HVP Update from Fermilab Lattice, HPQCD and MILC

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on behalf of the Fermilab, HPQCD and MILC Collaboration

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University of Connecticut

THE UNIVERSITY OF UTAH

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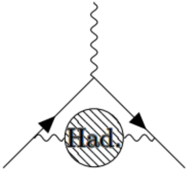
The HVP from Lattice QCD

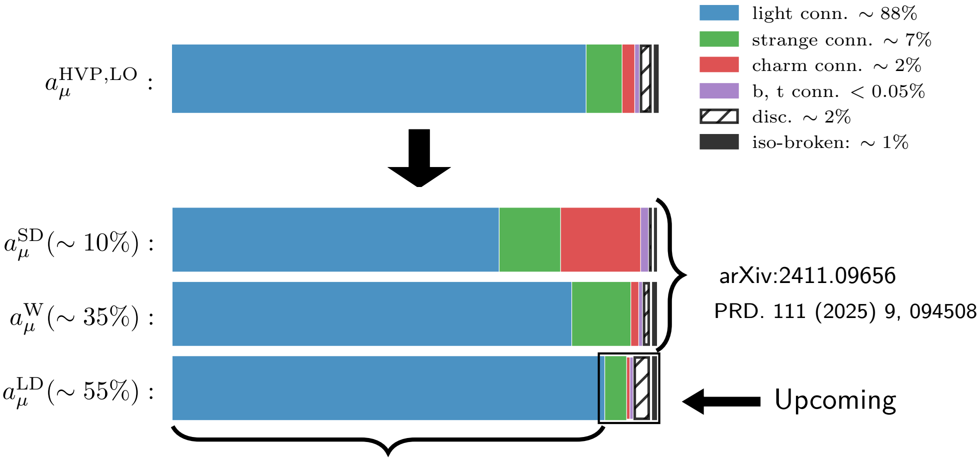


$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty dt\tilde{K}(t)\,C(t), \quad C(t) = \frac{1}{3}\sum_i^3 \int d^3x \langle J_i(x) J_i(0)\rangle$$

$$J_i(x) = \sum_f Q_{q_f}\bar{q}_f(x)\gamma_i q_f(x), \quad Q_u = +\frac{2}{3},\; Q_d = -\frac{1}{3},\; Q_s = -\frac{1}{3},\ldots$$

$C^{ff}_{\mathrm{conn.}}(t)$, $C^{ff}_{\mathrm{disc.}}(t)$ & $\Delta C_{\mathrm{I.B.}}(t)$ are computed on the lattice.

Fermilab/HPQCD/MILC Status



arXiv:2412.18491
PRL. 135 (2025) 1, 011901

This talk: Status of missing pieces

Separation scheme

$$a_\mu^{\text{HVP, LO}} = a_\mu^{\text{HVP, LO}}(\text{iso}) + \Delta a_\mu^{\text{HVP, LO}}(\text{SIB}) + \Delta a_\mu^{\text{HVP, LO}}(\text{QED})$$

isoQCD $\xrightarrow{+\text{SIB}}$ QCD $\xrightarrow{+\text{QED}}$ QCD+QED (FLAG-2024 / "Edinburgh Consensus")

	QCD	isoQCD
M_{π^+}	135.0	135.0
M_{K^+}	491.6	494.6
M_{K^0}	497.6	494.6
$M_{D_s^+}$	1967	1967
f_{π^+}(FLAG)	130.50	130.50
M_Ω(Preferred)	$1671.26(32)$	$1671.26(32)$
w_0 [fm] (TI)	0.17236	0.17236

masses in MeV; lower block = scale-setting choices

(M_Ω from Ref.[1]; w_0: WP25 + 12 w_0/a definitions[2])

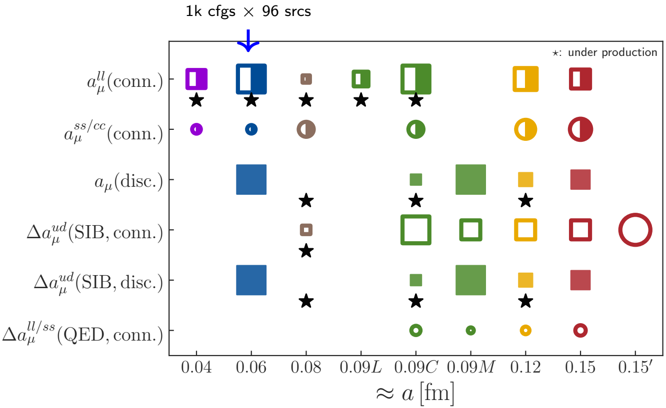
In progress:

- t_0-based scale setting
- M_Ω/f_π calculation

[1] High-Precision Scale Setting with the Omega-Baryon Mass and Gradient Flow, Phys. Rev. D 113 (2026) 5, 054501, arXiv:2509.14367

[2] 2 flows x 3 observables x w/ or w/o improvement

Datasets (HISQ physical-mass ensembles)

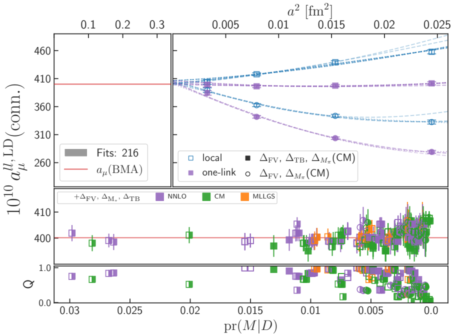


- Two vector current discretizations: local (unfilled) & one-link (filled).
- Size $\sim$ statistics: $N_{\text{cfg}} \times N_{\text{src}}$ (random-wall).
- Squares: LMA data
- $M_\pi L \in [3.7, 4.2]$[3]
- L : Large volume, C : Corrected, M : Mistuned, $0.15'$: SIB-sea

[3] Except 0.15 $M_\pi L = 3.4$, 0.09L $M_\pi L = 7.7$

Fermilab/HPQCD/MILC HVP analyses

- **Blinding**: Each analysis blinded independently.
- **Finite-volume**: EFT/EFT-inspired models → data-driven study
- **Discretization**:
  - Two vector discretizations (local and one-link)
  - I=1 contributions: w/ & w/o TB corrections
  - Secondary renormalization scheme
  - (Chiral) continuum fit variations based on symanzik EFT.
- **StN**: Correlator reconstruction for LD tail (fit & bounding).
- **Systematic uncertainties and covariances**[4]: Bayesian model averaging (BMA) with Bayesian AIC weight:



$$\mathrm{pr}(M \mid D) \equiv \mathrm{pr}(M) \exp\left[-\frac{1}{2}\left(\chi^2_{\rm data}\left(\mathbf{a}^\star\right) + 2k + 2N_{\rm cut}\right)\right].$$

[4] More details in appendices of arXiv:2411.09656

Outline

- I=0
  - Preliminary results based on new disconenected dataset at 0.06 fm
- I=1
  - Updating our light-quark connected contribution
  - Exclusive staggered two-pion reconstruction progress.
- Isospin breaking
- Outlook

I=0 / Disconnected

$I = 0$

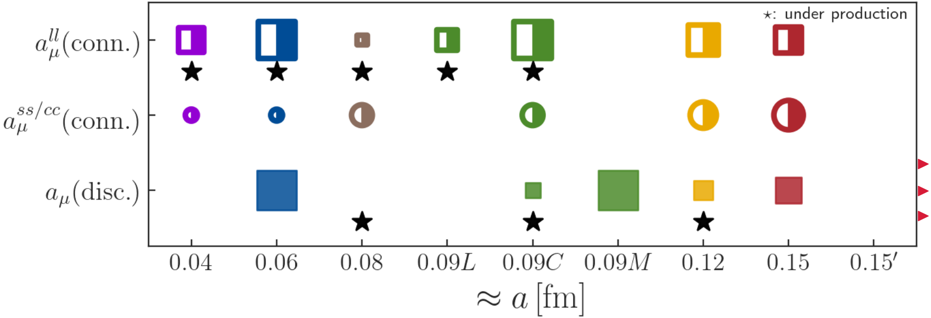
$a_\mu^{\mathrm{HVP,LO}}$:

$$a_\mu^{I=1} : \ \frac{9}{10} a_\mu^{ll}$$
$$a_\mu^{I=0} : \ \frac{1}{10} a_\mu^{ll} + a_\mu^{ss} + a_\mu^{cc} + a_\mu(\mathrm{disc.})$$

$a_\mu^{\mathrm{HVP,LO}}$:

- $a_\mu(\mathrm{disc.}) \sim a_\mu^{I=0}$ - $\frac{1}{9} a_\mu^{I=1}$
- Correlator reconstruction difficult & not easily testable $\Rightarrow$ construct $I = 0$ instead.
- Removes dominant finite-volume & taste-breaking effects.
- Rest of talk: Charm is separate

$I = 0$ / disconnected dataset

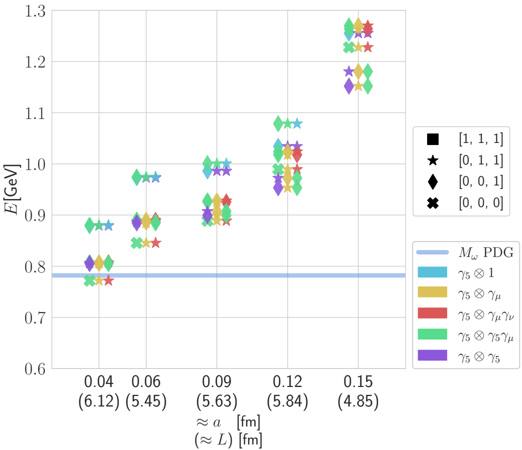


- M: Mistuned
- C: Corrected
- L: Large volume

- $I = 0$ constrained by the disconnected dataset (onelink discretization: taste singlet).
- New dataset generated at 0.06 fm with LMA:
  - 4000 exact low-modes (pairs)
  - 1000 tsm-improved deflated random-volume sources
  - Mass(frequency)-split technique
- Coarser ensembles also under production: retuned $0.09C$ and new 0.08 ($a \approx 0.077$ fm).

$I = 0$ lattice spectrum

- Continuum IV: $\omega \to \pi\pi\pi$, $\phi \to KK$
- Lattice group theory restricts $\pi\pi\pi$ states coupling to vector current.
  eg. No $\pi(0,0,1)\pi(0,0,-1)\pi(0,0,0)$ coupling.
- Lowest state in $I = 0$ channel on HISQ physical-mass lattices is dominated by (stable) ω not $\pi\pi\pi$ [5]
- KK states also above ω (and ϕ).
- This is non-interacting picture but expect binding energies to be small [6].



$$E_{\pi_\xi, \mathrm{free}} = \sqrt{\vec{p}^2 + M_\xi^2}$$

[5] Except at 0.04 fm

[6] Parrino/Lehner talk @ 8th g-2 TI workshop.

$I = 0$ & disconnected correlator reconstruction

$I = 0$:

$$C_{\text{fit}}^{I=0}(t) = \sum_{n=0}^{N_{\text{states}}} Z_n^2 \left(e^{-E_n t} + e^{-E_n (T-t)}\right) + (-1)^t \sum_{m=0}^{M_{\text{states}}} Z_{m,\text{osc}}^2 \left(e^{-E_{m,\text{osc}}\, t} + e^{-E_{m,\text{osc}}(T-t)}\right)$$

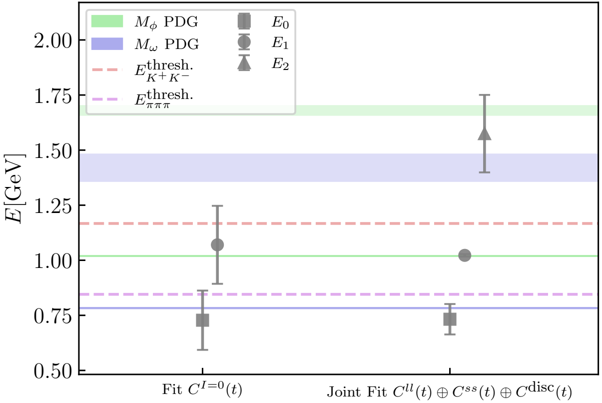
- Expect $E_{\text{fit},0} = M_\omega$
- Reconstruct tail with $C_{\text{recon}}^{I=0}(t) = Z_0^2 e^{-E_{\text{fit},0}\, t}$
- Also consider bounding $C^{I=0}(t)$ with $E_{\text{upper}} = M_\omega$, $E_{\text{lower}} = E_{\pi\pi\pi,0,\text{free}}$

Disconnected:

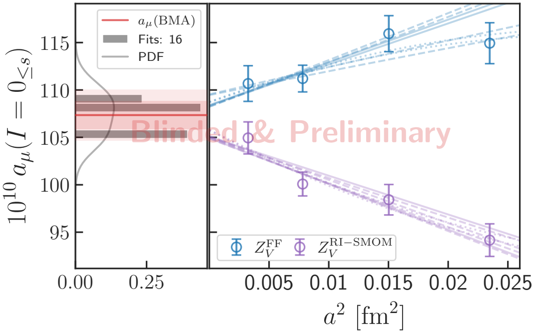
- $C^{\text{disc}}(t) = C^{I=0}(t) - \frac{1}{9}\, C^{I=1}(t)$
- Two channel fit to $C^{\text{disc}}(t)$ unconstrained $\Rightarrow$ Joint fit to $C^{\text{ll}}(t) \oplus C^{\text{ss}}(t) \oplus C^{\text{disc}}(t)$
- $I = 1$ amplitudes shared between light & disc (up to charge factors)

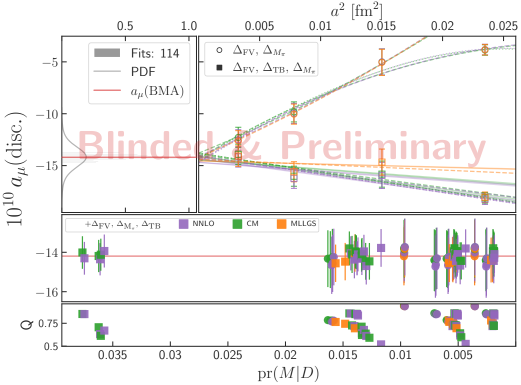
$I = 0$ spectrum (0.06 fm)

- Ground state E_0 consistent with ω, below the $\pi\pi\pi$ & K^+K^- thresholds.
- Only using the ground state, not doing spectroscopy!
- Final a_μ result cross-checked against bounding method



$I = 0$ & disconnected continuum extrap.



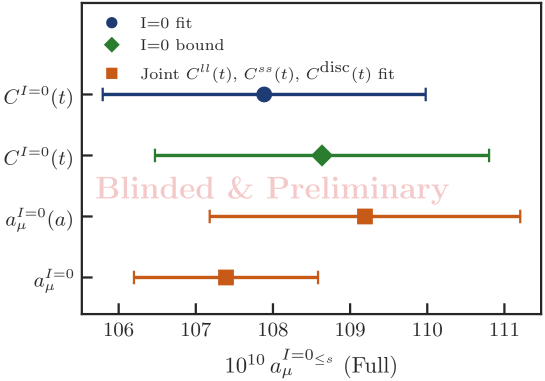


- $a_\mu^{I=0}$: FV & taste-breaking effects cancel by construction $\Rightarrow$ cleaner continuum, no TB correction.
- $a_\mu(\mathrm{disc.})$: LD FV & taste-breaking effects (from $I = 1$) $\Rightarrow$ corrections: $(\Delta_{\mathrm{FV}},\ \Delta_{\mathrm{TB}},\ \Delta_{M_\pi})$.

$I = 0$ & disconnected cross-checks

Cross-check: $a_\mu^{I=0}$ consistent if constructed from:

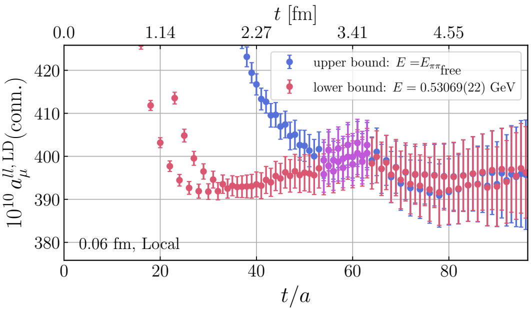
- $I = 0$ correlator w/ fit
- $I = 0$ correlator w/ bound
- flavor components on the lattice
- flavor components in the continuum

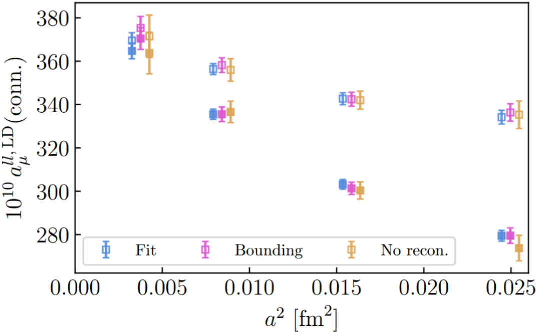


Smaller error if constructed in continuum but increased correlations with a_μ^{ll} ($a_\mu^{I=1}$)
$\Rightarrow a_\mu(\text{iso}) = a_\mu^{I=0} + a_\mu^{I=1}$ has similar errors regardless of approach. (backup slide)

l=1 / Light

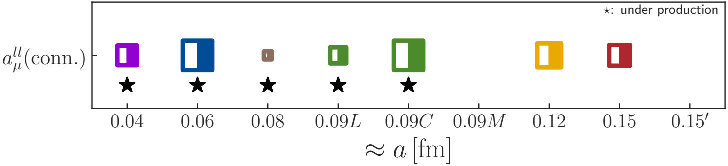
Correlator reconstruction recap





- Fit vs. bounding at 0.06 fm needs further investigation
  (fake-data test confirmed fit approach (backup slides), but plan to revisit with more statistics.)

Light update plans



- M: Mistuned
- C: Corrected
- L: Large volume

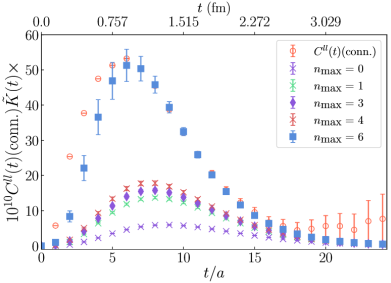
- Still the dominant source of uncertainty in our lattice HVP calculations
- Extensive light data generation ongoing on Aurora, Frontier & Perlmutter:
  - 300 $\to$ 1k configs @ 0.04
  - 1k $\to$ 3k configs @ 0.06
  - 0 $\to$ 1k configs @ 0.08
  - 1k $\to$ 3k configs @ $0.09C$ (primarily for FV study)
- Near-term: update the light contribution with $2\times$ statistics @ 0.06 fm.

Staggered two-pion reconstruction

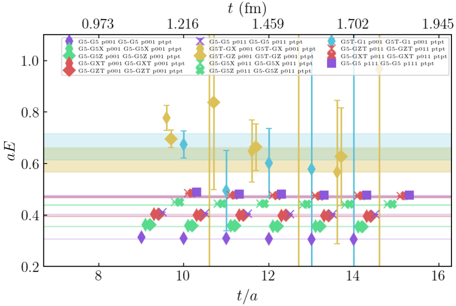
$$\mathbf{C}(t) = \begin{pmatrix} C_{\rho,\tilde{\rho}\to\rho,\tilde{\rho}}(t) & C_{\rho,\tilde{\rho}\to\pi\pi}(t) \\ C_{\pi\pi\to\rho,\tilde{\rho}}(t) & C_{\pi\pi\to\pi\pi}(t) \end{pmatrix}$$

$$C^{ll}(t)(\text{conn.}) = \frac{10}{9}\sum_{n=0} \left|\langle 0|\rho^0|n\rangle\right|^2 e^{-E_n t}$$

Established at 0.15 **fm** (8×8 basis):



Extending to 0.12 **fm:** More momenta, more tastes $\Rightarrow$ basis: $8 \times 8 \to 16 \times 16$.

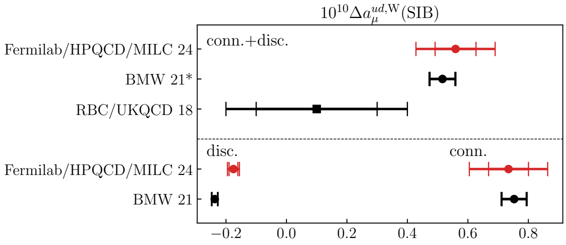


- $\gamma_5\gamma_0 \otimes \xi_A$ ops used over $\gamma_5 \otimes \xi_A$ to remove staggered time-links
- $\gamma_5\gamma_0$: bad ground-state overlap $\Rightarrow$ noisy, no clean plateau,
- GEVP won't diagonalise with full basis (not complete).
- **Plan:** Investigate time-link ops; modify matrix fit / GEVP.

Strong Isospin Breaking

$$\text{isoQCD} \xrightarrow{+\,\mathrm{SIB}} \text{QCD} \xrightarrow{+\,\mathrm{QED}} \text{QCD+QED}$$

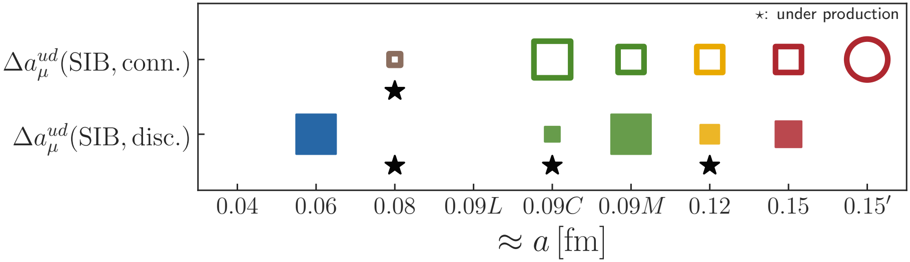
$$\Delta a_\mu^{ud}(\mathrm{SIB},\ \text{conn.}) \equiv a_\mu^{ud}(\text{conn.}) - a_\mu^{ll}(\text{conn.})$$
$$\Delta a_\mu^{ud}(\mathrm{SIB},\ \text{disc.}) \equiv a_\mu^{ud}(\text{disc.}) - a_\mu(\text{disc.})$$



- Correct for up-down mass splitting by computing the (correlated) differences shown
- **Done:** SD & W SIB (conn. & disc.)[7]
- **In progress:** the LD contribution with expanded LMA datasets

[7] arXiv:2411.09656, PRD 111 (2025) 9, 094508

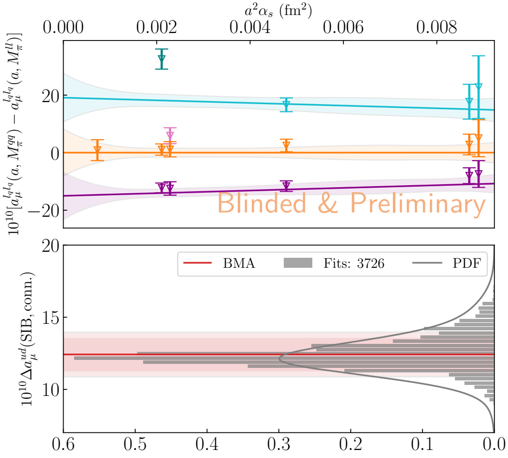
SIB datasets



- M: Mistuned
- C: Corrected
- L: Large volume

- SIB computed non-perturbatively: direct simulation at physical m_u, m_d valence masses.
- Sea isospin breaking estimated from the dedicated $0.15'$ (SIB-sea) ensemble.
- New finest ensembles: 0.06 fm (disc. only), 0.08 fm (in production) to address the continuum limit.

SIB: connected



Plot from Jake Sitison

- Correlators computed with valence mass $\mathrm{l_q} = \{h, \mathrm{u}, \mathrm{l}, \mathrm{d}\}$ (+ mistuned l on 0.09M.)
- Joint chiral-continuum fit to reconstruct SIB correction:

$$\Delta a_\mu^{ud}(\mathrm{SIB},\ \text{conn}) = a_\mu^{ud}(\text{conn}) - a_\mu^{ll}(\text{conn})$$

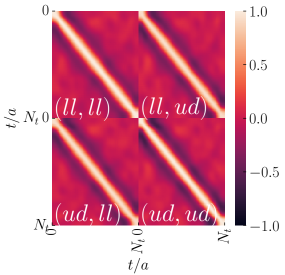
$$a_\mu^{ud} = \frac{q_u^2}{q_l^2}\, a_\mu^{l_u l_u} + \frac{q_d^2}{q_l^2}\, a_\mu^{l_d l_d}$$

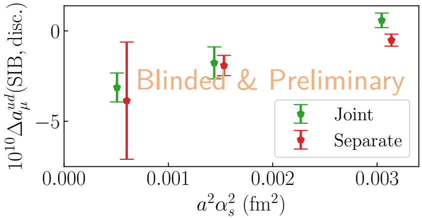
- Continuum extrapolation remains quite flat, mass dependence significant
- New error: $\approx 2 \times 10^{-10}$, >2x improvement from previous published conn SIB correction[8]

[8] arXiv:2411.09656, PRD 111 (2025) 9, 094508

SIB: disconnected

- Disconnected correlation function is computed directly as $C_{ud}^{\text{disc}}(t) \Rightarrow$ compute difference on lattice and extrapolate
- Off-diagonal correlations with $\mathrm{C_{ll}}$ are strong (top plot shows 0.09M); must include full covariance and do joint fits
- Fit function from isospin mixing in SIB,
  $$C_{ud}^{\text{disc}}(t) = C_{I=0}(t) - \frac{1}{9}\, C_{I=1}(t) + C_{\text{mix}}(t)$$
- Can expand $C_{ud} - C_{ll}$ spectrum in $Z \to Z + \delta Z$, $E \to E + \delta E$ in (isospin corrections $O(1\%)$ or smaller); work in progress



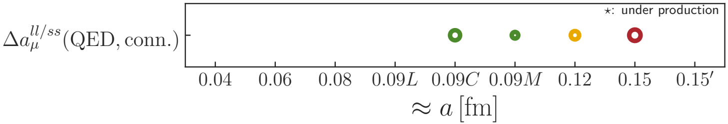


Plots from Jake Sitison

QED

$$\text{isoQCD} \xrightarrow{+\,\mathrm{SIB}} \text{QCD} \xrightarrow{+\,\mathrm{QED}} \text{QCD+QED}$$

Non-perturbative electroquenched QED_L calculation



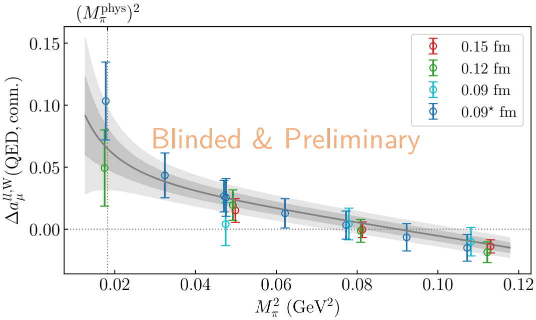
$$\Delta a_\mu^{ff}(\mathrm{QED}) = a_\mu^{ff}(m_f, Q_f) - a_\mu^{ff}(m_f, 0) + (\zeta_f - 1)\, a_\mu^{ff} \frac{\partial \ln a_\mu^{ff}}{\partial \ln m_f}, \qquad \zeta_f = \frac{v_f(m_f, 0)}{v_f(m_f, Q_f)} \Big/ \frac{v_{f,\mathrm{FLAG}}}{v_{f,\mathrm{PDG}}}$$

- Compute difference at fixed bare-quark mass + term that tunes to the FLAG-PDG difference
- Vector current in $a_\mu^{ff}(m_f, Q_f)$ renormalized with $Z_V^{\mathrm{QCD+QED}}$[9]
- Quark mass tuning: v parameterizes quark mass via GMOR:
  - $v_l = (\pi^0)^2$
  - $v_s = M^2_{\eta_{ss}} \ / \ M^2_{K^0} + M^2_{K^+} - M^2_{\pi^+}$

[9] HPCQCD, Phys. Rev. D 102 (2020) 054511 (arXiv:2005.01845)

QED light: Chiral dependence

- Chiral behaviour dominates over discretization effects [10]
- Extended mass range $[7m_l,\, 3m_l] \to [7m_l,\, m_l]$ to fix the chiral behaviour near the physical point
- $0.09M \to 0.09C$ ensemble (significant sea-mass dependence)
- Complementary EFT [11]: spurion matching of QCD diagrams to χPT (J. Sitison talk later today)



Full chiral-continuum extrapolation in progress

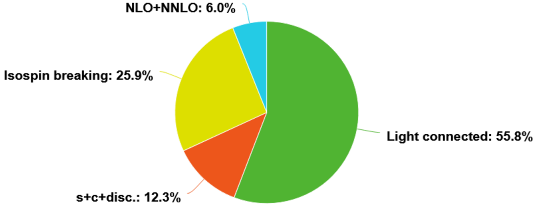
Plot from Jake Sitison

[10] Djukanovic *et. al.*, J. High Energ. Phys. 2025, 98 (2025), arXiv:2411.07969
[11] J. Parrino, V. Biloshytskyi, E.-H. Chao, H.B. Meyer, V. Pascalutsa, JHEP 07 (2025) 201, arXiv:2501.03192

Outlook

- **Near term**:
  - Increasing statistics on light datasets
  - FV study on lattice with $2\times$volume.
  - Lattice calculation of IB effects (Complete SIB & partial QED)
  - NLO calculation
  - Plan: Total HVP paper by end of the year
    - $a_\mu(\mathrm{iso.})$ uncertainty $\sim$ WP25 average
    - IB errors less well known at this point.

$\sigma_{a_\mu^{\mathrm{HVP}}}(\mathrm{WP25})$

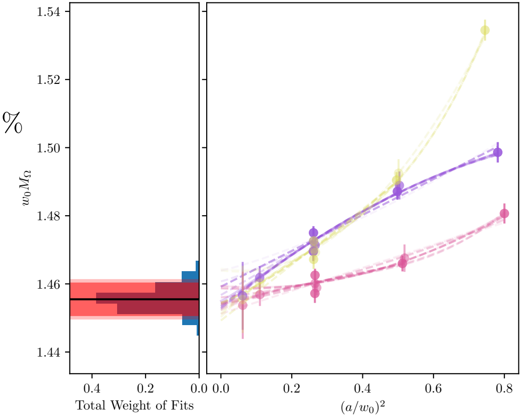


- **Longer term**:
  - Complete lattice calculation of QED contributions.
  - Light-quark tail using two-pion operators.

Thank you

Updated scale setting

- New scale setting based on M_Ω[12]
  - w_0 fm $= 0.17187(68)$, $w_0/a < 0.05\%$
  - $M_\Omega(\mathrm{QCD}) = 1671.01(43)$ MeV, $aM_\Omega < 0.2\%$
- Quantification of:
  - QED effects
  - Quark-mass dependence
  - Finite volume
  - w_0/a definition
- Updated cross-check with f_π determination soon.



[12] High-Precision Scale Setting with the Omega-Baryon Mass and Gradient Flow, Phys. Rev. D 113 (2026) 5, 054501, arXiv:2509.14367

Continuum Fits

$$a_\mu^{qq}(a, \{M_A\}) = a_\mu^{qq}\left(1 + F^{\text{disc.}}(a) + F^M(\{M_A\})\right), \tag{1}$$

where

$$F^a_{\text{local}}(a) = C_{a^2,n}(a\Lambda)^2\alpha_s^n + \sum_{k=2}^{4} C_{a^{2k}}(a\Lambda)^{2k}, \qquad \text{where } n = \{0\}, 1, 2, \tag{2}$$

$$F^a_{\text{one-link}}(a) = \left\{C_{a^2\log(a)}(a\Lambda)^2\log(a\Lambda)\right\} + \sum_{k=1} C_{a^{2k}}(a\Lambda)^{2k} \tag{3}$$

$$F^M(\{M_A\}) = C_{\text{sea}}\sum_{A=\pi,K,D_s}\delta M_A^2, \qquad \delta M_A^2 = \frac{M^2_{A,\,\text{phys.}} - M^2_{A,\,\text{latt.}}}{M^2_{A,\,\text{phys.}}}. \tag{4}$$

Chiral-Continuum Fits

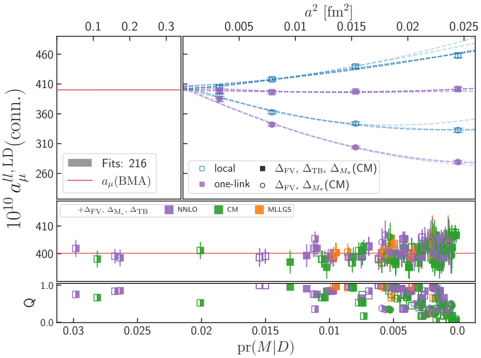
$$a_\mu\left(a, M_\pi\right) = a_{\mu,0}\left[b_{\mathrm{L}}(a)h_{\mathrm{L}}\left(M_\pi\Lambda_M\right) + \sum_{i=-1}^{2} b_i(a)h_i\left(M_\pi\Lambda_M\right)\right] \tag{5}$$

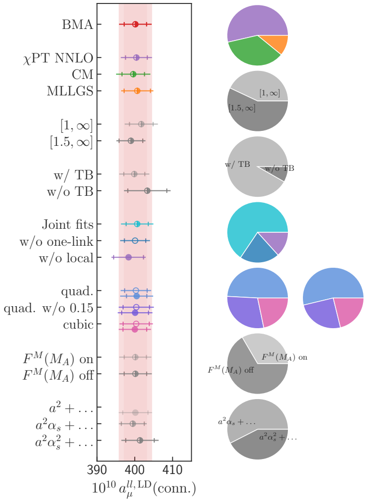
$$b_i(a) = \begin{cases} 1 + \sum_{j=1}^{4} c_{0j}\left(a\Lambda_a\right)^{2j}, & i = 0, \\ d_{i00} + \sum_{j=1}^{4} c_{ij}\left(a\Lambda_a\right)^{2j}, & \text{else}, \end{cases} \tag{6}$$

$$c_{ij} = \sum_{k=0}^{4} d_{ijk}\alpha_s^k, \tag{7}$$

$$h_i(y) = \begin{cases} \log y^2, & i = \mathrm{L}, \\ y^{2i}, & \text{else}, \end{cases} \tag{8}$$

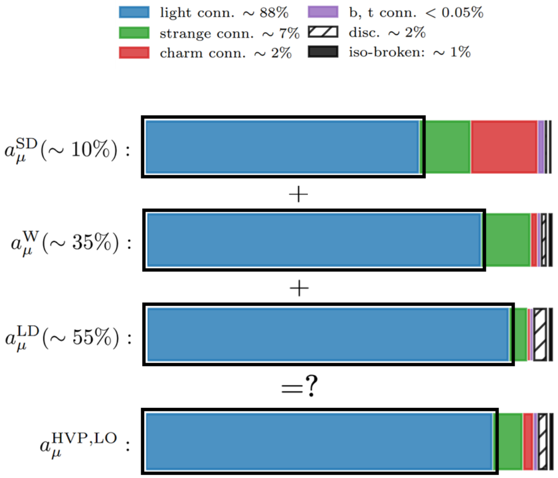
Light quark: Long-distance window (arXiv:2412.18491)





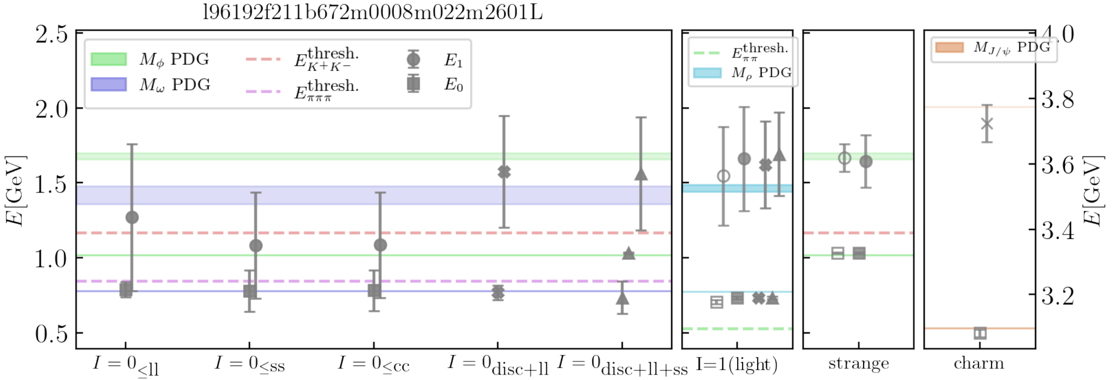
- Only four spacings (not enough statistics at 0.04).
- ‘Taste-breaking corrections’ to remove long-distance discretization effects (LQCD formulation specific).

$a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$ (arXiv:2412.18491)

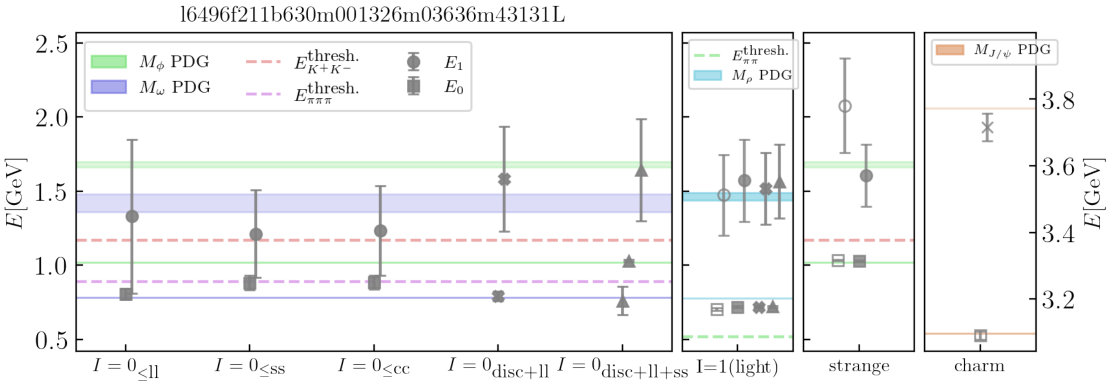


- Full integrand from: $a_\mu^{ll,\,\rm SD} + a_\mu^{ll,\,\rm W} + a_\mu^{ll,\,\rm LD}$
  - including stat. and syst. correlations.
- Performed independent analysis of $a_\mu^{ll,\,\rm Full}$
  - Have shown: $a_\mu^{ll,\,\rm Full} - \left(a_\mu^{ll,\,\rm SD} + a_\mu^{ll,\,\rm W} + a_\mu^{ll,\,\rm LD}\right) = 0$

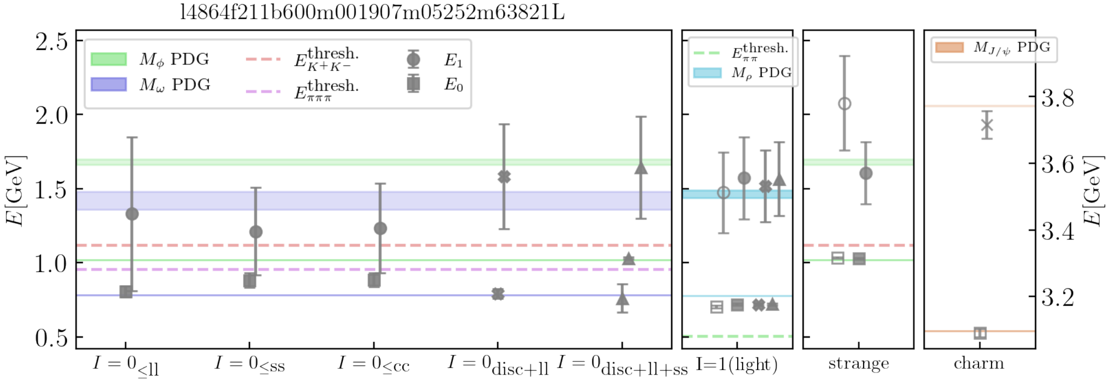
$I = 0$ spectrum (0.06 fm) — full detail



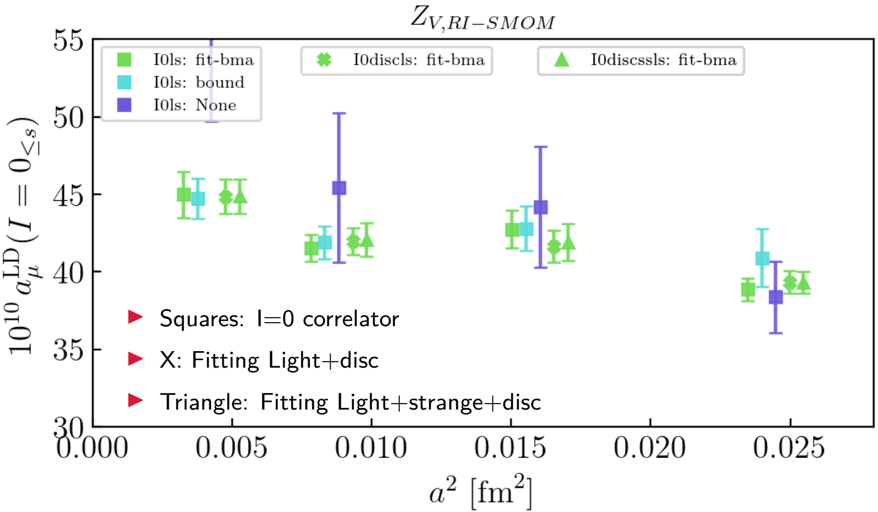
$I = 0$ spectrum (0.09 fm) — full detail



$I = 0$ spectrum (0.12 fm) — full detail

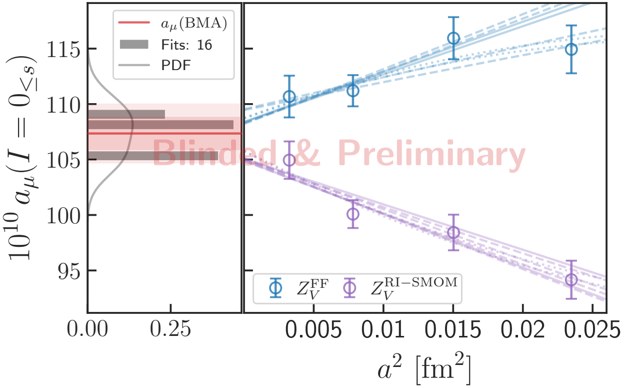


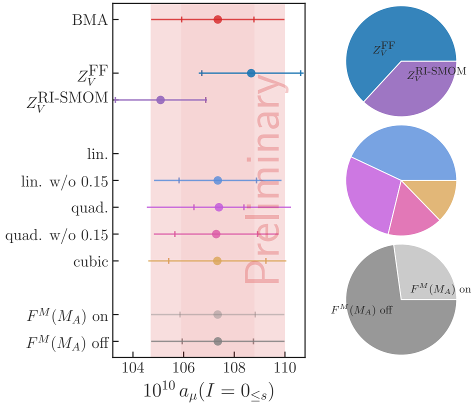
$I = 0$ noise reduction: fit vs. bounding



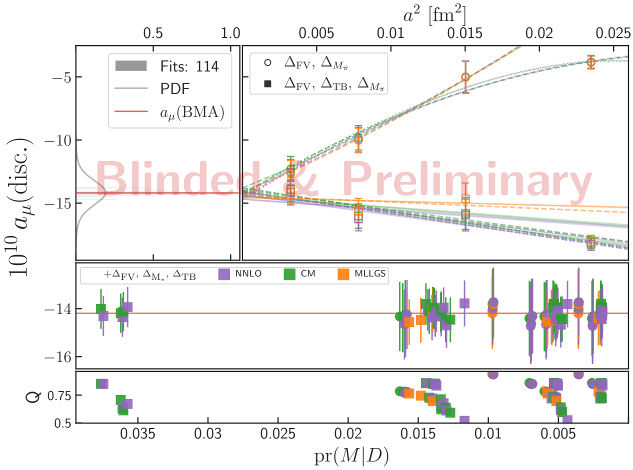
- Squares: I=0 correlator
- X: Fitting Light+disc
- Triangle: Fitting Light+strange+disc

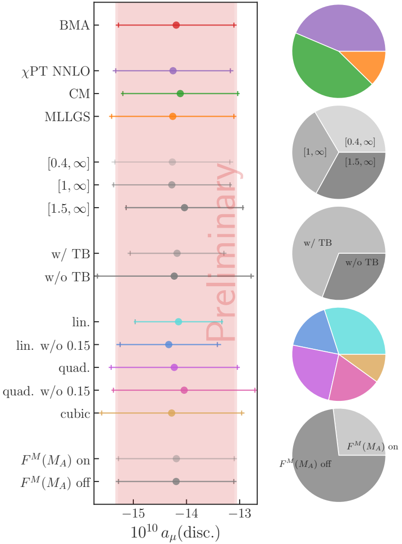
$I = 0$ continuum extrap. & model average





Disconnected continuum extrap. & model average

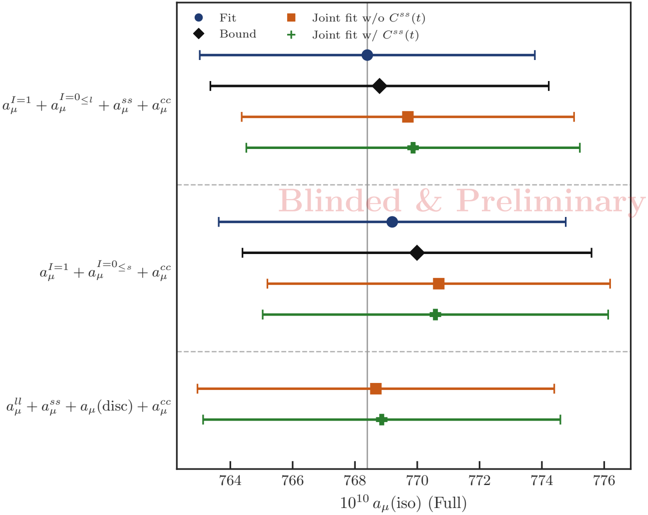




$a_\mu(\mathrm{iso})$ cross-checks

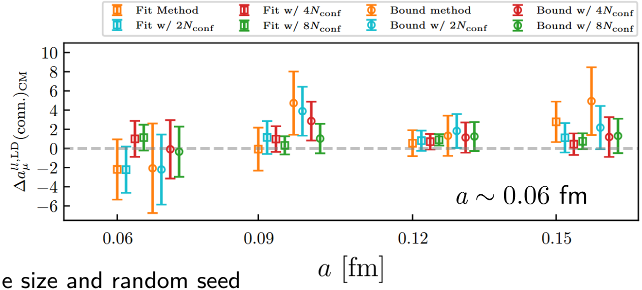
Cross-check: $a_\mu(\mathrm{iso}) = a_\mu^{I=0} + a_\mu^{I=1}$ consistent across every construction of the $I = 0$ piece:

- Constructing $a_\mu^{I=0}$ from $C^{I=0}$ w/ or w/o strange
- Constructing $a_\mu^{I=0}$ flavour components on the lattice
- Summing flavour components in the continuum



Fake data test

- Generate spectrum from EFT and EFT-inspired models (CM, MLLGS)
- Draw N_{conf} samples using (real) data covariance
- Reconstruct correlator with fit or bounding method

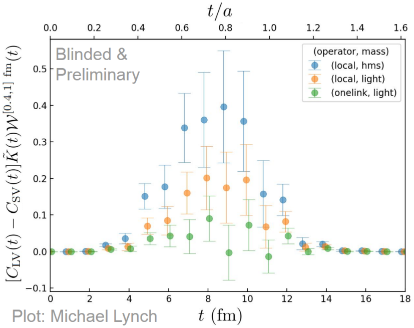


- Varied sample size and random seed
- Fit method consistent with truth
- Bounding method less robust to stat fluctuations

See [2412.18491] for details

FV Study

- Currently: FV effects estimated (χPT, CM, MLLGS, HP)
- Upcoming: Lattice FV determination with large volume (L$\sim$11 fm) ensemble at a$\sim$0.09 fm
- Data generation ongoing: light (local and one-link), "harmonic mean square[13]" mass (local)



- Light quark connected integrand on W
- Current number of configs (LMA):
  - $\sim$70 large volume (LV)
  - $\sim$1000 small volume (SV)

[13] S. Borsanyi et al., Nature, 593.7857, 51–55 (2021).

Backup: scale-setting scheme comparison

Light-quark connected $a_\mu^{\text{HVP, LO}} \times 10^{10}$ in three scale schemes (arXiv:2412.18491)

	Results			Correlated differences		
	M_Ω	f_π	WP25 w_0	Δ_{M_Ω,f_π}	Δ_{M_Ω,w_0}	Δ_{f_π,w_0}
$a_\mu^{ll,\,\mathrm{SD}}$	$48.139(92)$	$48.126(101)$	$48.115(109)$	$0.013(78)$	$0.024(83)$	$0.011(80)$
$a_\mu^{ll,\,\mathrm{W}}$	$206.90(63)$	$206.94(49)$	$207.40(42)$	$-0.03(43)$	$-0.50(42)$	$-0.47(27)$
$a_\mu^{ll,\,\mathrm{LD}}$	$400.2(4.3)$	$396.6(4.0)$	$401.3(3.8)$	$3.6(3.9)$	$-1.2(3.6)$	$-4.7(3.3)$
a_μ^{ll}	$655.2(4.5)$	$651.7(4.1)$	$656.9(3.9)$	$3.5(4.0)$	$-1.6(3.7)$	$-5.2(3.3)$