

# Overview on radiative corrections and Monte Carlo codes (an update on the RadioMonteCarlow2 activities)

F. Piccinini



INFN, Pavia and GGI

**Nineth Plenary Workshop of the Muon  $g - 2$  Theory Initiative**

Department of Physics, University of Connecticut, 3-7 August 2026

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⇒ the precision of theoretical simulations enters directly as a systematics in  $a_\mu$

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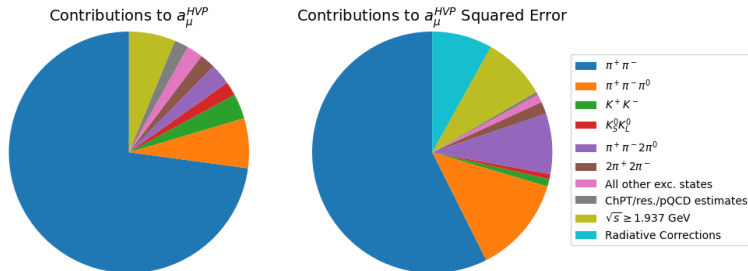
- in addition to  $\pi^+\pi^-$  and  $\pi^+\pi^-\gamma$  also other hadronic final states are relevant

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# Classification of QED radiative corrections

- perturbative series for a  $2 \rightarrow n$  cross section

$$\sigma \sim \alpha^n \sum_{m=0}^{\infty} \alpha^m \sum_{i=0}^m \sum_{j=0}^m c_m^{\{i,j\}} L^i \ell^j$$

$$L = \ln \left( \frac{Q^2}{m_e^2} \right) \quad \text{collinear log (for ISC)}$$

$$\ell = \ln \left( \frac{Q^2}{(\Delta E_\gamma)^2} \right) \quad \text{IR log}$$

- $\sqrt{Q^2}$  of the order of 1 – 10 GeV  $\implies L \sim 10 - 15$
- $\Delta E_\gamma$  is an effective scale for maximum photon radiation induced by the particular event selection

$\implies$  radiative corrections strongly dependent on the event selection and underlying physics

# Order of magnitude estimates

Loosely and schematically

- referring only to collinear logs

|      |                                               |                                                   |                       |
|------|-----------------------------------------------|---------------------------------------------------|-----------------------|
| LO   | $\alpha^0$                                    |                                                   |                       |
| NLO  | $\alpha L$                                    | $\alpha$                                          |                       |
| NNLO | $\frac{1}{2}\alpha^2 L^2$                     | $\frac{1}{2}\alpha^2 L$                           | $\frac{1}{2}\alpha^2$ |
| h.o. | $\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$ | $\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$ | $\dots$               |

Blue: Leading-Logs

# Order of magnitude estimates

Loosely and schematically

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|      |       |       |       |
|------|-------|-------|-------|
| LO   | 90%   |       |       |
| NLO  | 10%   | 0.5%  |       |
| NNLO | 0.5%  | 0.05% | 0.01% |
| h.o. | 0.01% | ...   | ...   |

Typically at flavour factories (on integrated Bhabha  $\sigma$ )

- possible additional enhancements from IR logs induced by
  - event selection
  - differential distribution under study

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  - typically reliable for observables sufficiently inclusive on radiation

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\* **assess the technical precision** through tuned comparisons on differential observables between different codes with the same physics input

\* **estimate the residual theoretical uncertainties** through detailed comparisons between codes adopting different approaches/approximations and in collaboration with the experimental colleagues

Established in 2006 and coordinated by H. Czyz and G. Venanzoni

| TOPIC                              | CONVENORS                  |
|------------------------------------|----------------------------|
| Luminosity                         | G. Montagna, F. Nguyen     |
| R Measurement                      | A. Arbuzov, G. Fedotovitch |
| ISR                                | H. Czyz, G. Venanzoni      |
| Tau                                | Z. Was, D. Epifanov        |
| Hadronic VP, $g-2$ and Delta alpha | T. Teubner, S. Eidelman    |
| Gamma-gamma physics                | S. Ivashyn, D. Moricciati  |
| FSR models                         | S. Gorini, A. Denig        |

- one/two meetings/year

Eur. Phys. J. C (2010) 66: 585–686  
DOI 10.1140/epjc/s10052-010-1251-4

THE EUROPEAN  
PHYSICAL JOURNAL C

Review

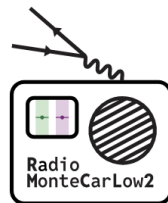
## Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

S. Actis<sup>38</sup>, A. Arbuzov<sup>9,e</sup>, G. Balossini<sup>32,33</sup>, P. Beltrame<sup>13</sup>, C. Bignamini<sup>32,33</sup>, R. Bonciani<sup>15</sup>, C.M. Carloni Calame<sup>35</sup>, V. Cherepanov<sup>25,26</sup>, M. Czakon<sup>1</sup>, H. Czyż<sup>19,a,f,i</sup>, A. Denig<sup>22</sup>, S. Eidelman<sup>25,26,g</sup>, G.V. Fedotov<sup>25,26,e</sup>, A. Ferroglia<sup>23</sup>, J. Gluza<sup>19</sup>, A. Grzebińska<sup>8</sup>, M. Guina<sup>19</sup>, A. Hafner<sup>22</sup>, F. Ignatov<sup>25</sup>, S. Jadach<sup>8</sup>, F. Jegerlehner<sup>3,19,41</sup>, A. Kalinowski<sup>29</sup>, W. Kluge<sup>17</sup>, A. Korchin<sup>20</sup>, J.H. Kühn<sup>18</sup>, E.A. Kuraev<sup>9</sup>, P. Lukin<sup>25</sup>, P. Mastrolia<sup>14</sup>, G. Montagna<sup>32,33,b,d</sup>, S.E. Müller<sup>22,f</sup>, F. Nguyen<sup>34,d</sup>, O. Nicrosini<sup>33</sup>, D. Nomura<sup>36,h</sup>, G. Pakhlova<sup>24</sup>, G. Pancheri<sup>11</sup>, M. Passera<sup>28</sup>, A. Penin<sup>10</sup>, F. Piccinini<sup>33</sup>, W. Placzek<sup>7</sup>, T. Przedzinski<sup>6</sup>, E. Remiddi<sup>4,5</sup>, T. Riemann<sup>41</sup>, G. Rodrigo<sup>37</sup>, P. Roig<sup>27</sup>, O. Shekhovtsova<sup>11</sup>, C.P. Shen<sup>16</sup>, A.L. Sibidanov<sup>25</sup>, T. Teubner<sup>21,h</sup>, L. Trentadue<sup>30,31</sup>, G. Venanzoni<sup>11,c,i</sup>, J.J. van der Bij<sup>12</sup>, P. Wang<sup>2</sup>, B.F.L. Ward<sup>39</sup>, Z. Was<sup>8,g</sup>, M. Worek<sup>40,19</sup>, C.Z. Yuan<sup>2</sup>

Coordinated by A. Kupsc, A. Signer, Y. Ulrich, G. Venanzoni

- 1<sup>st</sup> Meeting, Zürich, 5-9 June 2023
- 1<sup>st</sup> Satellite Meeting, Liverpool, 15 Nov. 2024
- 2<sup>nd</sup> Meeting, Pisa, 7-9 May 2025
- 2<sup>nd</sup> Satell. Meeting, Liverpool, 14 Nov. 2025
- 3<sup>rd</sup> Meeting, Torino, 3-5 June 2026



## The RadioMonteCarLow 2 Effort

<https://radiomontecarlow2.gitlab.io/>

- **main output of phase I**
  - a repository of benchmark results obtained through **tuned comparisons** for a set of standardized event selections and input parameters
  - a repository of the corresponding versions of the MC codes
  - in order to have a gauge of the impact of different physical effects/approximations, collect comparisons among the “best” predictions of the MC codes (**without any attempt to give estimates of theoretical uncertainties**)



## Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in $e^+e^-$ collisions

● Riccardo Aliberti<sup>1</sup>, ● Paolo Beltrame<sup>2</sup>, ● Ettore Budassi<sup>3,4</sup>,  
● Carlo M. Carloni Calame<sup>4</sup>, ● Gilberto Colangelo<sup>5</sup>, ● Lorenzo Cotrozzi<sup>2</sup>,  
● Achim Denig<sup>1</sup>, ● Anna Driutti<sup>6,7</sup>, ● Tim Engel<sup>8</sup>, ● Lois Flower<sup>2,9</sup>,  
● Andrea Gurgone<sup>3,6,7</sup>, ● Martin Hoferichter<sup>5</sup>, ● Fedor Ignatov<sup>2</sup>,  
● Sophie Kollatzsch<sup>10,11</sup>, ● Bastian Kubis<sup>12</sup>, ● Andrzej Kupść<sup>13,14\*</sup>,  
● Fabian Lange<sup>10,11</sup>, ● Alberto Lusiani<sup>7,15</sup>, ● Stefan E. Müller<sup>16</sup>, ● Jérémy Paltrinieri<sup>2</sup>,  
● Pau Petit Rosàs<sup>2</sup>, ● Fulvio Piccinini<sup>4</sup>, ● Alan Price<sup>17</sup>, ● Lorenzo Punzi<sup>7,15</sup>,  
● Marco Rocco<sup>10,18</sup>, ● Olga Shekhovtsova<sup>19,20</sup>, ● Andrzej Siódmok<sup>17</sup>,  
● Adrian Signer<sup>10,11\*</sup>, ● Giovanni Stagnitto<sup>21</sup>, ● Peter Stoffer<sup>10,11</sup>,  
● Thomas Teubner<sup>2</sup>, ● William J. Torres Bobadilla<sup>2</sup>,  
● Francesco P. Ucci<sup>3,4</sup>, ● Yannick Ulrich<sup>2,5\*</sup> and ● Graziano Venanzoni<sup>2,7\*</sup>  
(RadioMonteCarLow 2 working group)

# Processes, reference “simplified” event selections and MC tools

- Processes

- $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$
- $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$
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- Reference event selections

- energy scan
  - CMD-like
- radiative return
  - KLOE-LA (tagged)
  - KLOE-SA (untagged)
  - BES III
  - B factories

- Monte Carlo codes

- AFKQED
- BABAYAGA@NLO
- KKMC
- MCGPJ
- McMULE
- PHOKHARA
- SHERPA

Not discussed within RMCL2 up to now because it is not an issue at the 0.1% level  
by comparing NNLO (McMule) with NLOPS (BabaYaga@NLO) photonic corrections

$e^+e^- \rightarrow e^+e^-$

$\sqrt{s} = 1020 \text{ MeV}$

$20^\circ < \vartheta_\pm < 160^\circ$

$E_e > 408 \text{ MeV}$

$\zeta < 10^\circ$

|                | $\sigma/\mu\text{b}$ |          | $\delta K^{(i)}/\%$ |
|----------------|----------------------|----------|---------------------|
|                | McMule               | BABAYAGA | McMule              |
| $\sigma^{(0)}$ | 6.8557               | 6.8557   |                     |
| $\sigma^{(1)}$ | -0.7957              | -0.7957  | -11.606             |
| $\sigma^{(2)}$ | 0.0312               | 0.0267   | 0.515               |
| $\sigma_2$     | 6.0912               | 6.0868   |                     |

$e^+e^- \rightarrow \gamma\gamma$

$45^\circ < \vartheta_\gamma < 135^\circ$

$E_\gamma > 0.3\sqrt{s}$

$\zeta < 10^\circ$

| $\sqrt{s}$                       | 1 GeV       |           | 3 GeV       |           | 10 GeV      |           |
|----------------------------------|-------------|-----------|-------------|-----------|-------------|-----------|
|                                  | McMule [nb] | [12] [nb] | McMule [nb] | [12] [nb] | McMule [nb] | [12] [nb] |
| $\sigma_0$                       | 137.531     | 137.53    | 15.2812     | 15.281    | 1.37531     | 1.3753    |
| $\sigma_1$                       | 129.444     | 129.45    | 14.2099     | 14.211    | 1.26185     | 1.2620    |
| $\sigma_2$                       | 129.760     |           | 14.2570     |           | 1.26738     |           |
| $\sigma_{\text{exp}}$            |             | 129.77    |             | 14.263    |             | 1.2685    |
| $\sigma^{(2,\gamma)}$            | 0.383       |           | 0.0598      |           | 0.00738     |           |
| $\sigma_{\text{exp}} - \sigma_1$ |             | 0.32      |             | 0.053     |             | 0.0065    |
| $\sigma^{(2,\text{VPe})}$        | -0.069      |           | -0.0128     |           | -0.00186    |           |
| $\sigma^{(2,\text{LbLe})}$       | -0.0014     |           | -0.00016    |           | -0.000014   |           |
| $\sigma^{(2,\text{rLbLe})}$      | 0.0030      |           | 0.00033     |           | 0.000030    |           |
| $\sigma^{(2,\text{VP}\mu\tau)}$  | -0.000077   |           | -0.00018    |           | -0.000080   |           |
| $\sigma^{(2,\text{VPh})}$        | 0.000090    |           | -0.00010    |           | -0.000097   |           |

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E. Budassi et al., JHEP 05 (2026) 221

[see talk by M. Ghilardi](#)

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E. Budassi et al., JHEP 05 (2026) 221

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- towards NNLO accuracy in fixed order calculations

T. Dave, J. Paltrinieri, P.P.Rosàs, W.J. Torres Bobadilla, arXiv:2604.16251; M. Pozzoli and W.J. Torres Bobadilla, arXiv:2607.03343

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[see talks by S. Kollatzsch and W.J. Torre Bobabilla](#)

- **improvements on the simulation/theoretical description of the pion final states**

up to know the simulations for radiative processes with hadronic final states have been based on the NLO calculation of PHOKHARA

C.M. Carloni Calame et al., Phys. Lett. B879 (2026) 140666; arXiv:2607.24642; P. Petit Rosàs, O. Shekhovtsova and W.J. Torres Bobadilla, arXiv:2603.13171; Y. Fang et al.,

arXiv:2512.10709

[see talks by M. Ghilardi, S. Kollatzsch and W.J. Torre Bobabilla](#)

The typical model for calculating matrix elements  $e^+e^- \rightarrow \pi^+\pi^- + n\gamma$  ( $n = 0, 1, \dots$ ) is sQED, with pion form factors at the  $\gamma\pi^+\pi^-$  vertices, in various levels of approximation

the simplest approach:

- **F $\times$ sQED**: gauge-invariant subamplitudes are multiplied by  $F(q^2)$  where  $q^2 = s$  or  $q^2 = m_{\pi\pi}^2$



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- guaranteed IR cancellations between real and virtual corrections

# $\pi^+\pi^-$ final states in Monte Carlo simulations

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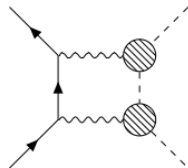
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cons

- ambiguities in virtual corrections involving pions in the loop diagrams



- **GVMD**: the form factor approximated by a sum of Breit-Wigner functions

$$F_{\pi}(q^2) = \sum_{v=0}^N c_v \frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$

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- the form factor does not respect the necessary requirements (the imaginary part below threshold different from zero)

# Inserting pion momenta in loop integrals

- **FsQED**: the form factor is represented dispersively:

$$\frac{F_\pi(q^2)}{q^2} = \frac{1}{q^2 - \lambda^2} - \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\text{Im}F(s')}{s'(q^2 - s')}$$

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cons

- the loop integration has to be organized partially numerically

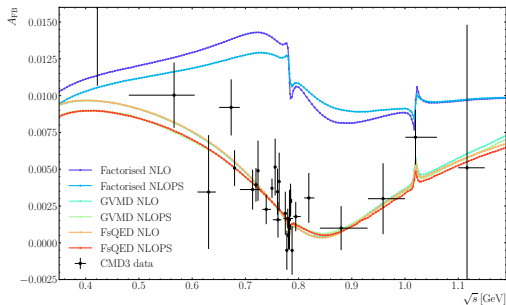
pros

- for the process  $e^+e^- \rightarrow \pi^+\pi^-$  it has been shown that FsQED is equivalent to the pion pole contribution to a model-independent dispersive

G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP 09 (2015) 074

## di-pion final state: charge asymmetry

$$A_{\text{FB}}(\sqrt{s}) = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

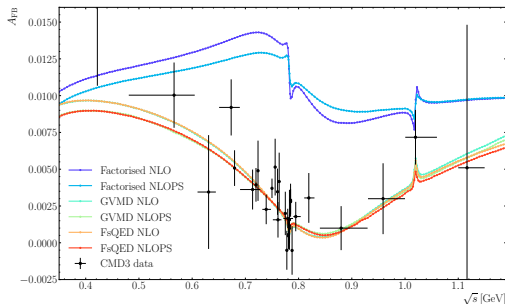


E. Budassi et al., JHEP05 (2025) 196 [arXiv:2409.03469]



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E. Budassi et al., JHEP05 (2025) 196 [arXiv:2409.03469]

- can we adopt the same treatment of the structure dependent corrections for the radiative process

$$e^+e^- \rightarrow \pi^+\pi^-\gamma?$$

C.M. Carloni Calame et al., arXiv:2607.24642; arXiv:2603.28621; P. Petit Rosàs O. Shekhovtsova, W.J. Torres Bobadilla, arXiv:2603.13171

see talks by M. Ghilardi and M. Hoferichner

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- possible future evolution within RMCL2
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  - More realistic event selections
  - additional background processes (e.g. additional  $e^+e^-$  pairs )