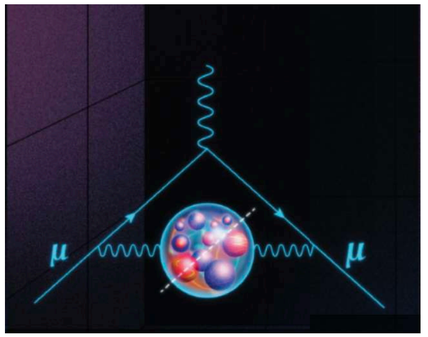
Cinvestav

9th Plenary Workshop on the muon g-2 Theory Initiative
University of Connecticut , 3-7 August 2026



Isospin breaking in pion form factors and consequences for $\Delta a_\mu^{\mathrm{HVP,LO}}[\tau, \pi\pi]$

G. López Castro,
Cinvestav, México

- **Flores-Baez, GLC, Toledo, PRD 113, 093002 (2026);**
- **GLC+K. Raya, in preparation**

Outline

Isospin Breaking (IB) corrections after WP2025: $S_{\rm EW}$, $\mathrm{G_{EM}}(s), \mathrm{FSR}(s), \Delta\Gamma_\rho[\pi\pi\gamma]$

- Summary of IB corrections using τ data (WP2025)
- Beyond sQED effects in $\Delta\Gamma_\rho[\pi\pi\gamma]$ (and FSR),
- Preliminary calculation of IB in $\rho\pi\pi$ couplings $\rightarrow \Delta\Gamma_\rho(g_{\rho\pi\pi})$
- Impact of IB corrections on $\Delta a_\mu^{\rm HVP,LO}[\tau, \pi\pi]$

HVP evaluation using $\tau \to \pi\pi\nu_\tau(\gamma)$ data

Exp < 0.2%

Theory ~ 5%

$$a_\mu^{\mathrm{HVP,LO}}[\pi\pi] = a_\mu^{\mathrm{HVP,LO}}[\tau,\pi\pi] + \Delta a_\mu^{\mathrm{HVP,LO}}[\tau,\pi\pi]$$

CVC, Alemany et al EPJC, 1998

$$\Delta a_\mu^{\mathrm{HVP,\ LO}}[\tau,\pi\pi] = \frac{\alpha^2 m_\tau^2}{6|V_{ud}|^2\pi^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\int_{4m_\pi^2}^{m_\tau^2} ds\,\frac{K(s)}{s}\,\frac{dN_{\pi\pi^0}}{N_{\pi\pi^0}ds}\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\left[\frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}}-1\right]$$

HVP evaluation using $\tau \to \pi\pi\nu_\tau(\gamma)$ data

Exp < 0.2%

Theory ~ 5%

$$a_\mu^{\mathrm{HVP,LO}}[\pi\pi] = a_\mu^{\mathrm{HVP,LO}}[\tau,\pi\pi] + \Delta a_\mu^{\mathrm{HVP,LO}}[\tau,\pi\pi]$$

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IB correction factors

Cirigliano et al, 2002

$$\frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}} = \frac{(\beta_0/\beta_+)^3}{S_{\mathrm{EW}}G_{\mathrm{EM}}(s)}\mathrm{FSR}(s)\left|\frac{F_\pi^V(s)}{f_+(s)}\right|^2$$

$$\frac{F_\pi^V(s)}{f_+(s)} = 1 + a\Delta m_\rho + b\Delta\Gamma_\rho + c\delta_{\rho\omega}$$

Status IB corrections in Δa_μ(in 10^{-10} units): White Paper 2025

(Phys. Rep. '25)

		Refs. [166,194]	Ref. [209]	Refs. [237,247]	Our estimate
Phase space		−7.88	−7.52	–	−7.7(2)
S_{EW}		−12.21(15)	−12.16(15)	–	−12.2(1.3)
G_{EM}		−1.92(90)	$(-1.67)^{+0.60}_{-1.39}$	–	−2.0(1.4)
FSR		4.67(47)	4.62(46)	4.42(4)	4.5(3)
ρ–ω mixing		4.0(4)	2.87(8)	3.79(19)	3.9(3)
$\frac{F_\pi^V}{f_+}$ (w/o ρ–ω)	ΔM_ρ	$0.20(^{+27}_{-19})(9)$	$1.95^{+1.56}_{-1.55}$	–	
	$\Delta\Gamma_\rho(\Delta M_\pi)$	4.09(0)(7)	3.37	–	
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	−5.91(59)(48)	−6.66(73)	–	
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	–	
	Total	−1.62(65)(63)	$(-1.34)^{+1.72}_{-1.71}$	–	−1.5(4.7)
Sum		−14.9(1.9)	$(-15.20)^{+2.26}_{-2.63}$	–	−15.0(5.1)

Exp: $\pm 1.45 \times 10^{-10}$

Uncertainties

- Matching and scheme-dependence in $S_{\rm EW}$, $G_{\rm EM}(s)$,
- Missing Str-Dep corrections in $G_{\rm EM}(s)$, FSR(s) & $\Delta\Gamma_\rho[\pi\pi\gamma]$
- Largest uncertainty due to IB in $\rho\pi\pi$, $\left| g_{\rho^+\pi^+\pi^0}/g_{\rho^0\pi^+\pi^-} - 1 \right| \approx 2\alpha/\pi$

[166] Davier et al, EPJC, '24
[194] Davier et al, EPJC, '10
[209] Castro et al, PRD, '25
[237] Colangelo et al, JHEP, '22
[247] Hoferichter et al, PRL, '23

Some recent (after WP25) developments in IB corrections

- $G_{\mathrm{EM}}(s)$: Structure-dependent corrections, Colangelo et al, PRL 136 (2026); JHEP 02, (2026)
- $S_{\mathrm{EW}} \cdot G_{\mathrm{EM}}(s)$: Scheme-independence, Cirigliano et al, PRL 137 (2026)
- $\mathrm{FSR}(s)$: Structure-dependent corrections, Flores-Baez et al, PRD 113 (2026)
- $F_V(s)/F_+(s)$: IB corrections in $\Delta\Gamma_\rho$, Flores-Baez et al 2026; GLC-KR ; Davier et al, 2504.13789

$$\frac{F_V(s)}{f_+(s)} = 1 + a\Delta m_\rho + b\Delta\Gamma_\rho$$

This talk

$$\Delta m_\rho = m_{\rho^+} - m_{\rho^0}, \qquad \Delta\Gamma_\rho = \Gamma_{\rho^0} - \Gamma_{\rho^+}$$

$$\frac{[\beta_0(s)/\beta_+(s)]^3}{S_{\rm EW}G_{\rm EM}(s)}$$

		Refs. [2, 19]	Ref. [33]	Refs. [64, 73]	Our estimate	
Phase space		−7.88	−7.52	–	−7.7(2)	
$S_{\rm EW}$	$-22.01(91)$	−12.21(15)	−12.16(15)	$-21.4(^{+0.6}_{-1.4})$	−12.2(1.3)	$-21.9(1.8)$
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After WP2025….see previous talks

$$(\mathrm{PS}, S_{\rm EW}, G_{\rm EM}) \to -24.9(5)\times 10^{-10}$$

Colangelo et al, PRL 136 (2026); JHEP(2026)

Cirigliano et al, PRL 137 (2026)

The width differente of rho mesons: $\Delta\Gamma_\rho = \Gamma_{\rho^0} - \Gamma_{\rho^+}$

- Data-based determination not precise enough, error $\sim 1\%$, needed $< 0.5\%$ **PDG, Davier et al 2504.1378**
- Theory-based $\Delta\Gamma_\rho = \Delta\Gamma_\rho[\pi\pi(\gamma)] + \Delta\Gamma_\rho(\text{rest})$ **Flores-Baez et al, PRD 76, 096010 (2007)**
- $\Delta\Gamma_\rho(\text{rest}) = +\,0.085(17)$ MeV, from channels other than $\pi\pi(\gamma)$ **PDG, 2024**
- IB dominated by $\rho^i \to \pi\pi(\gamma)$ channels, $(i = 0,\ +)$

$$\Gamma[\rho^i \to \pi\pi(\gamma)](s) = \frac{g_i^2\sqrt{s}}{48\pi}\beta_i^3(s)\,(1+\delta_i)\,, \qquad g_0 \equiv g_{\rho^0\pi^+\pi^-} \;\;,\;\; g_+ \equiv g_{\rho^+\pi^+\pi^0}$$

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Radcorr **IB in $\rho\pi\pi$** $m_{\pi^+} - m_{\pi^0}$

At leading order in IB

$$\Delta\Gamma_\rho[\pi\pi(\gamma)] = \Gamma(\rho^0 \to \pi^+\pi^-)\left[\delta_0 - \delta_+ - 2\left(\frac{g_+}{g_0} - 1\right) - \frac{3}{2}\left(\frac{1-\beta_0^2}{\beta_0^2}\right)\frac{\delta m_\pi}{m_\pi}\right]$$

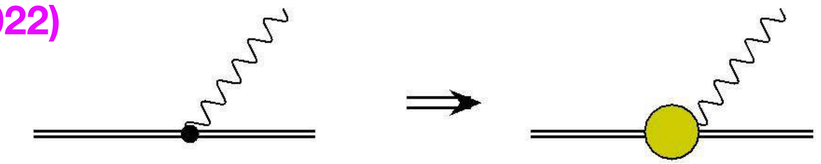
1. $\delta_{+,0}$ corrections to $\rho \to \pi\pi$ decays beyond sQED

sQED: Flores-Baez et al, PRD (2007)

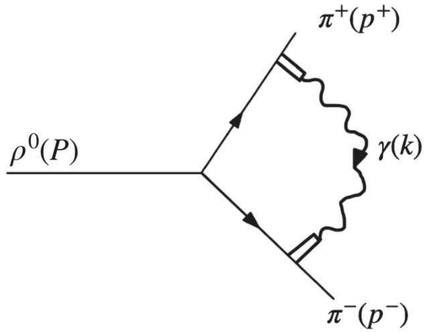
GVMD: Ignatov & Lee, PLB (2022)

Beyond sQED

$$\frac{1}{k^2} \Longrightarrow \frac{1}{k^2}\left[\sum_V a_V F_V(k^2)\right]^2$$



Scalar QED: $\sum_V a_V F_V(k^2) = \sum_V a_V F_V(0) = 1 \to \sum_V a_V = 1$



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- Asymptotic FF $1/k^4$, Ultraviolet Finite,
- Negligible structure-dependent real-photon corrections

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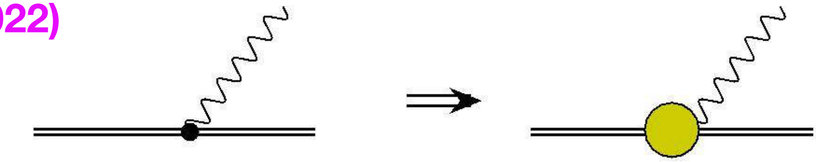

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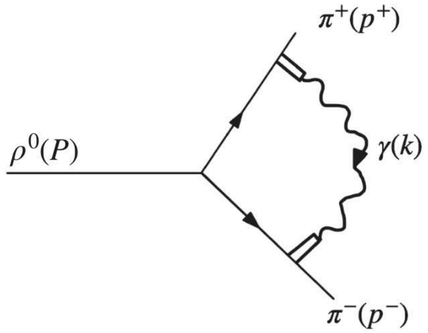

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At $m_\rho = 775$ MeV

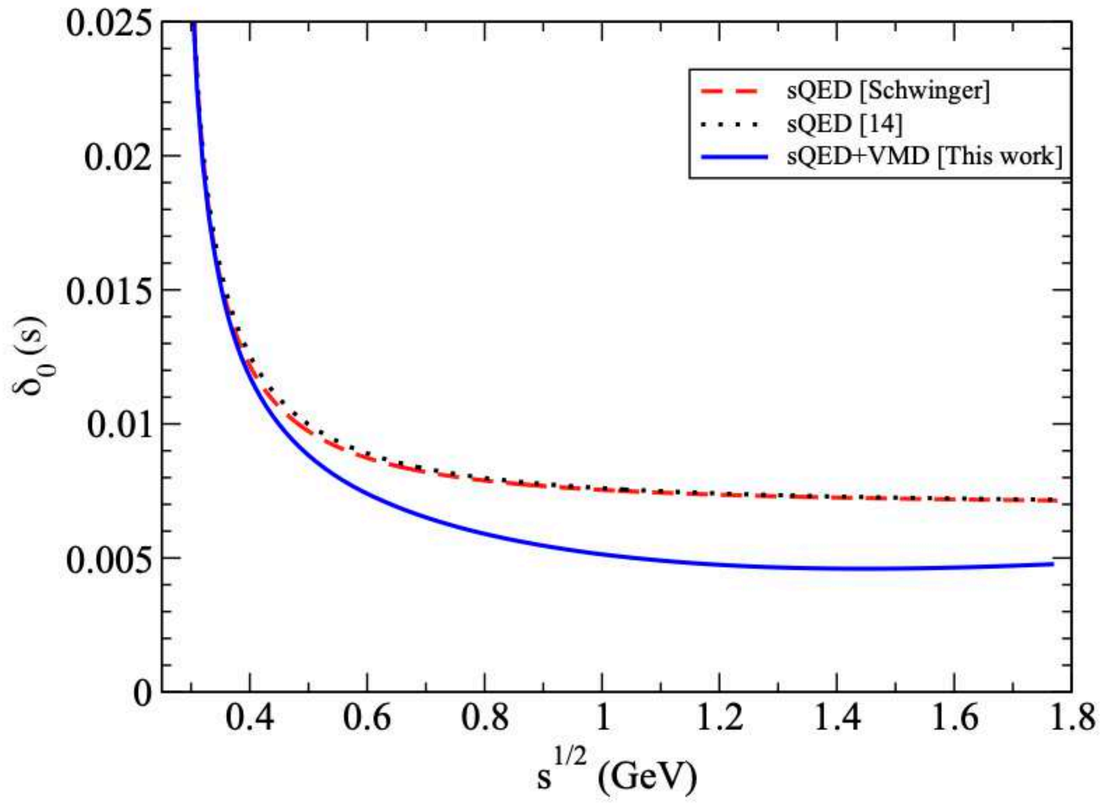
- $\delta_0 = +\,6.04(2) \times 10^{-3}$, for $\rho^0 \to \pi^+\pi^-(\gamma)$
- $\delta_+ = +\,1.44(3) \times 10^{-3}$, for $\rho^+ \to \pi^+\pi^0(\gamma)$
- $\delta_0 - \delta_+ = 4.60(4) \times 10^{-3}$,

Flores-Baez, GLC,Toledo PRD 113, (2026)

Energy-dependence of radiative corrections

Flores-Báez et al, PRD 113(2026)

$\rho^0 \to \pi^+\pi^-(\gamma)$



$$\mathrm{FSR}(s) = 1 + \delta_0(s)$$

sQED: Schwinger's Book; Drees-Hikasa, PLB (1990)
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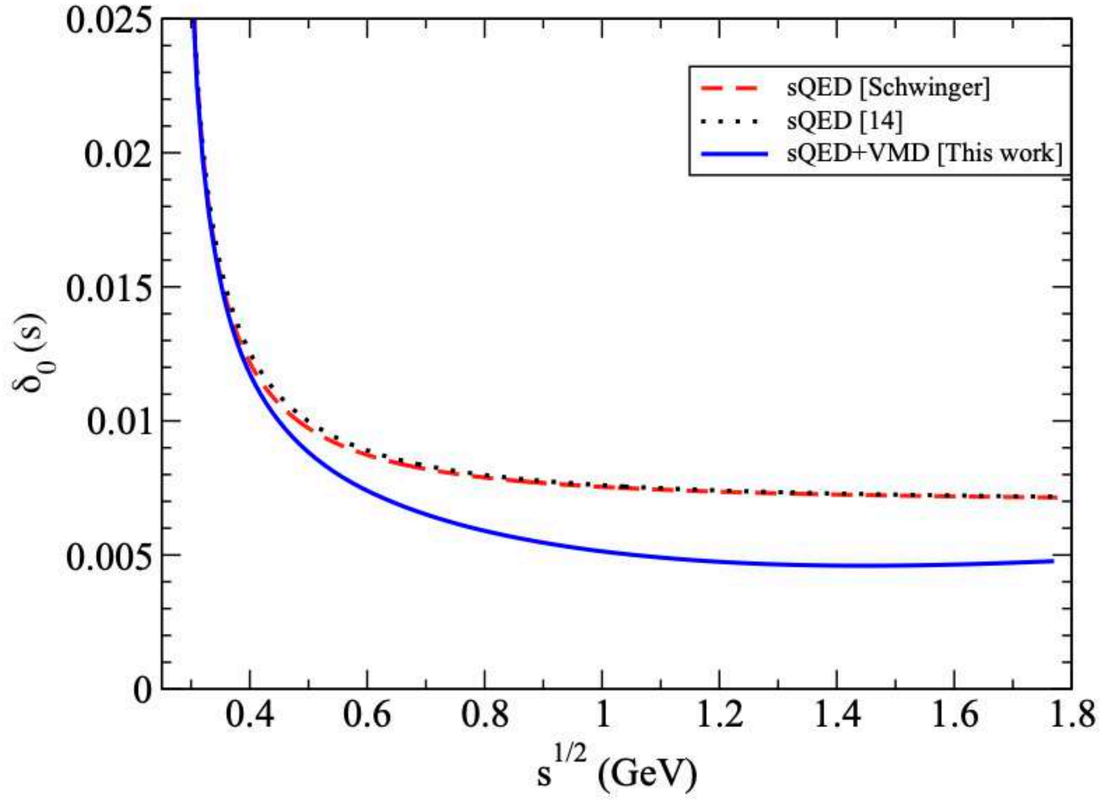

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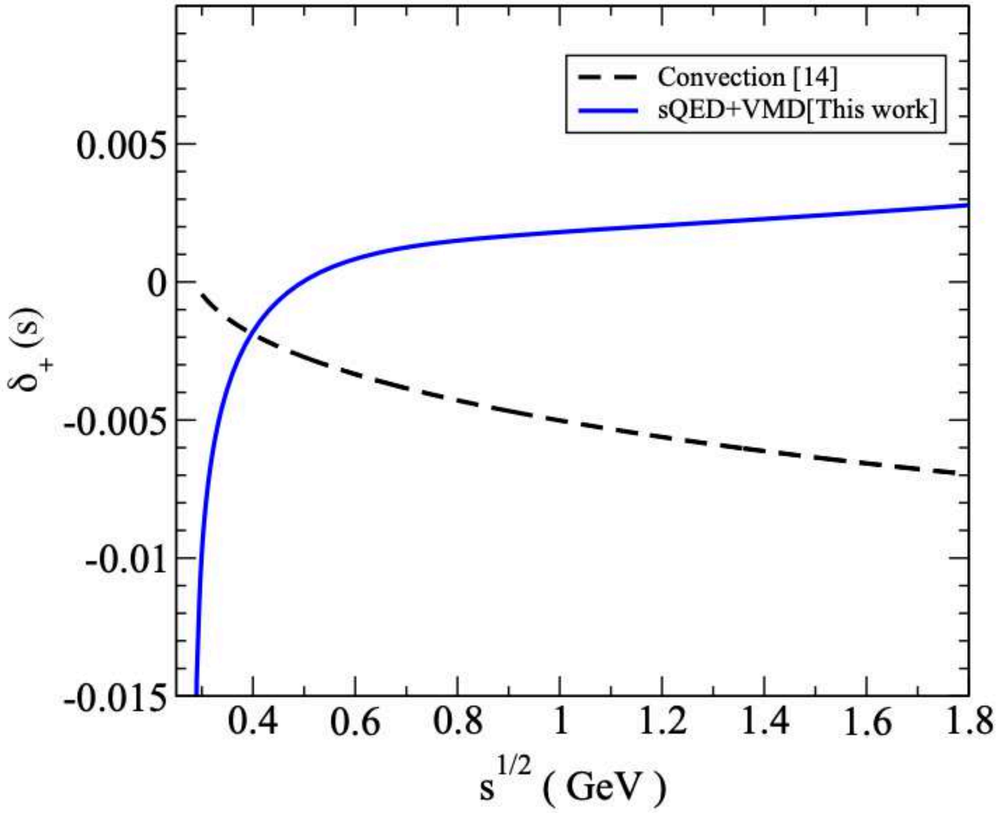

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2. Isospin breaking in $\rho\pi\pi$ couplings

- $|\delta_g| = \left|(g_+ - g_0)/g_0\right| \leq 2\alpha/\pi$, compromise guess , $|\Delta\Gamma_\rho| = 300\,|\delta_g|\ \text{MeV} \leq 1.4\ \text{MeV}$
- $\Longrightarrow \Delta a_\mu^{\text{LO,HVP}}[\tau, \pi\pi] = \pm\, 4.7 \times 10^{-10}$, **WP2025**
- Some model-dependence cancel in g_+/g_0 ratio (?)

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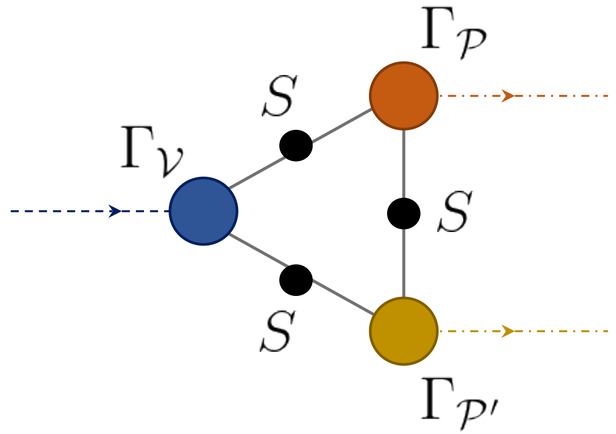
With Khepani Raya

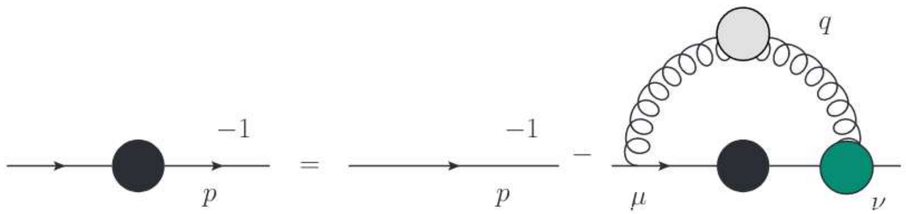
The $g_{\rm VPP}$ strong coupling is computed via Schwinger-Dyson in the impulse approximation:

$$\rho(p_1,\epsilon) \to \pi(p_2)\pi(q)$$

$$g_{\mathcal{V}\mathcal{P}\mathcal{P}'}\epsilon^\lambda\cdot q \;=\; N_c{\rm tr}\int\frac{d^4k}{(2\pi)^4}S^{f_3}(k_3)\epsilon^\lambda\cdot\Gamma_{\mathcal{V}}(k_V;p_1)S^{f_1}(k_1)$$
$$\times\;\bar\Gamma_{\mathcal{P}'}(k_{P'},-p_2)S^{f_2}(k_2)\Gamma_{\mathcal{P}}(k_P,-q)\,.$$

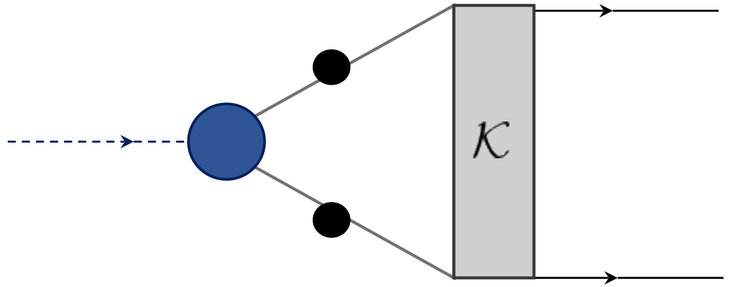


S^f : f-flavored quark propagator



Quark Dyson-Schwinger equation (DSE)

$\Gamma_{\mathcal{V},\mathcal{P}}$: Vector and pseudoscalar meson Bethe-Salpeter amplitudes (**BSA**)



Bethe-Salpeter equation (BSE)

Within a symmetry-preserving **contact interaction:**

Z. Xing, K. Raya, L. Chang, PRD 104, (2021)

$$g^2 D^{\text{eff}}_{\mu\nu}(p-q) \to \frac{1}{m_G^2}\delta_{\mu\nu}$$

The gluon propagator saturates at an infrared-enhanced fixed point

- The quark DSE thus reads:

$$S^{-1}(p) = [S^{(0)}(p)]^{-1} + \frac{4}{3m_G^2}\int_q \gamma_\mu S(q)\gamma_\mu$$

$$[S^{(0)}(p)]^{-1} = -i\gamma\cdot p + m$$

$$S(p)^{-1} = -i\gamma\cdot p + M$$

Non-perturbative quark mass function **M** becomes momentum-independent, playing the role of a constituent mass

- Meanwhile, the meson BSE:

$$\Gamma_H(P) = -\frac{4}{3m_G^2}\int_q \left[\gamma_\mu \chi_H(P)\gamma_\mu - \xi\tilde{\Gamma}_j \chi_H(P)\tilde{\Gamma}_j\right]$$

Rainbow-ladder (RL) piece

Beyond RL components

$$\{\mathbb{I},\ \gamma_5,\ i/\sqrt{6}\sigma_{\mu\nu}\}$$

$$\Gamma_{\mathcal{P}}(P) = i\gamma_5 E_{\mathcal{P}}(P) + \frac{1}{\bar{M}}\gamma_5\gamma\cdot P F_{\mathcal{P}}(P)$$

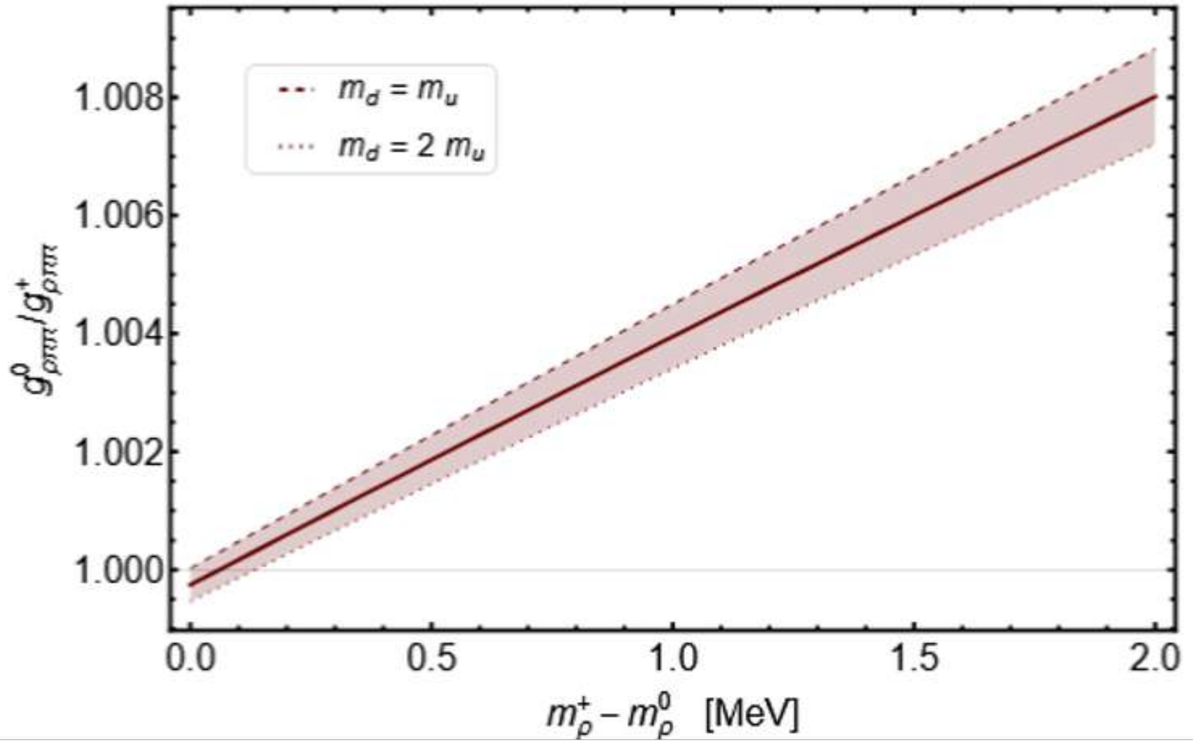
$$\Gamma^{\mathcal{V}}_\mu(P) = \gamma^T_\mu E_{\mathcal{V}}(P) + \frac{1}{\bar{M}}\sigma_{\mu\nu}P_\nu F_{\mathcal{V}}(P)$$

1) $m_\pi = 139.57$ MeV reproduced with

- $m_d = m_u = 7.00$ MeV, Isospin symmetry
- $m_d = 2m_u = 9.32$ MeV, Isospin breaking

2) with current quark masses fixed,

- $m_{\rho^+} = m_{\rho^0} = 775.19$ MeV, for $\xi = \xi_0 = 0.37$ in BSE,
- $m_{\rho^+} \neq m_{\rho^0}$, for varying $\xi_{u,d}$, keep $\xi = \sqrt{\xi_u \xi_d}$ fixed



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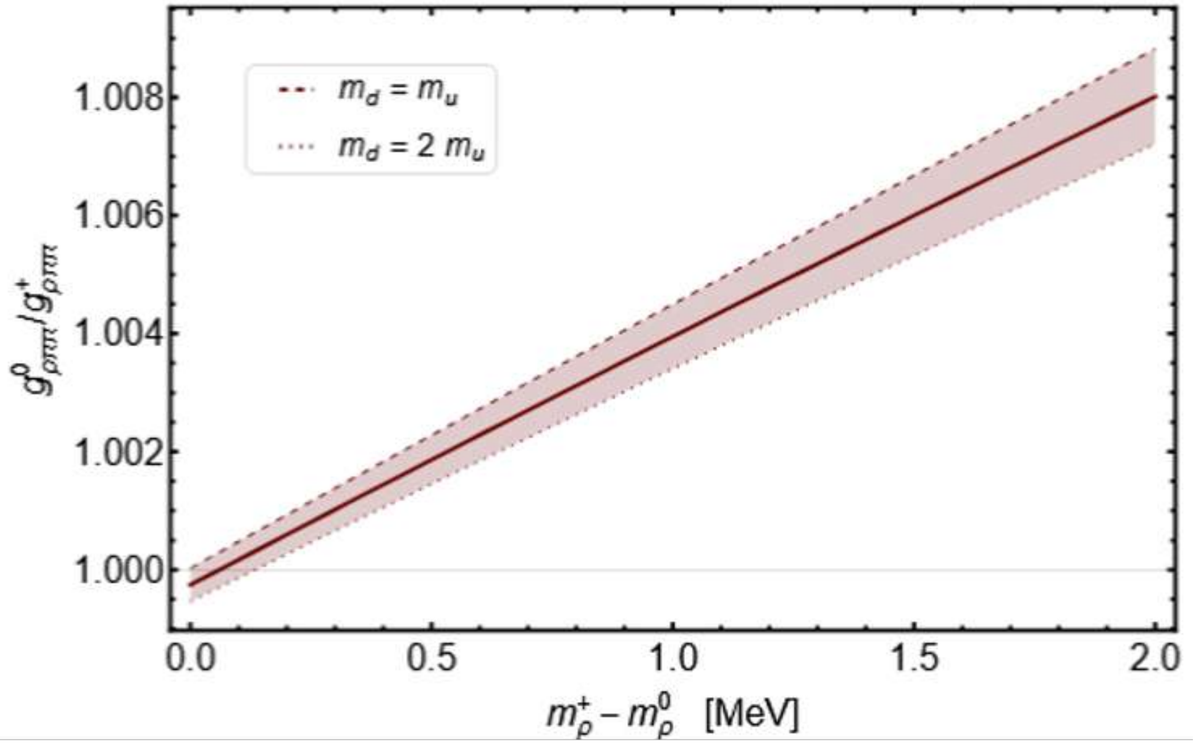

Using $m_{\rho+} - m_{\rho^0} = +0.7(8)$ MeV **PDG,** one gets

$$\frac{g_{\rho^0\pi^+\pi^-}}{g_{\rho^+\pi^+\pi^0}} = 1.0024(17)$$

$\delta g = -0.0024(17)$, compared to $\delta g = 0(\frac{2\alpha}{\pi}) = 0.0000(46)$

WP 2025

Summary of IB in rho meson widths

$$\Delta\Gamma_\rho = \Delta\Gamma_\rho[\pi\pi(\gamma)] + \Delta\Gamma_\rho(\text{rest})$$
$$= \Gamma(\rho^0 \to \pi^+\pi^-)\left[\delta_0 - \delta_+ - 2\left(\frac{g_+}{g_0} - 1\right) - \frac{3}{2}\left(\frac{1-\beta_0^2}{\beta_0^2}\right)\frac{\delta m_\pi}{m_\pi}\right] + \Delta\Gamma_\rho(\text{rest})$$
$$= [+0.690(6) + 0.72(51) - 1.104]\ \text{MeV} + 0.085(17)\ \text{MeV}$$
$$= +0.39(51)\ \text{MeV}$$

Compared to

- $\Delta\Gamma_\rho = (+0.3 \pm 1.3)$ MeV from **PDG2024**,
- $\Delta\Gamma_\rho = (-0.58 \pm 1.01)$ MeV (GS), from **Davier et al 2504.13789**

Summary: Estimated impact on $\Delta a_\mu^{\rm HVP,LO}[\tau,\pi\pi]$ (units 10^{-10})

		Refs. [a,b]	Ref. [c]	2026	
PS, $S_{\rm EW}$, $G_{\rm EM}$		$-22.01(91)$	$-21.4(^{+0.6}_{-1.4})$	$-24.9(5)$ [d,e]	
FSR		$+4.67(47)$	$+4.62(46)$	$+3.77(2)$ [f]	*
$\rho-\omega$ mixing		$+4.0(4)$	$+2.87(8)$	$+4.0(4)$	
$\frac{F_\pi^V}{f_+}$ (w/o $\rho-\omega$)	Δm_ρ	$+0.20(^{+27}_{-19})$	$+1.95(^{+1.56}_{-1.55})$	$+0.20(^{+27}_{-19})$	
	$\Delta\Gamma_\rho(\Delta m_\pi)$	$+4.09$	$+3.37$	$+4.09$	
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	$-5.91(59)$	$-6.66(73)$	$-2.22(3)$ [f]	*
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	$-2.32(1.64)$ [g]	
	Total	$-1.62(65)$	$-1.34(1.72)$	$-0.25(1.66)$	
$\Delta a_\mu^{\rm HVP,LO}[\tau,\pi\pi]$		$-14.9(1.9)$	$-15.2(^{+2.3}_{-2.6})$	$-17.4(1.8)$	

(GS)

[a] Davier et al, EPJ C84, 721 (2024),
[b] Davier et al, EPJC C66, 127(2010)
[c] Castro et al, PRD 111, 073004 (2025)
[d] Colangelo et al PRL 136, 10(2026); JHEP 02, 181 (2026)
[e] Cirigliano et al, arXiv 2602.11253
[f] Flores-Baez et al, PRD 113 , 093002 (2026)
[g GLC+K. Raya, preliminar

Summary: Estimated impact on $\Delta a_\mu^{\rm HVP,LO}[\tau,\pi\pi]$ (units 10^{-10})

		Refs. [a,b]	Ref. [c]	2026	
PS, $S_{\rm EW}$, $G_{\rm EM}$		$-22.01(91)$	$-21.4(^{+0.6}_{-1.4})$	$-24.9(5)$ [d,e]	
FSR		$+4.67(47)$	$+4.62(46)$	$+3.77(2)$ [f]	*
$\rho-\omega$ mixing		$+4.0(4)$	$+2.87(8)$	$+4.0(4)$	
$\frac{F_\pi^V}{f_+}$ (w/o $\rho-\omega$)	Δm_ρ	$+0.20(^{+27}_{-19})$	$+1.95(^{+1.56}_{-1.55})$	$+0.20(^{+27}_{-19})$	
	$\Delta\Gamma_\rho(\Delta m_\pi)$	$+4.09$	$+3.37$	$+4.09$	
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	$-5.91(59)$	$-6.66(73)$	$-2.22(3)$ [f]	*
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	$-2.32(1.64)$ [g]	
	Total	$-1.62(65)$	$-1.34(1.72)$	$-0.25(1.66)$	
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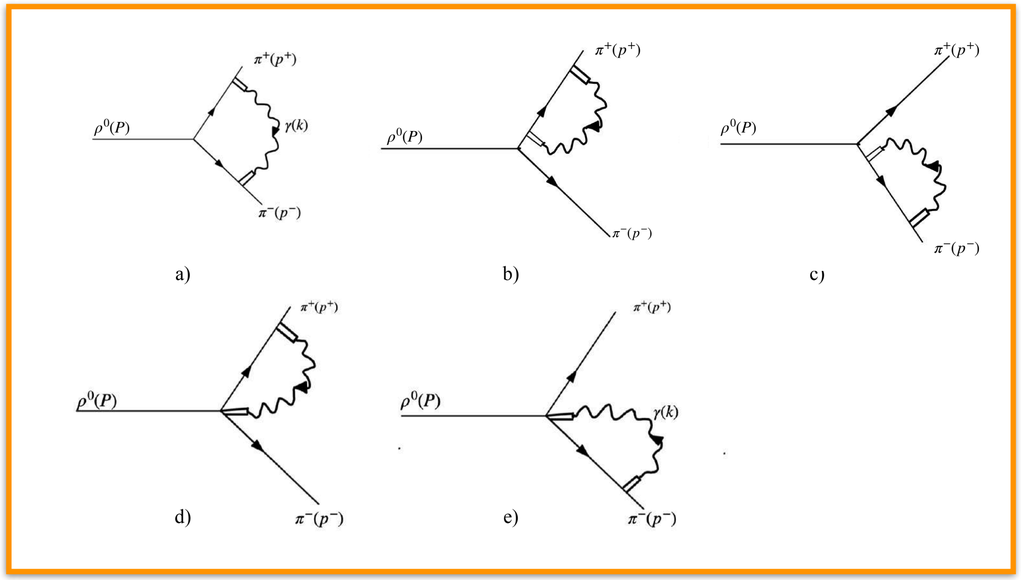
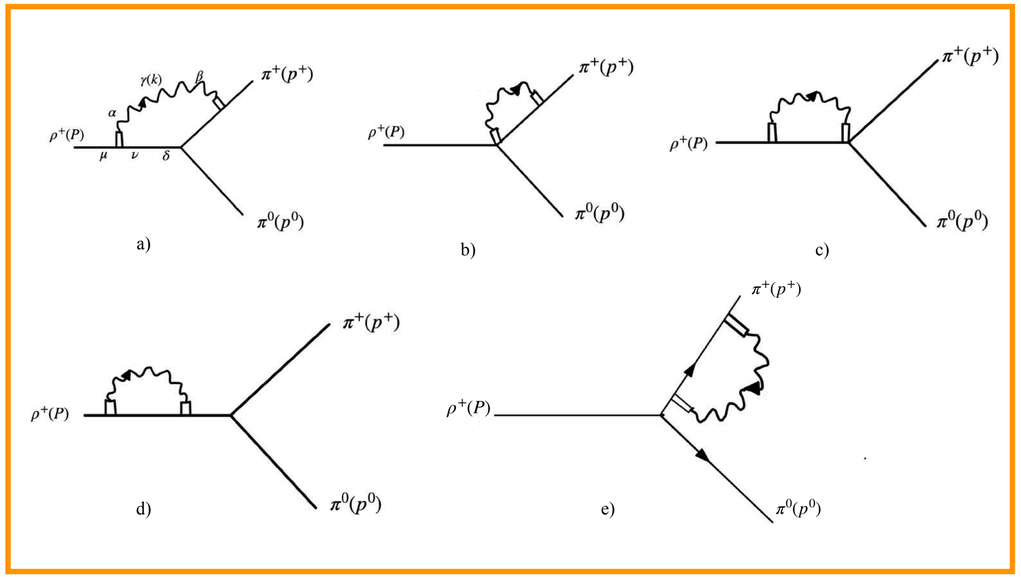
* Structure-dependent effects (*) in radcorr $\rho \to \pi\pi(\gamma)$
* Different results compatible within errors

Thank you for your attention!!

Backup

Neutral rho meson $\rho^0 \to \pi^+\pi^-(\gamma)$





- Real photon emission same as in the previous analysis (**PRD 2007**),
- IR finite upon inclusion of real photon emission contribution;
- UV finite

Conclusions

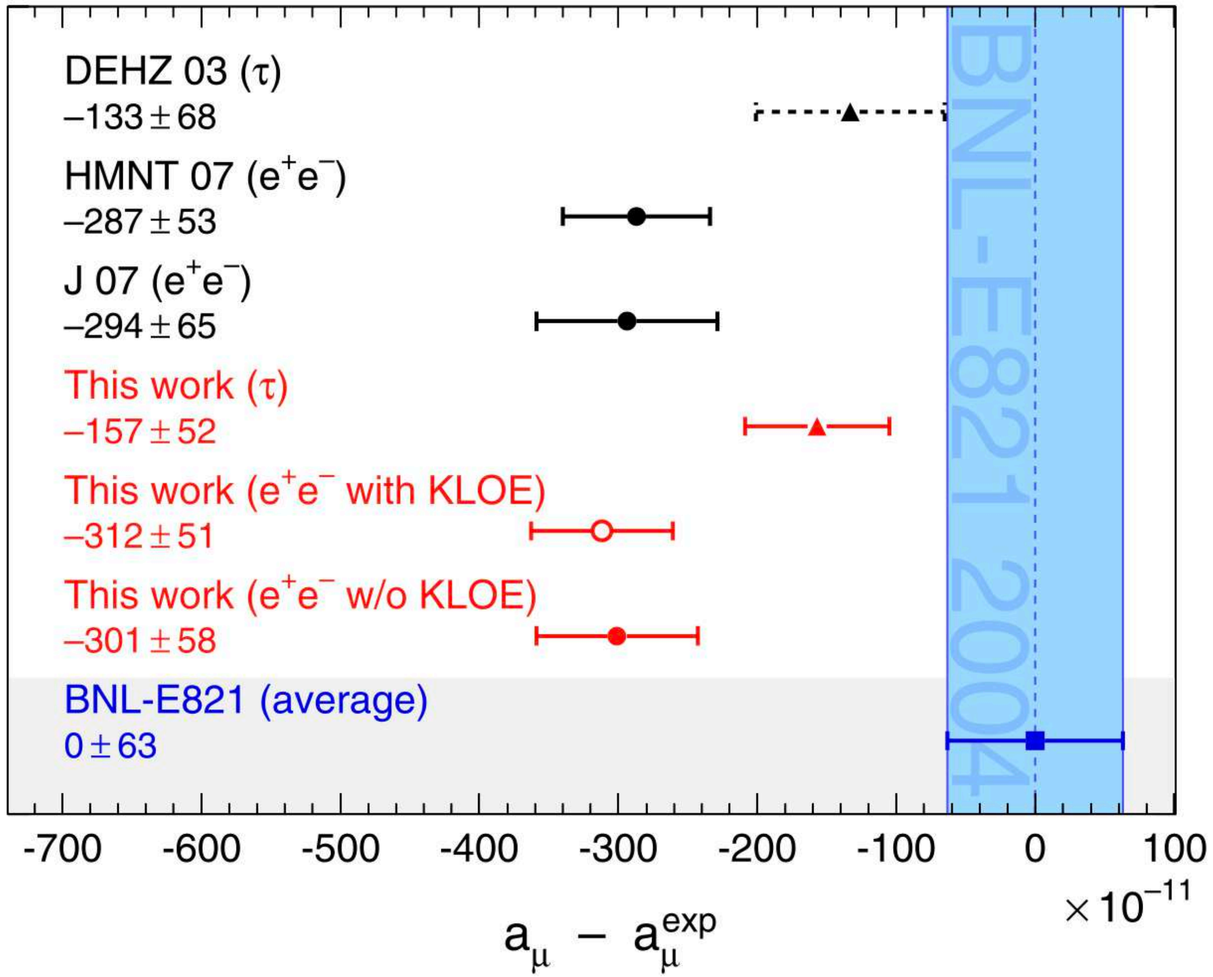
- Structure-dependent effects in FRS(s) and in the width difference $\Delta\Gamma_\rho[\pi\pi(\gamma)]$, calculated within a GVMD model, are larger than previously estimated (a 10% uncertainty).

- Taken together, these effects reduce the size of IB corrections to the HVP contribution based on tau data

$$\Delta a_\mu^{HVP}[\tau,\pi\pi] = \begin{cases} (-11.9 \pm 1.7) \times 10^{-10}, & 2510.02723\text{v2} \\ (-14.9 \pm 1.9) \times 10^{-10}, & \text{Davier et al, EPJC(2024)} \\ (-12.2 \pm 3.41) \times 10^{-10}, & \text{Davier et al, 2504.13189} \\ (-15.22^{+2.26}_{-2.63}) \times 10^{-10} & \text{CMR, PRD(2025)} \\ (-15.0 \pm 5.1) \times 10^{-10}, & \text{WP (2025)} \end{cases}$$

- $a_\mu^{\text{CVC},\pi\pi} = a_\mu^{\text{HVP}}[\tau,\pi\pi] + \Delta a_\mu^{\text{HVP}}[\tau,\pi\pi] \Rightarrow$ tau-based prediction of a_μ moves closer to experimental value.

M. Davier et al, EPJC 66, 127 (2010)



$$\Delta a_\mu^{HVP}[\tau,\pi\pi] = (-161 \pm 19)\times 10^{-11} \quad \textbf{DH…'10}$$

$$\Downarrow$$

$$(-149 \pm 19)\times 10^{-11} \quad \textbf{DHLMZ'23}$$

$$\Downarrow$$

$$(-152^{+23}_{-26})\times 10^{-11} \quad \textbf{CMR'25}$$

$$\Downarrow$$

$$(-150 \pm 50)\times 10^{-11} \quad \textbf{WP-TI'25}$$

$$\Downarrow$$

$$(-119 \pm 17)\times 10^{-11} \quad \textbf{FLT'25}$$

$$R_\phi \equiv \frac{B(\phi \to K^+K^-)}{B(\phi \to K^0\bar{K}^0)}$$

$$\left|\frac{g_{\phi K^+K^-}}{g_{\phi K^0\bar{K}^0}}\right| = \sqrt{\frac{R_\phi}{1+\delta_{QED}}\left(\frac{\beta_0}{\beta_+}\right)^3}$$

$$\delta_{QED} = 4.09\,\%$$

$R_\phi^{\rm exp}$	$\lvert g_{\phi K^+K^-}/g_{\phi K^0\bar{K}^0}\rvert$	Ref.
1.637 ± 0.062	1.019 ± 0.019	BESIII 25-26
1.351 ± 0.057	0.925 ± 0.019	PDG Average
1.449 ± 0.029	0.958 ± 0.010	PDG Fit

BESIII-25: PRL 136, (2025);
BESIII-26: arXiv 2605.11464

Premilinar result Schwinger-Dyson: $\left|\frac{g_{\phi K^+K^-}}{g_{\phi K^0\bar{K}^0}}\right| = 0.97 \pm 0.01$

The ρ width difference

Source	$\Delta\Gamma_\rho \equiv \Gamma_{\rho^0} - \Gamma_{\rho^+}$ (MeV)	$\Delta m_\rho \equiv m_{\rho^+} - m_{\rho^0}$ (MeV)	Ref.
Data based PDG	$+0.3 \pm 1.3$ (-1.7 ± 1.1)	$+0.7 \pm 0.8$ (-0.15 ± 0.41)	PDG '24
Data based	-0.58 ± 1.04 *	$+0.30 \pm 0.53$	arXiv: 2504.13789
Th. rad corr.	$+0.76 \pm 0.20$ **	–	Flores-Baez ...PRD'07 Davier et al, EPJC'10

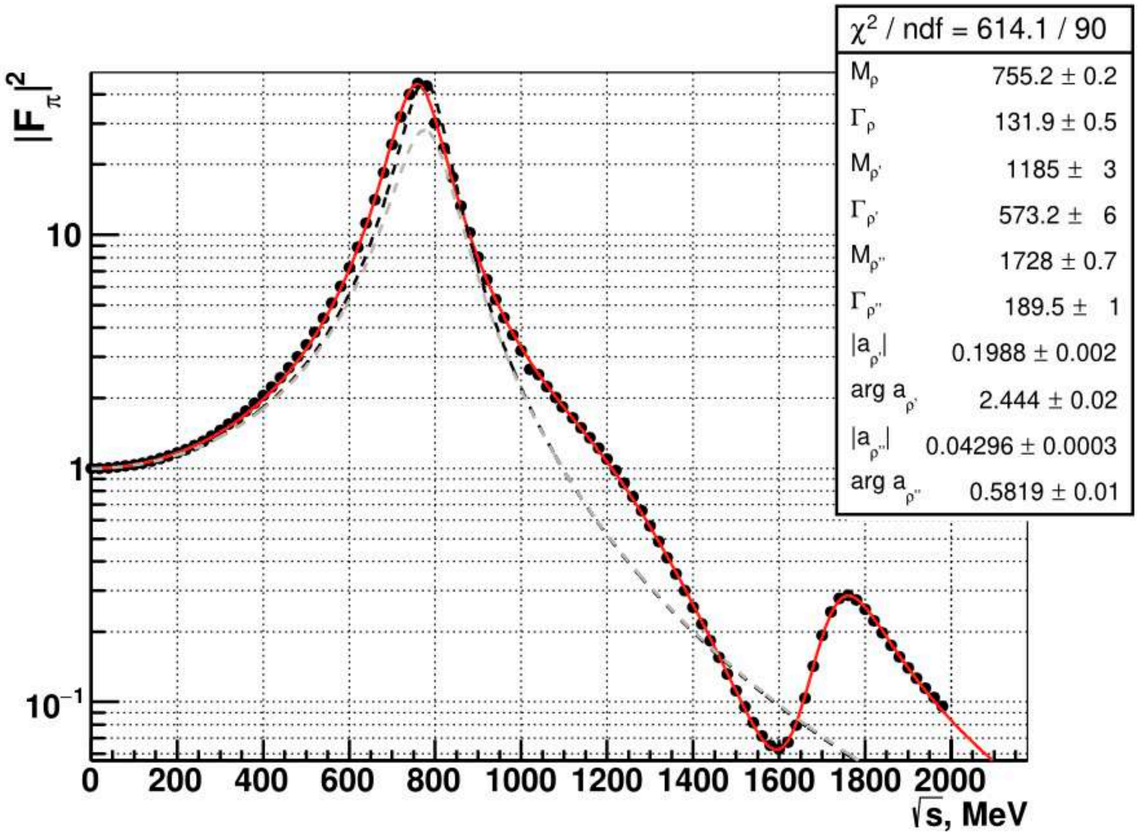
Induced uncertainty

$$\delta(\Delta a_\mu^{HVP}[\Delta\Gamma_\rho]) = \begin{cases} \pm 2.92 \times 10^{-10} & * \text{ arXiv : 250413789} \\ \pm 0.59 \times 10^{-10} & ** \text{ Davier et al, EPJC, '10} \end{cases}$$

** **1**) sQED approximation and, 2) incomplete $\rho\rho\gamma$ electromagnetic vertex

Pion charge asymmetry in $e^+e^- \to \pi^+\pi^-$ measured by CMD3

$$F_\pi(s) = \sum_{V=\rho,\rho',\rho''} a_V F_V(s)$$



Ignatov+Lee, PLB, '22

$$A = \frac{N_{\theta<\pi/2} - N_{\theta>\pi/2}}{N_{\theta<\pi/2} + N_{\theta>\pi/2}}$$