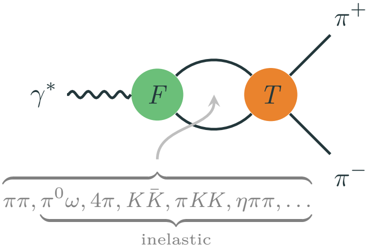
Improved parameterisations of pion form factors including inelastic effects

9th Plenary Workshop of the Muon $g-2$ Theory Initiative

Leon A. Heuser, G. Chanturia, C. Hanhart, M. Hoferichter, B. Kubis

Motivation: premise

- Tension in the 2π data
  $\implies$ no prescription for data-driven $a_\mu^{\mathrm{HVP,\,LO}}$ Aliberti et al., 2025
- Differences in datasets are almost exclusively contained in inelastic contributions Stoffer et al., 2023
- Inelastic effects are combined as a multiplicative factor in the current dispersive treatment Leplumey et al., 2025
- No unified fits available for multi-channel treatments N. N. Achasov et al., 2013



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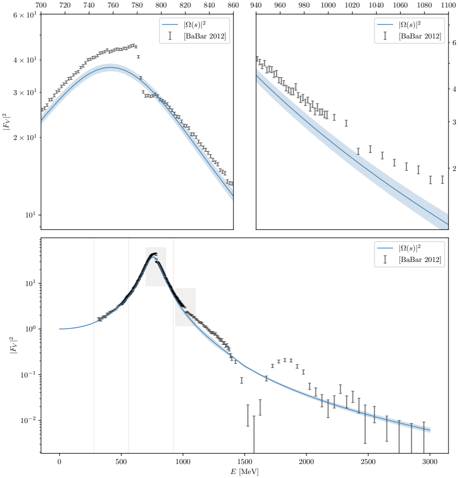
The goal:

- Model 2-pion production + inelastic contributions
- Fit to exclusive data from multiple channels simultaneously

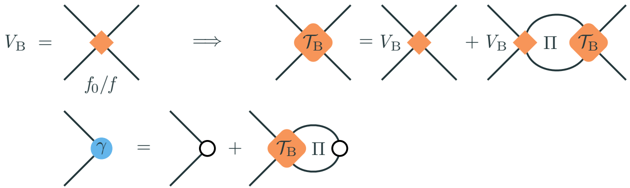
Framework

Two-potential model: why only resonances are not enough



Chanturia, 2022

- Pole with channel enough for main features
- Precision not high enough
- Non-resonant contributions improve description
- Unitarity and analyticity necessary



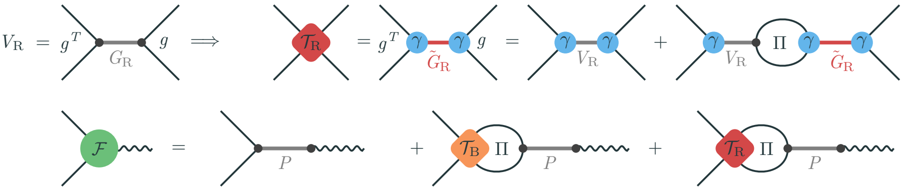
Background T-matrix

$$V_B(s)_{ab} = \left(\frac{f_0}{f(s)}\right)_{ab} \quad \Longrightarrow \quad \mathcal{T}_B = (1 - V_B \Pi)^{-1}\, V_B$$

$$\text{with} \quad \Pi(s) = \frac{s}{\pi}\int_{s_{\text{thr}}}^{\infty} \frac{\mathrm{d}s'}{s'} \frac{\rho(s')\xi(s')^2}{s' - s - i\epsilon} + \text{const.}$$

$$\gamma(s) = 1 + \mathcal{T}_B(s)\Pi(s) \mathrel{\widehat{=}} p(s)\exp\left(\frac{s}{\pi}\int_{s_{\text{thr}}}^{\infty} \frac{\mathrm{d}s'}{s'} \frac{\delta_B(s')}{s' - s - i\epsilon}\right)$$

Two-potential model: resonance potential



$$V_{\rm R}(s)_{ab} = \sum_{k=1}^{n_{\rm R}} \frac{g_a^k g_b^k}{m_k^2 - s} \quad \Longrightarrow \quad \mathcal{T}_{\rm R} = \gamma^\dagger \left(1 - V_{\rm R}\Sigma\right)^{-1} V_{\rm R}\gamma$$

Resonance T-matrix

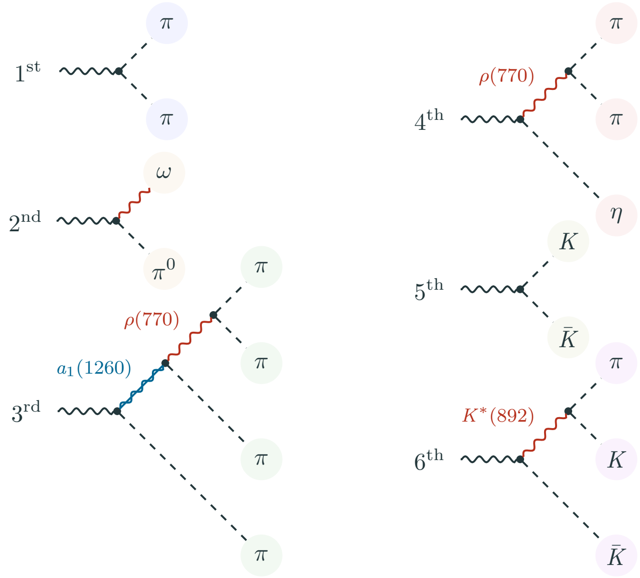
$$\text{with} \quad \Sigma(s) = \frac{s}{\pi}\int_{s_{\rm thr}}^{\infty} \frac{{\rm d}s'}{s'} \frac{\rho(s')\xi(s')^2|\gamma(s')|^2}{s' - s - i\epsilon} + \text{const.}$$

Form factor

$$\mathcal{F} = \gamma\left(1 - V_{\rm R}\Sigma\right)^{-1} P \quad \text{with} \quad P(s)_b = \sum_{k=1}^{n_{\rm R}} \frac{g_b^k a_k}{m_k^2 - s}$$

Application and results

Application: open channels ($J = 1,\ I = 1$)



Application: data

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$\pi\pi$ p-wave phase shift: Bern [Colangelo, 2004, Stoffer et al., 2023, ...]

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Application: challenges and strategies

- Large number of parameters

  $\Longrightarrow$ must be careful with local minima

  $\Longrightarrow$ both gradient-based and global minimization routines

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  $\implies$ lowest number of resonances (ρ, ρ', ρ'')

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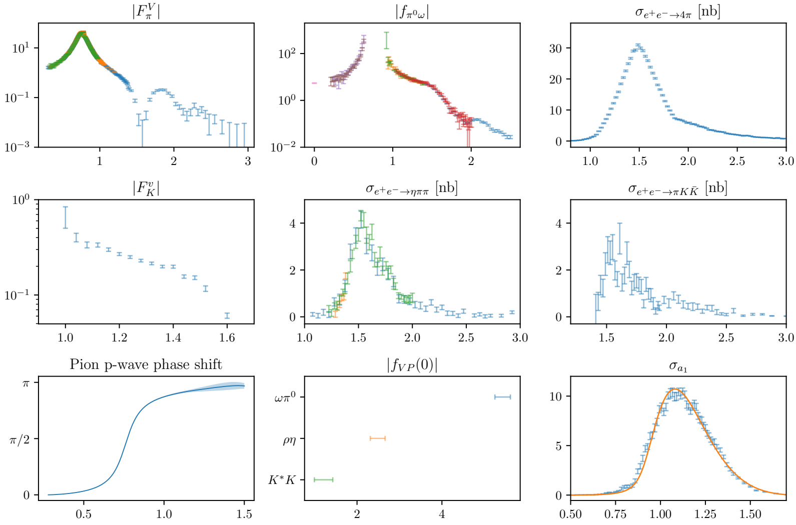
- Data quality not uniform
  $\Longrightarrow$ must be compensated for during fitting
- Pion VFF sensitive to phase shifts, ω parameters
  $\Longrightarrow$ individual input for different datasets
- Need to correct for bias
  $\Longrightarrow$ d'Agostini procedure (see e.g. Ball et al., 2010)

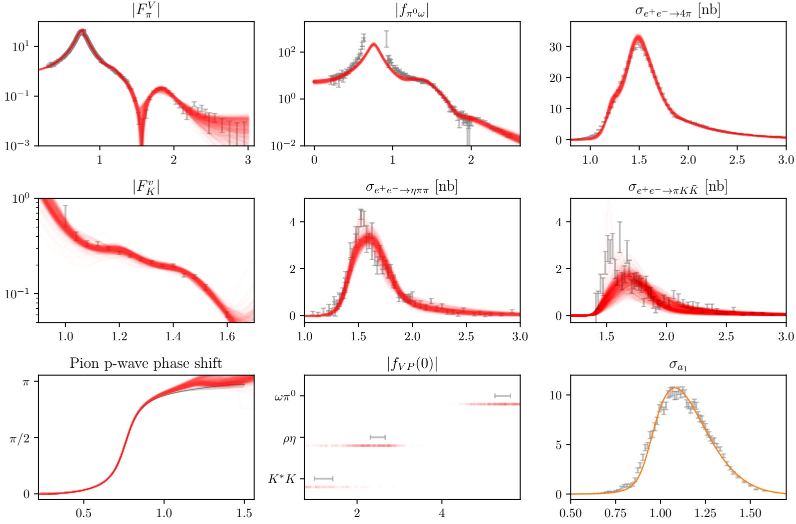
Application: different fit configurations

Fit	**Background**	**Weights**	**Cov. matrices**	
Fit #1	full	equal	decorrelated	next slides
Fit #2	diagonal	equal	decorrelated	backup
Fit #3	full	unequal	decorrelated	backup
Fit #4	full	equal	correlated	backup

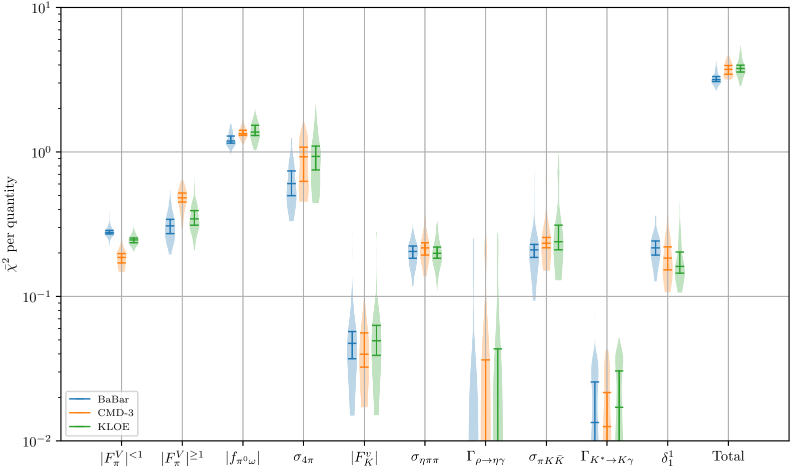
Application: data



Results: model performance



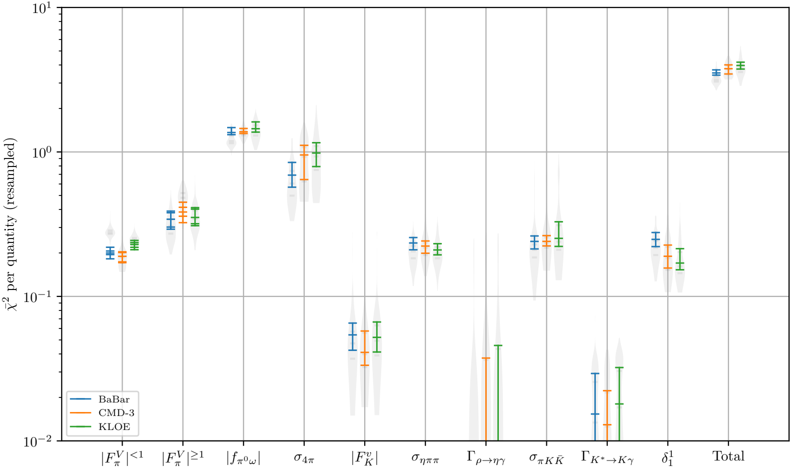
Results: contributions to the global $\bar{\chi}^2$ (fit #1)



Caution

The $\pi\pi$ data sets have different number of points

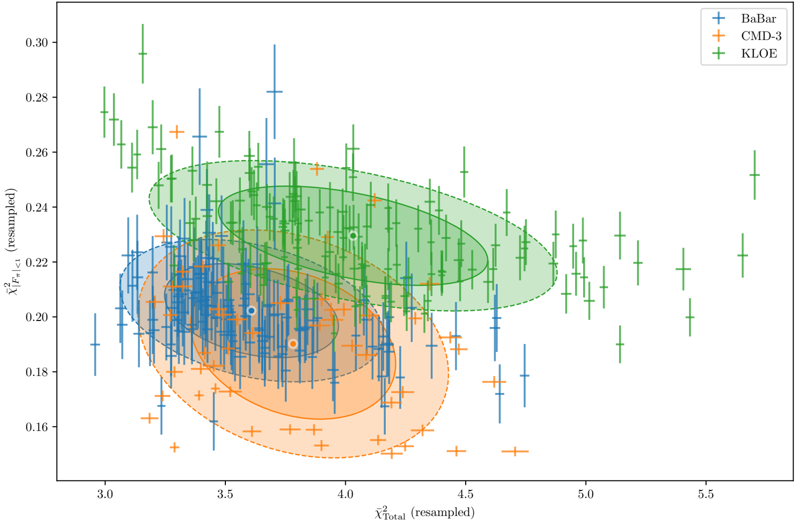
Results: resampled contributions to $\bar{\chi}^2$ (fit #1)



Solution

Sample same amount of points for each $\pi\pi$ data and calculate χ^2

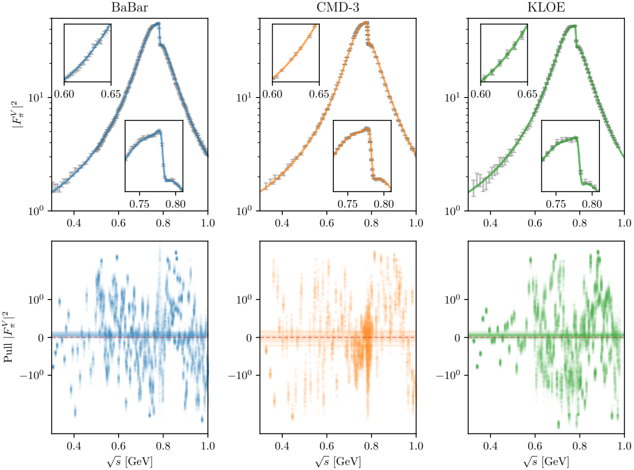
Results: resampled contributions to $\bar{\chi}^2$ (fit #1)



Caution

Not enough for proper quantification; actual d.o.f. unknown

Results: $\pi\pi$ pulls below 1 GeV (fit #1)



$$\mathrm{Pull}(F_i) = (F_{\mathrm{fit},i} - F_{\mathrm{data},i})\,/\Delta F_i$$

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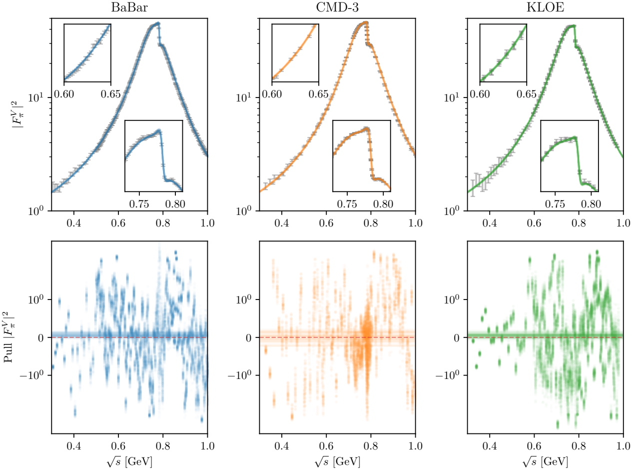
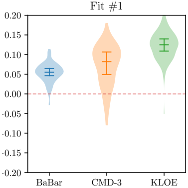




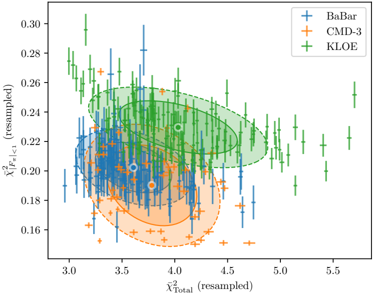
$$\frac{1}{N}\sum_{i=1}^{N}\mathrm{Pull}(F_i)$$

$$\mathrm{Pull}(F_i) = \left(F_{\mathrm{fit},i} - F_{\mathrm{data},i}\right)/\Delta F_i$$

Results: quantification of differences

- Need no. of effective degrees of freedom of each $|F_\pi^V|^{<1}$
  $\implies$ use bootstrapping procedure
  $\implies$ generate pseudo data around fit and refit
- Peak of χ^2 distribution is at N_{eff}
- Use fit from center of ellipses

$N_{\text{BaBar}} = 270 \quad N_{\text{CMD-3}} = 172 \quad N_{\text{KLOE}} = 193$



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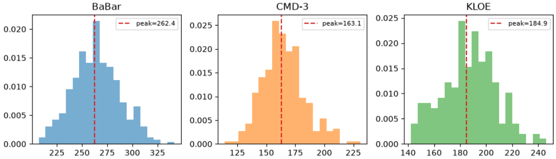
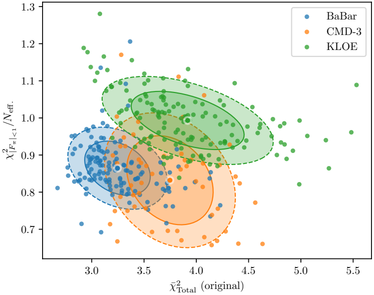




$\implies$ p-value/significance calculation to be done

Conclusions

Summary

- Global fits performed to data from multiple channels
- Dispersive constraints imposed through $\pi\pi$-specific phases
- Two-potential formalism ensures unitarity and analyticity
- To be submitted to arXiv soon!

Qualitative:

- Differences between BaBar and CMD-3 small, KLOE noticeably worse
- All fits have positive pulls

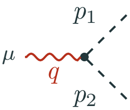
Quantitative:

- Proper significance calculation to be done

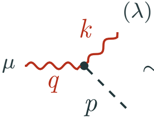
Thank you for your attention!

Backup slides

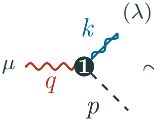
Two-potential model: vertex structures



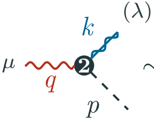
$\sim \xi^{\mu}_{VPP} = (p_1 - p_2)^{\mu}$ (e.g. $\gamma^* \to \pi\pi, K\bar{K}$)



$\sim \xi^{(\lambda)\mu}_{VVP} = \epsilon^{\mu\nu\alpha\beta} n^{(\lambda)}_{\nu} p_{\alpha} q_{\beta}$ (e.g. $\gamma^* \to \omega\pi^0, \rho\eta, K^*K$)



$\sim \xi^{(\lambda)\mu}_{VAP,1} = (q \cdot k)\, n^{(\lambda)\mu} - (n^{(\lambda)} \cdot q)\, k^{\mu}$ (e.g. $\gamma^* \to a_1\pi$)



$\sim \xi^{(\lambda)\mu}_{VAP,2} = q^2 k^2 n^{(\lambda)\mu} + (q \cdot k)(n^{(\lambda)} \cdot q) q^{\mu} - k^2 (n^{(\lambda)} \cdot q) q^{\mu}$

Application: semistable channels

Spectral function for an unstable state A via dressed propagator
$\to$ fix parameters g, M_A like in Heuser et al., 2024 via pole position

$$\sigma_A(q^2) = -\frac{1}{\pi}\mathrm{Im}\left(G_A(q^2)\right) , \quad \text{where} \quad G_A(q^2) = \frac{1}{q^2 - M_A^2 + g^2\Pi_i(q^2)}$$

Imaginary part of the self-energy Π_{AB} with A being a (broad) resonance

$$\mathrm{Im}\Pi_{AB} = \int_{s_{\mathrm{thr,decay}}}^{(\sqrt{s}-M_B)^2} \mathrm{d}q_A^2 \rho(s, q_A^2, M_B^2)\xi^2(s, q_A^2, M_B^2)\sigma_A(q_A^2)$$

$$\mathrm{Im}\Pi_{\pi\pi\eta}(s) = \int_{(2M_\pi)^2}^{(\sqrt{s}-M_\eta)^2} \mathrm{d}q_\rho^2\, \sigma_\rho(q_\rho^2)\rho(s, q_\rho^2, M_\eta^2)\, \xi_{VVP}^2(s, q_\rho^2, M_\eta^2)$$

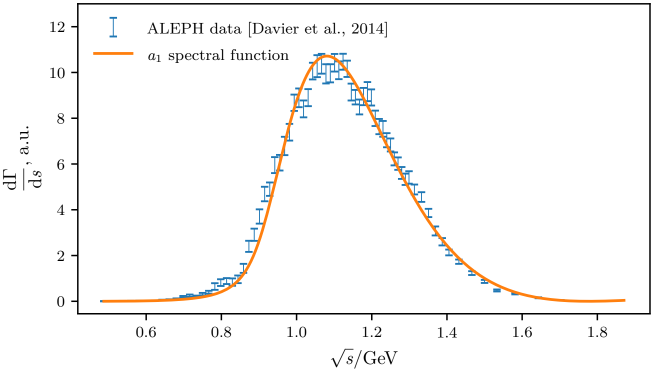
$$\mathrm{Im}\Pi_{\pi KK}(s) = \int_{(M_\pi+M_K)^2}^{(\sqrt{s}-M_K)^2} \mathrm{d}q_{K^*}^2\, \sigma_{K^*}(q_{K^*}^2)\rho(s, q_{K^*}^2, M_K^2)\xi_{VVP}^2(s, q_{K^*}^2, M_K^2)$$

Application: 4π and the a_1 spectral function

$a_1\pi$ problematic:

a_1 pole might be model dependent + three-body effects

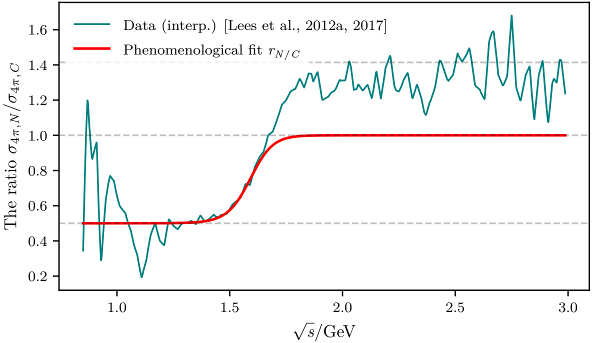
$\rightarrow$ use τ-decay data for a_1 and fit $g_{a_1\rho\pi}$, M_{a_1}



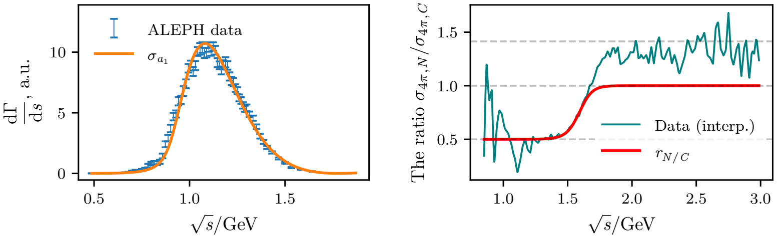
Application: 4π and the a_1 spectral function

$a_1\pi$ problematic:
difference between interference effects in $2\pi^0\pi^+\pi^-$ and $2(\pi^+\pi^-)$ not accounted for within the model
$\rightarrow$ fit multiplicative factor to ratio of corresponding cross sections

$$r_{N/C}(s) = \frac{1}{2}\left(1 + \exp^{-1}\left(\frac{a^2 - s}{b^2}\right)\right)$$



Application: 4π and the a_1 spectral function



$$\mathrm{Im}\Pi_{\rho\pi}(q_{a_1}^2) = \int_{(2M_\pi)^2}^{(\sqrt{q_{a_1}^2}-M_\pi)^2} \mathrm{d}q_\rho^2\, \sigma_\rho(q_\rho^2)\, \rho(q_{a_1}^2, q_\rho^2, M_\pi^2)\, \xi_{VVP}^2(q_{a_1}^2, q_\rho^2, M_\pi^2)$$

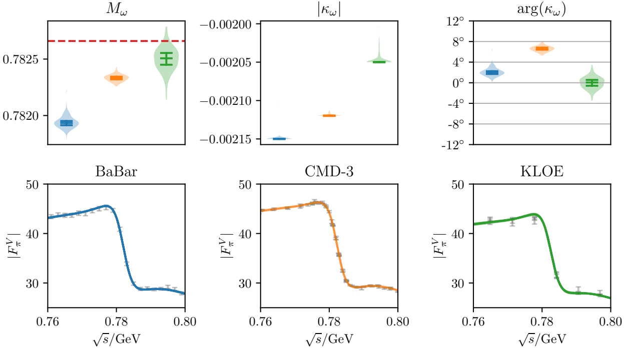
$$\sigma_{a_1}(q_{a_1}^2) = -\frac{1}{\pi}\mathrm{Im}\left(\frac{1}{q_{a_1}^2 - M_{a_1}^2 + g_{a_1\rho\pi}^2 \Pi_{\rho\pi}(q_{a_1}^2)}\right)$$

$$\mathrm{Im}\Pi_{i,4\pi}(s) = \int_{(3M_\pi)^2}^{(\sqrt{s}-M_\pi)^2} \mathrm{d}q_{a_1}^2\, \sigma_{a_1}(q_{a_1}^2)\, \rho(s, q_{a_1}^2, M_\pi^2)\, \xi_{i,VAP}^2(s, q_{a_1}^2, M_\pi^2)\, r_{N/C}(s)$$

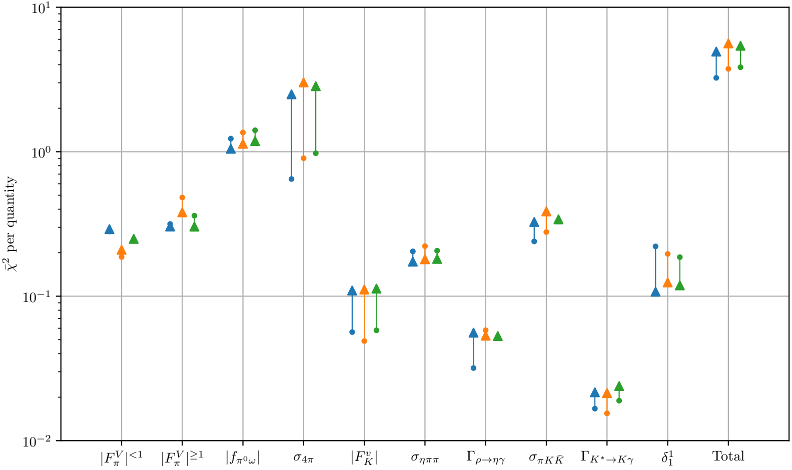
Results: ω parameters from 2-pion data

$$F_\pi^V(s) = \left(1 + \kappa_\omega \frac{s}{s - M_\omega^2 + iM_\omega\Gamma_\omega} + \dots\right) F_\pi^{V,I=1}(s)$$

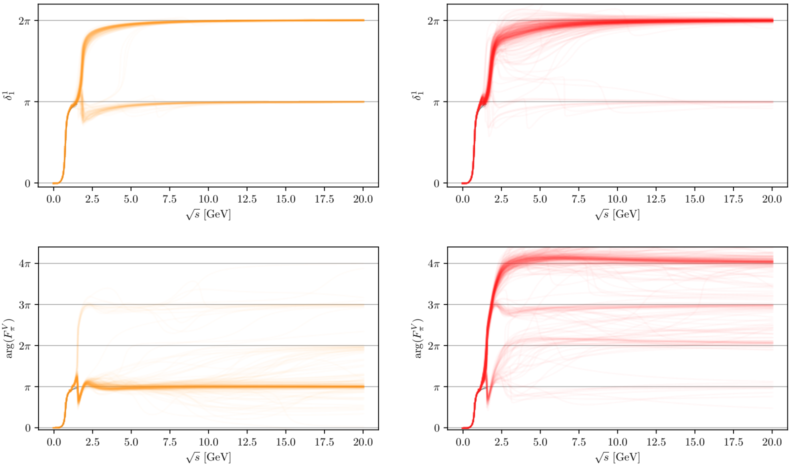
- Γ_ω fixed from 3π Hoferichter et al., 2023
- Bounds for M_ω and κ_ω were selected according to the $\pi\pi$ dataset [Stoffer et al., 2023]



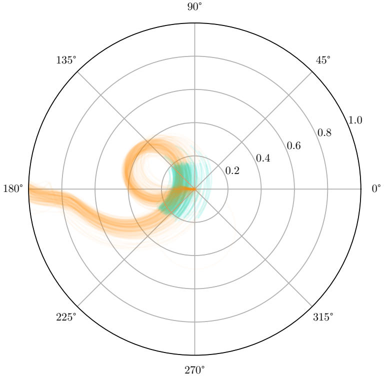
Results: fit #2 (diagonal background)

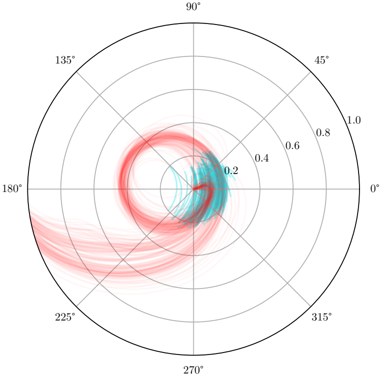


Results: fit #2 (diagonal background)

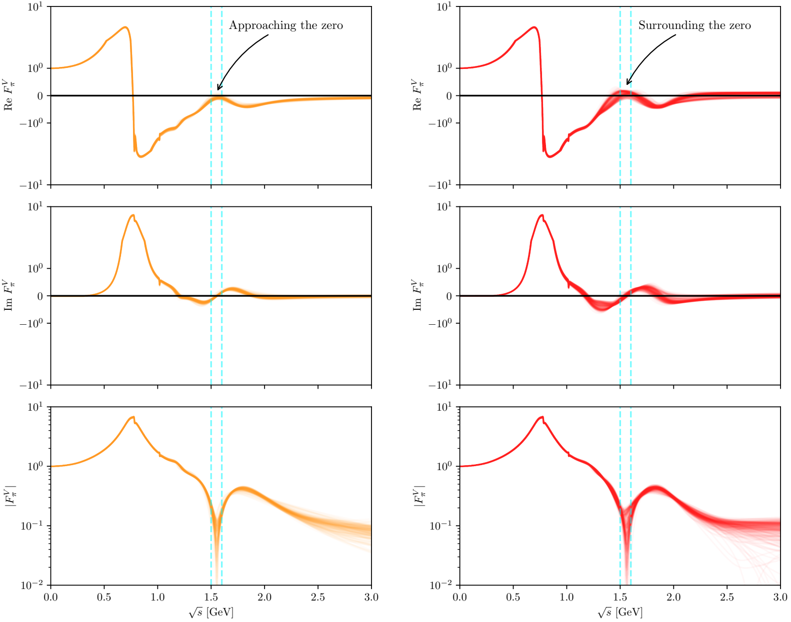


Results: fit #2 (diagonal background)

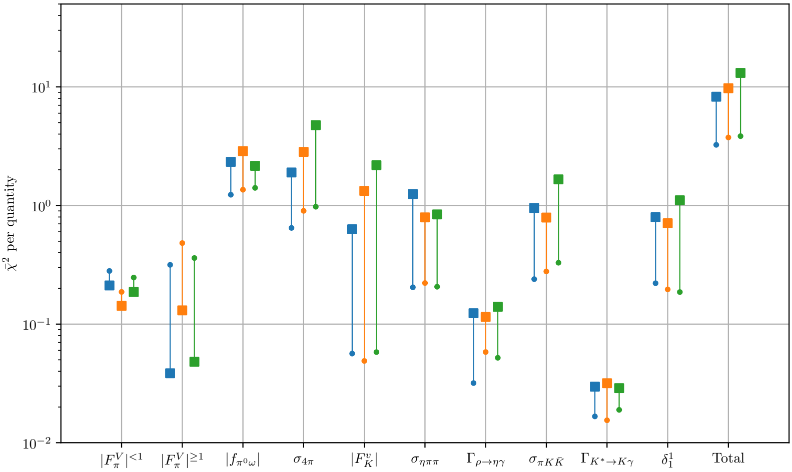




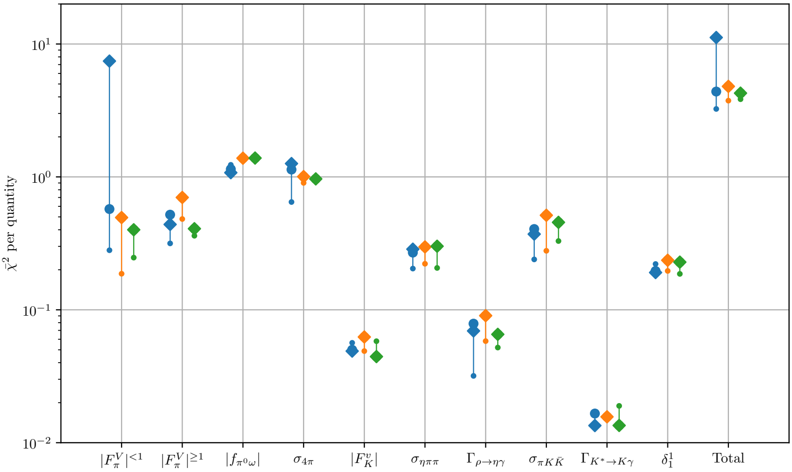
Results: fit #2 (diagonal background)



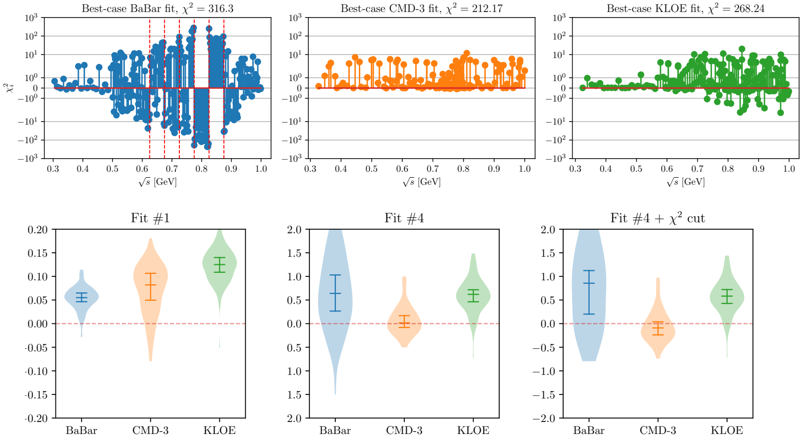
Results: fit #3 (exaggerated $\pi\pi$)



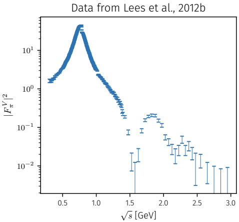
Results: fit #4 (correlation included)

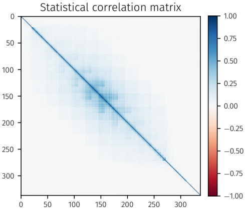


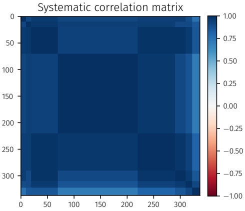
Results: fit #4 (correlation included)

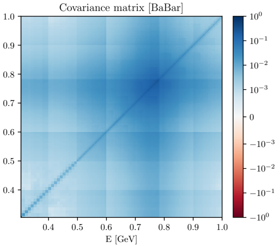


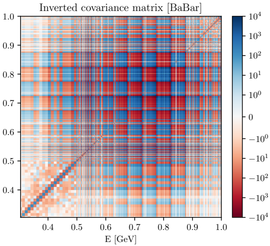
Covariance matrices: BaBar



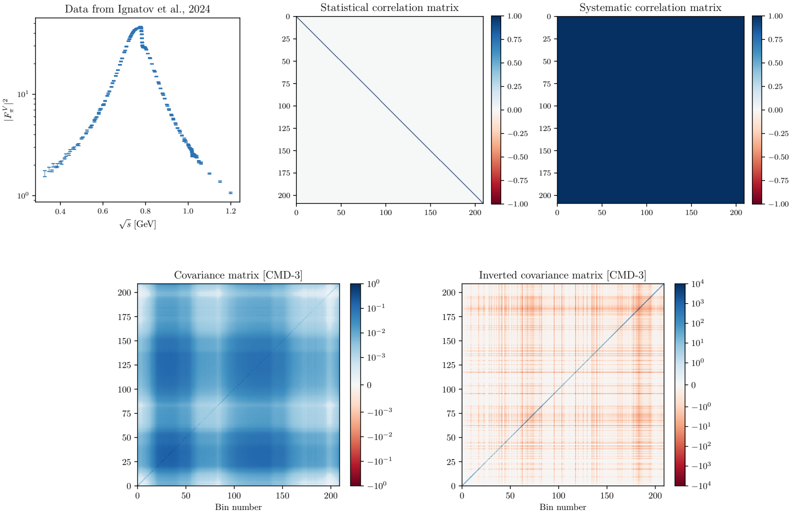




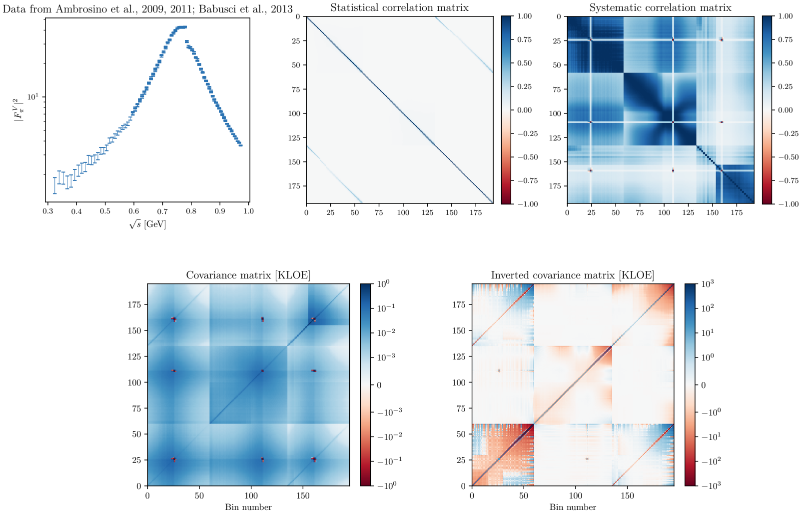




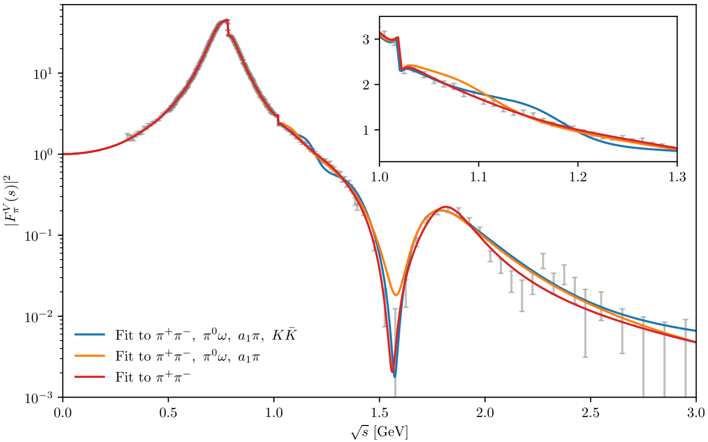
Covariance matrices: CMD-3



Covariance matrices: KLOE



Emergent structure after the ϕ



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