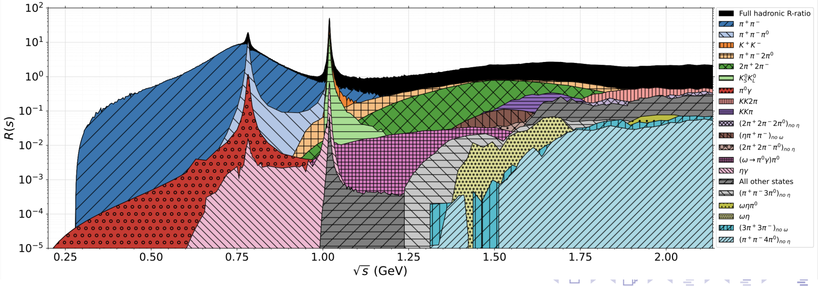
Dispersive a_μ^{HVP} (and $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$) – KNTW update

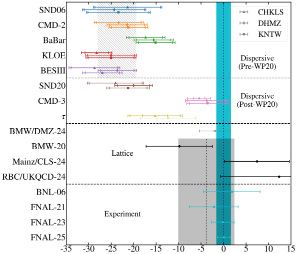
[Muon $g-2$ Theory Initiative Workshop #9 @ UConn, Storrs]

Aidan Wright



HVP Contributions to $g-2$: Present Status & KNTW Approach

- Still face significant tensions in data-driven HVP.
- Presently no explanation...
- While dispersive situation is unclear, so are the ultimate implications of FNAL $g-2$!



- Work ongoing to:
  - Understand HVP discrepancies,
  - Develop next-generation analysis tools,
  - Produce new data inputs with unprecedented precision.
- **KNTW philosophy**: be as data-driven as possible; minimise model dependence.
- Combination procedure (in PYTHON):
  - All non-defective data used.
  - Robust radiative corrections.
  - Clustering - dynamic data-driven combination.
  - Fitting - incorporate full correlation information whilst avoiding bias.
  - Numerically integrate for observables.
- Key input to a_μ^{HVP}; increasingly important to consider e.g. $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ also...

Mid/long-term goal:
Reliable, independent and precise dispersive evaluation from all data inputs to compare to experiment & lattice.

KNTW New Analysis

- Choices during combination:
  - Radiative correction routines;
  - Re-binning procedure;
  - Fitting procedure - correlations;
  - Additional constraints;
  - Error inflation;
  - Interpolation/integration;
  - Additional systematic uncertainties...
- KNTW-DHMZ difference - these clearly influence the central value!
- KNT19/DHMZ19 $\sim 1\sigma$ $a_\mu^{\pi\pi}$ difference mirrors KLOE/BaBar data tension.

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(1.0)(3.5)(1.6)(0.1)$_\psi$(0.7)$_{\text{DV+QCD}}$	692.8(2.4)	1.2

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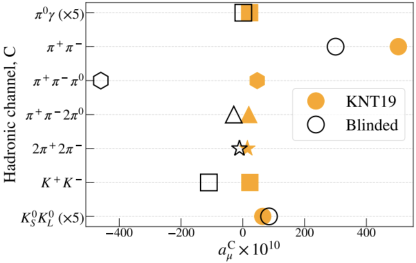
- Mitigate and/or quantify these effects $\Longrightarrow$ KNTW new analysis.
- Want to avoid biases in updated procedure $\Longrightarrow$ KNTW **blinded** analysis.

$$a_\mu^{\text{blind}}[i] = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} ds \left\{ \sigma_i^0(s) K_\mu(s)\, B_i(s) \right\}$$

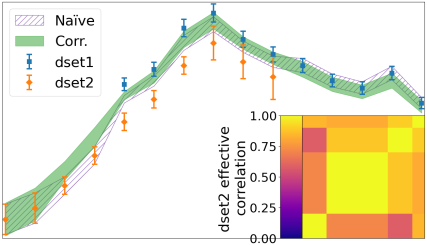
- Well underway: re-baselining, **method systematics**, FSR improvements...

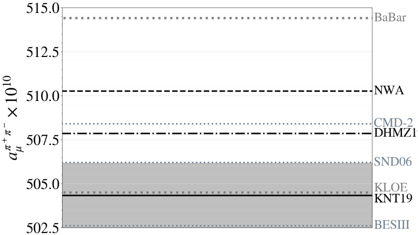


KNTW - arXiv:2409.02827, Phys. Rev. D 111, L011901

Fits to Correlated Data

- Correlations strongly affect averages/fits of spectra.
- Right: 'naïve' weighted average (NWA) ignoring correlations jumps between datasets.
- NWA ignores information: systematics on normalisations affect the entire dataset.
- KNTW baseline: except for explicit info. otherwise, *each* systematic is fully correlated.
- Correlated fits are unable to locally optimise (to the same extent).
- How to use systematic correlations is a key analysis choice:
  - Has been understood to be primary cause of DHMZ/KNTW tensions - significant!
  - WP20/DHMZ19 'KLOE-BaBar' systematic – in part an ad-hoc cover of this.
  - **∴ Procedurally quantify syst uncertainty on combination for new analysis.**





Data Uncertainties and Correlations

- Experimental σ_{hadrons} input features statistical and systematic uncertainties.
- Both can be and often are correlated between data points in the energy range.

Statistical Uncertainties/Correlations

- Stat. uncertainties derived directly from event counts in measurements.
- Typically uncorrelated – bin-by-bin event numbers.
- Experimental procedures – e.g. unfolding – can induce correlations.
- Uncertainties on unfolding: systematic.

Systematic Uncertainties/Correlations

- Estimated size of effects potentially affecting the measurement.
- Often normalisations (e.g. luminosity), evaluated in broad regions.
- Uncertainty on true magnitude $\Longrightarrow$ uncertain correlations. Multiple syst prescriptions valid.

The statistical uncertainties of the measured cross sections and their correlations are generally known rather precisely,[19] both for measurements performed at fixed $\sqrt{s}$ values and for ISR measurements involving unfolding techniques. However, even if they are published together with the nominal values of the measurement, the systematic uncertainties and their correlations are never really measured, but rather *estimated*. For this reason, the size of the systematic uncertainties of the various measurements, the correlations of a systematic uncertainty component impacting several measurements, as well as the correlations between the systematic uncertainties impacting a given measurement, are not perfectly known. Furthermore, the uncertainties impacting all th

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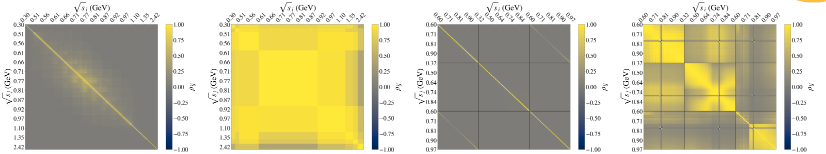
KNTW Correlations Framework – Motivation



BaBar: stat syst KLOE: stat syst

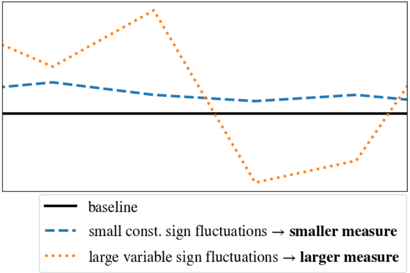
So...

- Use of correlations in data combinations significantly affects the output fit.
- Systematic uncertainties are less well-known than statistical, and their correlations are inherently uncertain.
- KNTW in their new analysis aim to assess the impact of major analysis choices on their data combination.

$\Longrightarrow$ **Goal:** develop a systematic procedure to assess the methodological uncertainty on a fit due to a chosen implementation of data systematic correlations.

KNTW Correlations Framework – Measure

- Fundamental KNTW object: σ_{had} data compilation – observables *from this*.
- Compute uncertainty consistently: evaluate at cross section level.
- Quantify spectrum deviation with a measure $\mathcal{M}$. Necessary properties:
  - **(i)** Depend on absolute deviations - signs of deviations are irrelevant.
  - **(ii)** Be dimensionless i.e. normalised.
  - **(iii)** Be finite everywhere.
  - **(iv)** Be symmetric in the baseline and modification so as to prefer neither.



- Take:

Default combination (KNTW ⇔ fully correlated)

Default channel covariance matrix

$$\mathcal{M}(\tilde{\sigma}_i) = \sum_{i,j} \left[(\tilde{\sigma}_i - \sigma_i)(\tilde{C}_{ij} + C_{ij})^{-1}(\tilde{\sigma}_j - \sigma_j) \right]$$

Modified (decorrelated) covariance matrix

Combination from different correlation assumption

and generate the $\tilde{\sigma}$ using a **decorrelation procedure**.

- If $\mathcal{M}(\hat{\sigma}) := \max \mathcal{M}$, take a new systematic covariance matrix:

$$C^{\rho}_{ij} = (\hat{\sigma}_i - \sigma_i)(\hat{\sigma}_j - \sigma_j).$$

KNTW Correlations Framework – Decorrelation Procedures

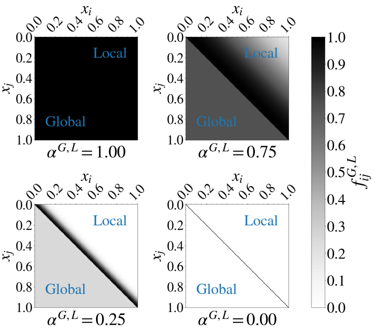
- Propose simple decorrelation procedures $f_{ij}(\alpha) : \tilde{C}_{ij} = f_{ij}(\alpha) \times C_{ij}$.
- Vary parameter $0 \leq \alpha \leq 1$ to alter correlation assumption in covariance matrix.
- **Global (G):** Systematics still treated as global norm.s; decrease correlation influence.

$$f_{ij}^G(\alpha^G) = \alpha^G + (1 - \alpha^G)\delta_{ij}$$

- **Local (L):** Limit correlations range – isolate distinct spectra regions with different properties.

$$f_{ij}^L(\alpha^L) = \exp\left[-\left(\frac{1-\alpha^L}{\alpha^L} \times \frac{\sqrt{s}_i - \sqrt{s}_j}{\mathcal{E}}\right)^2\right]$$

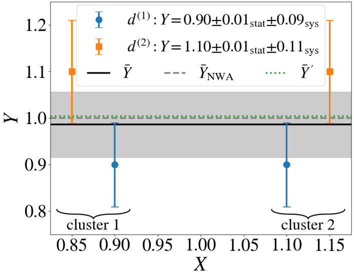
  - Based on Leplumey & Stoffer, arXiv:2501.09643.
  - Note: $\mathcal{E}$ is not a variable; any change of value is equivalent to a rescaling of α^L.

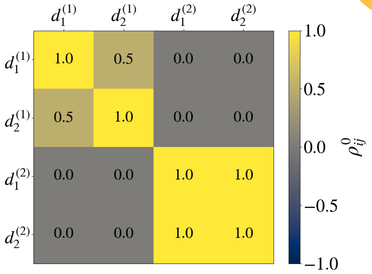


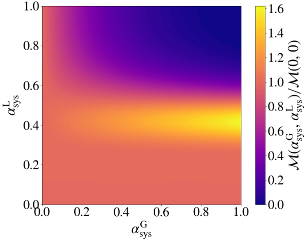
KNTW - arXiv:2604.25004

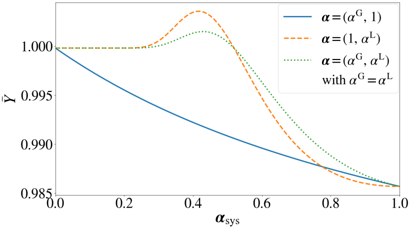
- Could imagine more complicated (e.g. multi-variate) decorrelation procedures but these are likely to provide a sufficient initial description.
- Can apply 1-D decorrelation functions subsequently, e.g. $\tilde{C}_{ij} = f_{ij}^G(\alpha^G) f_{ij}^L(\alpha^L) \times C_{ij}$.

KNTW Correlations Framework – Toy Model







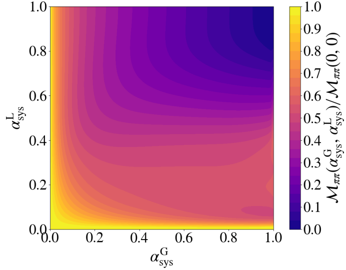


Correlation Study Results

Channel	$d^\rho a_e^{\mathrm{HVP}} \times 10^{14}$	$d^\rho a_\mu^{\mathrm{HVP}} \times 10^{10}$	$d^\rho a_\tau^{\mathrm{HVP}} \times 10^{8}$	$d^\rho \Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) \times 10^4$	$d^\rho \Delta\nu_{\mathrm{Mu}}^{\mathrm{HVP}}$ (Hz)
$\pi^+\pi^-$	0.14	0.45	0.06	0.01	0.13
$\pi^+\pi^-\pi^0$	0.15	0.58	0.28	0.07	0.20
$\pi^+\pi^-2\pi^0$	0.07	0.30	0.21	0.07	0.11
$2\pi^+2\pi^-$	0.04	0.17	0.14	0.05	0.06
K^+K^-	0.16	0.61	0.29	0.06	0.21
Inclusive ($\sqrt{s} \gtrsim 2$ GeV)	0.08	0.34	0.50	0.81	0.15
Total d^ρ uncertainty	0.29	1.08	0.72	0.82	0.38
KNT19 uncertainty	0.66	2.42	1.39	1.12	0.82
Updated uncertainty	0.72	2.65	1.57	1.39	0.90
% error contribution	16.2	16.6	21.1	34.9	17.7

- Proposed decorrelation procedures put into measure; applied to KNT19 **systematics**.
- max $\mathcal{M}$ at $\alpha_{\mathrm{sys}}^G = \alpha_{\mathrm{sys}}^L = 0$.
- Example right: 2D scan of $\pi^+\pi^-$ channel.
- Uncertainties on channel combination $\rightarrow$ uncertainties on integrated observables.
- Table: largest uncertainty for inclusive channel dominated $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$ (see later slides).
- Follows from relatively small correlation strength (d^ρ) uncertainty in $\pi^+\pi^-$ channel...

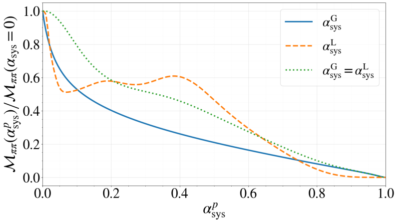
KNTW - arXiv:2604.25004

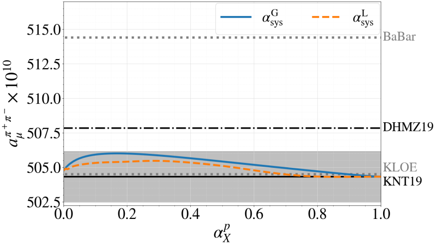


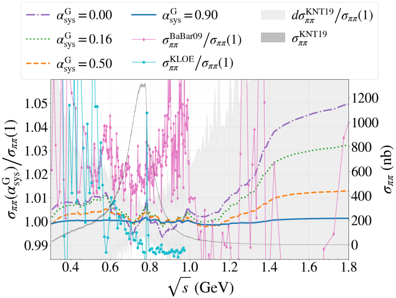
KNTW - arXiv:2604.25004

Correlation Study Results – $a_\mu^{\pi\pi}$ Systematic

- Point-to-point uncertainty on cross section maximised at $\alpha_{\text{sys}}^{G,L} = 0$.
- $\delta a_\mu^{\pi\pi}$ is however maximised at $\alpha^{\text{G}} = 0.16$.[a]
- $\alpha = 0 \implies$ significant anticorrelations around ρ-peak $\implies$ reduced *integrated* uncertainty.
- Consistency & enforcing conservative spectrum uncertainties requires d^ρ from $\alpha^G = 0$.
- What about KLOE-BaBar systematic: $>> d^\rho$?



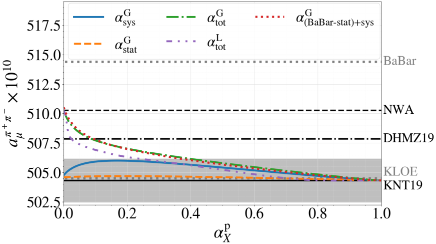


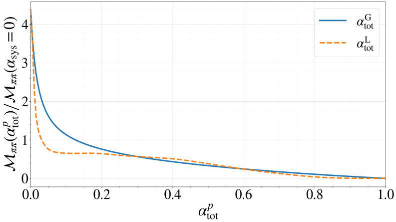


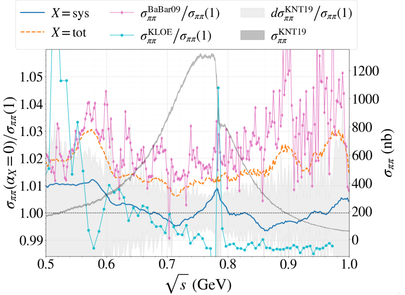
[a] Different integrated obs. diff.s maximised at different α^G.

Correlation Study Results – $a_\mu^{\pi\pi}$ Statistical + Systematic

- Much larger effects seen when decorrelating statistics and systematics together.
- Much larger NWA recovered.
- BaBar statistical covariances $\implies$ fits can consistently take lower values.[a]
- **NOT** recommending use of this as uncertainty – earlier notes on stat correlation reliability.
- KLOE-BaBar $\therefore$ overestimates corrs. impact.







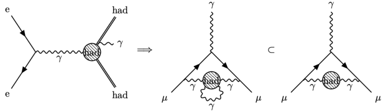
[a] Mid-range correlations arise from broader lumi. binning.

FSR Improvements

Above: Correlations framework – past informs future.

Below: Entirely new studies – FSR and updated **blinded** fits!

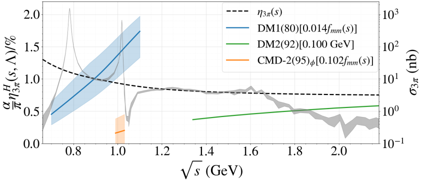
- Need FSR inclusive measurements for dispersive HVP evaluation.



- Some cuts remove hard (but always keep soft + virtual) $\implies$ we add hard back.
- KNT19 considered FSR for two scalar channels (limited for $\pi^+\pi^-$).
- Other channels given 1% uncertainty
- Significantly reduce theory uncertainty by accounting for more channels' FSR.

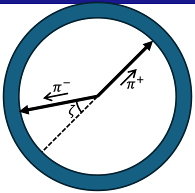
Example: $\pi^+\pi^-\pi^0$ Channel

- Previously: standard 1% uncertainty.
- Selection criteria $\implies$ 18/21 FSR inclusive, including all modern datasets.
- Hard corrections for remainder provided by MH et al – appreciated!
- Correction/uncertainty now $\mathcal{O}(10^{-5})$.



FSR Improvements – $\pi^+\pi^-$ Acolinearity

- Most FSR exclusive $\pi^+\pi^-$ (pre-2000) datasets rely on an 'acolinearity cut' ζ.
- Seek only $\mathcal{O}(\alpha)$ real emissions correction so can freely calculate in $F\times$sQED.

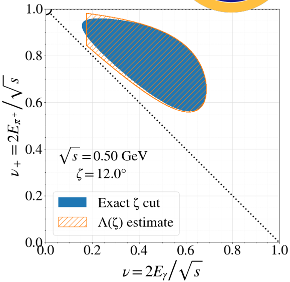


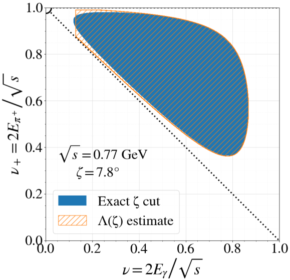
- Derived acolinearity cut (hard) correction factor depends on modified region of integration $\mathcal{R}$:

$$\eta^H_{2\pi}(s,\zeta) \sim \iint_{\mathcal{R}(\beta,\zeta,\nu,\nu^+)\leq 0} d\nu d\nu^+ \left|M(\pi^+\pi^-\gamma)\right|^2,$$

$$\sigma^0_{\rm exp} \to \sigma^0_{\rm exp}\left(1+\frac{\alpha}{\pi}\eta^H\right) \qquad [a_\mu^{\pi\pi} \text{ off by } \lesssim 10^{-7}]$$

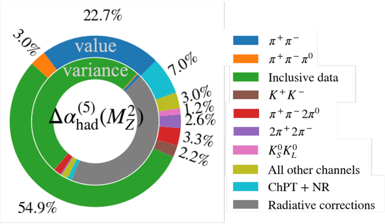
- Superior to KNT19's $E_\gamma = \frac{1}{2}\sqrt{s}\beta\times\zeta$ and $(0.5\pm0.5)\times$(soft+hard+virtual) estimates.
- ζ known for required datasets $\implies$ ~exact correction.
- Can take reduced uncertainty: generated from conservative $\pm\frac{1}{3}\zeta$ variation. Results in backup.



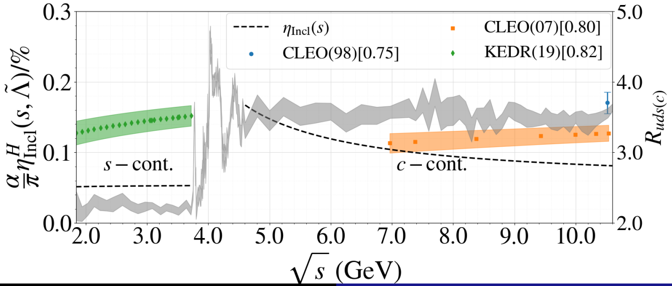


FSR Improvements – Inclusive Channel

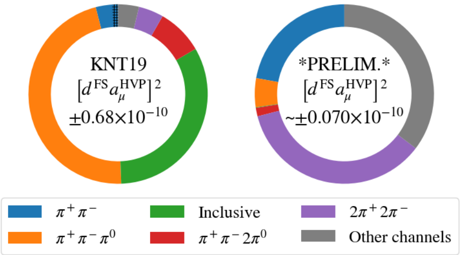
- Dominating KNT19 $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ uncertainty – unknown inclusive FSR.



- Cuts on very hard FSR by three continuum measurements.
- Necessary correction for missing hard FSR well known: pQCD applicable!
- Shown: full NLO QCD correction and relevant hard corrections ($\tilde{\Lambda} = 2E_\gamma^{\rm max}/\sqrt{s}$).
- Includes mixed QED-QCD effects. $\mathcal{O}(\alpha_s)$ part of correction: conservative error.
- Uncertainty becomes **inconsequential**.

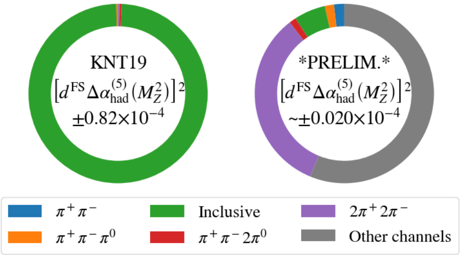


FSR Improvements – Potential Impacts



- 4π FSR uncertainties also decreased.
- FSR inclusive datasets make up
  - 90% of $\pi^+\pi^-2\pi^0$
  - 70% of $2\pi^+2\pi^-$

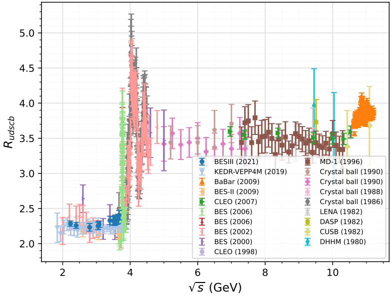
  by effective weight – only take uncertainty on remainder.
- One step better - *crude* model-based estimate for FSR in $\pi^+\pi^-2\pi^0$ reduces error further.



- FSR uncertainty no longer relevant to error budget.
- Even wedges which grow (left) have $4-5\times$ smaller values.
- Further reduction superfluous – values likely not significantly affected.
- Together with VP improvements, theory uncertainties on KNT19 near eliminated.

Updates to the Inclusive Channel – Note

Next few slides: **updated Inclusive fit.**



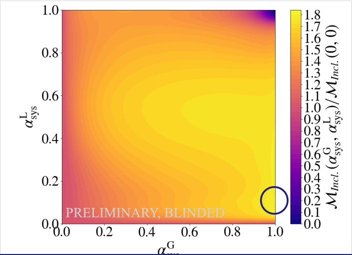
All shown results are **blinded** and **preliminary**.

Blinded quantities are indicated with a tilde, i.e. $a \rightarrow \tilde{a}$.

Updates to the Inclusive Channel (incl. d^ρ Systematic)

- Inclusive channel important to a_μ^{HVP}, integral for $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$.
- New datasets since KNT19:
  - BESIII(21) at lower energies.
  - Missed precise CLEO-c(09) R_c data in open c – supplement like BaBar $b\bar{b}$.
  - Near future: excited for KEDR 4.5→7.0 GeV scan data
- Fit improves despite mild BESIII tensions.
- Generally good agreement with pQCD (except BESIII, see D. Boito et al.– arXiv:2509.12956.)
- Charm continuum:
  - MD-1 shows signs of overbinning (recognised in paper!).
  - Rebin data to improve fit quality.
  - Different cluster widths in s- vs c-continua – 'localised clustering'.
  - Test-bed for other channels where data properties change over energy range.
  - $\implies$ Better fits, smoother descriptions.

- Relevant that $\mathcal{M}$ maximised at $\alpha^{\text{L}} = 0.11$!
- Retains correlations over ~ 30 MeV.
- Significant anticorrelation: c-continuum decorrelation has opposite sign.
- Point-to-point **most conservative uncertainty**...
  - Corresponds to $\sim$largest uncertainty on $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$: 20% $>$ full decorrelation.
  - Slightly smaller than $\alpha = 0$'s $d^\rho a_\mu^{\text{HVP}}$.



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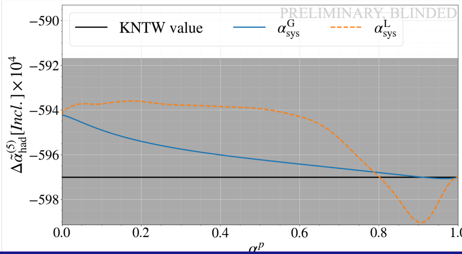
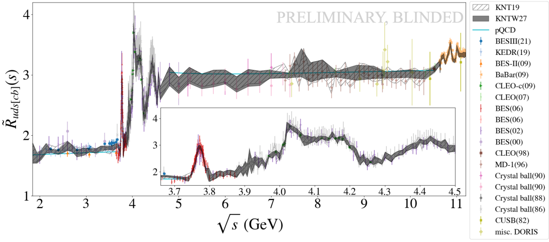
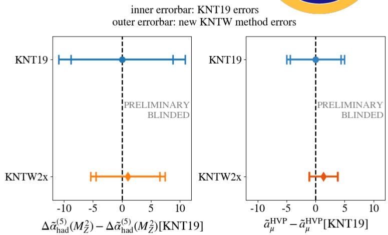
    - Corresponds to $\sim$largest uncertainty on $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$: 20% $>$ full decorrelation.
    - Slightly smaller than $\alpha = 0$'s $d^\rho a_\mu^{\mathrm{HVP}}$.



NEW (Blinded) Inclusive Channel





$$\chi^2/\text{ndof} = 1.47_{\text{KNT19}} \rightarrow 1.36_{\text{KNTW2x}}$$

$$\Delta\tilde{\alpha}^{(5)}_{\text{incl}}(M_Z^2)[\sim \text{KNT19}] = -598.02(1.46)_{\text{st}}(6.29)_{\text{sy}}(0.08)_{\text{VP}}(5.98)_{\text{FSR}}(0.86)_{\text{meth}}(6.40)_{\rho}[10.91]_{\text{tot}}$$

$$\Delta\tilde{\alpha}^{(5)}_{\text{incl}}(M_Z^2)[\text{KNTW2x}] = -597.01(1.31)_{\text{st}}(5.32)_{\text{sy}}(0.04)_{\text{VP}}(0.04)_{\text{FSR}}(0.55)_{\text{meth}}(3.30)_{\rho}[\ 6.42]_{\text{tot}}$$

$$\tilde{a}_\mu^{\text{incl}}[\sim \text{KNT19}] = -265.39(0.90)_{\text{st}}(3.34)_{\text{sy}}(0.01)_{\text{VP}}(2.65)_{\text{FSR}}(0.36)_{\text{meth}}(2.38)_{\rho}[4.98]_{\text{tot}}$$

$$\tilde{a}_\mu^{\text{incl}}[\text{KNTW2x}] = -264.03(0.60)_{\text{st}}(2.34)_{\text{sy}}(0.01)_{\text{VP}}(0.02)_{\text{FSR}}(0.42)_{\text{meth}}(0.37)_{\rho}[2.48]_{\text{tot}}$$

Potential $\times 2$ improved precision for KNTW2x $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$!
(putting aside potential $\pi^+\pi^-$ impact...)

Conclusions & Outlook

A lot of work remains:

→ Must understand dispersive tensions to fully understand FNAL $g-2$ in the SM.

KNTW blinded analysis:

→ Substantial improvements to unbiased new analysis.

→ Important step – quantify method uncertainties.

Key analysis choice: use of systematic correlations.

→ Modular correlation strength framework quantifies d^ρ uncertainties on KNT19 result – see arXiv:2604.25004.

→ Results smaller than expected in $\pi^+\pi^-$. Larger results when varying known statistics.

Substantial precision improvements possible in new analysis:

→ A posteriori FSR studies to eliminate leading KNT19 theory uncertainty.

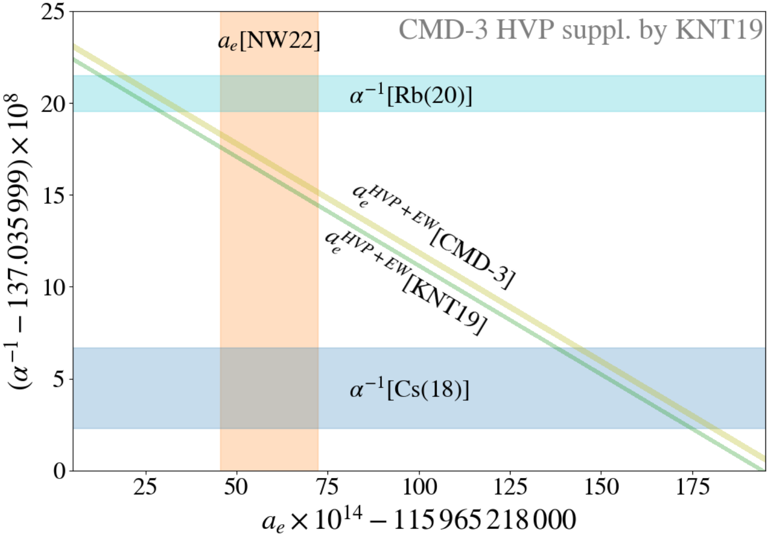
- → Exact treatment of acolinearity cuts in $\pi^+\pi^-$.
- → Substantial pQCD-based improvements in Inclusive data – critical for $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$.

→ New Incl. data $\implies$ doubled $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ precision (tensions permitting...).

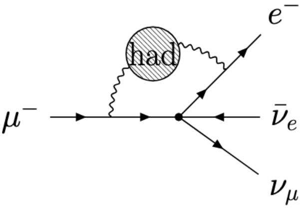
All this and more in the next KNTW update...

BACKUP

Backup: a_e^{HVP}

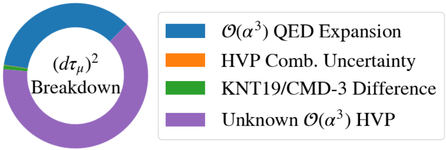


Backup: Muon Lifetime HVP



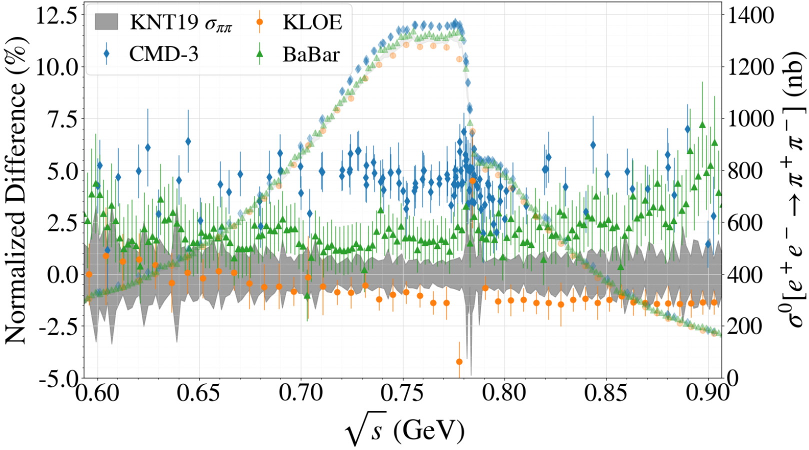
$$\propto G_F^2 \int_{m_0^2}^{\infty} \frac{ds}{3s} R(s) K_\Gamma(s)$$

- HVP contributions to muon lifetime τ_μ very relevant at current precision.
- Evaluated in Fermi theory; QED known to $\mathcal{O}(\alpha^3)$.
- Dispersion integral for LO HVP (i.e. α^2).
- Even CMD-3/KNT19 difference is irrelevant to current τ_μ error budget.
- Theory τ_μ estimate contributes only $\sim 0.11\%$ to uncertainty on extracted G_F.



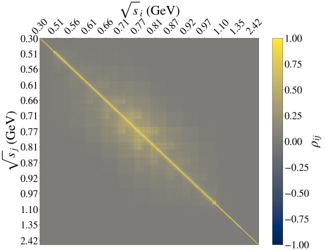
see A. Eberhart, M. Fael, A. Keshavari, D. Nomura, K. Schönwald, M. Steinhauser, T. Teubner, A. Wright – arXiv:2607.02657, *Muon lifetime and Fermi constant: an update*, for more details...

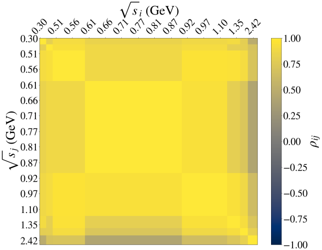
Intro Backup: CMD-3 Tension



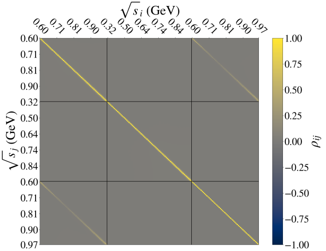
Intro Backup: BaBar/KLOE Covariance Matrices

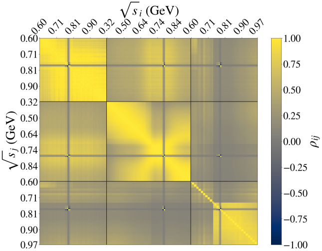
BaBar Correlation Matrices





KLOE Correlation Matrices





Corrs. Backup: α^L Rescaling

Consider a change $\mathcal{E} \to \mathcal{E}'$ in

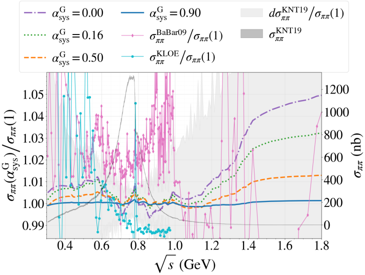
$$f_{ij}^L(\alpha^L) = \exp\left[-\left(\frac{1-\alpha^L}{\alpha^L} \times \frac{\sqrt{s}_i - \sqrt{s}_j}{\mathcal{E}}\right)^2\right].$$

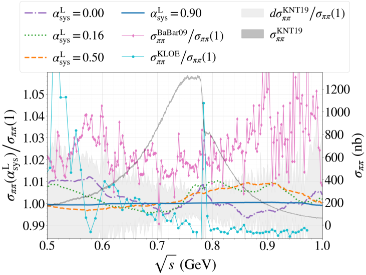
Then

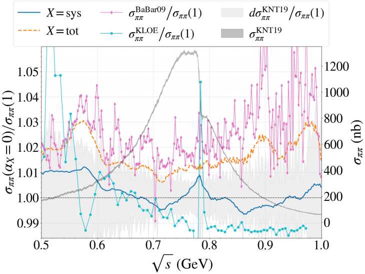
$$\frac{1-\alpha^L}{\alpha^L} \times \frac{1}{\mathcal{E}} = \frac{1-\alpha^{L\prime}}{\alpha^{L\prime}} \times \frac{1}{\mathcal{E}'}$$
$$\implies \alpha^{L\prime} = \left(1 + \frac{1-\alpha^L}{\alpha^L} \times \frac{\mathcal{E}'}{\mathcal{E}}\right)^{-1}.$$

We have mapping $(\alpha^L = 0, 1) \to (\alpha^{L\prime} = 0, 1)$ as required (remains bounded). Corresponds to shrinking and stretching of regions of α^L. Sufficient dense sampling of α^L for any appropriate $\mathcal{E}$ provides full description.

Corrs. Backup: Local Decorrelation $\pi^+\pi^-$





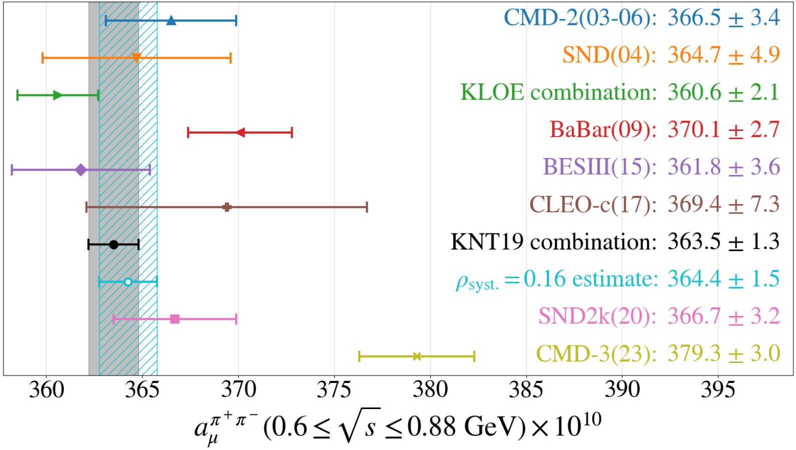


Upper left: global syst decorrelation

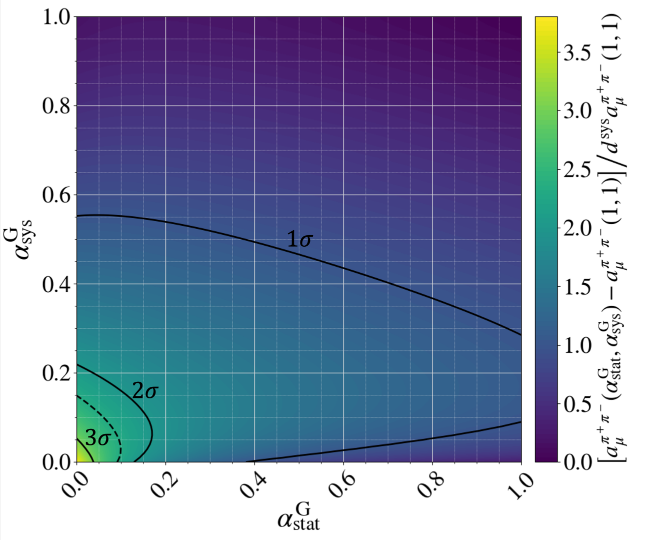
Upper right: local syst decorrelation

Lower left: full syst, stat+syst decorrelation

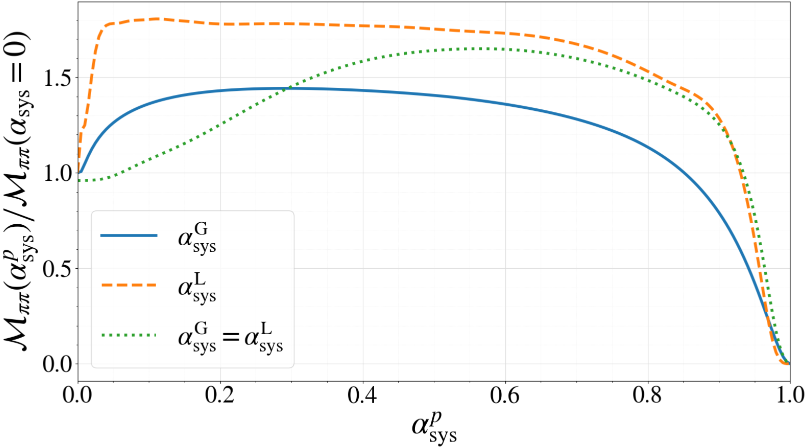
Corrs. Backup: Central Region Averages



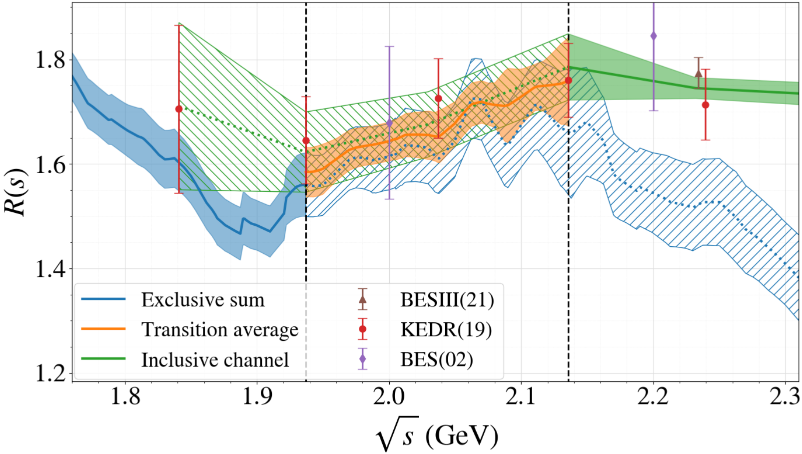
Corrs. Backup: Stat./Syst. Decorrelation of $\pi^+\pi^-$



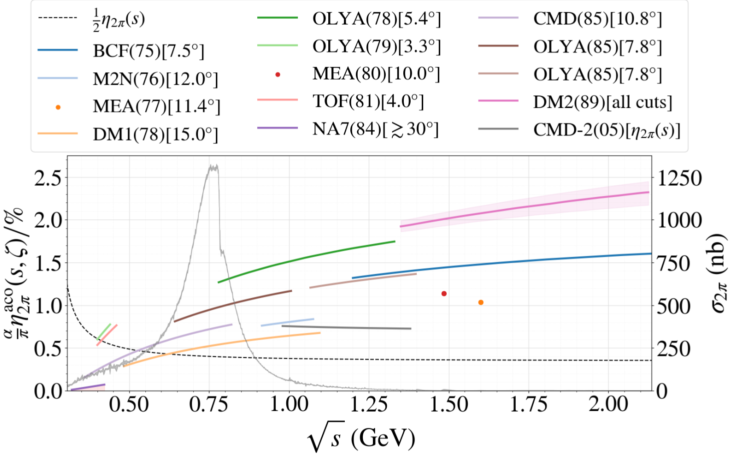
Fit Backup: New Inclusive Measure 1D



Fit Backup: Updated Transition Region

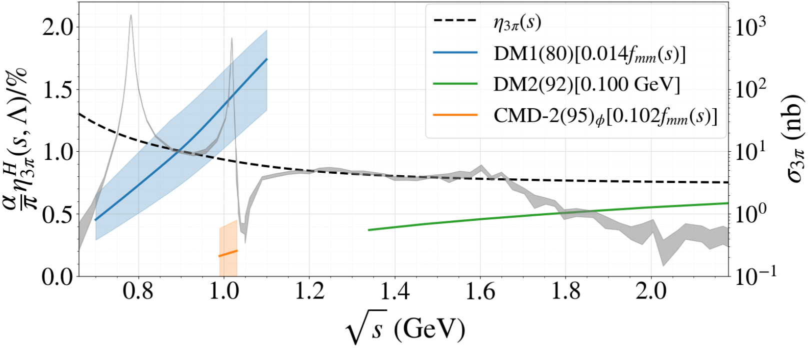


FSR Backup: $\pi^+\pi^-$ FSR Acolinearity Correction



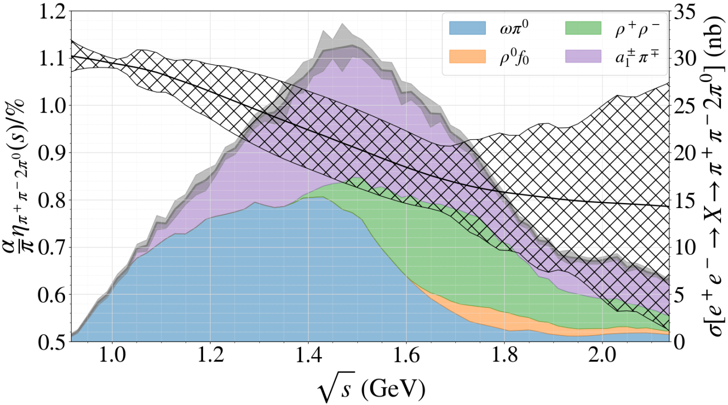
- KNT19: $(\frac{1}{2} \pm \frac{1}{2})\eta_{2\pi}(s)$ for all FSR exclusive datasets.
- Update analytic up to final two numerically evaluated integrals.
- NA7 fixed target config. $\Longrightarrow$ effectively FSR inclusive.
- DM2 cuts on E_γ and ζangle $\Longrightarrow$ larger uncertainty. 'Stitched' contours.

FSR Backup: $\pi^+\pi^-\pi^0$ FSR Correction



- Hard 3π sQED-based correction kindly provided by Martin Hoferichter, Bai-Long Hoid and Bastian Kubis.
- See arXiv:2307.02546 for details.
- DM2 cuts on additional energy in detector.
- DM1, CMD-2$_\phi$ cut on 'missing mass' to reconstruct neutral pion.

FSR Backup: $\pi^+\pi^-2\pi^0$ FSR Correction



- Rough decomposition of $\pi^+\pi^-2\pi^0$ into intermediates.
- Rough estimates for process FSR using functions $\eta_{2,3\pi}$.
- Only an inclusive estimate.
- Conservative errors on approximations.
- Overall $\sim 125\%$ uncertainty on (1/2) correction
- $\implies \sim 0.06\%\ d^{FSR}a_\mu^{\pi^+\pi^-2\pi^0}$, down from $\sim 0.1\%$.