathrm{Im}F_\pi(s')}{q^2 - s'}$$

$$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\mathrm{Im}F_\pi(s') = 1$$



F×sQED

2601.19530

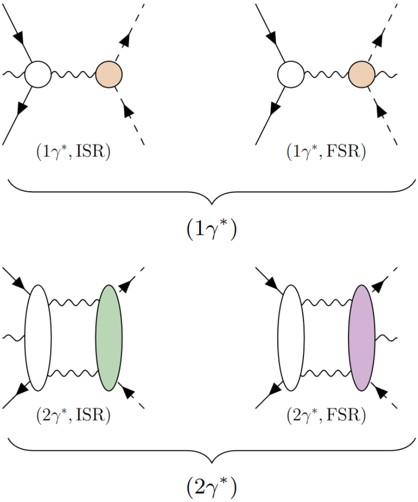
GVMD

2603.28621

FsQED

2607.24642

Subsets of corrections



Scenarios of arxiv:2410.22882

(a) KLOE I Large-Angle (LA)	**(b) KLOE II Small-Angle (SA)**
$\sqrt{s} = 1.02\ \mathrm{GeV}$	$\sqrt{s} = 1.02\ \mathrm{GeV}$
$50^\circ \leq \theta^\pm \leq 130^\circ$	$50^\circ \leq \theta^\pm \leq 130^\circ$
$p_z^\pm \geq 90\ \mathrm{MeV} \ \vee\ p_\perp^\pm \geq 160\ \mathrm{MeV}$	$p_z^\pm \geq 90\ \mathrm{MeV} \ \vee\ p_\perp^\pm \geq 160\ \mathrm{MeV}$
$50^\circ \leq \theta_\gamma \leq 130^\circ \ \wedge\ E_\gamma \geq 20\ \mathrm{MeV}$	$\theta_{\tilde\gamma} \leq 15^\circ \ \vee\ \theta_{\tilde\gamma} \geq 165^\circ$
$0.1\ \mathrm{GeV}^2 \leq M_{XX}^2 \leq 0.85\ \mathrm{GeV}^2$	$0.35\ \mathrm{GeV}^2 \leq M_{XX}^2 \leq 0.95\ \mathrm{GeV}^2$
(c) BES III	**(d) B**
$\sqrt{s} = 4\ \mathrm{GeV}$	$\sqrt{s} = 10\ \mathrm{GeV}$
$\lvert\cos\theta^\pm\rvert \leq 0.93 \ \wedge\ p_\perp^\pm \geq 300\ \mathrm{MeV}$	$0.65\ \mathrm{rad} \leq \theta^\pm \leq 2.75\ \mathrm{rad} \ \wedge\ p^\pm \geq 1\ \mathrm{GeV}$
$\left[\lvert\cos\theta_\gamma\rvert \leq 0.8 \ \wedge\ E_\gamma \geq 25\ \mathrm{MeV}\right]$	$0.6\ \mathrm{rad} \leq \theta_\gamma \leq 2.7\ \mathrm{rad} \ \wedge\ E_\gamma \geq 3\ \mathrm{GeV}$
$\vee\ \left[0.86 \leq \lvert\cos\theta_\gamma\rvert \leq 0.92 \ \wedge\ E_\gamma \geq 50\ \mathrm{MeV}\right]$	$\theta_{\tilde\gamma,\gamma^{(h)}} \leq 0.3\ \mathrm{rad}$
$\exists!\,\gamma:\ E_\gamma \geq 400\ \mathrm{MeV}$	$M_{XX\gamma} \geq 8\ \mathrm{GeV}$

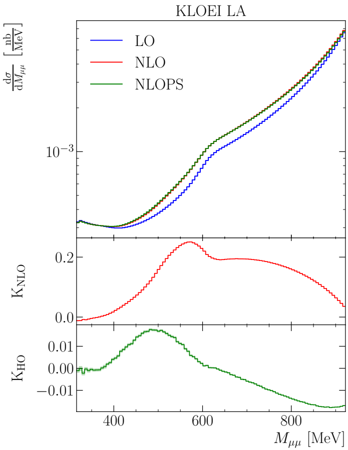
Scenarios of arxiv:2410.22882

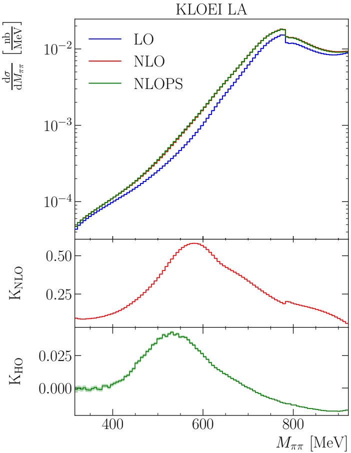
(a) KLOE I Large-Angle (LA)	(b) KLOE II Small-Angle (SA)
$\sqrt{s} = 1.02$ GeV	$\sqrt{s} = 1.02$ GeV
$50^\circ \le \theta^\pm \le 130^\circ$	$50^\circ \le \theta^\pm \le 130^\circ$
$p_z^\pm \ge 90$ MeV $\vee$ $p_\perp^\pm \ge 160$ MeV	$p_z^\pm \ge 90$ MeV $\vee$ $p_\perp^\pm \ge 160$ MeV
$50^\circ \le \theta_\gamma \le 130^\circ \wedge E_\gamma \ge 20$ MeV	$\theta_{\tilde{\gamma}} \le 15^\circ \vee \theta_{\tilde{\gamma}} \ge 165^\circ$
$0.1\ \mathrm{GeV}^2 \le M_{XX}^2 \le 0.85\ \mathrm{GeV}^2$	$0.35\ \mathrm{GeV}^2 \le M_{XX}^2 \le 0.95\ \mathrm{GeV}^2$
(c) BES III	**(d) B**
$\sqrt{s} = 4$ GeV	$\sqrt{s} = 10$ GeV
$\lvert\cos\theta^\pm\rvert \le 0.93 \wedge p_\perp^\pm \ge 300$ MeV	$0.65\ \mathrm{rad} \le \theta^\pm \le 2.75\ \mathrm{rad} \wedge p^\pm \ge 1$ GeV
$\left[\lvert\cos\theta_\gamma\rvert \le 0.8 \wedge E_\gamma \ge 25\ \mathrm{MeV}\right]$	$0.6\ \mathrm{rad} \le \theta_\gamma \le 2.7\ \mathrm{rad} \wedge E_\gamma \ge 3$ GeV
$\vee \left[0.86 \le \lvert\cos\theta_\gamma\rvert \le 0.92 \wedge E_\gamma \ge 50\ \mathrm{MeV}\right]$	$\theta_{\tilde{\gamma},\gamma^{(h)}} \le 0.3$ rad
$\exists!\,\gamma:\ E_\gamma \ge 400$ MeV	$M_{XX\gamma} \ge 8$ GeV

$M_{\mu\mu}$, $M_{\pi\pi}$ - F×sQED

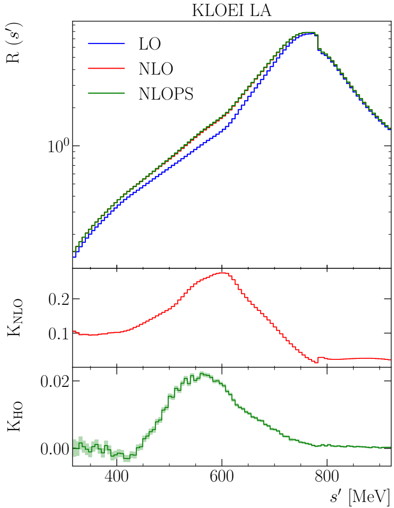
INFN UNIVERSITÀ DI PAVIA

$$\mathrm{K_{NLO}} = \frac{d\sigma_{\mathrm{NLO}} - d\sigma_{\mathrm{LO}}}{d\sigma_{\mathrm{LO}}} \qquad \mathrm{K_{HO}} = \frac{d\sigma_{\mathrm{NLOPS}} - d\sigma_{\mathrm{NLO}}}{d\sigma_{\mathrm{LO}}}$$





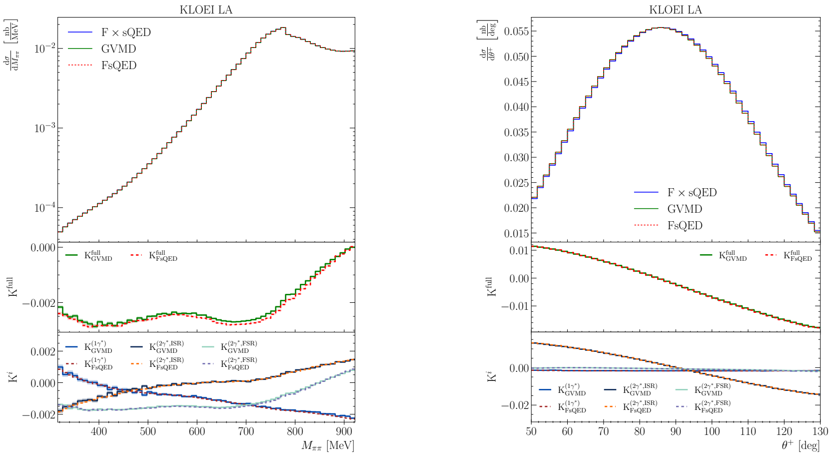
R-ratio



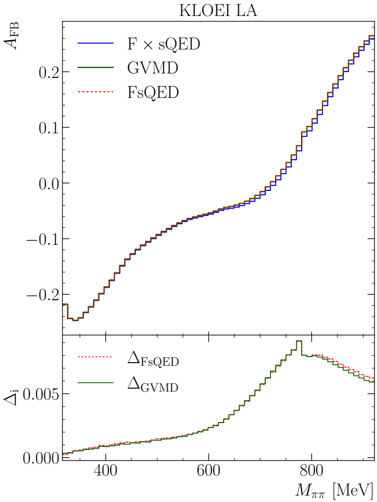
$$R(s') = \frac{d\sigma(e^+e^- \to \pi^+\pi^-\gamma)/ds'}{d\sigma(e^+e^- \to \mu^+\mu^-\gamma)/ds'},$$

$M_{\pi\pi}, \theta_{\pi^+}$ - Structure Dependent

$$\mathrm{K_i} = \frac{\mathrm{d}\sigma_\mathrm{i} - \mathrm{d}\sigma_{\mathrm{F\times sQED}}}{\mathrm{d}\sigma_{\mathrm{F\times sQED}}} \qquad i = \mathrm{GVMD, FsQED}$$



Asymmetry



$$A_{\rm FB}\left(M_{\pi\pi}\right) = \frac{d\sigma_{\rm F} - d\sigma_{\rm B}}{d\sigma_{\rm F} + d\sigma_{\rm B}}\,, \qquad (1)$$

where

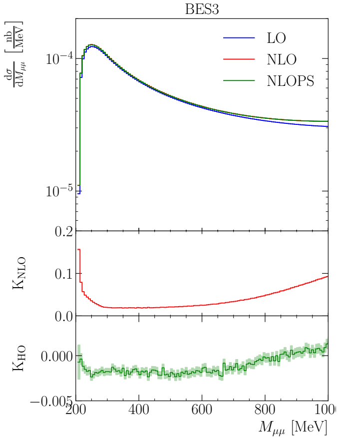
$$\begin{aligned} d\sigma_{\rm F} &= \int_0^1 \frac{d\sigma}{d M_{\pi\pi}\ d\cos\theta^+} d\cos\theta^+\,, \\ d\sigma_{\rm B} &= \int_{-1}^0 \frac{d\sigma}{d M_{\pi\pi}\ d\cos\theta^+} d\cos\theta^+\,. \end{aligned} \qquad (2)$$

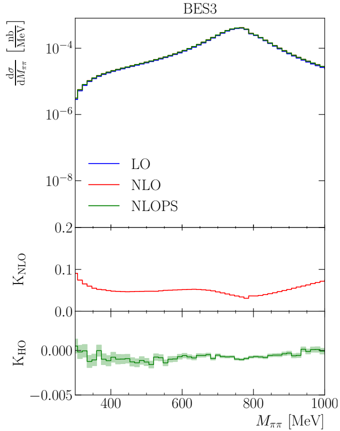
Scenarios of arxiv:2410.22882

(a) KLOE I Large-Angle (LA)	(b) KLOE II Small-Angle (SA)
$\sqrt{s} = 1.02\ \mathrm{GeV}$	$\sqrt{s} = 1.02\ \mathrm{GeV}$
$50^\circ \leq \theta^\pm \leq 130^\circ$	$50^\circ \leq \theta^\pm \leq 130^\circ$
$p_z^\pm \geq 90\ \mathrm{MeV} \ \vee\ p_\perp^\pm \geq 160\ \mathrm{MeV}$	$p_z^\pm \geq 90\ \mathrm{MeV} \ \vee\ p_\perp^\pm \geq 160\ \mathrm{MeV}$
$50^\circ \leq \theta_\gamma \leq 130^\circ \ \wedge\ E_\gamma \geq 20\ \mathrm{MeV}$	$\theta_{\tilde{\gamma}} \leq 15^\circ \ \vee\ \theta_{\tilde{\gamma}} \geq 165^\circ$
$0.1\ \mathrm{GeV}^2 \leq M_{XX}^2 \leq 0.85\ \mathrm{GeV}^2$	$0.35\ \mathrm{GeV}^2 \leq M_{XX}^2 \leq 0.95\ \mathrm{GeV}^2$

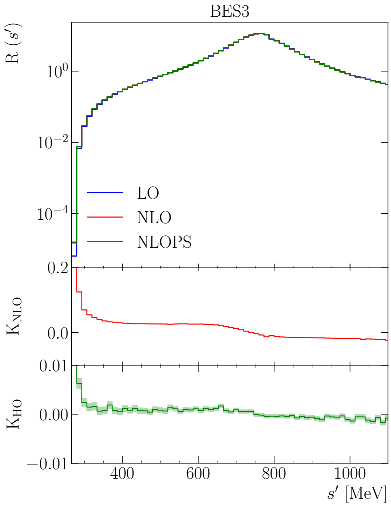
(c) BES III	(d) B
$\sqrt{s} = 4\ \mathrm{GeV}$	$\sqrt{s} = 10\ \mathrm{GeV}$
$\lvert\cos\theta^\pm\rvert \leq 0.93 \ \wedge\ p_\perp^\pm \geq 300\ \mathrm{MeV}$	$0.65\ \mathrm{rad} \leq \theta^\pm \leq 2.75\ \mathrm{rad} \ \wedge\ p^\pm \geq 1\ \mathrm{GeV}$
$\left[\lvert\cos\theta_\gamma\rvert \leq 0.8 \ \wedge\ E_\gamma \geq 25\ \mathrm{MeV}\right]$	$0.6\ \mathrm{rad} \leq \theta_\gamma \leq 2.7\ \mathrm{rad} \ \wedge\ E_\gamma \geq 3\ \mathrm{GeV}$
$\vee\ \left[0.86 \leq \lvert\cos\theta_\gamma\rvert \leq 0.92 \ \wedge\ E_\gamma \geq 50\ \mathrm{MeV}\right]$	$\theta_{\tilde{\gamma},\gamma^{(h)}} \leq 0.3\ \mathrm{rad}$
$\exists!\,\gamma:\ E_\gamma \geq 400\ \mathrm{MeV}$	$M_{XX\gamma} \geq 8\ \mathrm{GeV}$

$M_{\mu\mu}$, $M_{\pi\pi}$ - F×sQED



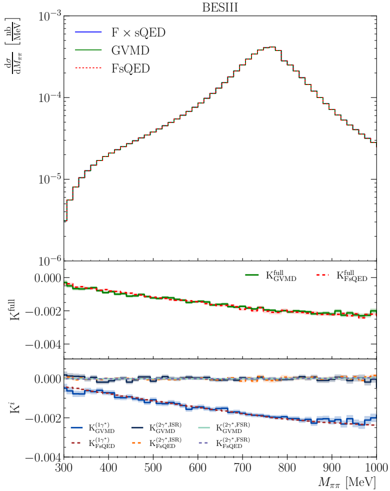


R-ratio

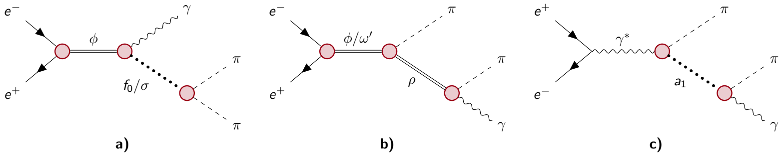


$$\mathrm{R}\,(s') = \frac{d\sigma(e^+e^- \to \pi^+\pi^-\gamma)/ds'}{d\sigma(e^+e^- \to \mu^+\mu^-\gamma)/ds'},$$

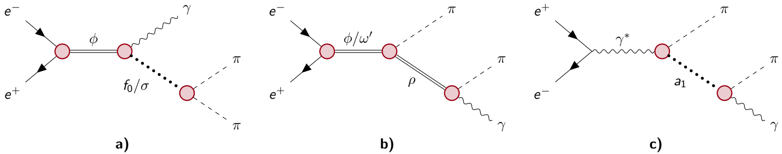
$M_{\pi\pi}, \theta_{\pi^+}$ - Structure dependent

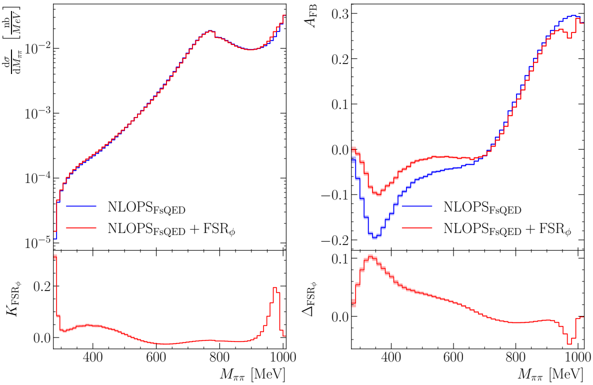


Around the ϕ resonance

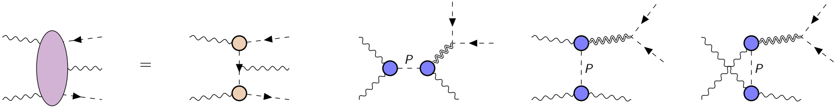


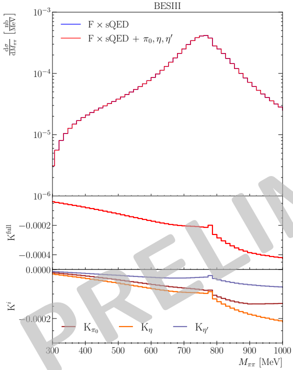
Around the ϕ resonance

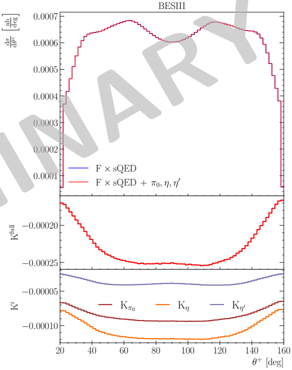




Beyond FsQED - π_0, η, η' poles







Summary and Outlook

- Multiple photon contributions reach the **percent level** across various key observables in radiative return experiments.

Summary and Outlook

- Multiple photon contributions reach the **percent level** across various key observables in radiative return experiments.
- Structure-dependent corrections are of the same order as higher-order (HO) radiative effects:
  - Angular variables are dominated by $(2\gamma^*, \mathrm{ISR})$ contributions.
  - FsQED and GVMD results agree at the 10^{-4} level, despite GVMD theoretical limitations.
  - Additional P-wave rescattering effects beyond FsQED remain well under control ($\mathcal{O}(10^{-4})$).

Summary and Outlook

- Multiple photon contributions reach the **percent level** across various key observables in radiative return experiments.
- Structure-dependent corrections are of the same order as higher-order (HO) radiative effects:
    - Angular variables are dominated by $(2\gamma^*, \mathrm{ISR})$ contributions.
    - FsQED and GVMD results agree at the 10^{-4} level, despite GVMD theoretical limitations.
    - Additional P-wave rescattering effects beyond FsQED remain well under control ($\mathcal{O}(10^{-4})$).
- Future Developments
    - $e^+e^-\gamma$ at NLOPS accuracy.
    - Extension to other hadronic channels ($K^+K^-, \ldots$).
    - NNLOPS matching for ISC on $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \gamma\gamma$.

Thank you !