Data-driven HVP in atoms, a_μ (±) a_e

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Theory g—2 Initiative
9th Plenary Workshop
U Connecticut

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I. atoms

with

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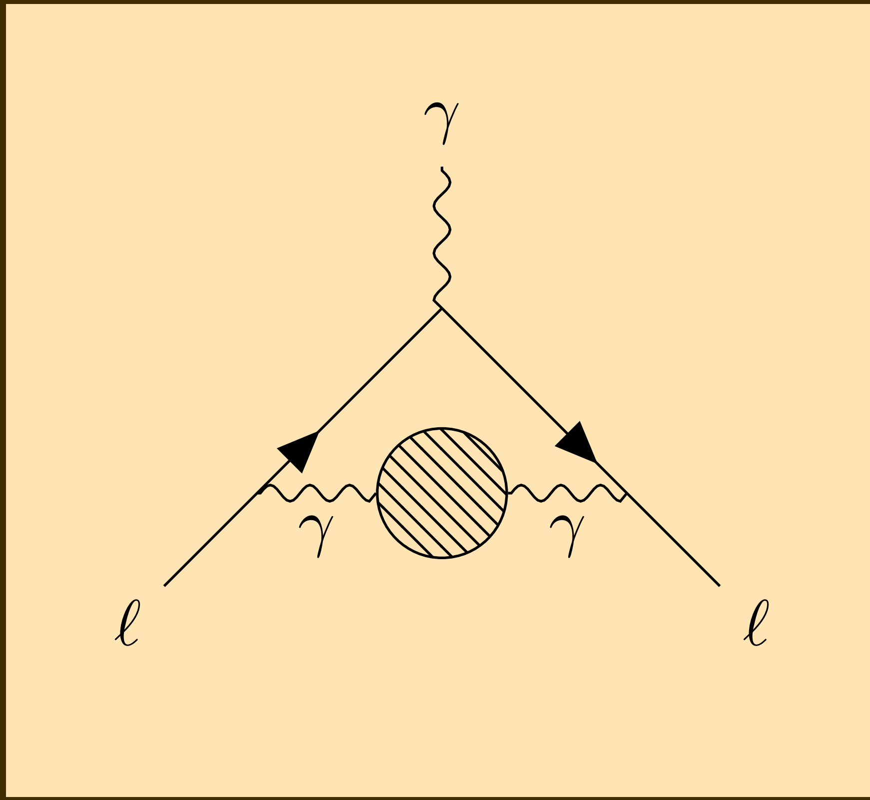
2607.07658 [physics.atom-ph]

II. lepton $g-2$



with
Siyuan Li
Maxim Pospelov

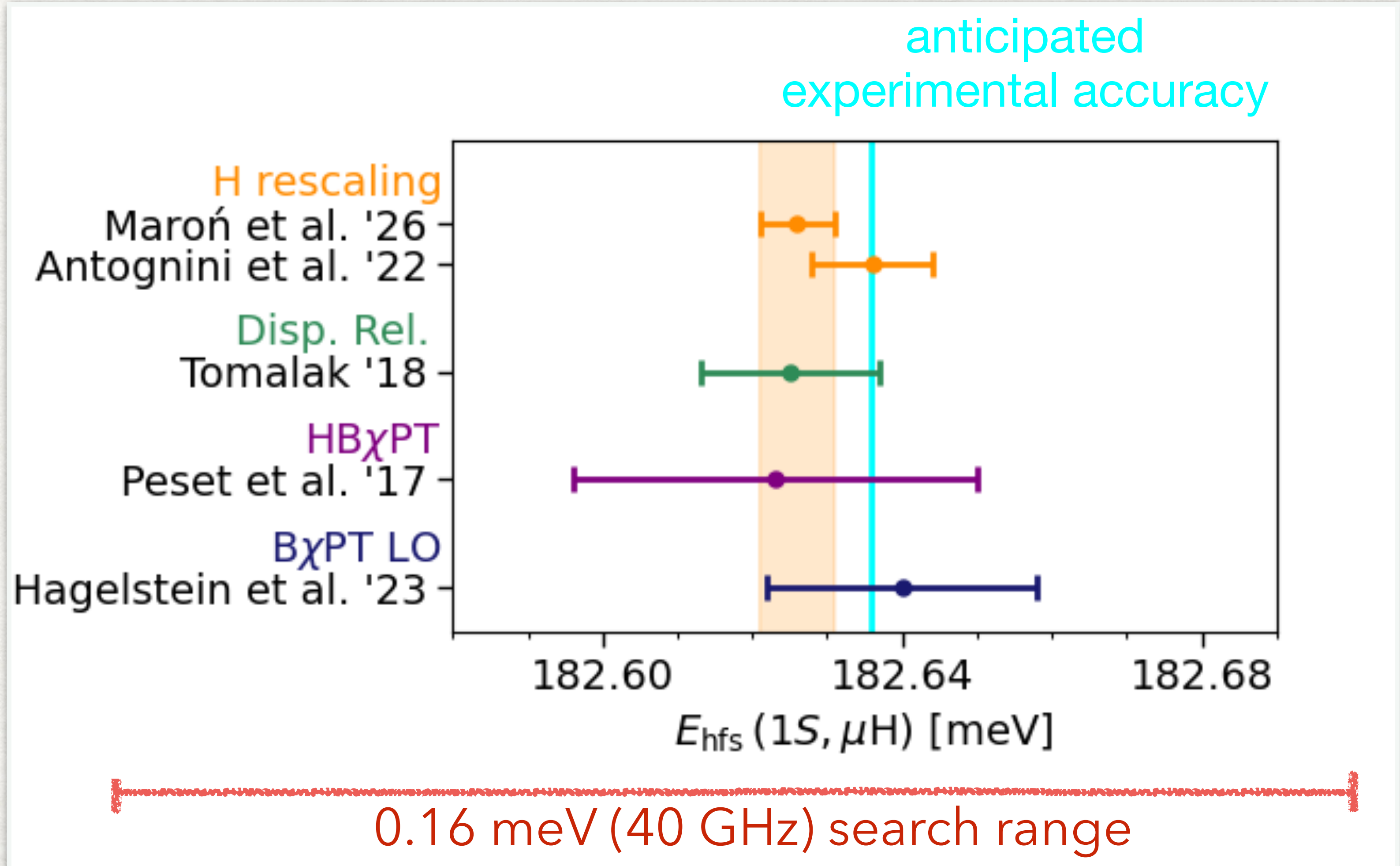
2606.08329 [hep-ph]

Bogdan Malaescu

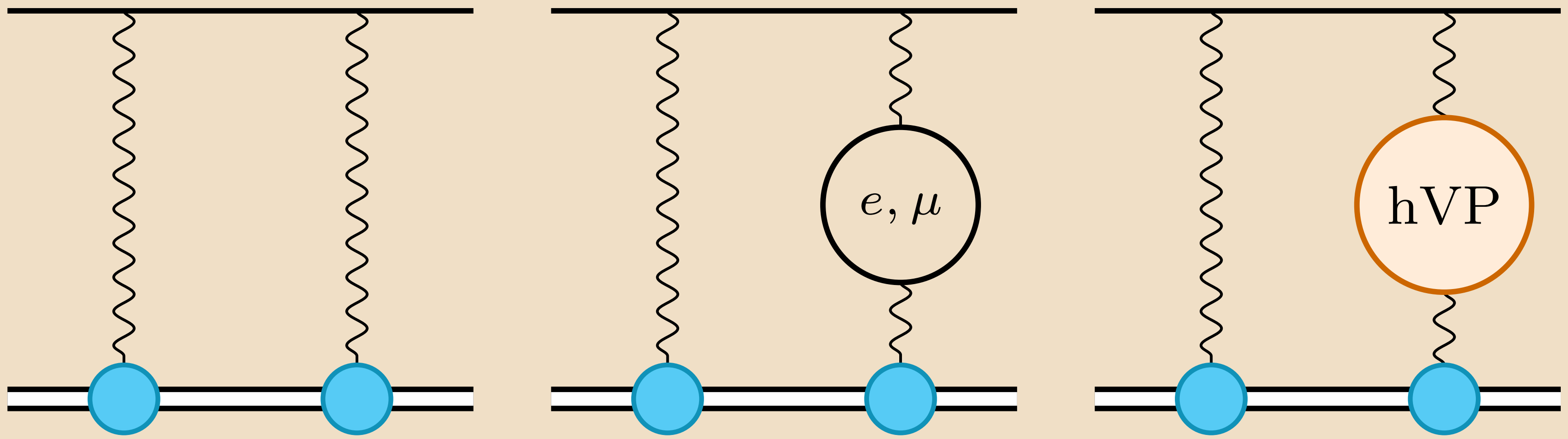
1S HFS SEARCH

by FAMU and CREMA Collaborations

- Ongoing search for ground-state hyperfine splitting in μH
- When found — ground breaking (1ppm exp. precision)
- Theory is reaching 30 ppm at best (with H rescaling)



TWO-PHOTON EXCHANGE



$$E^{\text{fs-VP}}_{nS\text{-HFS}} = -E_{\text{F}} \frac{2Z\alpha}{\pi^2 n^3} \frac{mM}{M^2 - m^2} \int\limits_{t_0}^{\infty} \frac{\mathrm{d}t}{t} W(t) \, \mathrm{Im}\,\Pi(t),$$

$$W(t) \sim \int_0^\infty \frac{dQ^2}{t + Q^2} \mathscr{W}\left[G_E(Q^2), G_M(Q^2), Q^2\right]$$

Muonium — nuclear structure-free ppb cross-check

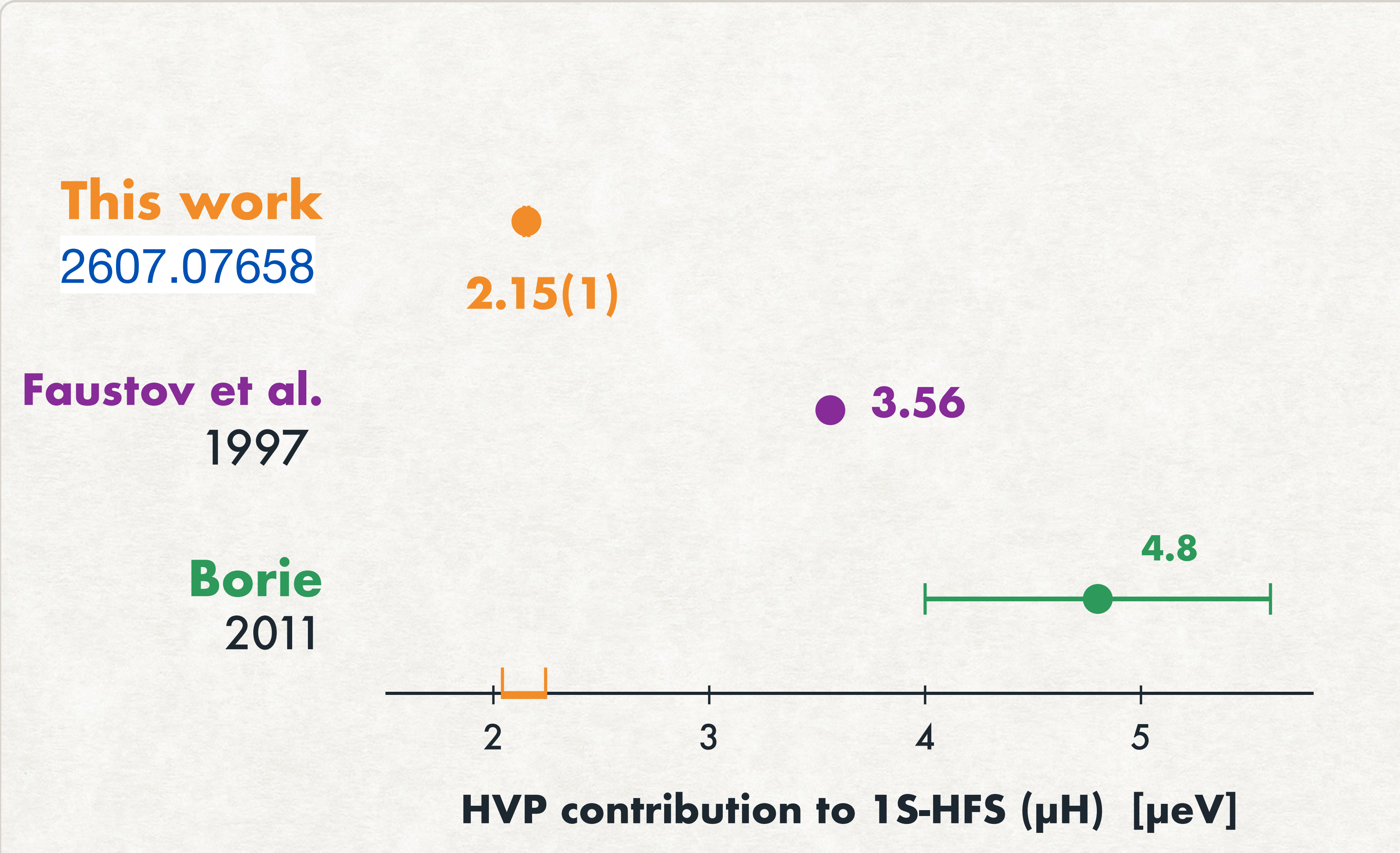
	Mu [kHz]
Previous calculations:	
Sapirstein *et al.* [25]	0.22(3)
Faustov *et al.* [53]	0.239 7(70)
Czarnecki *et al.* [54]	0.233(3)
Nomura and Teubner [55]	0.232 68(144)
Keshavarzi *et al.* [56]	0.232 04(82)
Karshenboim and Shelyuto [1]	0.236(5)
This work	0.233 3(11)

Current experimental precision: 0.051 kHz

H-like atoms

	H [kHz]	^{3}He$^+$ [kHz]	μH [µeV]	μ^3He$^+$ [µeV]
Previous calculations:				
Karshenboim [65]				
finite size	0.04(1)			
pointlike	0.19(8)			
Faustov and Martynenko [9]			3.561 0	
Borie [66]			4.8(8)	−72.8
This work:				
finite size (10)	0.086 0(4)	−0.476(17)	2.153(11)	−15.19(57)
pointlike (13)	0.273 6(12)	−1.860(10)	7.04(3)	−61.0(3)

MUONIC HYDROGEN μH HFS



CREMA target

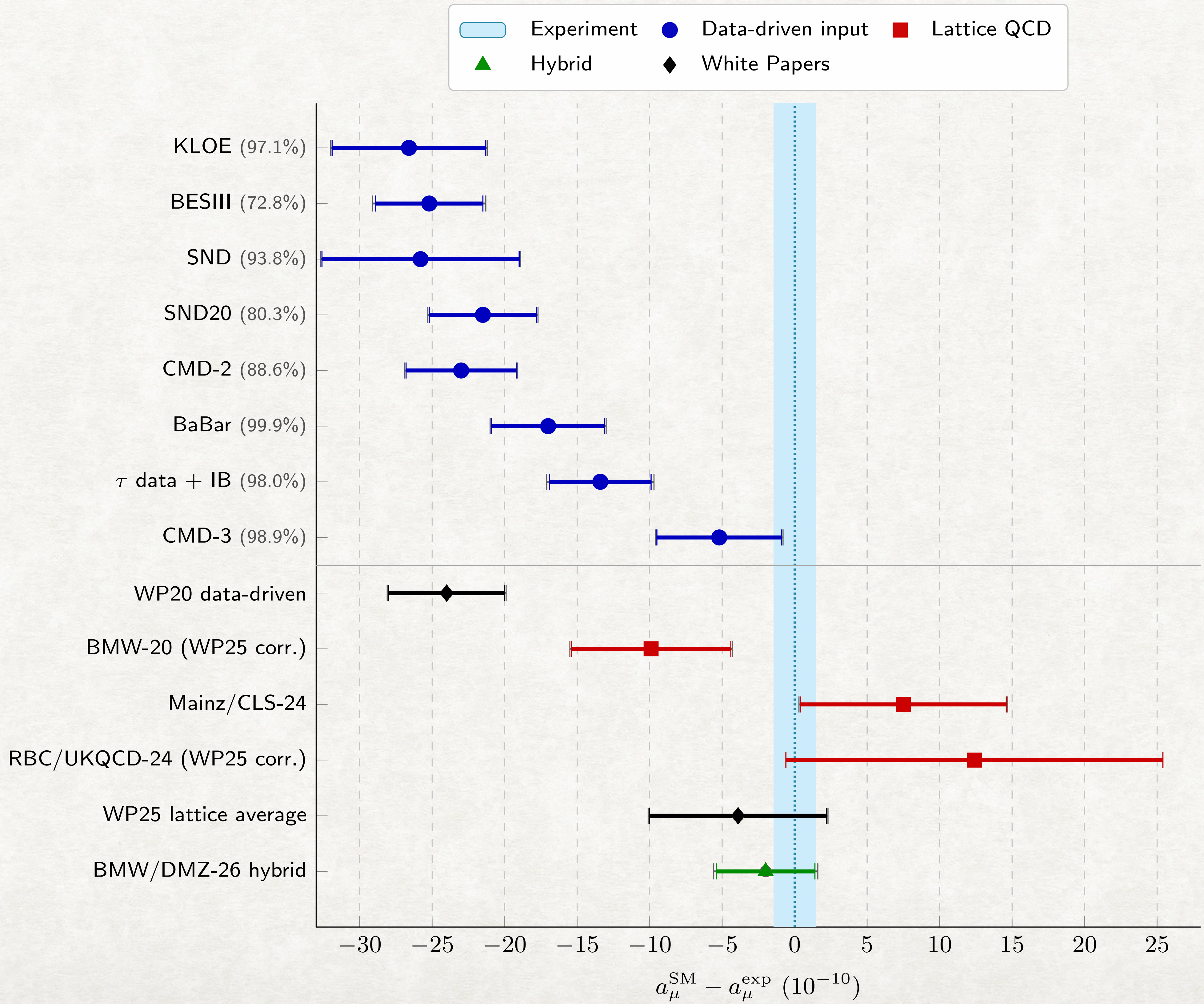
1 ppm
≈ 0.2 μeV

Earlier estimates differ by 1.4–2.6 μeV → O(10) × the anticipated experimental precision

PART II

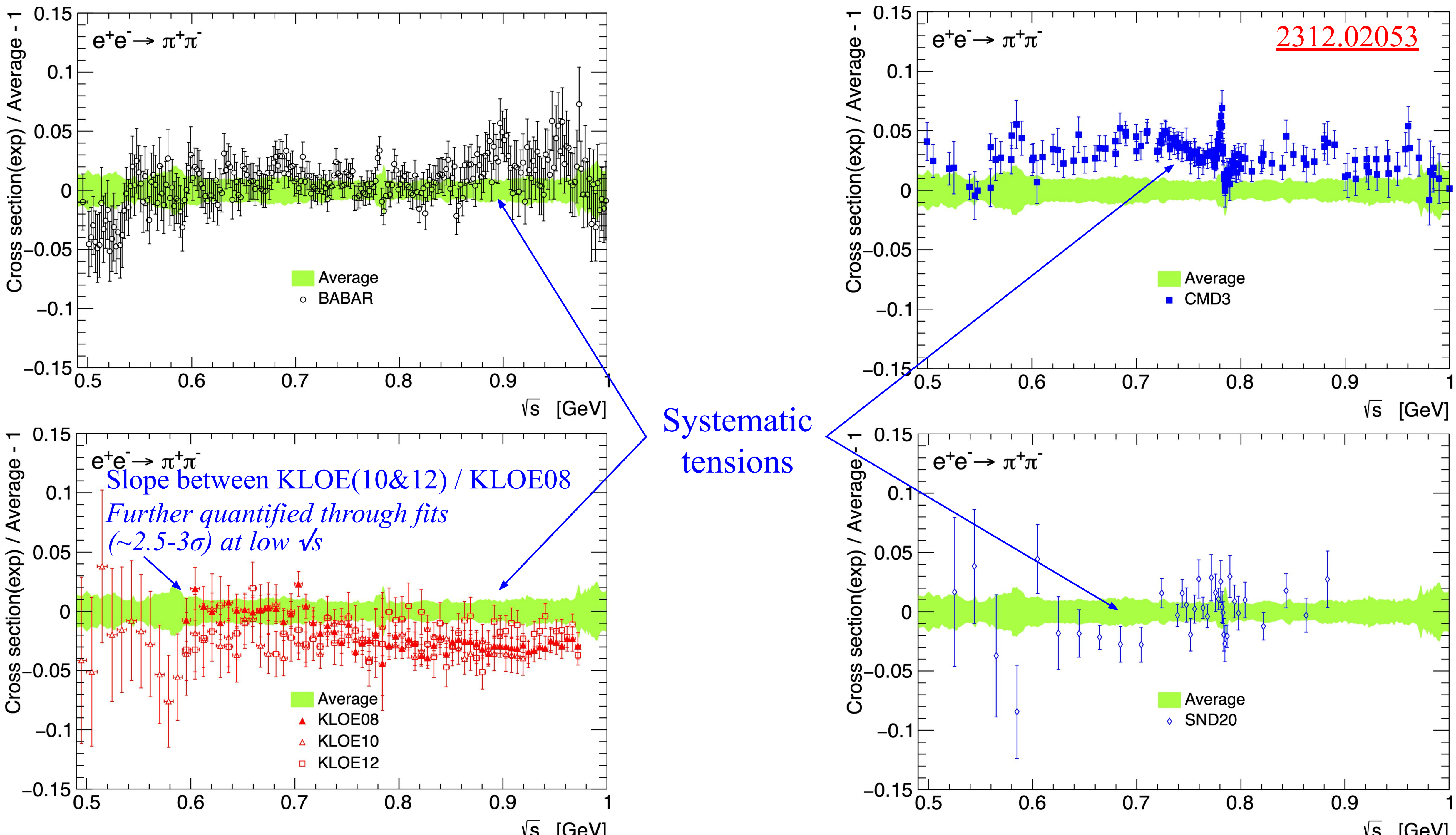
MASS SCALING IN LEPTON $g-2$

DATASET DISCREPANCIES: MUON

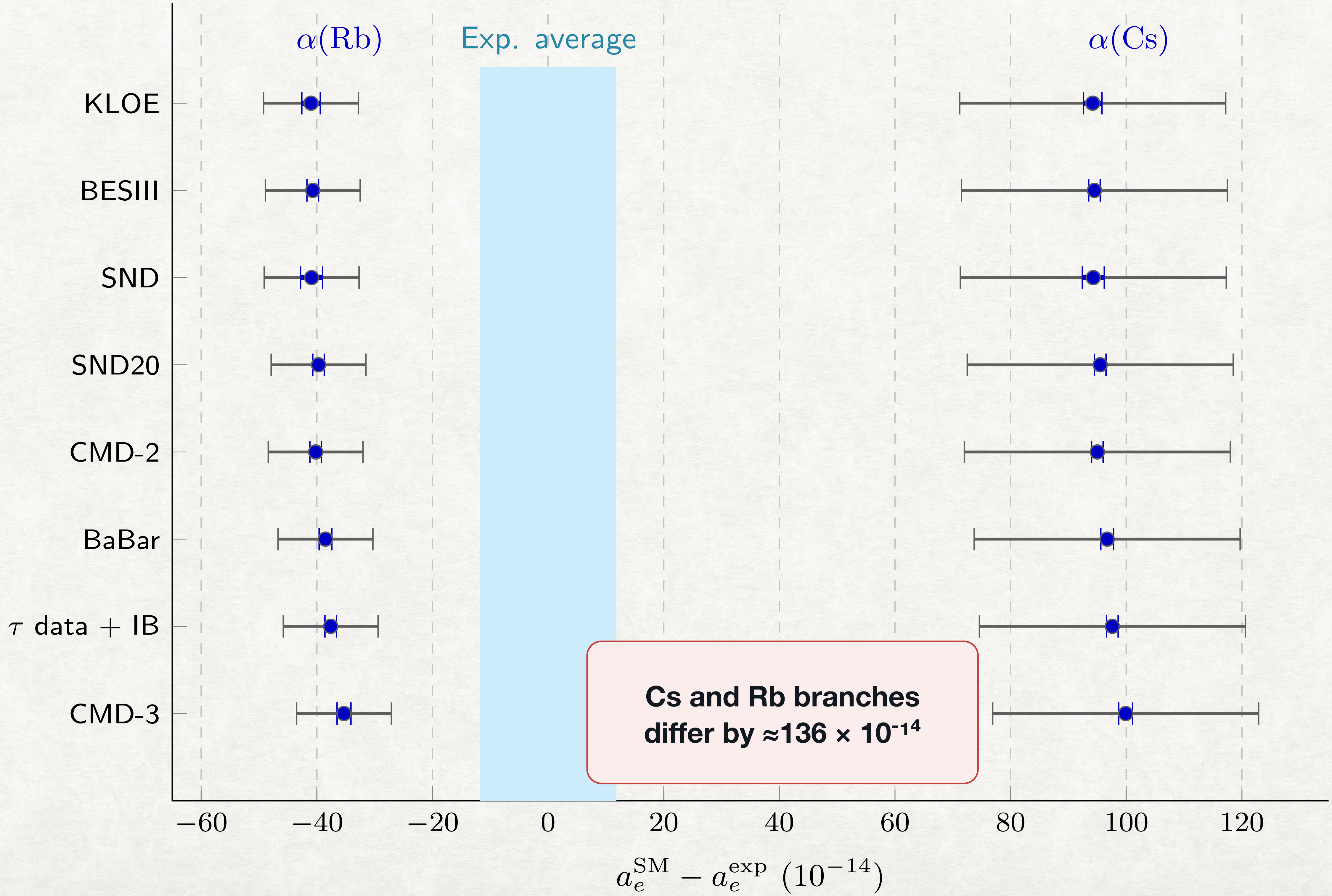


DATASET DISCREPANCIES

Combining the $e^+e^- \rightarrow \pi^+\pi^-$ data: relative differences



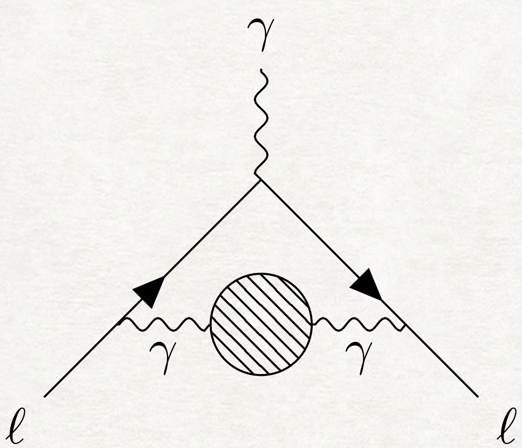
DATASET DISCREPANCIES: ELECTRON



Cs and Rb branches differ by ≈136 × 10^{-14}

Combined quantity (presently a diagnostic tool?):

$$a_{\mu-e} \equiv a_\mu - \frac{m_\mu^2}{m_e^2} a_e$$

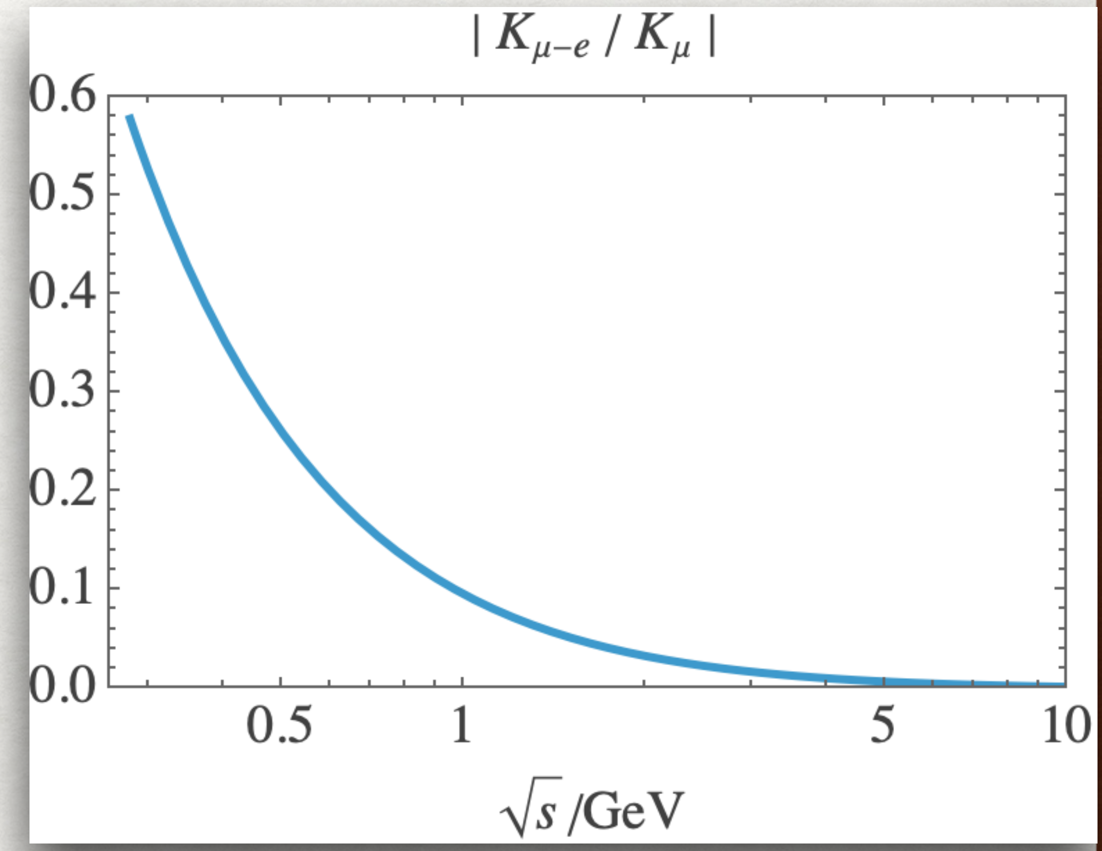


$$a_\ell^{\mathrm{HVP}} = \frac{\alpha}{\pi^2} \int_{s_0}^{\infty} \mathrm{d}s \, \frac{\mathrm{Im}\,\Pi(s)}{s} K_\ell(s)$$

$$a_e^{\mathrm{HVP}} \cong \frac{\alpha m_e^2}{3\pi} \Pi'(0) \equiv \frac{\alpha}{3\pi^2} m_e^2 \int_{s_0}^{\infty} \mathrm{d}s \, \frac{\mathrm{Im}\,\Pi(s)}{s^2}$$

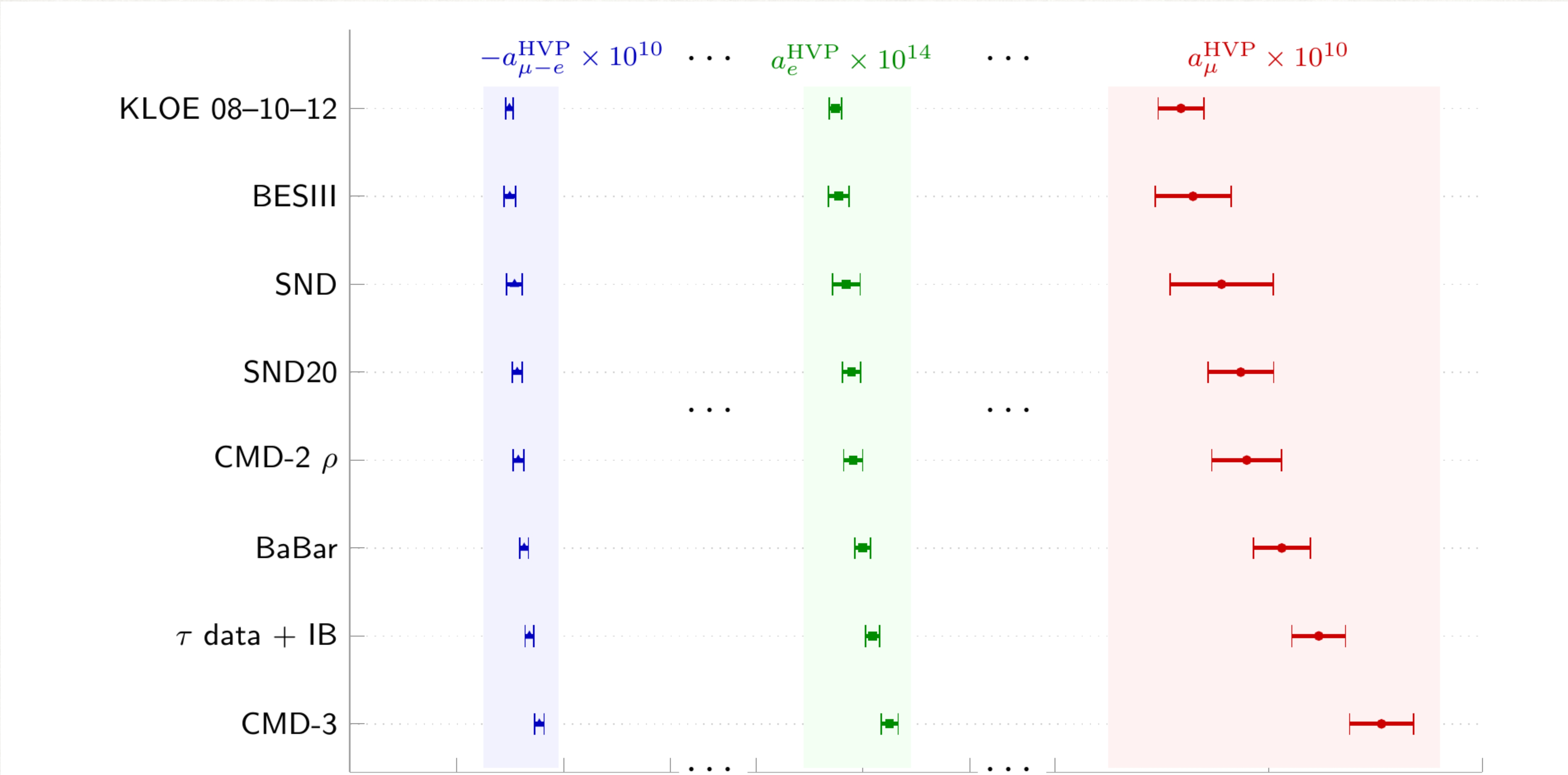
$$K_{\mu-e} \sim \frac{K_\ell}{s}$$

High-energy contributions are strongly suppressed



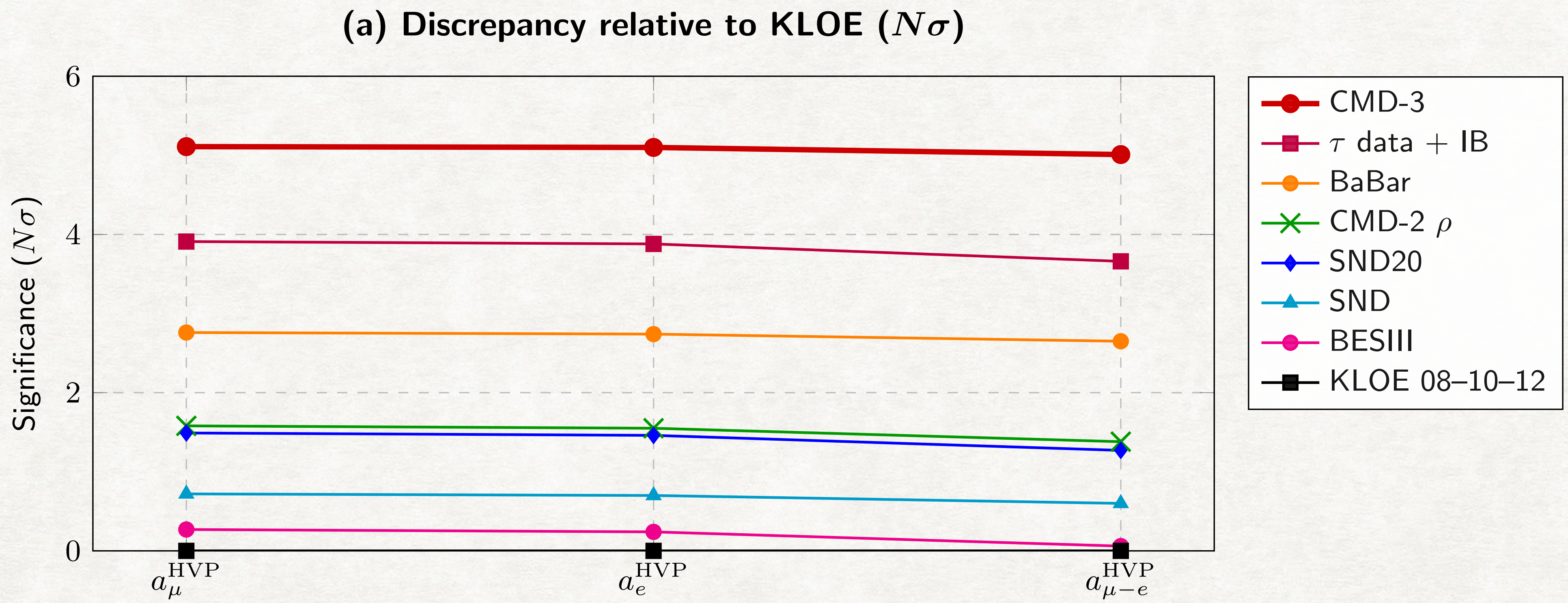
COMMON WINDOW: 0.600–0.8832 GeV

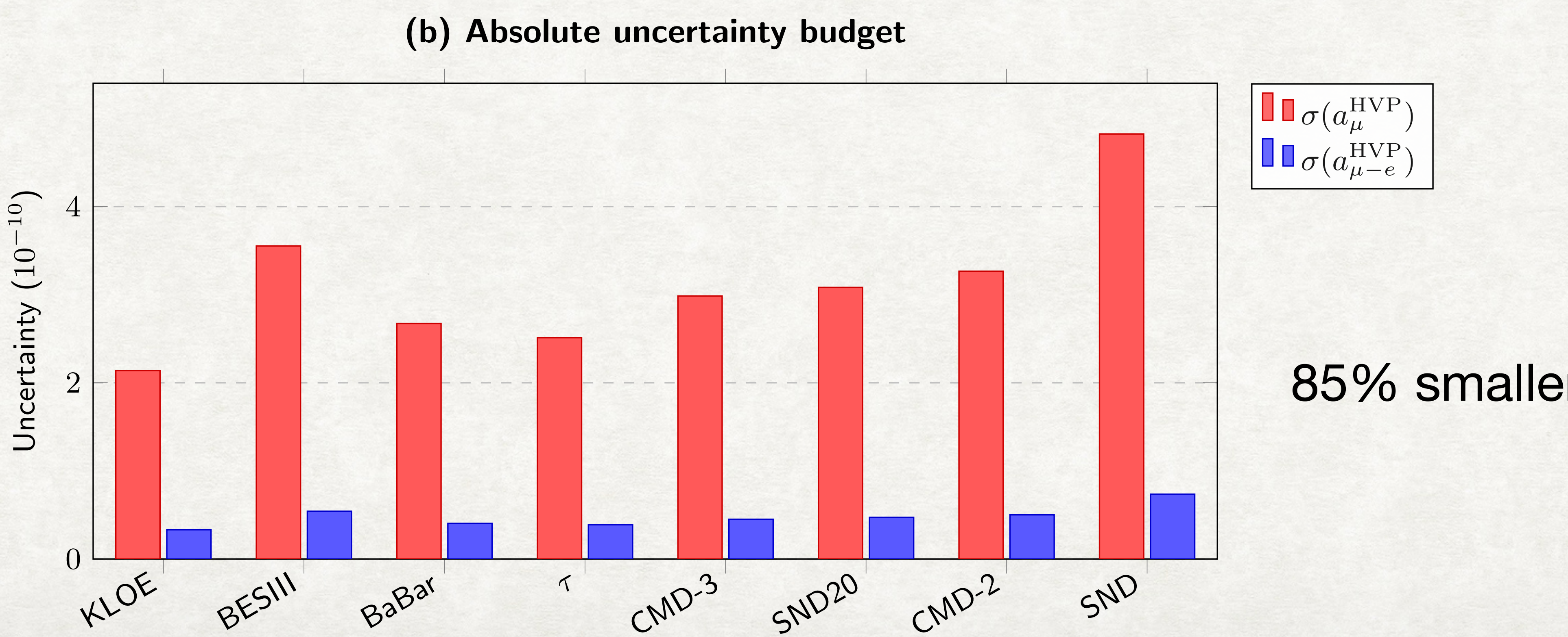
All three kernels preserve the experimental ordering



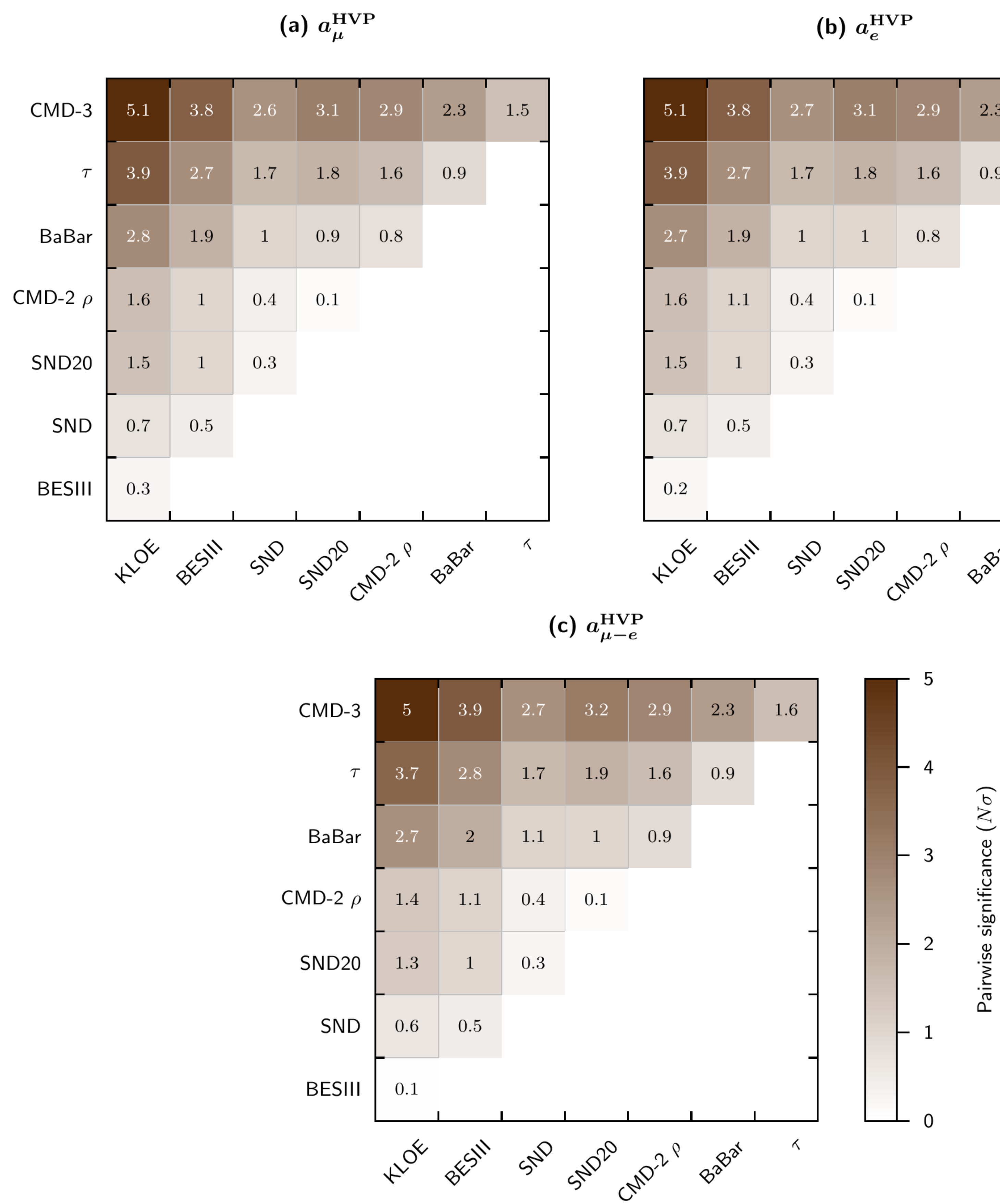
KLOE and BESIII are low; CMD-3 is high; BaBar and τ lie between

The tensions survive the change of kernel





85% smaller errors



CMD-3 – KLOE

5.1σ	**5.1σ**	**5.0σ**
aμHVP	aeHVP	aμ−eHVP

Dataset replacements against data-driven benchmarks

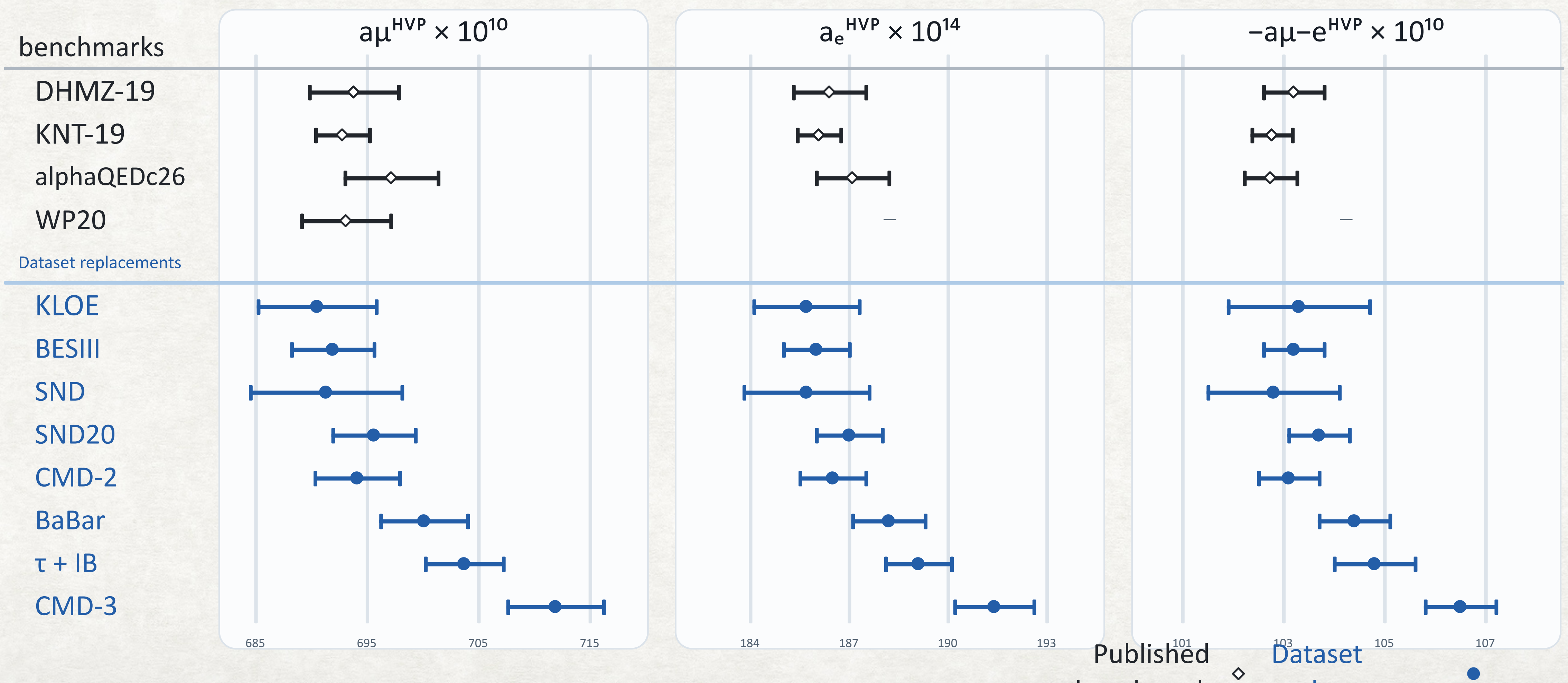
PRELIMINARY

Data input, references, and native interval [GeV]	$a_\mu^{\mathrm{HVP}} \times 10^{10}$	$a_e^{\mathrm{HVP}} \times 10^{14}$	$-a_{\mu-e}^{\mathrm{HVP}} \times 10^{10}$
	Dataset evaluations		
KLOE [7, 8, 11] $(0.316, 0.975)$	690.5 ± 5.3 (70.9%)	185.7 ± 1.6 (72.8%)	103.3 ± 1.4 (85.3%)
BESIII [9] $(0.600, 0.900)$	691.9 ± 3.7 (53.2%)	186.0 ± 1.0 (53.3%)	103.2 ± 0.6 (53.9%)
SND [12] $(0.390, 0.970)$	691.3 ± 6.8 (68.6%)	185.7 ± 1.9 (69.9%)	102.8 ± 1.3 (78.8%)
SND20 [13] $(0.525, 0.883)$	695.6 ± 3.7 (58.8%)	187.0 ± 1.0 (59.3%)	103.7 ± 0.6 (62.5%)
CMD-2 [14–16] $(0.370-0.520;\ 0.600-1.380)$	694.1 ± 3.8 (64.9%)	186.5 ± 1.0 (65.8%)	103.1 ± 0.6 (72.2%)
BaBar [6] $(0.300, 1.800)$	700.1 ± 3.9 (73.4%)	188.2 ± 1.1 (75.1%)	104.4 ± 0.7 (87.2%)
τ data + IB [4] $(0.360, 1.775)$	703.7 ± 3.5 (72.1%)	189.1 ± 1.0 (73.5%)	104.8 ± 0.8 (83.1%)
CMD-3 [5] $(0.327, 1.199)$	711.9 ± 4.3 (73.1%)	191.4 ± 1.2 (74.8%)	106.5 ± 0.7 (85.9%)
	Data-driven benchmarks		
DHMZ [17]	693.8 ± 4.0	186.4 ± 1.1	103.2 ± 0.6
KNT [18]	692.78 ± 2.42	186.08 ± 0.66	102.77 ± 0.40 [a]
alphaQEDc26 [19, 20]	697.17 ± 4.18	187.1 ± 1.1	102.74 ± 0.52 [a]
WP20 [2]	693.1 ± 4.0	—	—
	Lattice benchmarks		
BMW [21]	711.1 ± 18.9	189.3 ± 6.2	98.2 ± 7.4 [a]
ETMC [22]	692.1 ± 16.3	185.8 ± 4.2	102.3 ± 1.7 [a]
BMW [3, 23]	707.2 ± 5.5	—	—
Mainz/CLS [3, 24]	724.5 ± 7.1	—	—
RBC/UKQCD [3]	729.4 ± 13.0	—	—
WP25 [3]	713.2 ± 6.1	—	—
	Hybrid		
BMW/DMZ [25]	715.1 ± 3.4	—	—

[a] Derived from the previous two columns assuming full correlation.

COVERAGE-MATCHED

Dataset replacements against data-driven benchmarks



Each dataset replaces the common reference only over its native interval; outside it the reference is unchanged.

Data-driven and lattice HVP benchmarks

Data driven

Collaboration	a_μ $[10^{-10}]$	a_e $[10^{-14}]$	$-a_{\mu-e}$ $[10^{-10}]$
DHMZ-19	693.8 ± 4.0	186.4 ± 1.1	103.2 ± 0.6
KNT-19	692.78 ± 2.42	186.08 ± 0.66	102.77 ± 0.40*
alphaQEDc26	697.17 ± 4.18	187.1 ± 1.1	102.74 ± 0.52*
WP20	693.1 ± 4.0	–	–

Lattice QCD

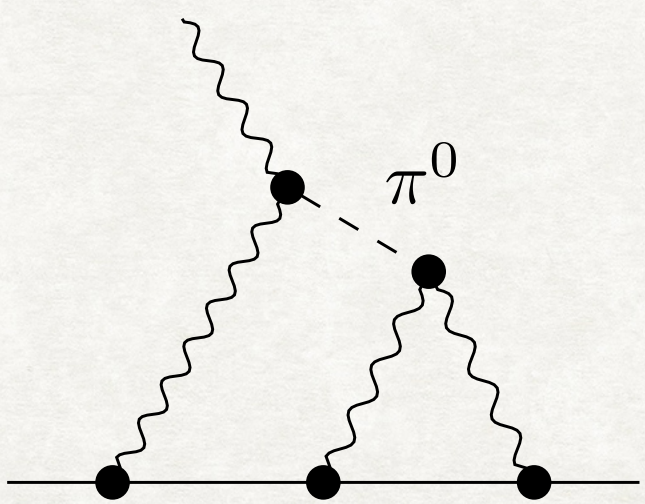
Collaboration	a_μ $[10^{-10}]$	a_e $[10^{-14}]$	$-a_{\mu-e}$ $[10^{-10}]$
BMW-17	711 ± 19	189 ± 6	98 ± 7*
ETMC-19	692 ± 16	186 ± 4	102 ± 2*
BMW-20	707.2 ± 5.5	–	–
Mainz/CLS	724.5 ± 7.1	–	–
RBC/UKQCD	729 ± 13	–	–
WP25	713.2 ± 6.1	–	–

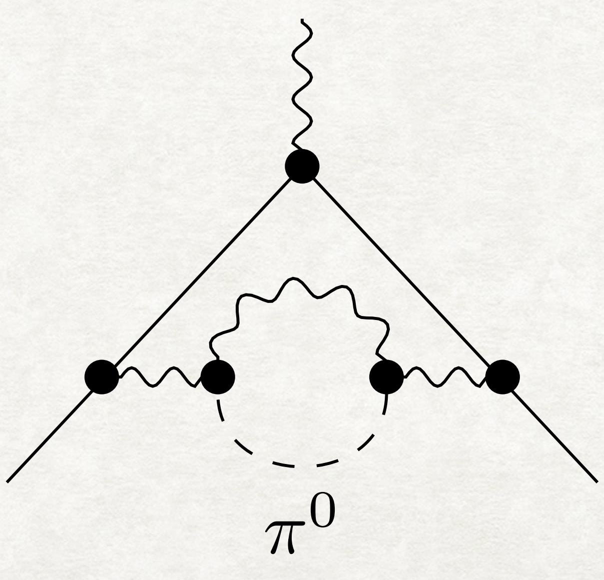
* Derived from the preceding two columns assuming full correlation.

A direct lattice calculation of the combined moment is missing

EFFECTIVE-FIELD-THEORY (EFT) PREDICTIONS FOR $a_{\mu-e}$

- π^0, η, η' contributions via light-by-light
- $\pi^0\gamma$ contr. (isospin breaking)





Compare the same infrared observable across methods

5σ tension survives

85% smaller errors

Dispersive data-driven

- Binned ππ spectra
- Full covariance matrices
- Nested and cumulative windows

Lattice QCD

- Direct a(μ–e)
- Long-distance tail
- Finite-volume and isospin tests

Low-energy EFT

- Threshold and ρ-region control
- Isospin-breaking corrections
- Energy-dependent shape tests