$e^+e^- \to \pi^+\pi^-$ using KKMC

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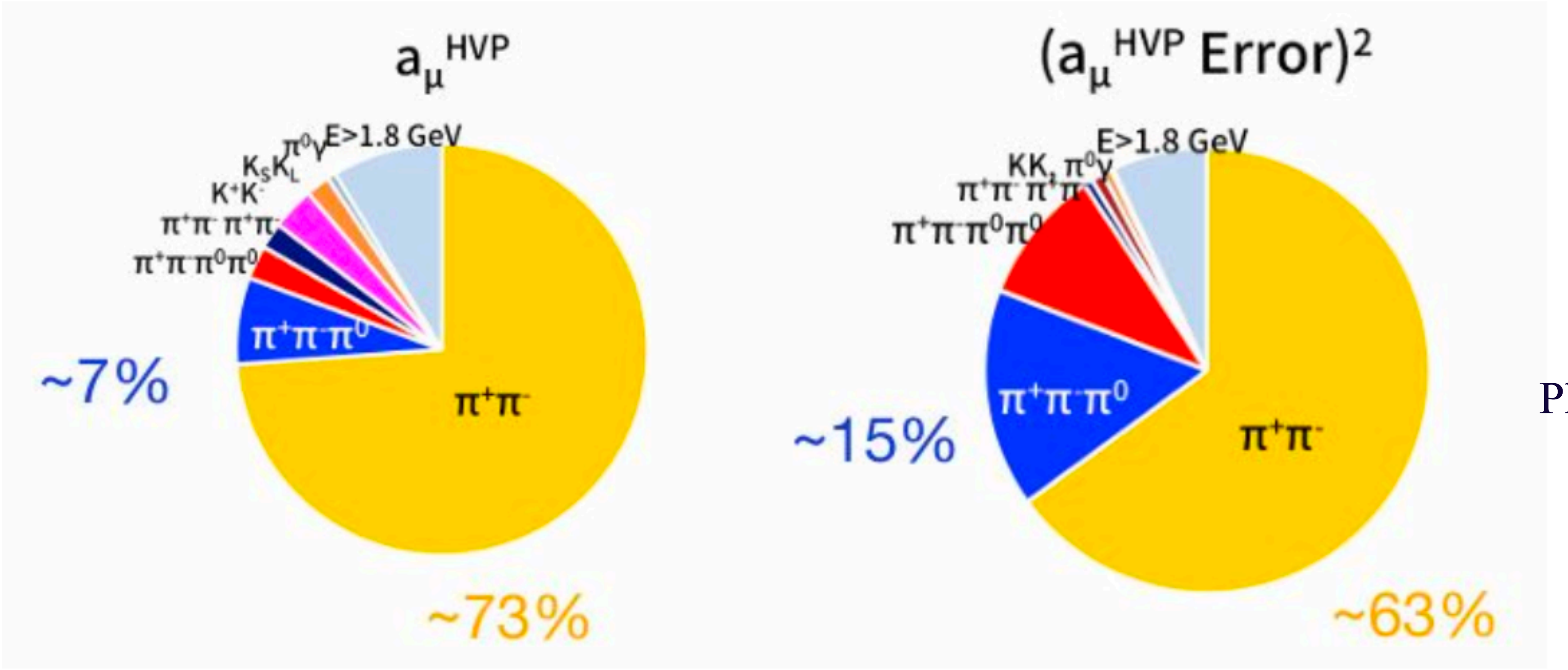
Introduction

- Magnetic moment $\vec{\mu}$ of the muon $= g_\mu \dfrac{e}{2m} \vec{S}$, where $\vec{S}$ is its spin.
- Anomalous magnetic moment: $a_\mu = \dfrac{g_\mu - 2}{2}$.
- $a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{HVP} + a_\mu^{HLbL}$.
- a_μ^{HVP} contributes to 83% of the error on a_μ.
- $\pi\pi$ contributes to ~73% of the value of a_μ^{HVP} and to ~63% of its (error)².



Phys. Rev. D 101, 014029 (2020)

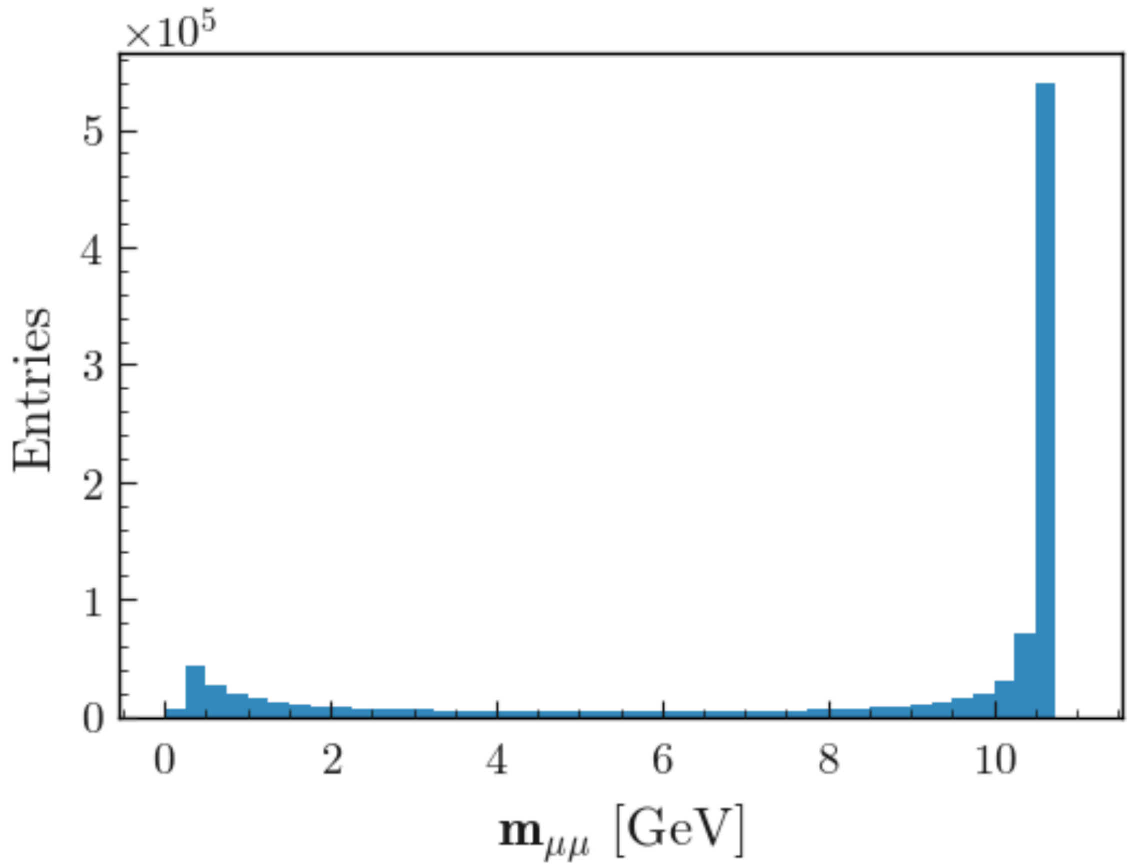
- $\pi\pi$ is an important decay channel to measure.

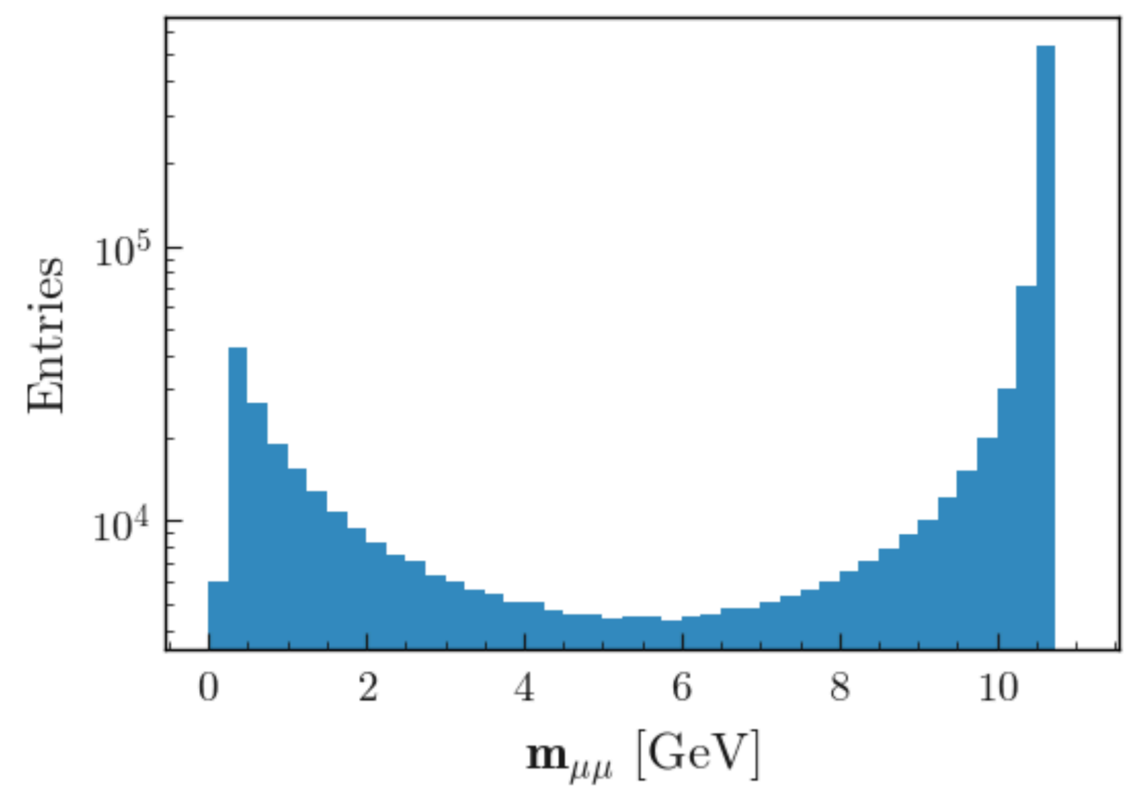
$e^+e^- \to \mu^+\mu^-$

At $\sqrt{s} = 10.58$ GeV, more abundant $e^+e^- \to \mu^+\mu^-$ events tend to eclipse $e^+e^- \to \pi^+\pi^-$ events. But $e^+e^- \to \mu^+\mu^-$ events are cleanly identifiable at the Belle II experiment, which plans to collect $\simeq 50\ ab^{-1}$ of data over the next decade.

μ-pair production in e^+e^- annihilation well described by:

- KK2F [S.Jadach, B.F.L.Ward, Z.Was, Comput.Phys.Commun. 130 (2000) 260]





$e^+e^- \to \mu^+\mu^-$

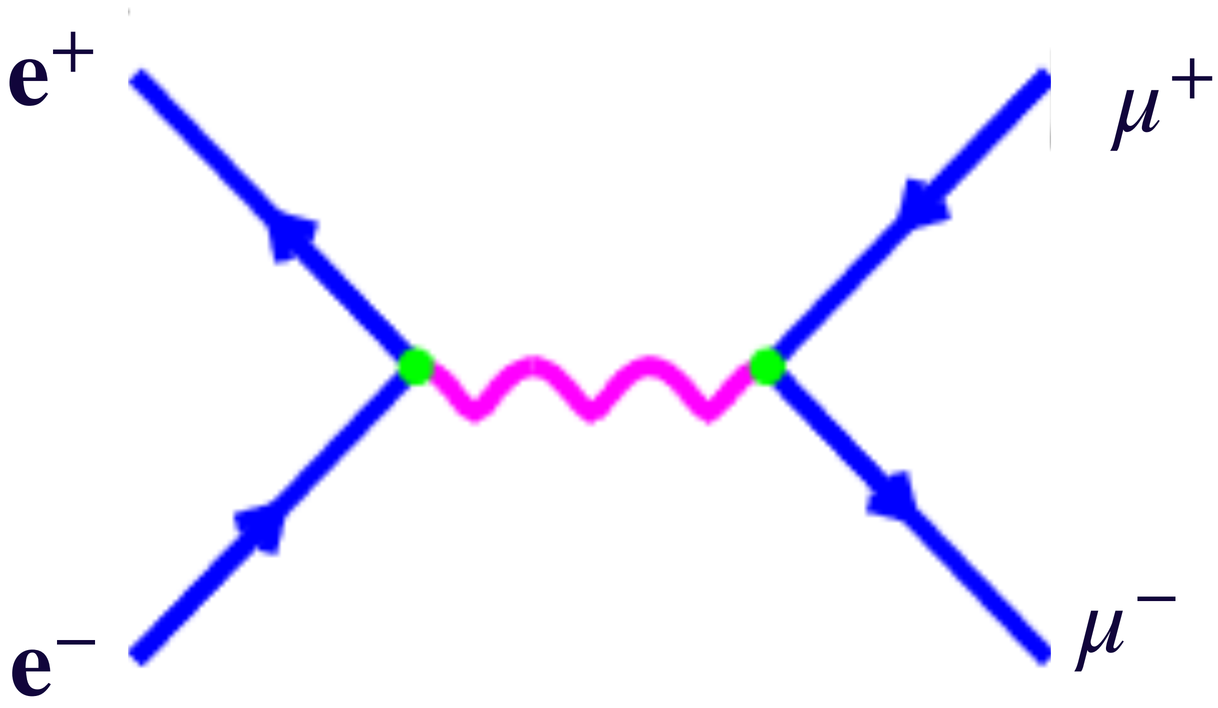
	$\sigma(\mu\mu)$	$\sigma_{cuts}(\mu\mu)$
Vacuum Polarization	0.22%	0.22%
Bremsstrahlung	0.2%	0.2%
Interference	0.04%	0.04%
Vertex Corrections	0.15%	0.15%
Vector Resonances	0.12%	0.1%
Total	0.36%	0.35%

- $\sigma(e^+e^- \to \mu^+\mu^-) = (1.147 \pm 0.004)\,\text{nb}$
- $\sigma_{cuts}(e^+e^- \to \mu^+\mu^-) = (0.835 \pm 0.003)\,\text{nb}$

Cuts: $\theta^{CM}_{\mu^\pm} \in [22°,158°]$, $\sqrt{s'/s} > 0.1$ closer to geometric acceptance.

SwB, B. Pietrzyk, J.Roney, Z.Was Phys.Rev.D 77 (2008) 054012

$e^+e^- \to \mu^+\mu^-$



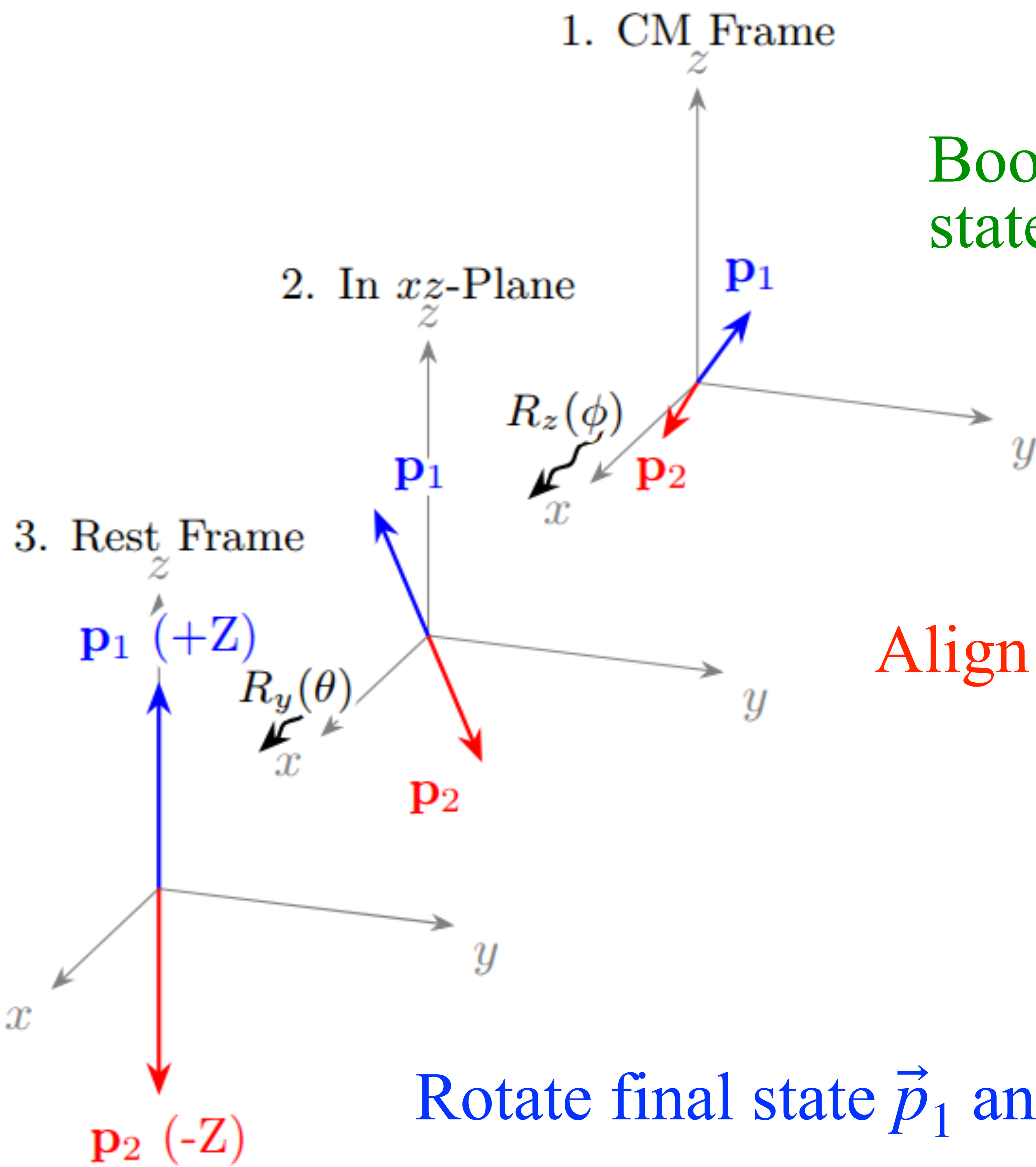
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2\theta)$$

However, there are always photons radiated, which may be soft, or collinear, or not seen in the detector due to fiducial acceptance or reconstruction inefficiency.

KKMC rigorously and unambiguously includes effects of Bremsstrahlung from Initial State Radiation (ISR) using Coherent Exclusive Exponentiation (CEEX) & Matrix Element up to 2nd order.

One can approximate & look at the process as a Born-level cross section with photon emissions separated to define reference frames.

Coordinate transformations



Boost to a frame where final state $\vec{p}_1$ and $\vec{p}_2$ are back-to-back.

Align x-z plane to contain beams.

Rotate final state $\vec{p}_1$ and $\vec{p}_2$ to be along new z-axis.

Choice of frames

Based on the cosine of angle of the e^- (c_1) and e^+ (c_2) in this rest frame of the final state $\mu-$pair oriented along the z-axis, and the energy of beams e^- (E_3') and e^+ (E_4'):

- Collins-Soper frame [average]:

$$\cos\theta_1 = \frac{c_1 + c_2}{2}$$

- Collins-Soper frame [weighted average]:

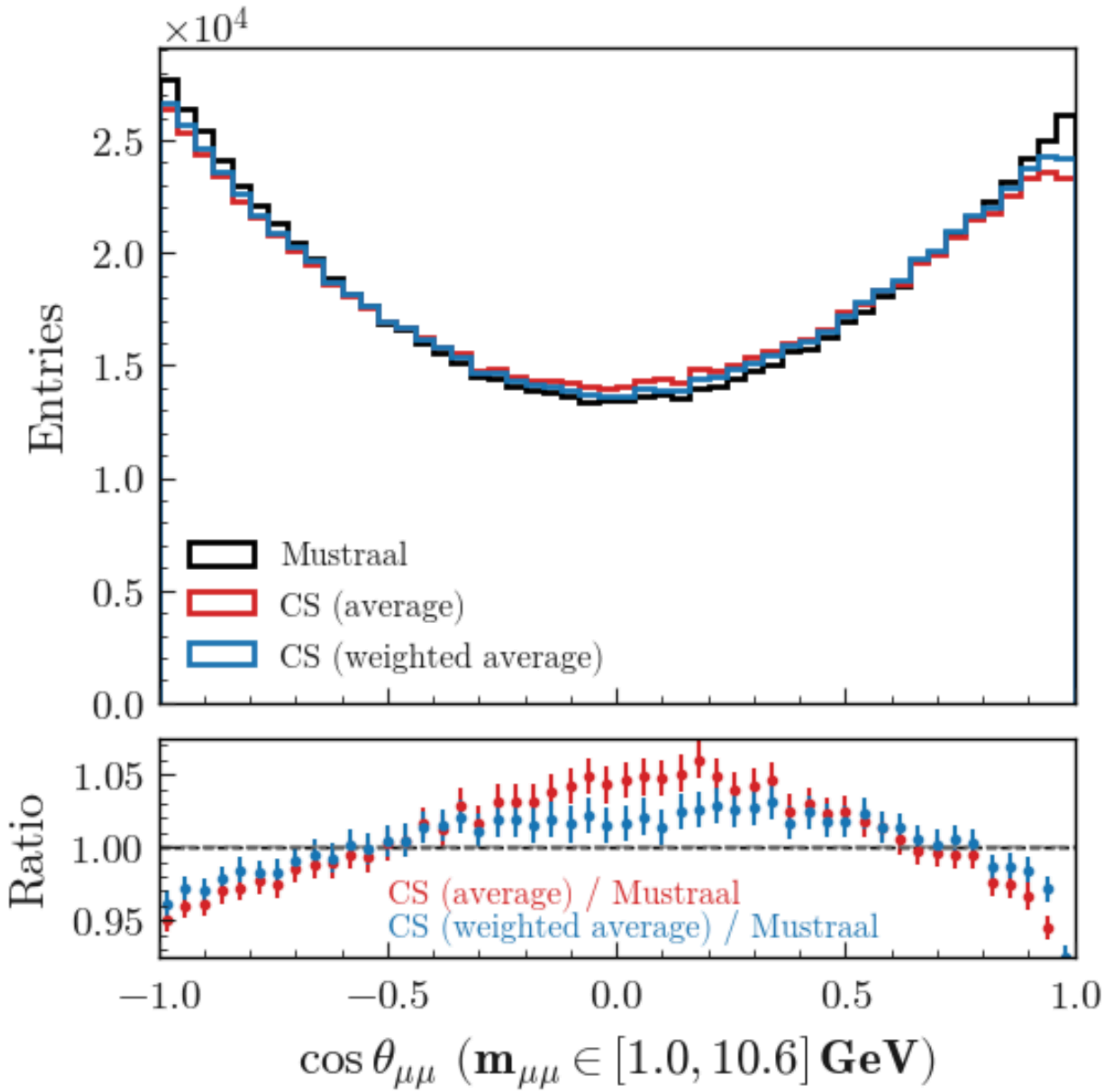
$$\cos\theta_0 = \frac{c_1 \sin\theta_2' + c_2 \sin\theta_1'}{\sin\theta_2' + \sin\theta_1'}$$

- Mustraal frame: choose $c_1$or c_2 based on

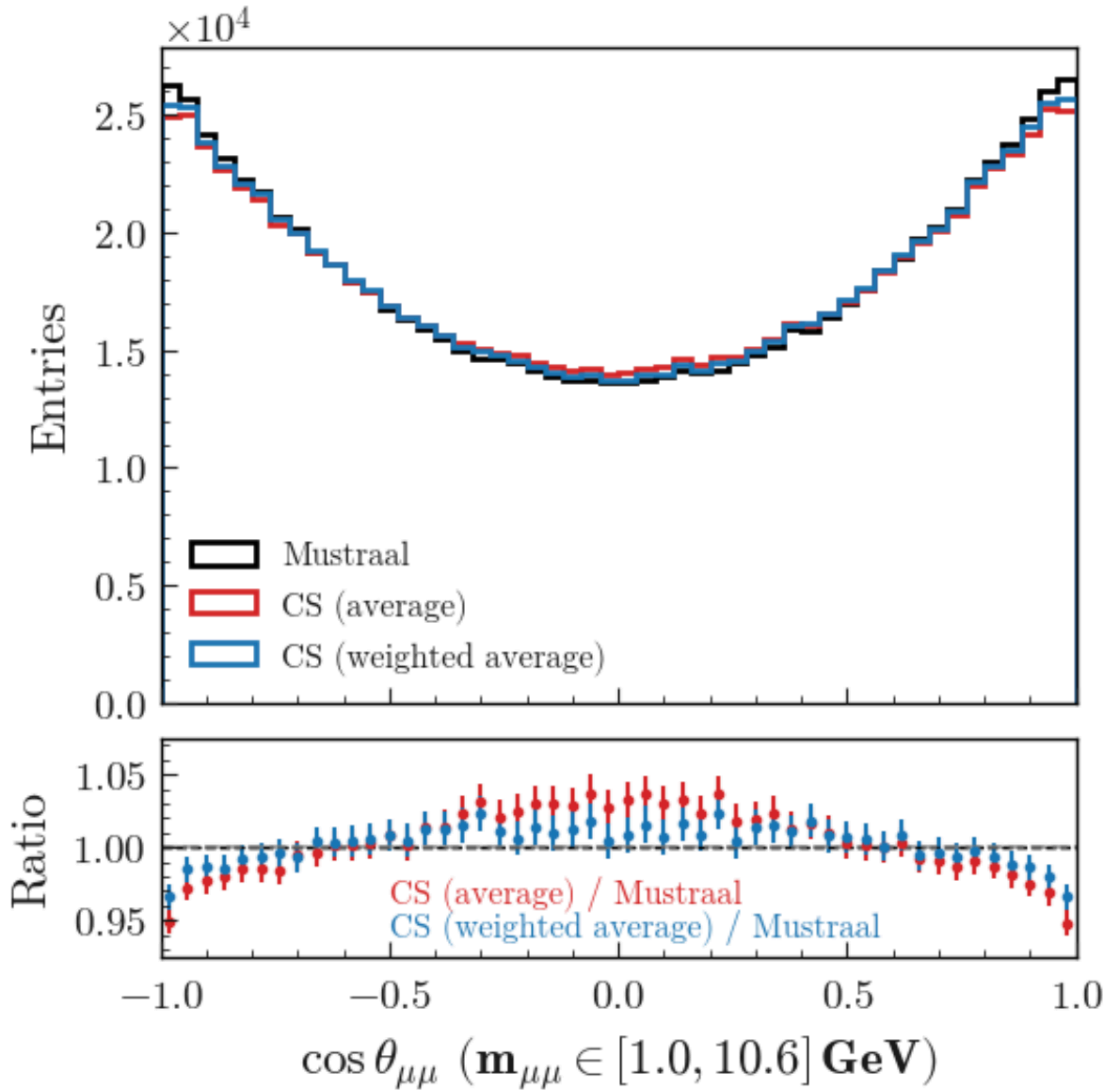
$$wt_1 = \frac{(E_3')^2(1+c_1^2)}{(E_3')^2(1+c_1^2) + (E_4')^2(1+c_2^2)}$$

$$wt_2 = 1 - wt_1$$

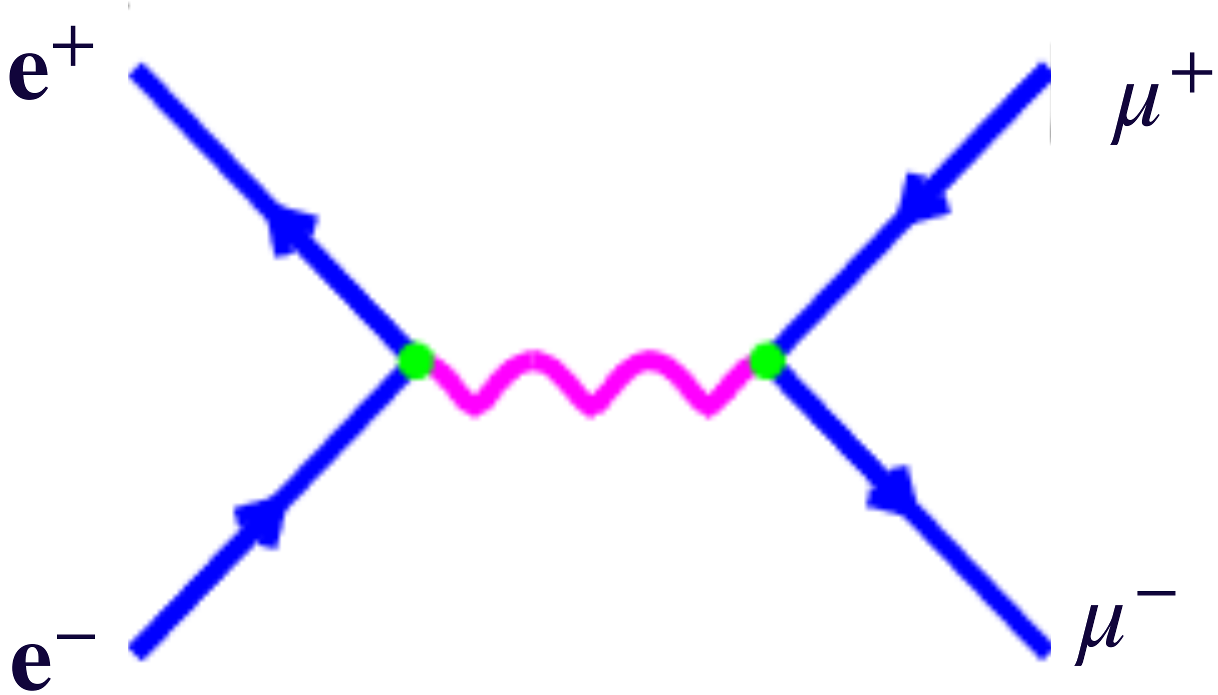
$e^+e^- \to \mu^+\mu^-$: ISR-FSR interference gives asymmetry



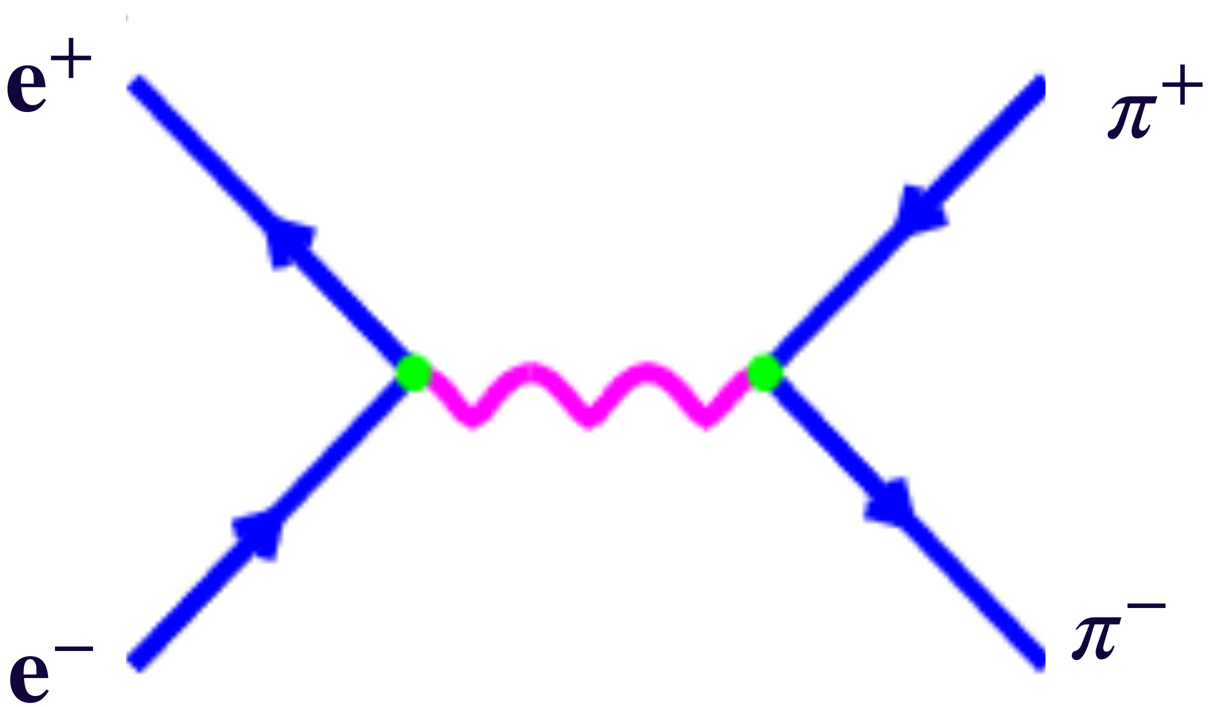
$e^+e^- \to \mu^+\mu^-$ (No FSR)



$e^+e^- \to \mu^+\mu^-$ to $e^+e^- \to \pi^+\pi^-$



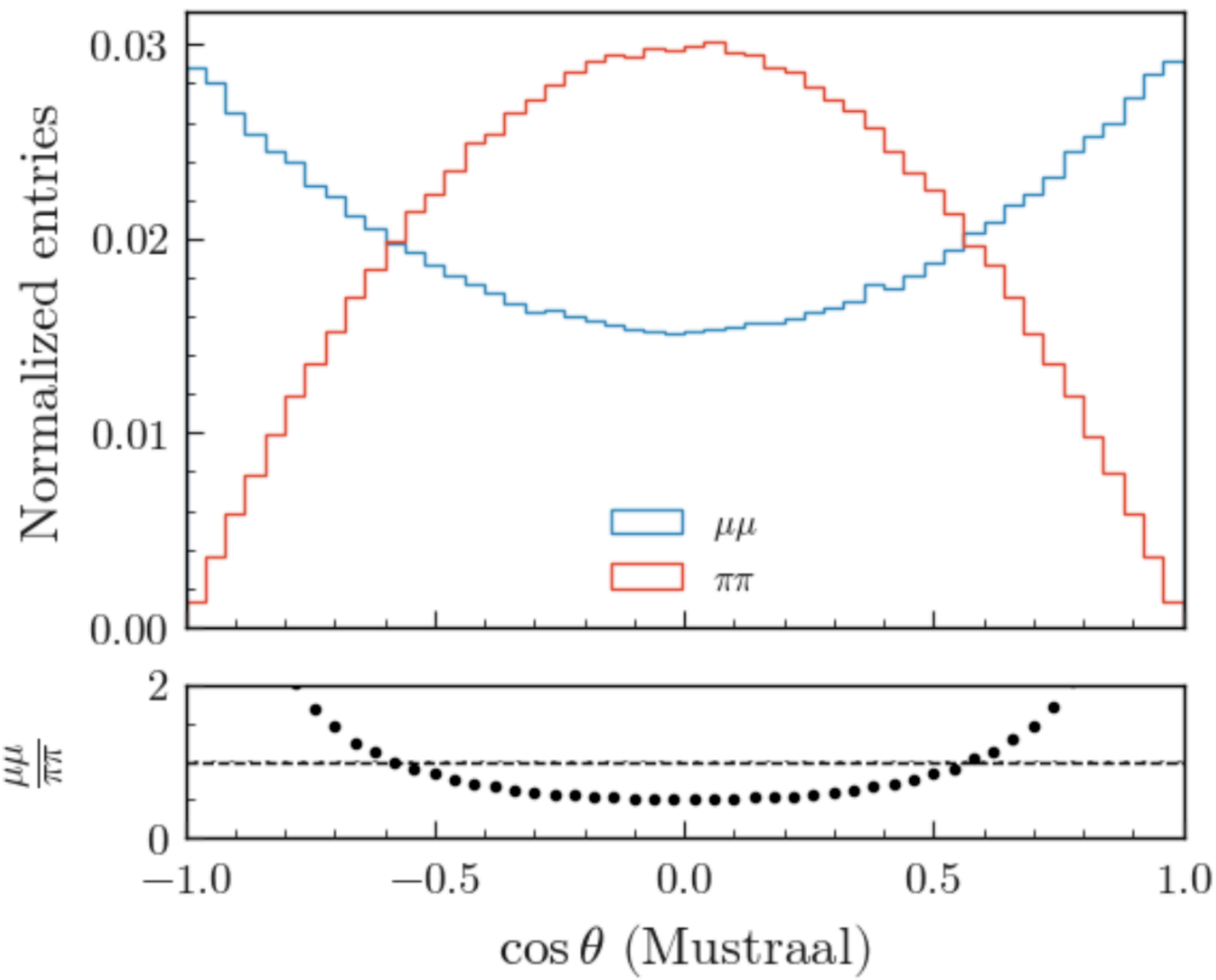
$\propto (1 + \cos^2\theta)$



$\propto (\sin^2\theta)$

$e^+e^- \to \mu^+\mu^-$ to $e^+e^- \to \pi^+\pi^-$

- Spin re-weighting factor: $\mathrm{wt} = \dfrac{\frac{\pi\alpha^2\beta^3}{4s}|F_\pi(s)|^2\sin^2\theta}{\frac{\pi\alpha^2\beta}{2s}\left(2-\beta^2\sin^2\theta\right)}$



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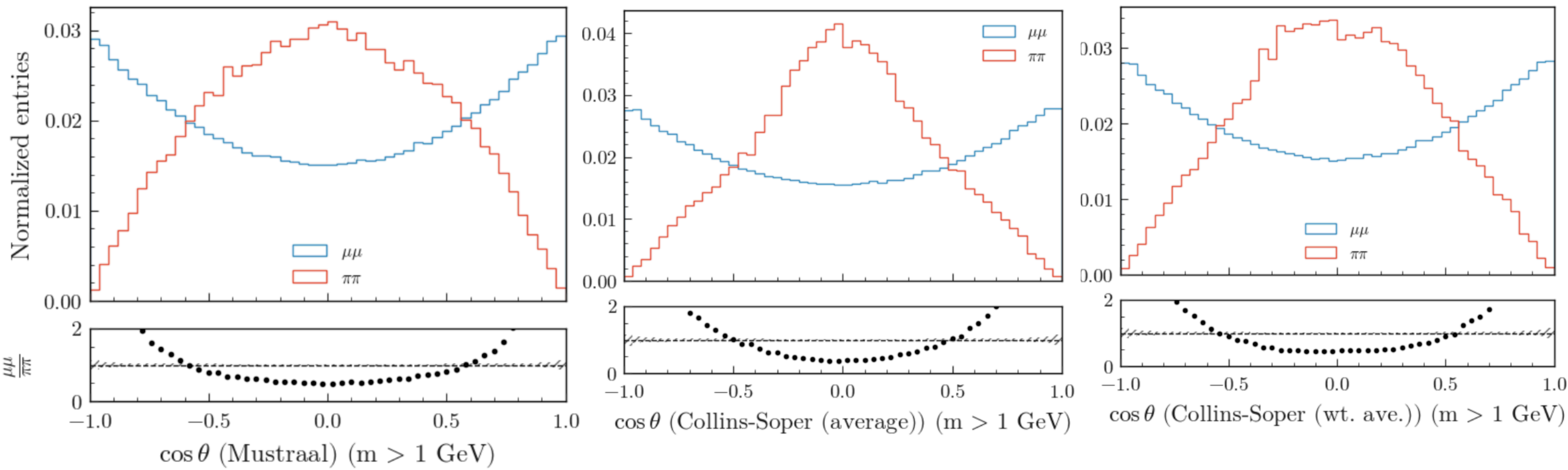

- Mustraal frame gives robust treatment for radiation
- CS (weighted average) closer to Mustraal than CS (average)

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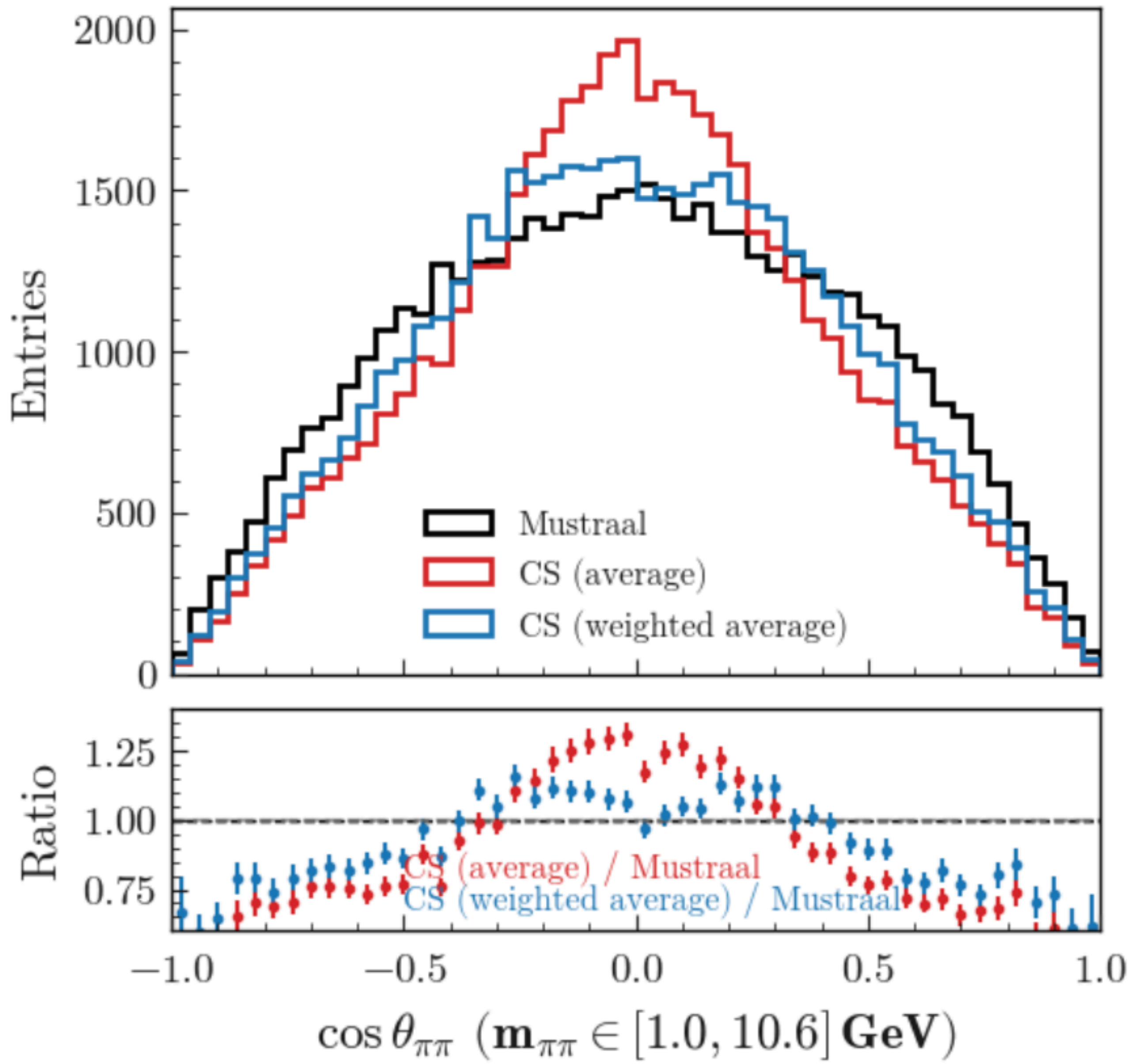

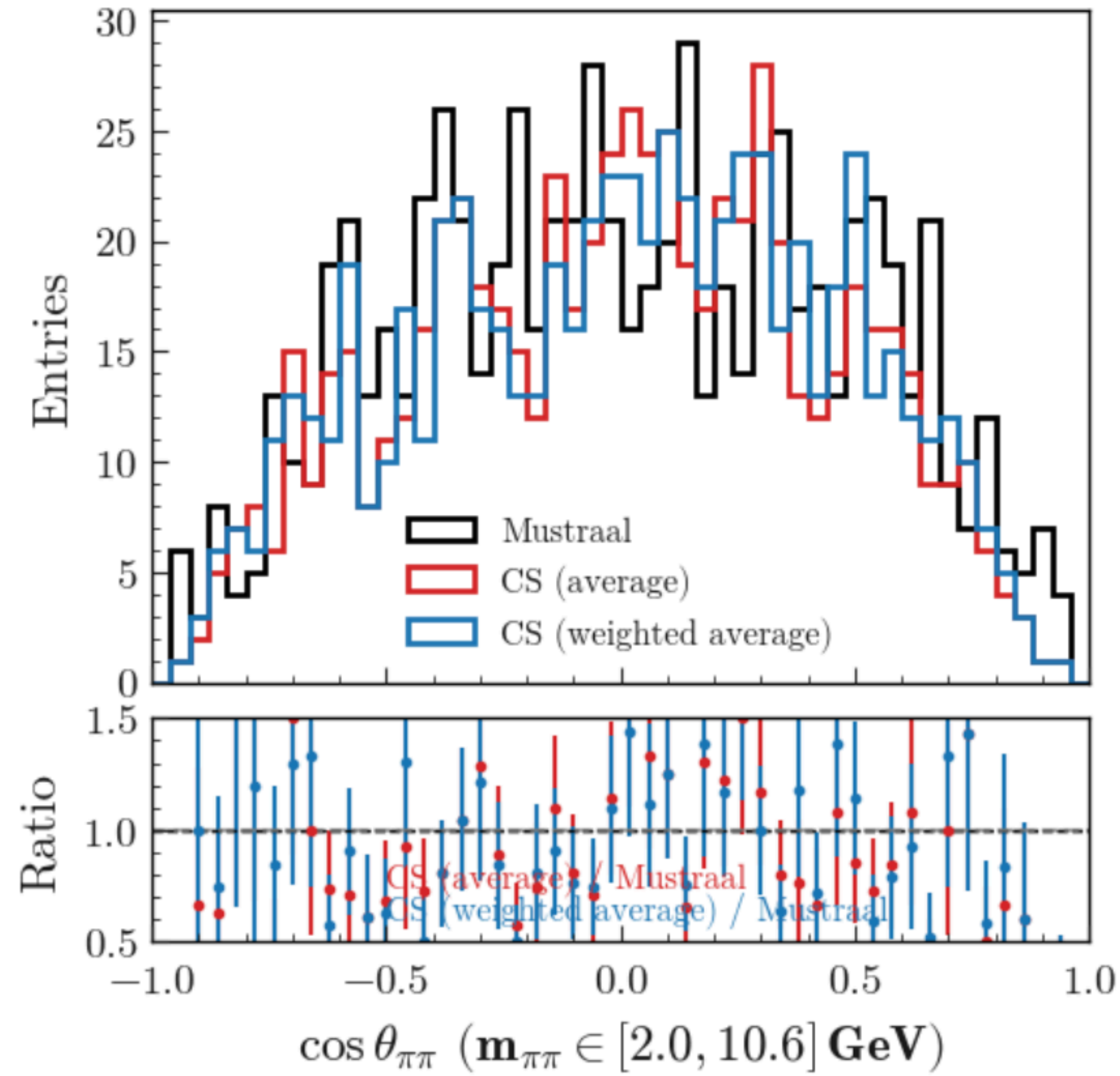

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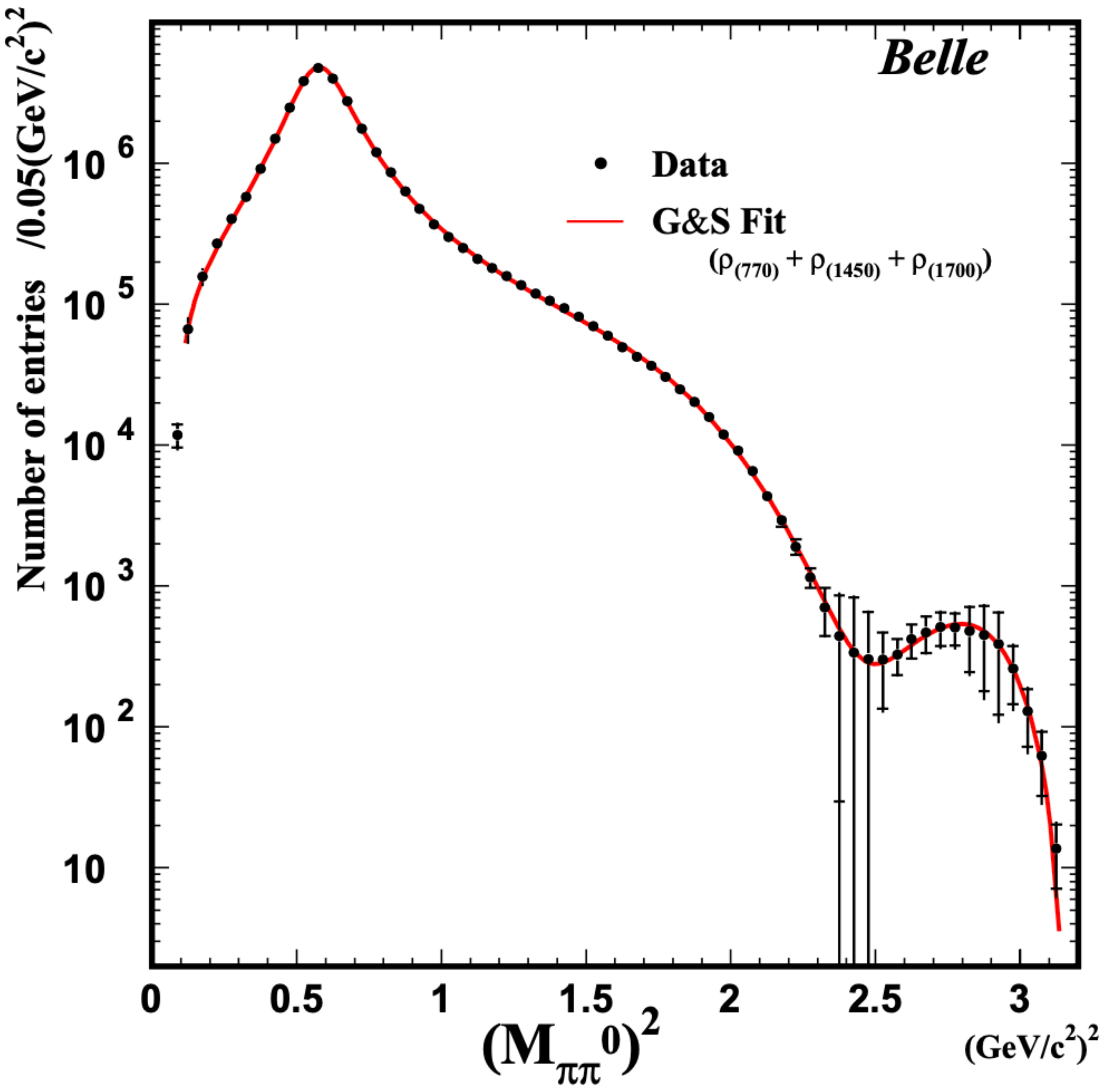

$$F_\pi(s) = \frac{1}{1+\beta+\gamma}(\mathrm{BW}_\rho + \beta \cdot \mathrm{BW}_{\rho'} + \gamma \cdot \mathrm{BW}_{\rho''}),$$

$\mathscr{F}_\pi(s)$ from $\tau^- \to \pi^-\pi^0\nu_\tau$ decays [Belle Collaboration, Phys.Rev.D 78 (2008) 072006]

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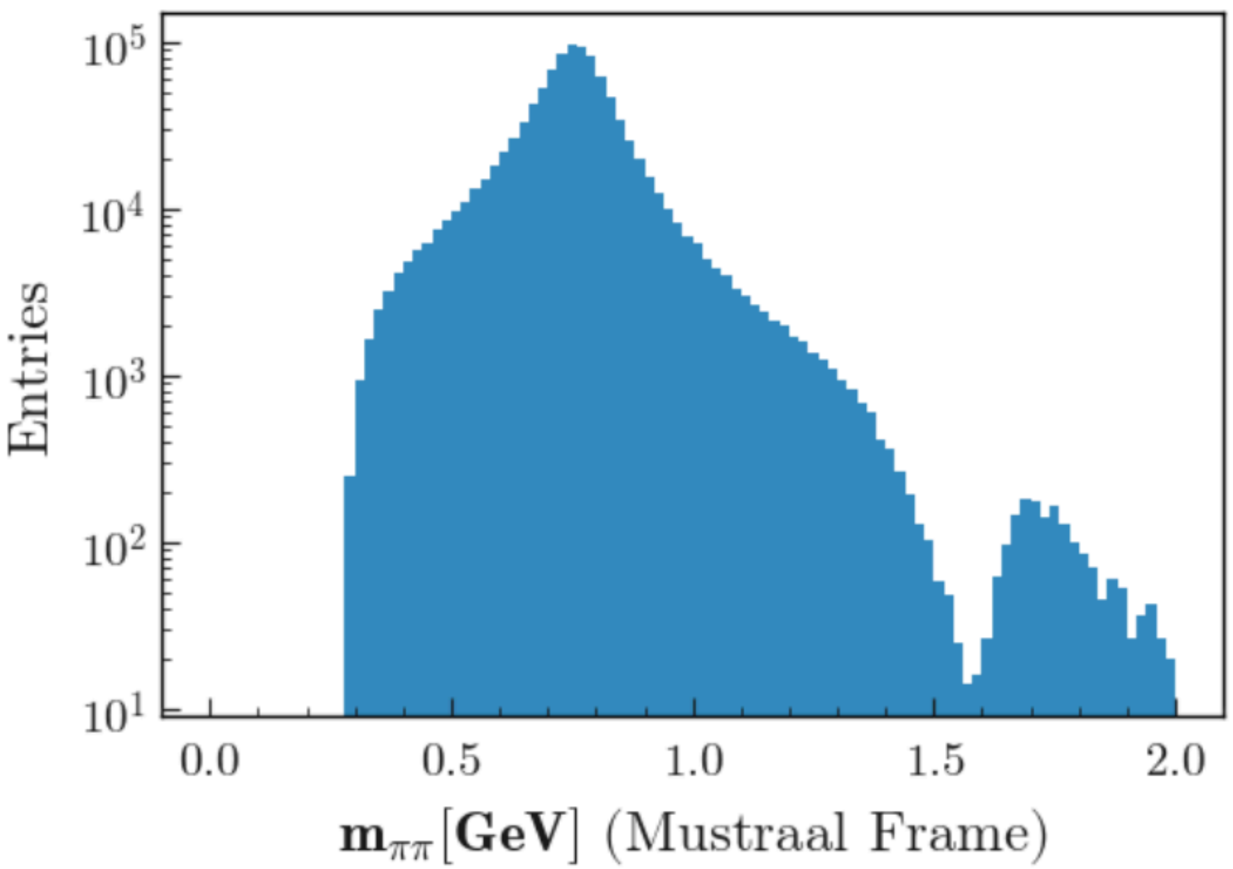

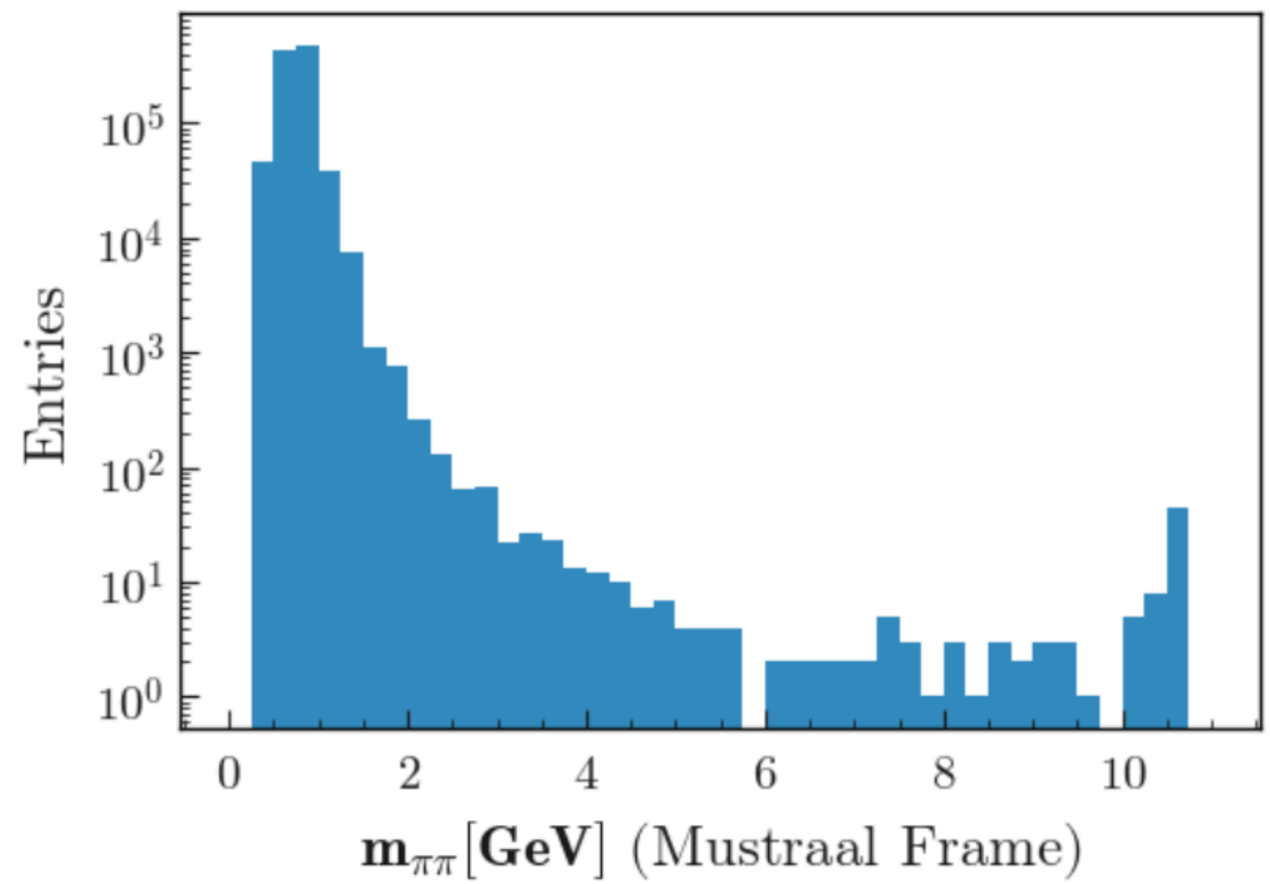

- Started with 1M $e^+e^- \to \mu^+\mu^-$ events with ISR, but no FSR
- After spin re-weighting, FSR added to $e^+e^- \to \pi^+\pi^-$ events using PHOTOS

[E.Barberio, Z.Was, Comput.Phys.Commun. 79 (1994) 291]

$e^+e^- \to \mu^+\mu^-$ to $e^+e^- \to \pi^+\pi^-$

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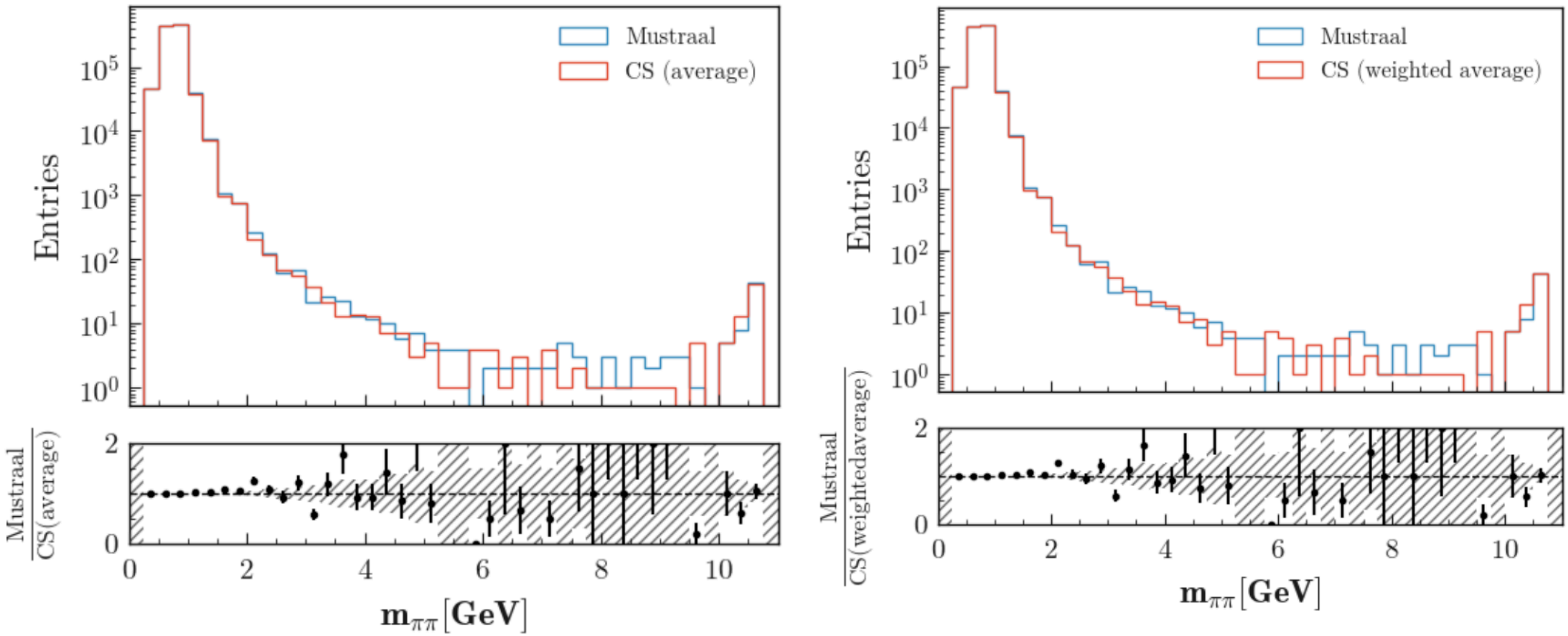
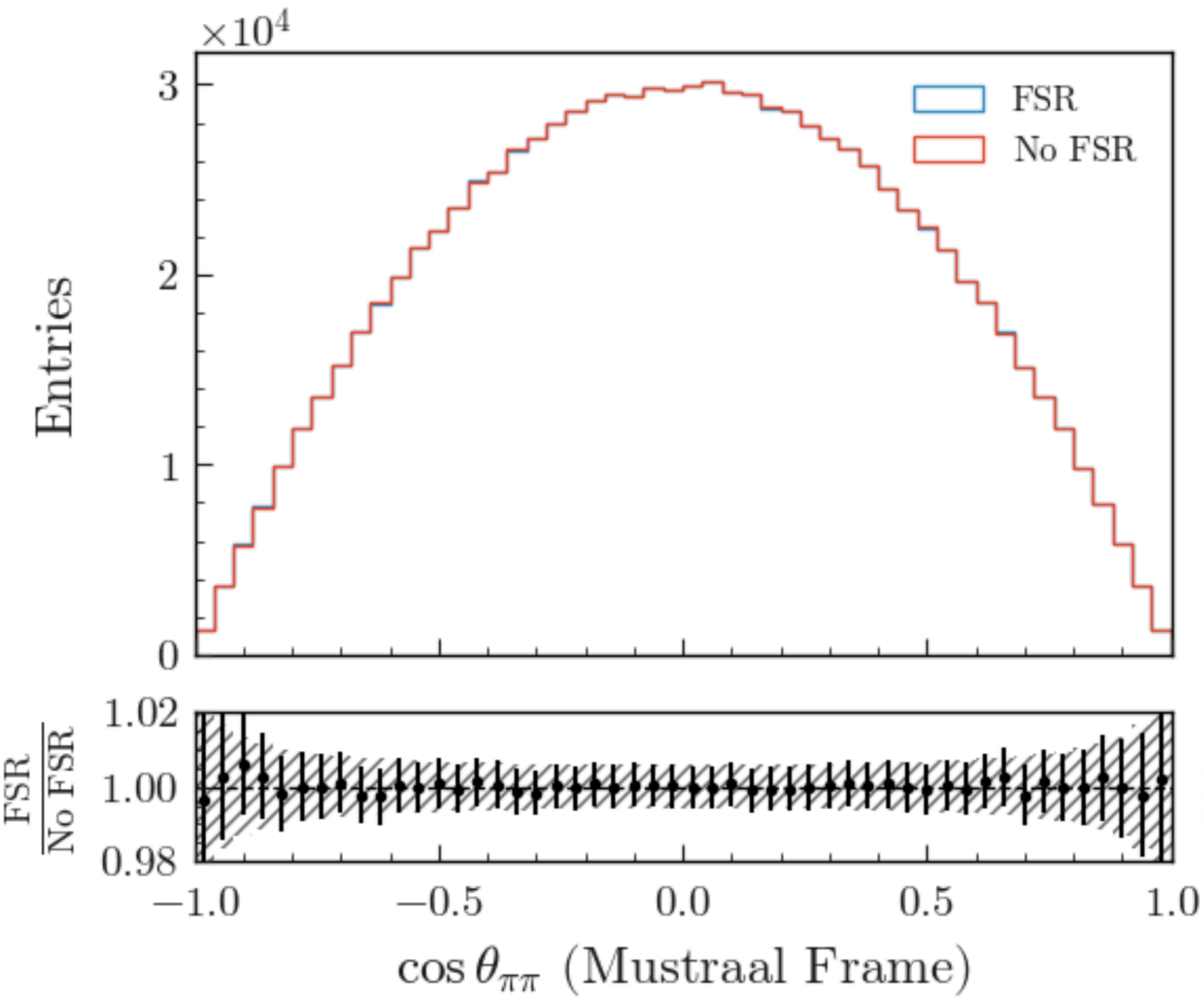
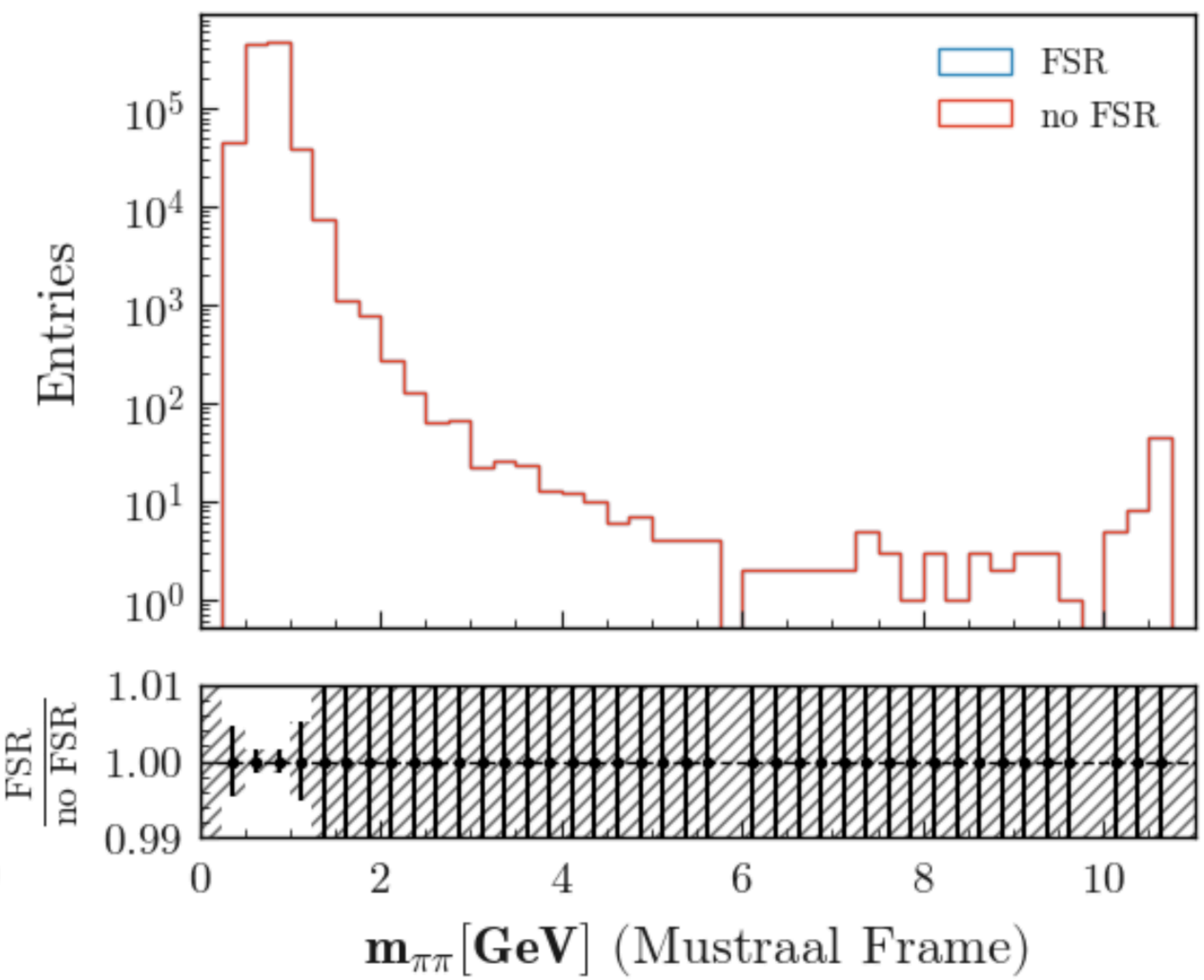


- CS (weighted average) and CS (average) work well for large $m_{\pi\pi}$

$e^+e^- \to \mu^+\mu^-$ to $e^+e^- \to \pi^+\pi^-$

- Effect of turning FSR on vs off at the end is quite small.





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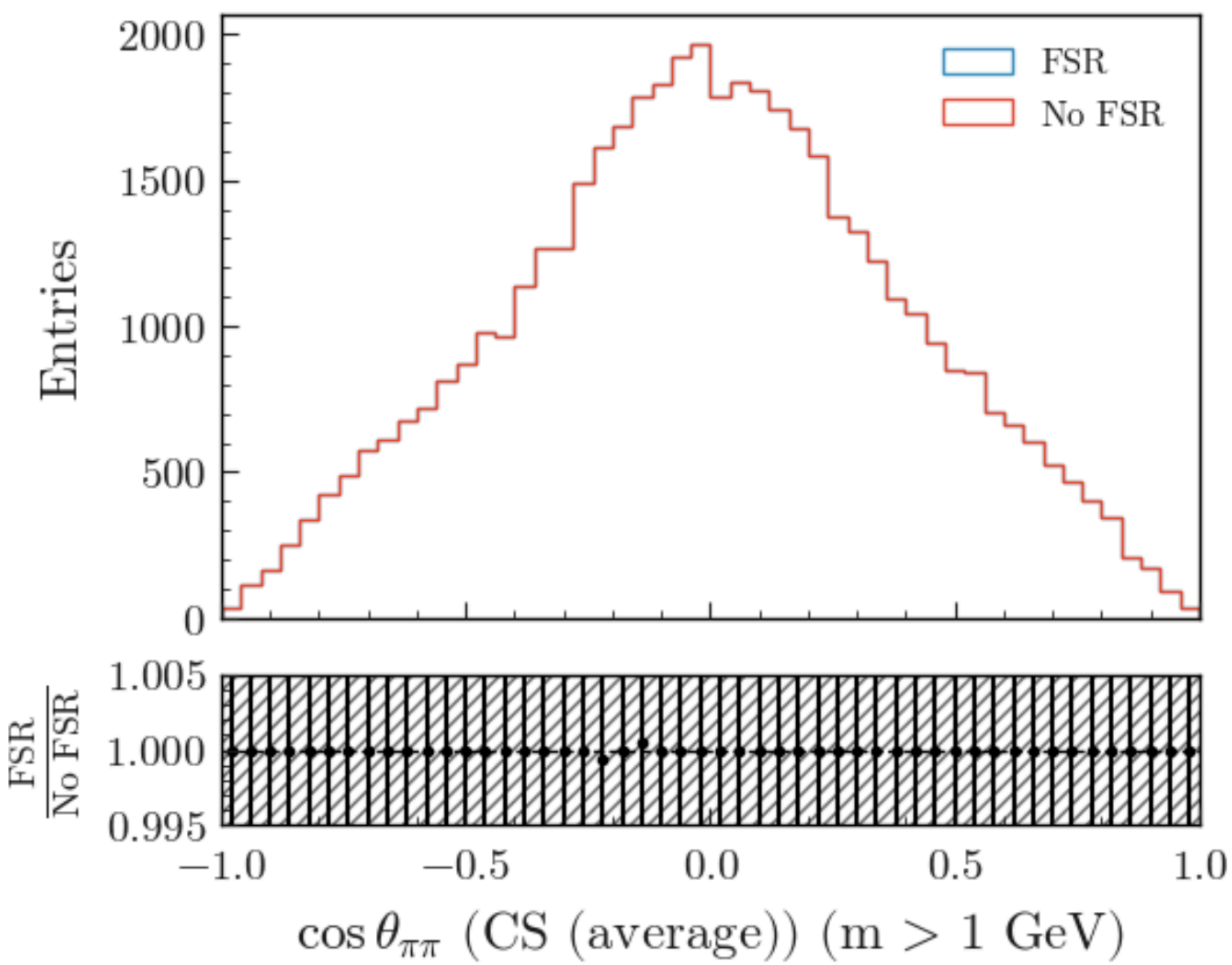

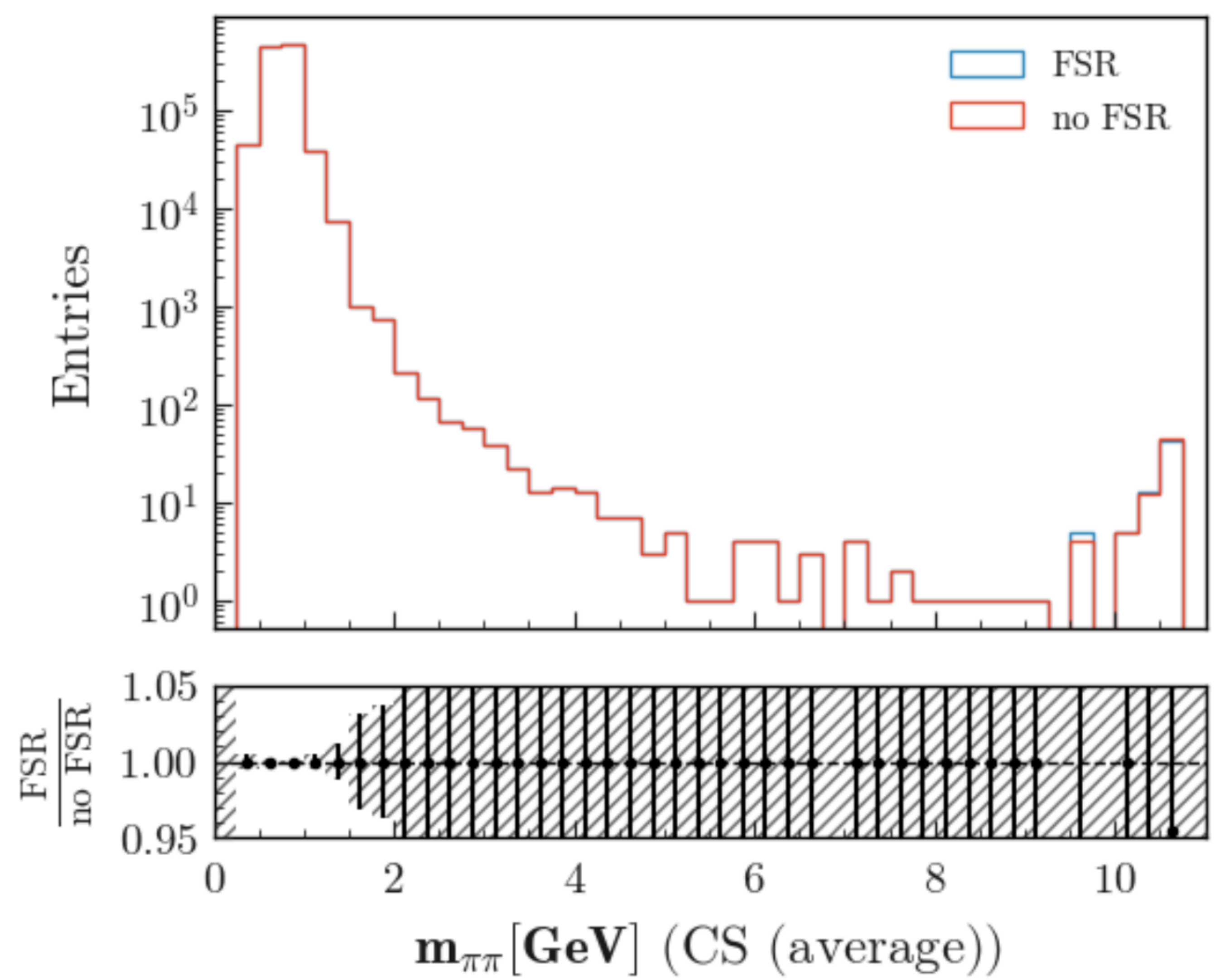

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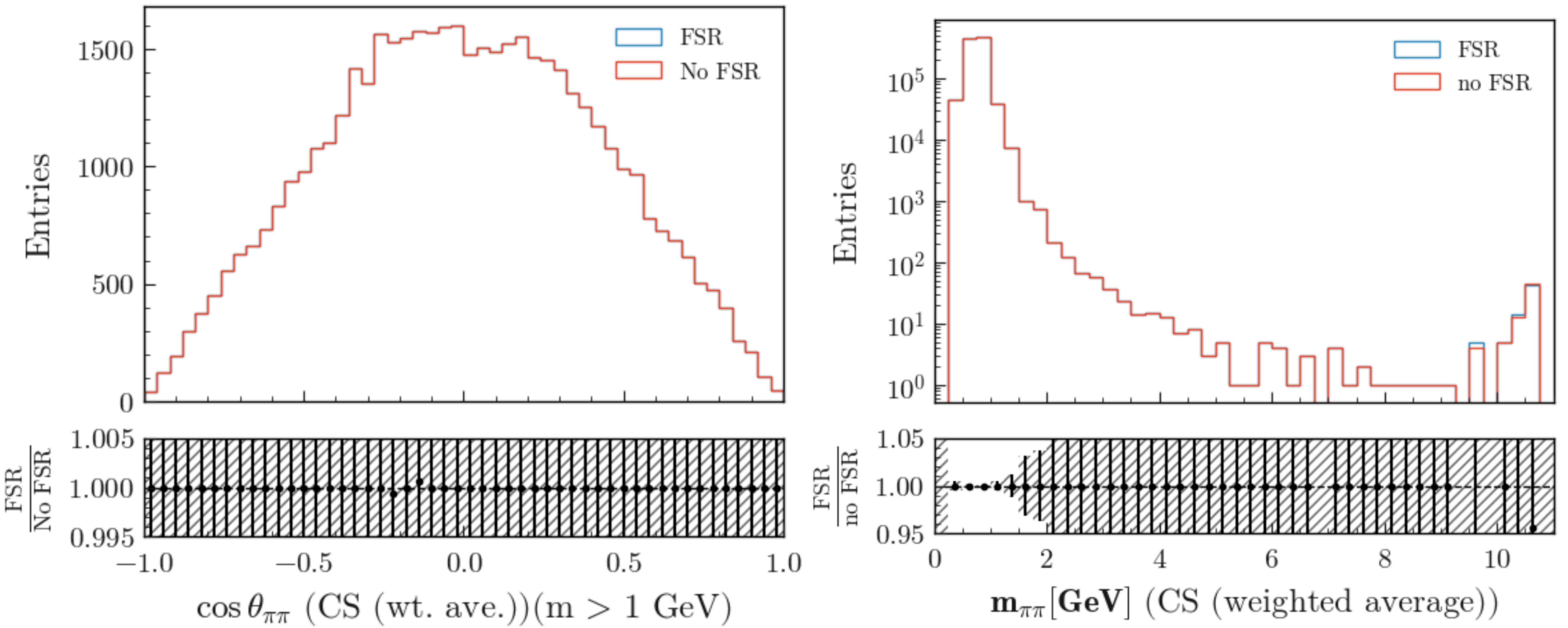

$e^+e^- \to \mu^+\mu^-$ to $e^+e^- \to \pi^+\pi^-$

Phys.Rev.D 63 (2001) 113009

- Effect of Coherent Exclusive Exponentiation
  - CEEX0: leading order
  - CEEX1: next-to-leading order
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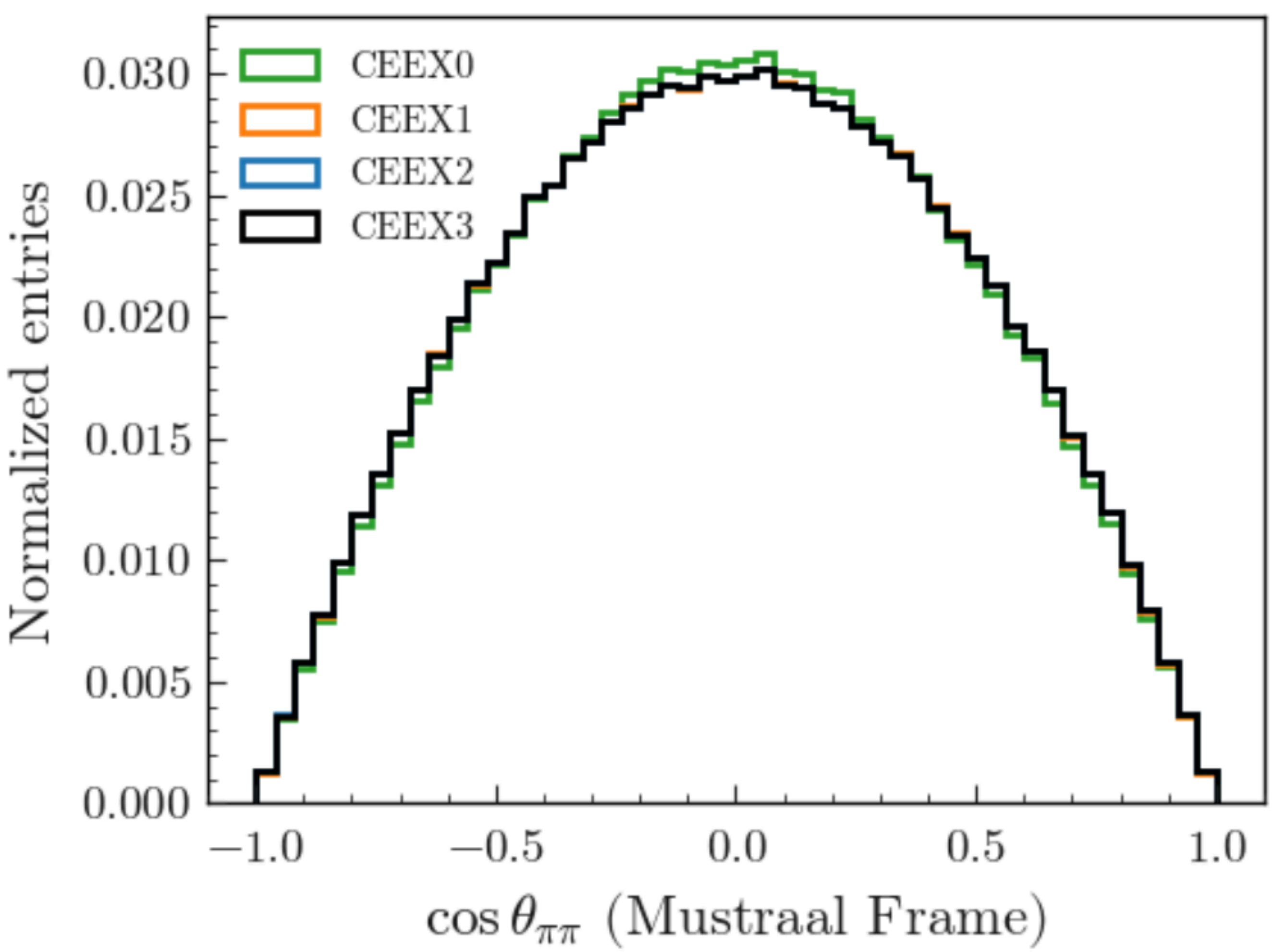

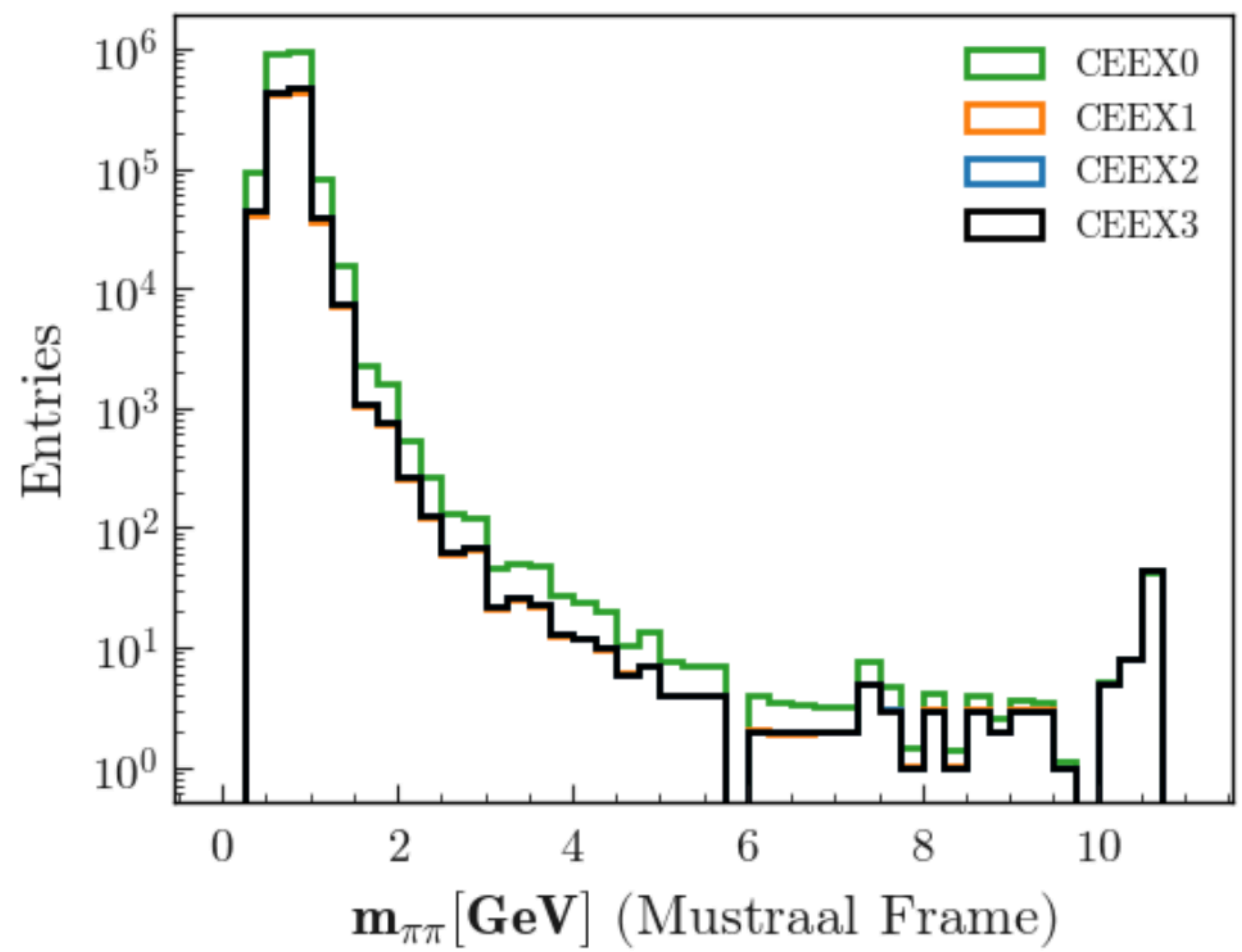

Systematic uncertainty from Bremsstrahlung are well under control.

Phys.Rev.D 63 (2001) 113009

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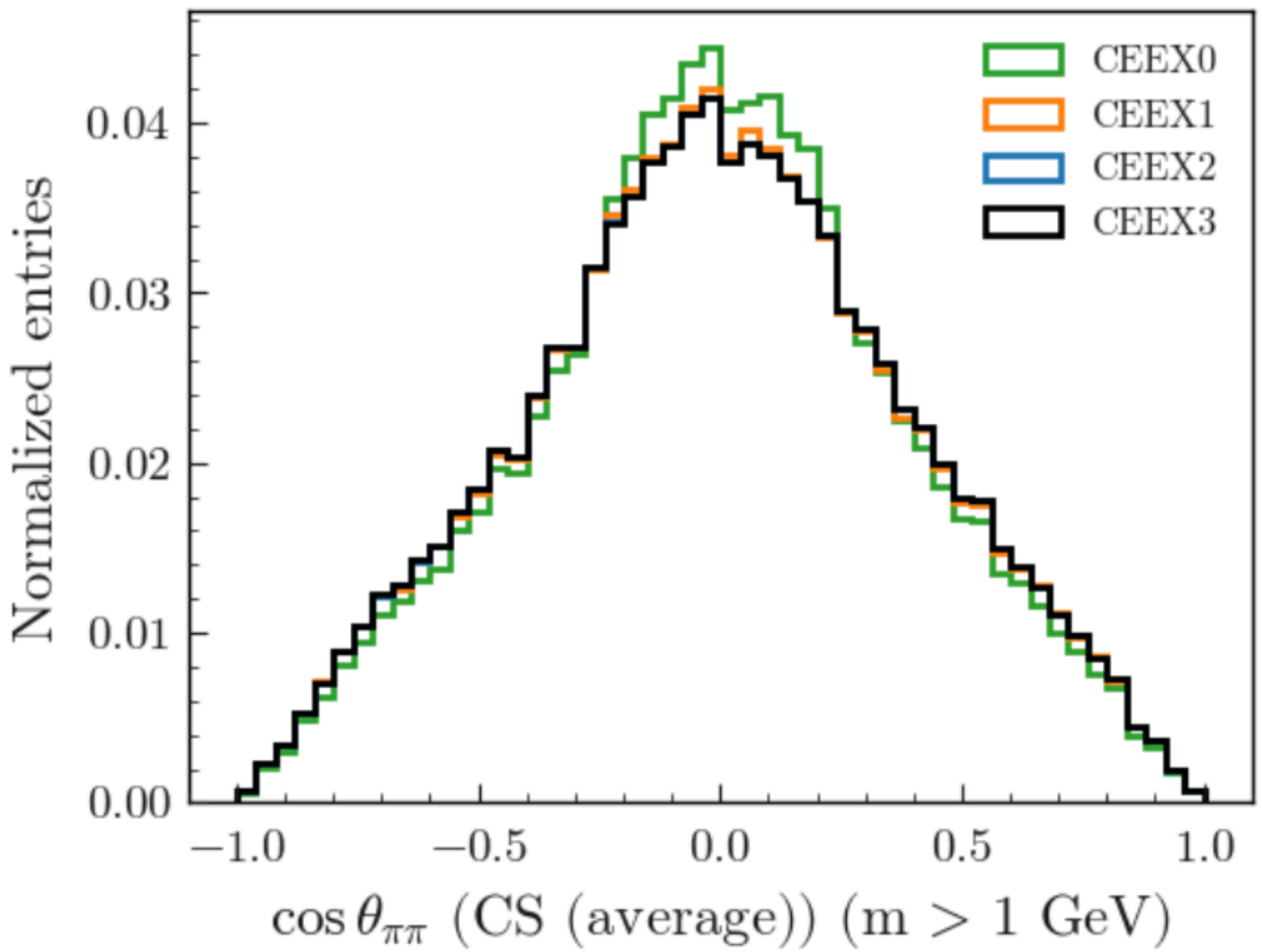

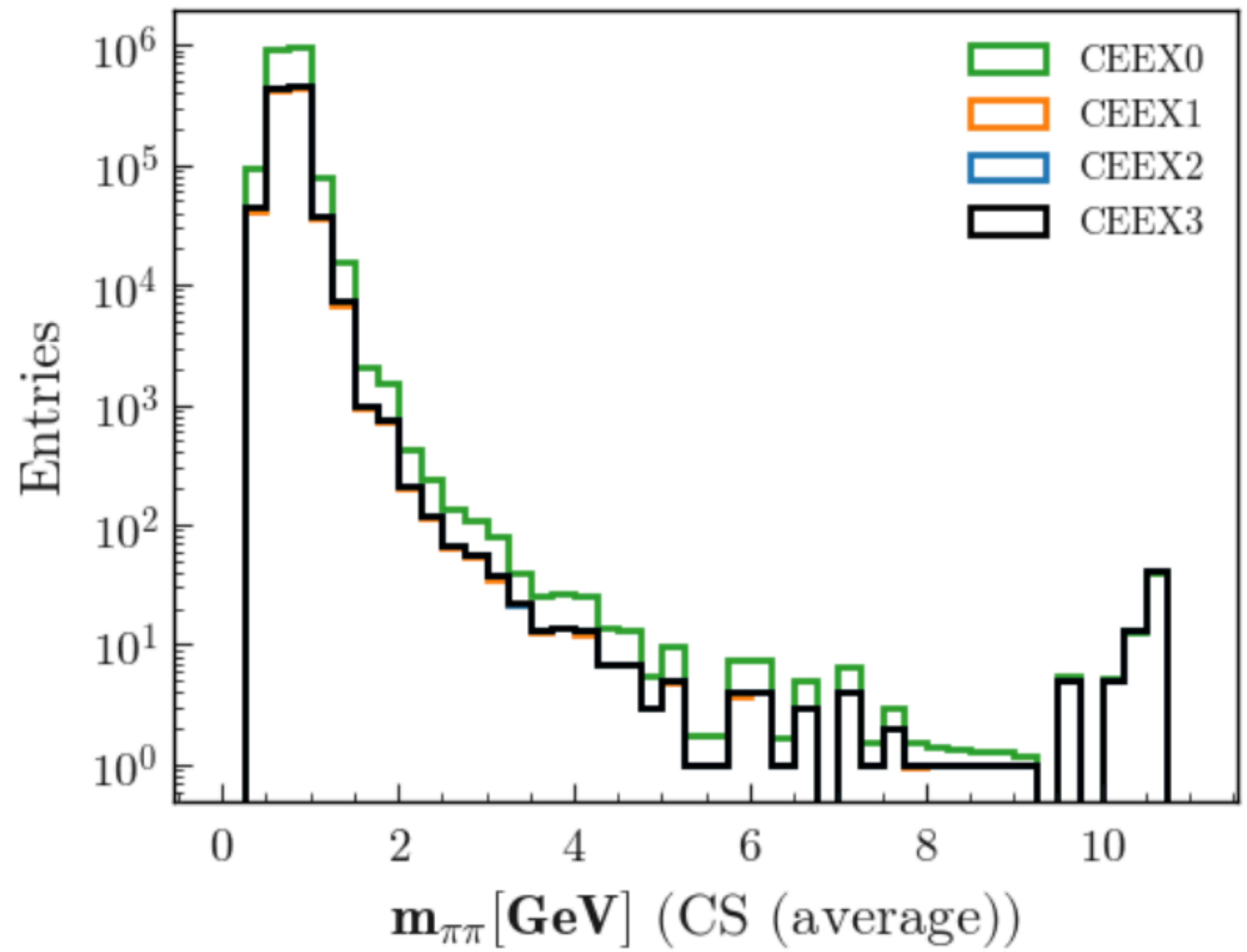

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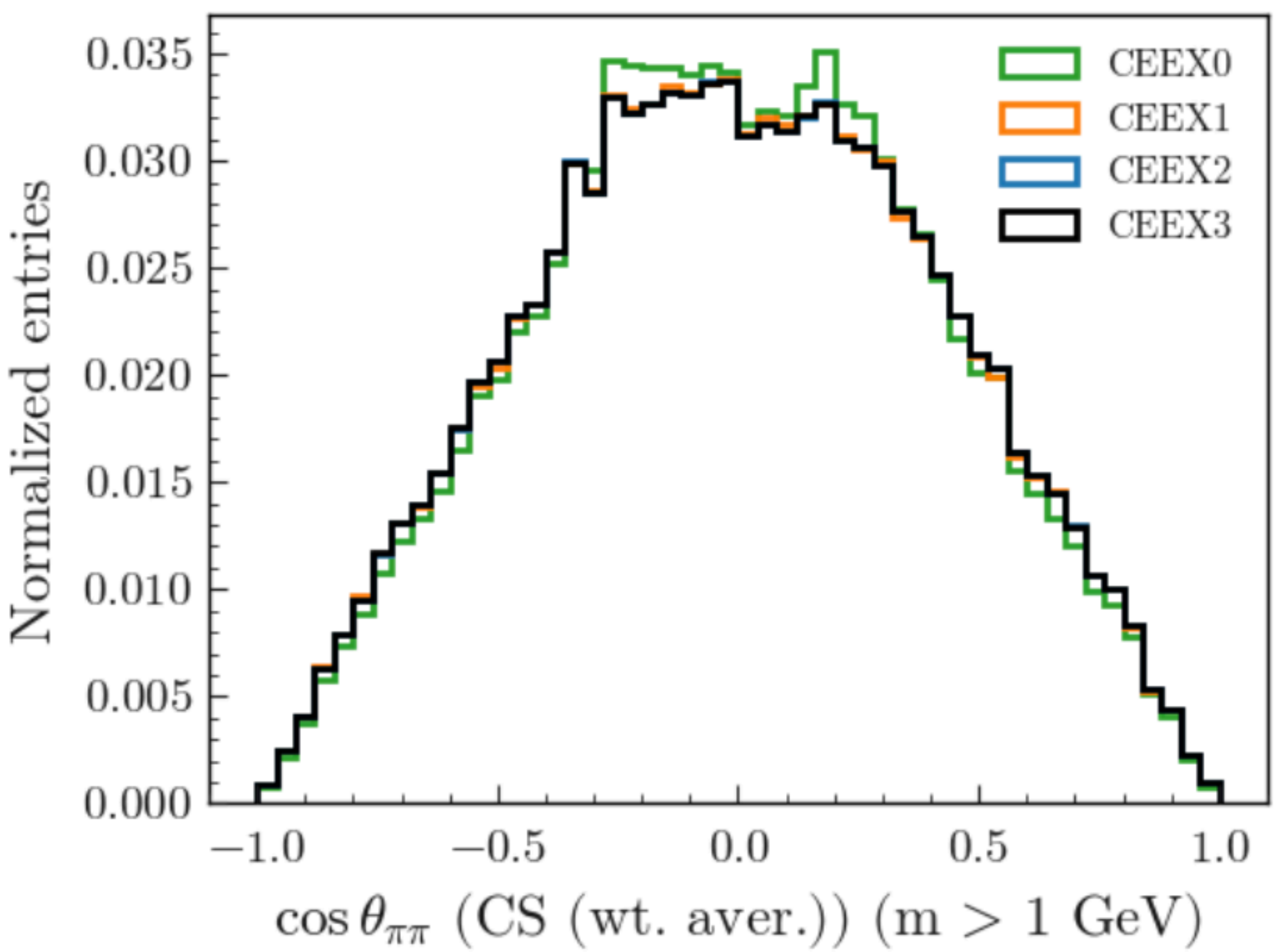

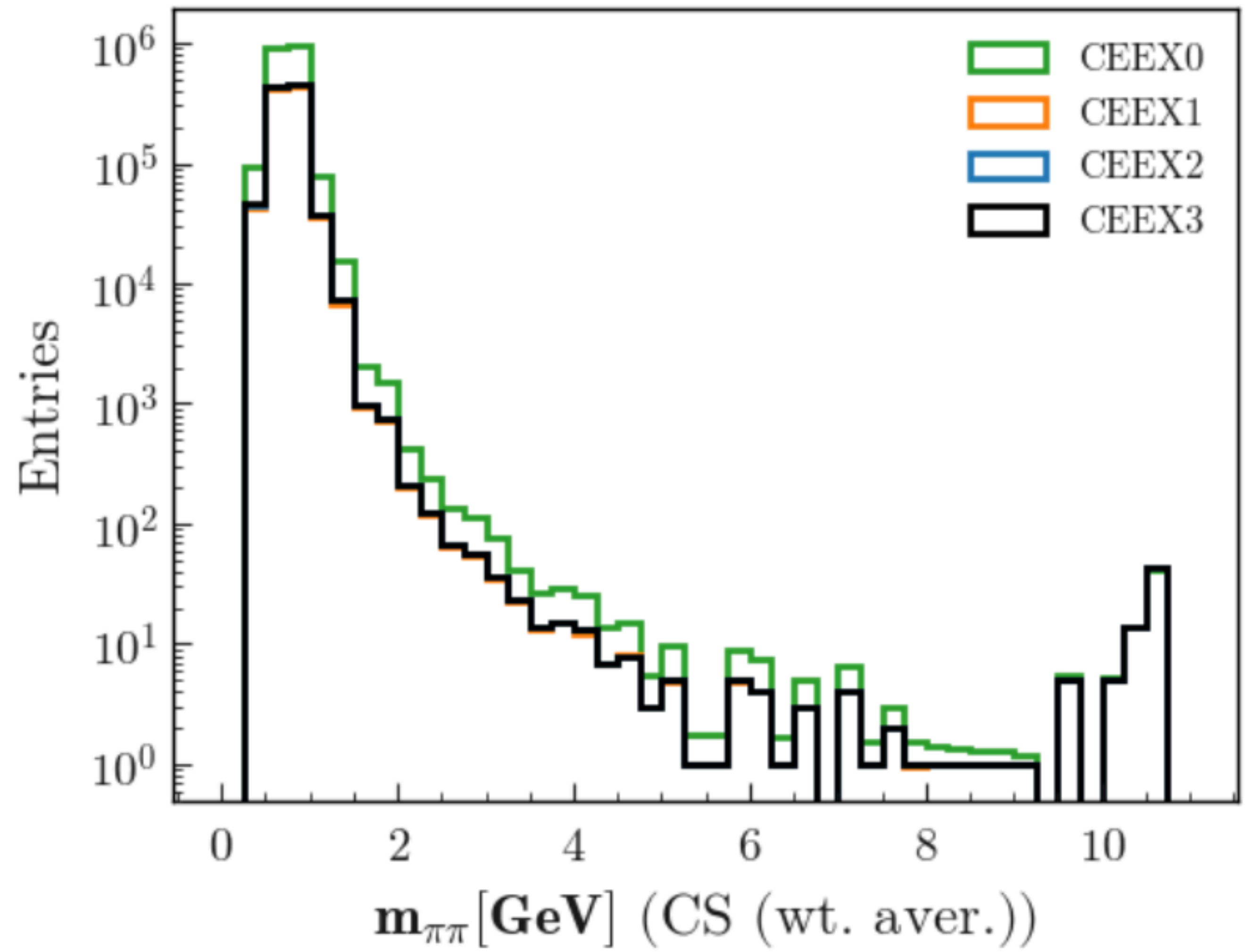

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Summary & Outlook

- $e^+e^- \to \pi^+\pi^-$ events play an important role in the measurement of $(g-2)_\mu$.

- $e^+e^- \to \mu^+\mu^-$ events are the dominant background.

- $e^+e^- \to \mu^+\mu^-$ events have $(1 + \cos^2\theta)$ dependence, but $e^+e^- \to \pi^+\pi^-$ events have $(\sin^2\theta)$ dependence. This can be used to re-weight the background to model our signal.

- KKMC provides state of the art description of Bremsstrahlung and interfaced with PHOTOS to model FSR $\Longrightarrow$ good alternative to generate $e^+e^- \to \pi^+\pi^-$.

- Mustraal frame good choice; Collins-Soper frame provides systematics.