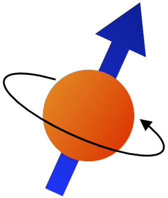
Recent Progress on the Hadronic Vacuum Polarization Contribution from RBC/UKQCD

Julian Parrino

Universität Regensburg



9th Plenary Workshop of the Muon (g-2) Theory Initiative

University of Connecticut, 5th July, 2026

The RBC & UKQCD collaborations

BNL & RBRC
Peter Boyle
Taku Izubuchi
Chulwoo Jung
Christopher Kelly
Patrick Oare
Amarjit Soni (emeritus)
Xin-Yu Tuo
Tianle Wang
Shuhei Yamamoto

CERN
Matteo Di Carlo
Felix Erben
Andreas Jüttner (Southampton)

Columbia University
Ryan Abbott
Norman Christ
Jonah Eick
Rajiv Eranki
Sarah Fields
Ceran Hu
Yikai Huo
Erik Lundstrum
Robert Mawhinney

University of Connecticut
Thomas Blum
Jonas Hildebrand
Luchang Jin
Vaishakhi Moningi
Anton Shcherbakov
Douglas Stewart
Joshua Swain

DESY Zeuthen
Raoul Hodgson

University of Edinburgh
Matthew Black
Luigi Del Debbio
Vera Gülpers
Maxwell T. Hansen
Nils Hermansson Truedsson
Ryan Hill
Rajnandini Mukherjee
Antonin Portelli
Teseo San Jose

Johannes Gutenberg University of Mainz
Alessandro Barone

KEK
Shigemi Ohta (emeritus)

Liverpool Hope
Nicolas Garron (Liverpool)

University of Liverpool
Tobias Tsang
Aurelia Zuchanke

Massachusetts Institute of Technology
En-Hung Chao

University of Milano Bicocca
Francesca A. Bresciani
Mattia Bruno
Gabriele Morandi

Peking University
Xu Feng

University of Regensburg
Daniel Knüttel
Christoph Lehner
Julian Parrino
Andreas Völklein
Haoyu Wang

RIKEN R-CCS
Masaaki Tomii

University of Siegen
Oliver Witzel

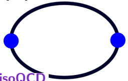
University of Southampton
Bipasha Chakraborty
Ahmed Elgaziari
Jonathan Flynn
Ollie Heald
Joe McKeon
Chris Sachrajda

Stony Brook University
Sergey Syritsyn (RBRC)

HVP diagram uncertainties normalized by muon $(g-2)$ target accuracy

isoQCD

(c)



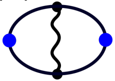
SD: 1.16
ID: 0.57
LD: 3.40
RBC/UKQCD 2023/2024

(d)

SD: <0.01
ID: <0.04
LD: <2.79
RBC/UKQCD 2018

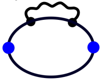
QED isospin breaking

(V)



SD: <0.01
ID: 0.01
LD: 0.43

(S)

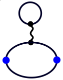


SD: <0.01
ID: 0.03
LD: 3.64

(F)

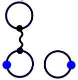
SD: <0.01
ID: <0.01
LD: 0.21

(T)



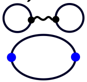
SD: <0.01
ID: <0.01
LD: 0.12

(Td)



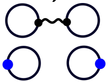
SD: <0.01
ID: <0.01
LD: 0.25

(D2)



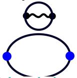
SD: <0.01
ID: <0.01
LD: 0.04

(D2d)



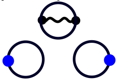
SD: <0.01
ID: <0.01
LD: <0.01

(D1)



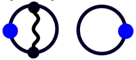
SD: ?
ID: ?
LD: ?
→ Talk by Christoph Lehner

(D1d)



SD: ?
ID: ?
LD: ?
→ Talk by Christoph Lehner

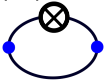
(D3)



SD: <0.01
ID: 0.09
LD: 51.91

Strong isospin breaking

(M)

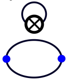


SD: <0.01
ID: 0.03
LD: 0.48

(O)

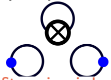
SD: <0.01
ID: 0.04
LD: 16.54

(R)



SD: <0.01
ID: <0.01
LD: 0.04

(Rd)



SD: <0.01
ID: <0.01
LD: 0.04

Current Status of HVP Calculation

- For intermediate-distance window, target precision is already reached
- For short-distance window, IB corrections at target precision. But for isoQCD result improve continuum limit $\rightarrow$ New ensemble 128I
- Strategy for IB corrections in long-distance window
  - Long-distance reconstruction in QED+QCD [Lehner, JP, Völklein, Phys.Rev.D 113 (2026) 5, 054503]

Long-distance reconstruction of QED corrections to the hadronic vacuum polarization for the muon g-2

C. Lehner,[1,*] J. Parrino,[1] and A. Völklein[1]

[1] *Fakultät für Physik, Universität Regensburg, Universitätsstraße 31, 93040 Regensburg, Germany*

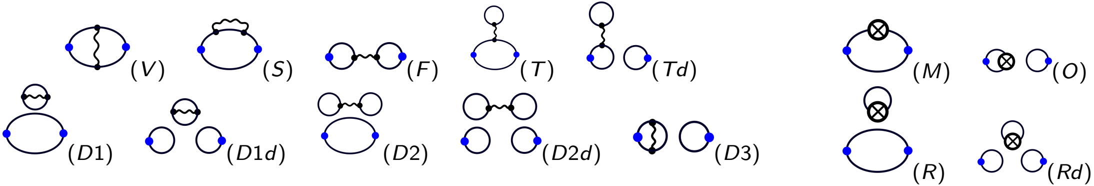
(Dated: September 1, 2025)

  - New noise reduction sampling strategies (Talk by Christoph Lehner, We 4:45 pm)
- Reuse IB diagrams for final-state corrections in τ decay

Topics

1. Current status of the RBC/UKQCD HVP calculation
2. Update on the isospin breaking corrections to the HVP contribution
    1. Complete calculation of all QED and SIB diagrams at the physical point
    2. Valence mass study for the leading SIB diagram (M)
    3. Determination of the quark mass shifts from IB effects in pion and kaon
    4. Strange component for diagrams (V), (S) and (F)
3. Reconstructing the HVP above a timelike cut in lattice QCD
    1. Necessary for fully inclusive determination of HVP from τ decay
    2. Application of lattice inverse methods
    3. First results for intermediate-distance window observable

Isospin Breaking Corrections: Status



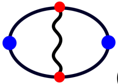
- Computed correlators for all QED and SIB diagrams at the physical point on 48I ensemble ($a^{-1} = 1.7312(28)$ GeV, $m_\pi L = 3.9$) using stochastic coordinate sampling

  [Bruno et al., arxiv:2602.24132]
- Calculate for all necessary external operators, e.g. $\{(\gamma_i, \gamma_i), (\gamma_5, \gamma_5)\}$
- Take into account all necessary flavor combinations, e.g. light, strange, light-strange, ...

Isospin Breaking Corrections: Stochastic Coordinate Sampling (SCS)

Use stochastic coordinate sampling of the all-to-all propagator, e.g. for diagram (V):

$$\int_{x,y,\boldsymbol{z}} G_{\nu\rho}(x,y)\Big\langle \mathrm{Tr}\Big[\gamma_5 S(z,x)\gamma_\rho S(x,(\mathbf{0},t))\gamma_5 S((\mathbf{0},t),y)\gamma_\nu S(y,z)\Big]\Big\rangle_U =$$



(V)

- Propagators are calculated on stochastic subset of points (Introduced in calculation of hadronic light-by-light contribution [Blum et al. , 1510.07100])
- Source positions: $O(10^3)$; Sink positions: $O(V/16)$
- For diagram (F), no sparsening in sink
- For diagram (S), (M), (D3) long-distance contribution with SCS too noisy for target accuracy

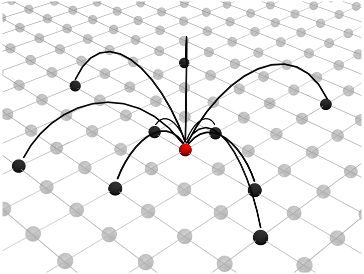


Fig. 1: Propagator for (red) source point to several (black) sink points

Isospin Breaking Corrections: Valence Mass Dif. Diagram (M)

Instead of SCS use different approach for diagram (M)

- Obtain diagram (M) from difference of leading order connected diagram (c) for two different valence quark masses on ensemble 48I

$$(c)_m = \frac{1}{3}\sum_{\boldsymbol{x},i}\langle \mathrm{Tr}[D_m^{-1}(0;\boldsymbol{x},t)\gamma_i D_m^{-1}(\boldsymbol{x},t;0)\gamma_i]\rangle_U$$

$$\begin{aligned}(M) &= \frac{1}{2}\frac{d}{dm}(c)_m\Big|_{m=m_l} \\ &\approx \frac{1}{2}\frac{(c)_{m_l}-(c)_{m_1}}{m_l-m_1}\end{aligned}$$

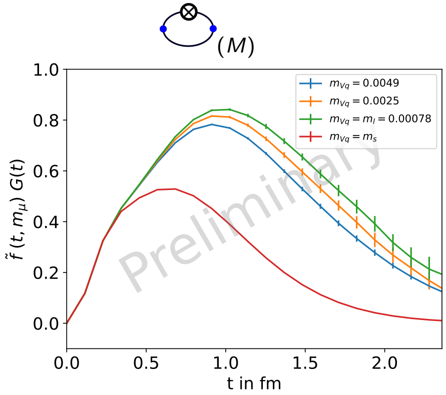


Fig. 2: Blinded HVP integrand for different valence quark masses

Isospin Breaking Corrections: Valence Mass Dif. Diagram (M)

Comparison on 48I ensemble between

- Finite difference

$$(M)^{\text{fin. dif.}} = \frac{1}{2}\frac{(c)_{m_l} - (c)_{m_1}}{m_l - m_1}$$

- Scalar insertion (SCS)

$$(M)^{\text{sca. ins.}} = \frac{1}{3}\sum_{\boldsymbol{x},i}\langle \mathrm{Tr}[D_m^{-1}(0;y)\mathbf{1}D_m^{-1}(y;\boldsymbol{x},t)\gamma_i D_m^{-1}(y,\boldsymbol{x},t;0)\gamma_i]\rangle_U$$

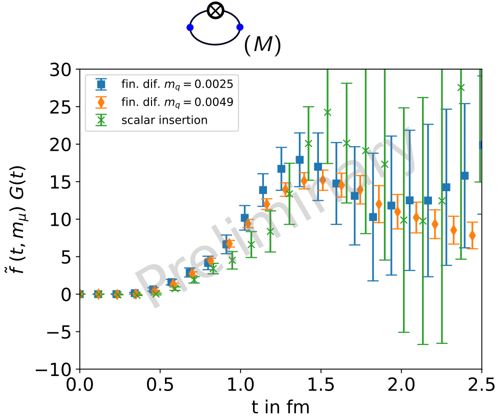


Fig. 3: Comparison of diagram (M) blinded integrand

Isospin Breaking Corrections: Quark Mass Shifts

- Use physical hadron mass shifts masses to set scale PDG 2024 [PhysRevD.110.030001]

$$\begin{aligned} m^0_{\pi^0} + \Delta m_{\pi^0} &= m^{phys}_{\pi^0} = 134.9768(5)\ \text{MeV}, \\ m^0_{K^0} + \Delta m_{K^0} &= m^{phys}_{K^0} = 497.611(13)\ \text{MeV}, \\ \Delta_K &= \Delta^{phys}_K = (m_{K^0} - m_{K^+})^{phys} = 3.9340(2)\ \text{MeV}, \\ m_\Omega &= m^{phys}_\Omega = 1672.45(29)\ \text{MeV} \end{aligned}$$

- The masses in isoQCD $m^0_{\pi^0}, m^0_{K^0}, m^0_{K^+}$ are scheme dependent
- Electromagnetic coupling does not renormalize at leading order

$$(\alpha_{\text{QED}})^{-1} = 137.035999177(21)$$

- IB corrections to Ω^- are found to be below lattice spacing uncertainty (RBC/UKQCD 2018)

Isospin Breaking Corrections: Quark Mass Shifts

- For the Kaon mass shift, one obtains

$$\begin{aligned}\Delta_K^{phys} &= \Big[e^2(Q_d - Q_u)Q_s\Delta m_K^{(V)} \\ &\quad + e^2(Q_d^2 - Q_u^2)\Delta m_K^{(S)} \\ &\quad + e^2(Q_u - Q_d)\sum_f \Delta m_K^{(T)_f} \\ &\quad + (\Delta m_d - \Delta m_u)\Delta m_K^{(M)}\Big] + \Delta m_K^{FV}\end{aligned}$$

- Apply universal FV correction Δm_K^{FV}
- Obtain, e.g. $\Delta m_K^{(V)}$ from a fit to the ratio $(V)/(c)$
- Solve for $(\Delta m_d - \Delta m_u)$

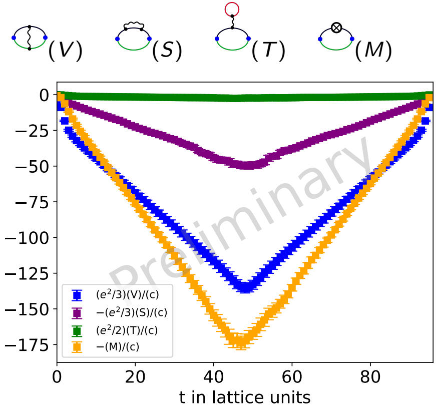


Fig. 4: Comparison of the correlator ratios to compute Δ_K

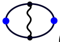
Isospin Breaking Corrections: Quark Mass Shifts

- Determine $(\Delta m_d + \Delta m_u)$ from fixing $m^0_{\pi^0} + \Delta m_{\pi^0} = m^{phys}_{\pi^0}$
- Determine Δm_s from $m^0_{K^0} + \Delta m_{K^0} = m^{phys}_{K^0}$
- The values for the quark mass shifts $\Delta m_u, \Delta m_d, \Delta m_s$ depend on the isoQCD scheme
- In the WP25 scheme, we find

$$\left| \frac{(\Delta m_d + \Delta m_u)}{(\Delta m_u)} \right| < 0.01$$

- We obtain for the quark mass ratio on the 48I ensemble

$$\frac{m_u}{m_d} = 0.455 \pm 0.0123$$

- Additionally, obtain pion mass splitting from  (V)  (F)

$$M_{\pi^+} - M_{\pi^0} = 4.31 \pm 0.113\,\mathrm{MeV}$$

agrees with earlier determination on 48I [Feng, Jin, Riberdy, 10.1103/PhysRevLett.128.052003]

Isospin Breaking Corrections: Strange Components

- Computation of strange components for leading diagrams (V), (S), (F) at the physical point
- Strange connected is however suppressed with charge factor 1/17 compared to light
- For diagram (F), strange-light, light-strange and strange-strange components are negligible compared to light-light, even without suppression from charge factor

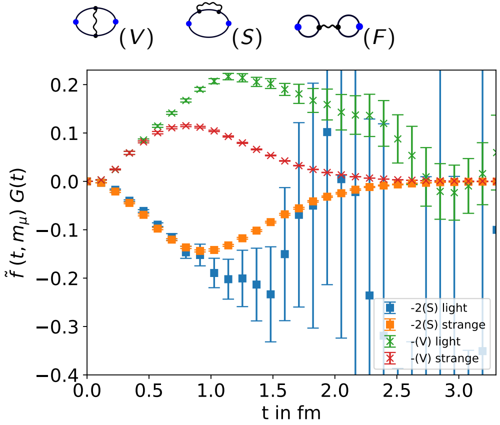


Fig. 5: Comparison of light and strange connected contributions.

Preliminary Results: Leading QED Contributions

Comparison of partial sum, with blinded TMR kernel on ensemble 48I

- Partial sums for diagrams (V) and (F) reach plateau
- Tail of diagram (S) affected by noise $\rightarrow$ QED+QCD Long-distance reconstruction
- Up to 2 fm, uncertainty of (S) is still below target uncertainty

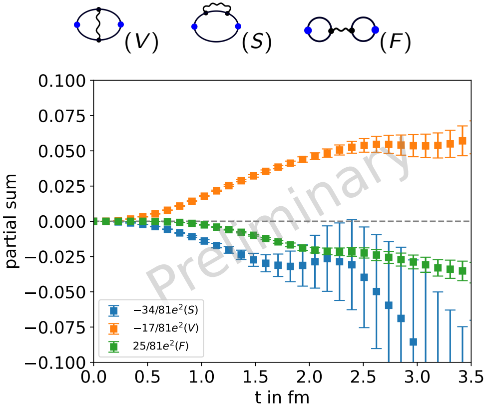


Fig. 6: Leading QED IB diagrams (V), (S), (F)

Leading QED Contributions: QED+QCD Long-distance Reconstruction

- Long-distance reconstruction of $\pi^0\gamma$ and ρ state on ensemble 4 ($m_\pi = 280$ MeV)
- For application at physical point, it is necessary to include $\pi\pi\gamma$ state

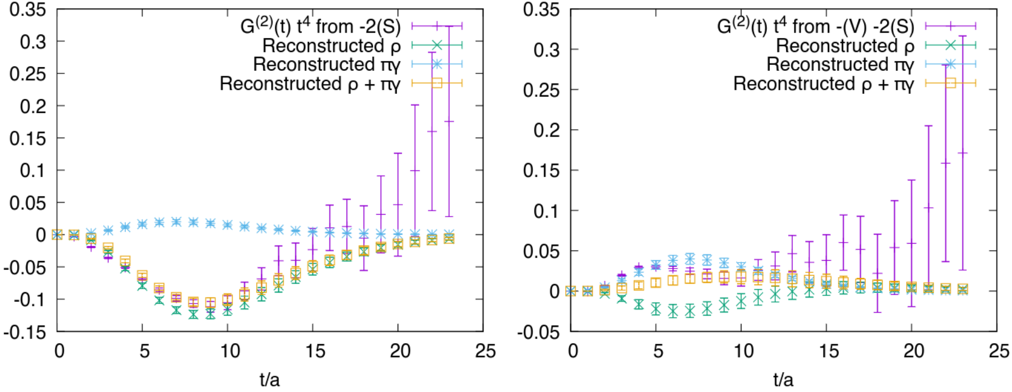


Fig. 7: [Lehner, JP, Völklein, Phys.Rev.D 113 (2026) 5, 054503]

Preliminary Results: SIB Contributions

Comparison of partial sum, with blinded TMR kernel on ensemble 48I

- Partial sums for diagrams (M), (R) and (Rd) reach plateau, tail of diagram (O) still noisy (above 2fm)
- Cancellation between (M) and (O) predicted by LO χPT [Lehner, Meyer, 10.1103/PhysRevD.101.074515]
- Since (R) and (Rd) multiply $(\Delta m_d + \Delta m_u)$ contribution is negligible

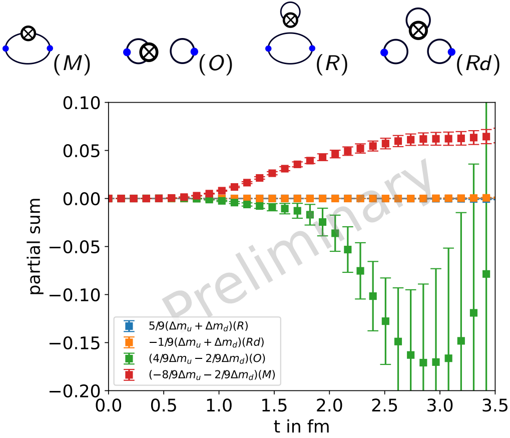


Fig. 8: SIB diagrams (M), (O), (R), (Rd)

Preliminary Results: Subleading QED Contributions

Comparison of partial sum, with blinded TMR kernel on ensemble 48I

- Partial sums for diagrams (T), (Td), (D2) reach plateaus
- Contribution well below leading contributions
- For diagrams (D1), (D1d), (D3) developing new noise reduced sampling strategy (Talk by Christoph Lehner)

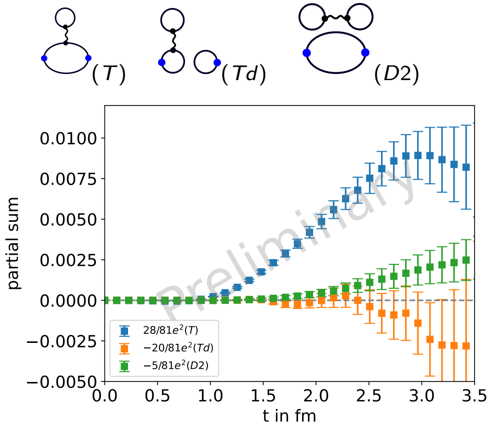
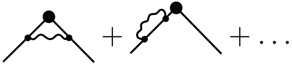


Fig. 9: Leading IB diagrams (T), (Td), (D2)

Isospin Breaking Corrections: Outlook

- Apply noise reduction techniques for diagrams (D1), (D1d), (D3)
- Perform long-distance reconstruction at the physical point
- Calculation of QED corrections to renormalization factor for local vector current Z_V in RI/SMOM scheme (Talk by Mattia Bruno, Tu, 2:25PM)



  or using PCAC relation, assuming $Z_A = Z_V$ for Domain-Wall fermions
- Additional lattice spacing to constrain cutoff effects
- Additional volume to constrain FV effects, already done for diagram (F)

Topics

1. Current status of the RBC/UKQCD HVP calculation
2. Update on the isospin breaking corrections to the HVP contribution
   1. Complete calculation of all QED and SIB diagrams at the physical point
   2. Valence mass study for the leading SIB diagram (M)
   3. Determination of the quark mass shifts from IB effects in pion and kaon
   4. Strange component for diagrams (V), (S) and (F)
3. Reconstructing the HVP above a timelike cut in lattice QCD
   1. Necessary for fully inclusive determination of HVP from τ decay
   2. Application of lattice inverse methods
   3. First results for intermediate-distance window observable

Reconstructing the HVP Above a Timelike Cut

- For a fully inclusive determination of HVP from τ-decay [Bruno et al, 2607.00564], [Allen et al, 2605.12205], it is necessary to estimate the contribution above the τ mass

$$a_\mu^{>}(\omega_{\text{cut}}) = \int_{m_\pi^2}^{\infty} d\omega^2 K(\omega) R(\omega^2) \theta(\omega - \omega_{\text{cut}})$$

- Another window for scrutinizing the difference of the HVP from lattice QCD and dispersive approach

- Can be evaluated in perturbative QCD, but its $\omega_{\text{cut}} = m_\tau$ is just at the threshold where pQCD can be safely applied

Reconstructing the HVP Above a Timelike Cut: Inverse Problem

$$a_\mu^{>}(\omega_{\text{cut}}) = \int_{m_\pi^2}^{\infty} d\omega^2 K(\omega) R(\omega^2) \theta(\omega - \omega_{\text{cut}})$$

- Can be obtained in a hybrid approach
- From isovector lattice QCD correlator

$$G^W(t) = \frac{1}{2}(c)(t)$$

- And using the e^+e^- data to estimate systematic uncertainty from reconstruction

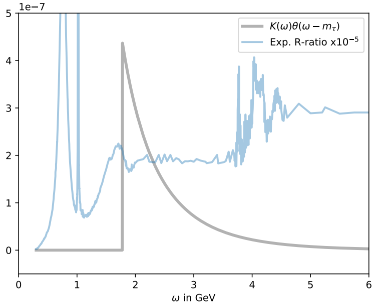


Fig. 10: Timelike kernel for $a_\mu^{>}(\omega_{\text{cut}})$

Reconstructing the HVP Above a Timelike Cut: Inverse Problem

- Define target kernel

$$T(\omega) = K(\omega)\theta(\omega - \omega_{\text{cut}})$$

- We approximate this kernel by

$$\widehat{T}^{\Delta,\lambda}(\omega) = \sum_n q_n^{\Delta,\lambda} e^{-\omega t_n}$$

- The approximated observable is then obtained by

$$a_\mu^{>}(\omega_{\text{cut}}) = \sum_n q_n^{\Delta,\lambda} G^W(t_n)$$

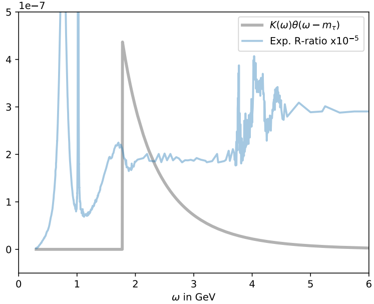


Fig. 11: Timelike kernel for $a_\mu^{>}(\omega_{\text{cut}})$

Reconstructing the HVP Above a Timelike Cut: Inverse Problem

$$a_\mu^>(\omega_{\mathrm{cut}}) = \sum_n q_n^{\Delta,\lambda} G^W(t_n)$$

- To calculate the coefficients $q_n^{\Delta,\lambda}$, we employ a HLT inspired method to approximate the sharp target kernel $T(\omega)$ by a smeared kernel $\widehat{T}^{\Delta,\lambda}(\omega)$.
- Δ : smearing width,
  λ: regulator for stat. noise
- As systematic uncertainty, consider

$$\int_{\omega_0}^{\infty} d\omega \left[\widehat{T}^{\Delta,\lambda}(\omega) - T(\omega)\right] R[e^+e^- \to had.](\omega^2)$$

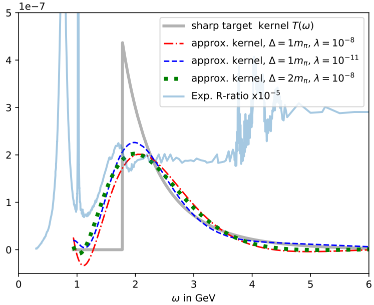


Fig. 12: Approximated kernel for $a_\mu^>(\omega_{\mathrm{cut}})$

Reconstructing the HVP Above a Timelike Cut: Pion-state Subtraction

- Balance systematic and statistical uncertainties by tuning λ, Δ
- Subtract two-pion states below 1 GeV obtained from GEVP, to increase stability

$$G^{W,sub}(t) = G^{W}(t) - \sum_{n=0}^{N} V_n e^{-E_n t}.$$

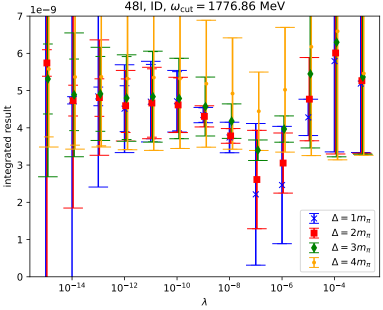


Fig. 13: Using $G^{W}(t)$ for reconstruction

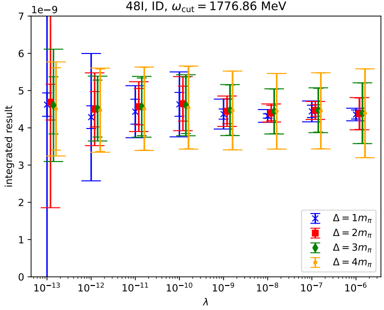


Fig. 14: Using $G^{W,sub}(t)$ for reconstruction

Reconstructing the HVP Above a Timelike Cut

- Balance systematic and statistical uncertainties by tuning λ, Δ

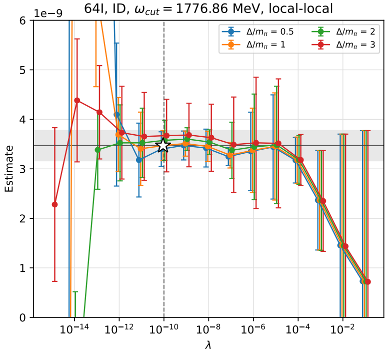


Fig. 15: Selection of λ, Δ

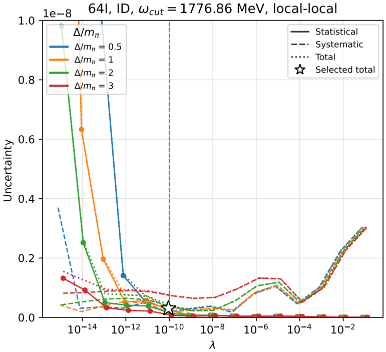


Fig. 16: Comparison of systematic and statistical uncertainty

Reconstructing the HVP Above a Timelike Cut: Continuum Extrapolation

- Focus on intermediate-distance window contribution above a timelike cut $a_\mu^{>,ID}(\omega_{\text{cut}} = m_\tau)$ first
- Analysis on 4 physical point ensembles, with 3 different lattice spacings (48I, 64I, 96I, C)
- Continuum extrapolation for local-local and local-conserved discretization using AIC model averaging

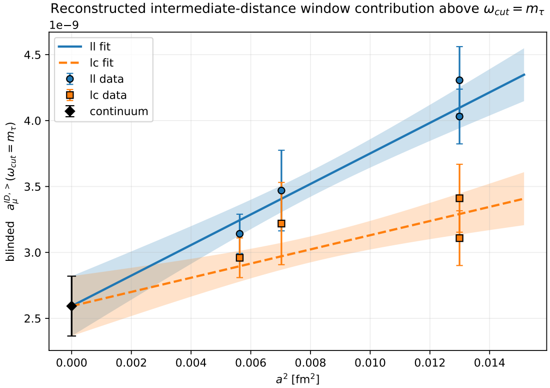


Fig. 17: Continuum extrapolation for ID window (blinded)

Summary and Outlook

Isospin breaking corrections to HVP:

- First principle calculation of all diagrams contributing to HVP at $O(e^2, \frac{m_u - m_d}{\Lambda_{QCD}})$
- IB effects for SD and ID windows already compatible with target precision
- LD window: Most diagrams already at target precision
- For some diagrams still work to do: long-distance reconstruction or more efficient sampling

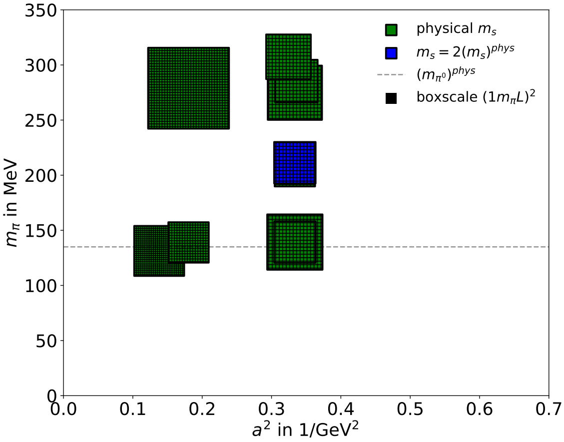
Reconstructing HVP above a timelike cut

- Inverse Laplace transform obtained from lattice QCD
- Only syst. unc. depends on e^+e^- data
- Final checks before unblinding and publishing

Backup

Backup: Ensemble Overview

- $N_f = 2+1$ Möbius domain wall ensembles
- 4 ensembles at physical pion mass in use so far
- Ensembles up to volumes of $m_\pi L \approx 8$; also new physical pion ensemble with $L^3 = (11\mathrm{fm})^3$
- Generation of new finer physical pion mass ensemble ($128^3 \times 288$): a^2 GeV$^2 = 0.08$ and $m_\pi L = 4.9$



Backup: Tadpole fields

- For quark-disconnected diagram (d) but also for many QED diagrams, we need estimators of the tadpole field

$$T_x^{(f)} = \langle \psi_f(x) \bar{\psi}_f(x) \rangle$$

with quark flavor $f \in \{u, d, s\}$.

- Employ method of our first quark-disconnected physical pion mass paper [Blum et al., PRL116(2016)232002]:
  - Compute $T^{(ud)} - T^{(s)} = T^{(ud,low)} + T^{(ud,high)} - T^{(s)}$ with $T^{(ud,high)} = T^{(ud)} - T^{(ud,low)}$
  - Use multi-grid Lanczos [Clark, Jung, Lehner, 1710.06884] to compute exact low-mode field $T_x^{(ud,low)}$ over 2000-5000 lowest preconditioned Dirac modes
  - Estimate $T^{(ud,high)} - T^{(s)}$ using a random sparse Z_2 grid

Backup: Quark Mass Shifts

- To calculate $(\Delta m_d + \Delta m_u)$, we require

$$\begin{aligned} m_{\pi^0}^{phys} &= m_{\pi^0}^0 + \Big[- e^2(Q_u^2 + Q_d^2)/2\Delta m_\pi^{(V)} \\ &-e^2(Q_u^2 + Q_d^2)\Delta m_\pi^{(S)} - \Big(\Delta m_d + \Delta m_u\Big)\Delta m_\pi^{(M)} \\ &+(Q_u + Q_d)e^2 \sum_f Q_f \Delta m_\pi^{(T)} + (Q_u - Q_d)^2/2e^2\Delta m_\pi^{(F)}\Big] \end{aligned}$$

- Fix Δm_s, by

$$\begin{aligned} m_{K^0}^{phys} &= m_{K^0}^0 + \Big[- \Delta m_d \Delta m_K^{(M)} - Q_d Q_s e^2 \Delta m_K^{(V)} - Q_d^2 e^2 \Delta m_K^{(S)} \\ & e^2 Q_d \sum_f Q_f \Delta m_K^{(T)_f} + e^2(Q_s) \sum_f Q_f \Delta m_K^{(T_s)_f} - Q_s^2 e^2 \Delta m_K^{(S_s)} - \Delta m_s \Delta m_K^{(M)_s}\Big] \end{aligned}$$

Backup: Quark Mass Shifts

- Obtain the mass shift of a certain diagram, i.e. $\Delta m^{(V)}$ from fitting the ratio $(V)/(c)$ to

$$f(A, \Delta m^{(V)}, m^0) = A + \Delta m^{(V)}(T/2 - t)\tanh(m^0(T/2 - t)).$$

- Use finite volume correction $(P = \pi, K)$

$$\Delta(m_P^{FV}(L))^2 = e^2 m_P^2 \Big[\frac{c_2}{4\pi^2 m_P L} + \frac{c_1}{2\pi(m_P L)^2} - \frac{c_0}{(m_P L)^3}\Big(\frac{\langle r_P^2\rangle m_P^2}{3} + C\Big) + O\Big(\frac{1}{(m_P L)^4}\Big)\Big]$$