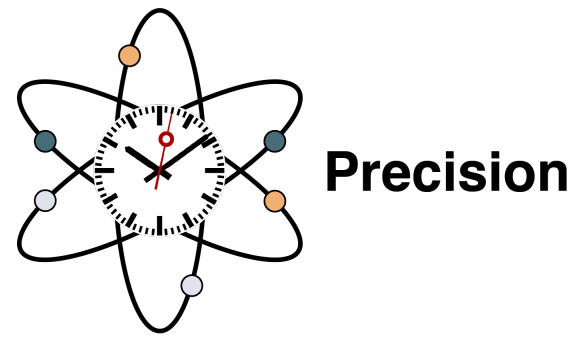




Towards HVP with C* Boundary Conditions

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[RC*: Altherr et al., 2506.19770 JHEP10(2025)158

[CERN lattice seminar](#), P. Tavella, Oct 28, 2025

[8th Plenary workshop of the Muon g-2 TI](#), P. Tavella, Sep 10, 2025,

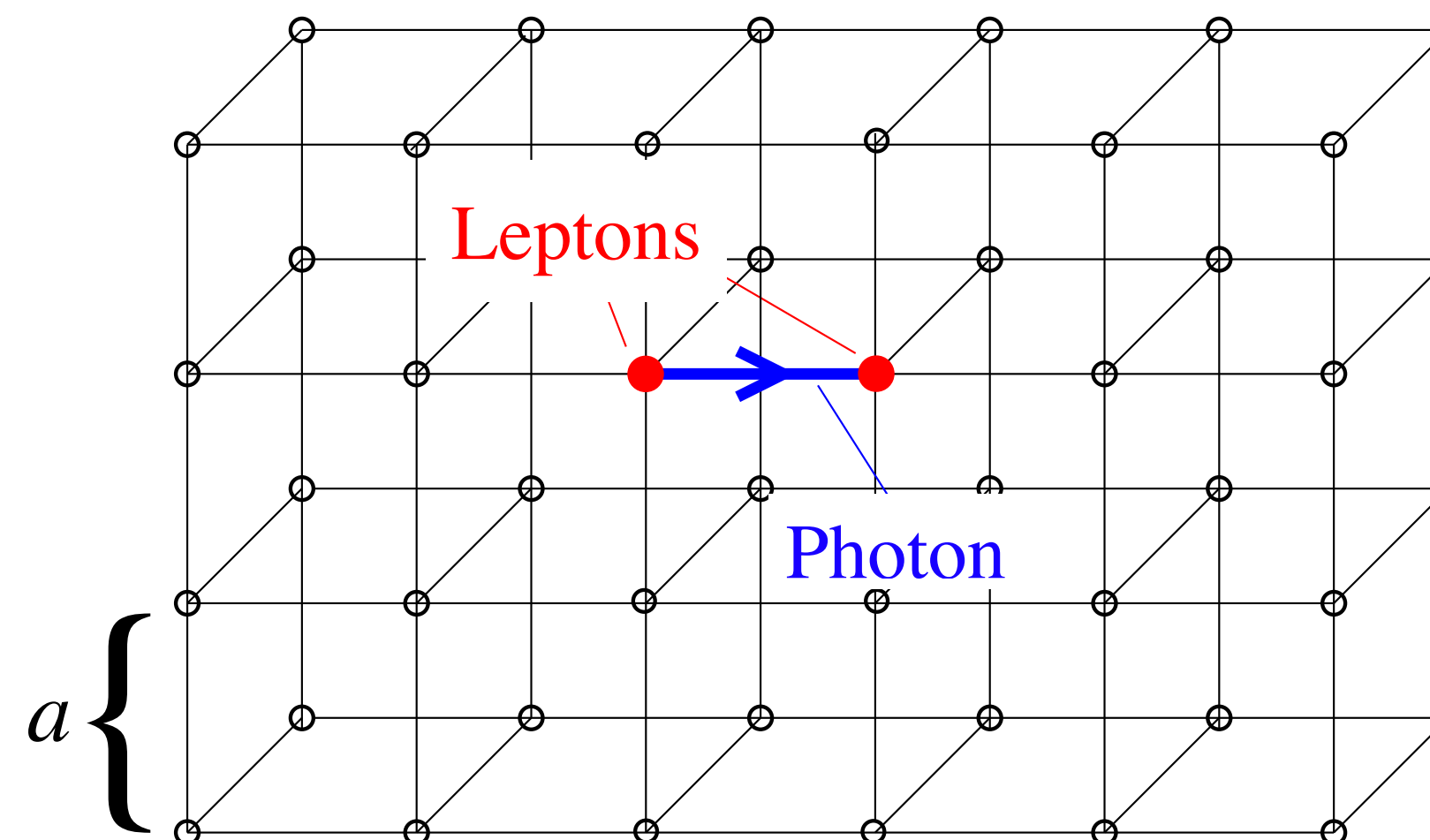
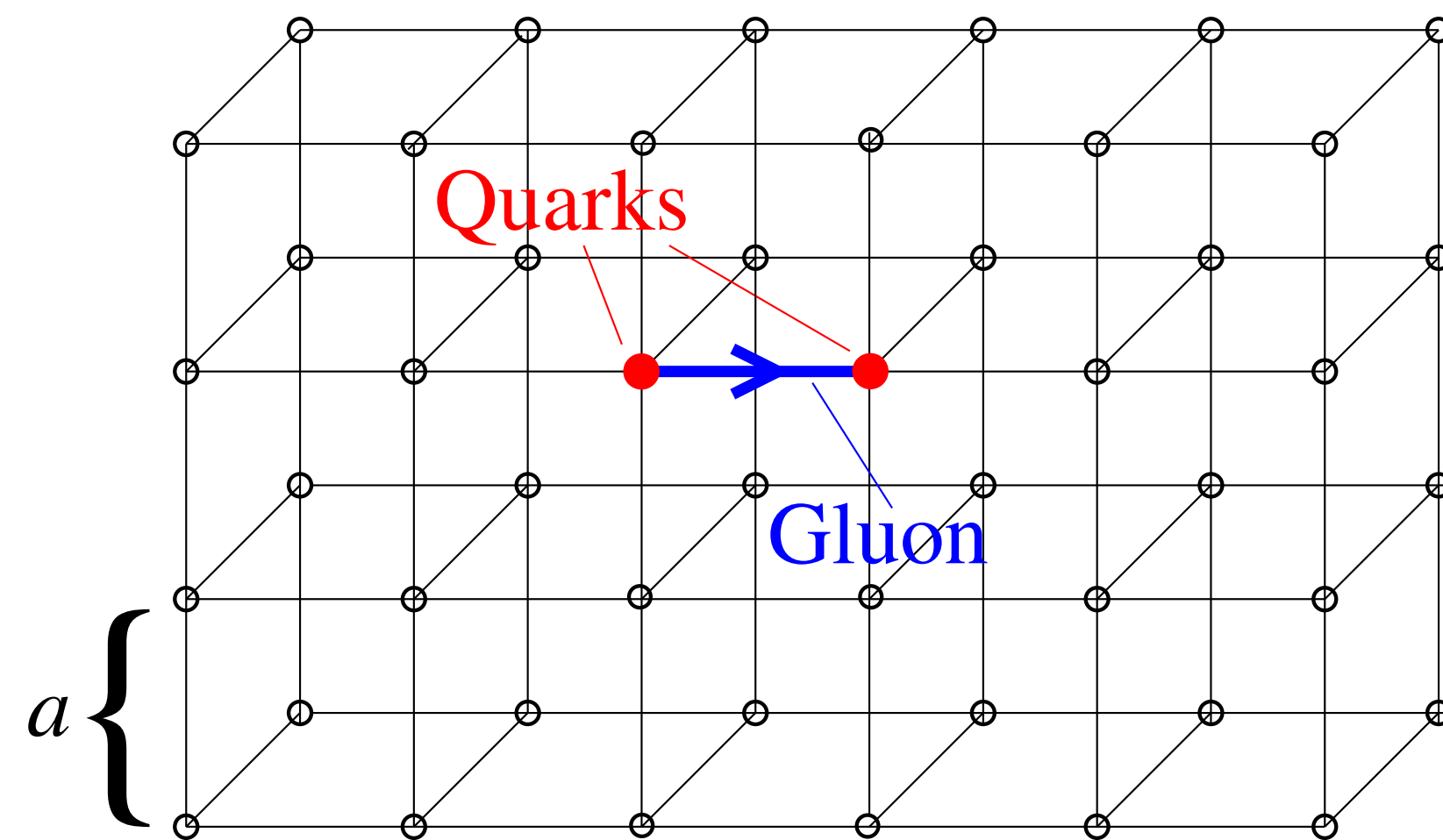
RC*: Alther et al. 2605.01085 PoS(LATTICE2025)198

Paola Tavella, Klemen Kersic @LAT2026]

Two approaches to HVP in QCD+QED by RC*

- ✦ Probing the SM through muon g-2 hadronic contributions [Aguillard et al., arXiv:2506.03069 Phys.Rev.Lett. 135 (2025)]
- ✦ IB corrections important next step for improved precision [WP25: Aliberti et al., 2505.21476 [hep-ph], Phys. Rept. 1143 (2025) 1]
- ✦ Compare the two approaches by computing a U-spin symmetric HVP-related observable [Altherr et al., 2506.19770 JHEP10(2025)158]
- ✦ Use C-periodic (C*) boundary conditions [Kronfeld and Wiese Nucl. Phys. B 357 (1991) 521, Lucini et al. JHEP02(2016)076]

2026 Breakthrough Prize in Fundamental Physics for the Muon g-2 experiment



$$P(U, A) \propto \sum_f \text{Pf}(C K D_f[U, A]) e^{S_g^{SU(3)}[U] + S_g^{U(1)}[A]}$$

❖ dynamical QCD+QED with C* bc's

❖ IsoQCD (C* bc's) + RM123

- ➡ expand about iso-symmetric theory [RM123: Divitiis et al. 1303.4896 PRD 87 (2013) 11]

RC*OR compute hadronic observables in full QCD+QED using C* boundary conditions as FV QED prescription [Lucini et al. 1509.01636 JHEP02(2016)076, Campos et al. 1908.11673 EPJC 80 (2020) 3, Bushnaq et al. 2209.13183 JHEP03, 012 (2023)]

QED_C approach to lattice QCD+QED

C[★] boundary conditions: preserve locality, gauge-invariance and translation-invariance at finite volume

$$A_\mu(x + L_k \hat{k}) = -A_\mu(x),$$
$$\psi(x + L_k \hat{k}) = C^{-1} \bar{\psi}(x)^\top,$$

$$U_\mu(x + L_k \hat{k}) = U_\mu^*(x),$$
$$\bar{\psi}(x + L_k \hat{k}) = -\psi^\top(x) C.$$

- zero mode of A_μ absent by construction .

Gauge action:

- Lüscher-Weisz discretization of the action for SU(3)
- Wilson plaquette action in the compact formulation for U(1)

Fermionic action:

$$S_F = - \sum_f a^4 \sum_x \frac{1}{2} \chi_f^\top(x) K C (D_{w,f} + \delta D_{sw,f} + m_f) \chi_f(x).$$

$$\chi = \begin{pmatrix} \psi \\ C^{-1} \bar{\psi}^\top \end{pmatrix}, \quad K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

QED_C approach to lattice QCD+QED

C^{*} boundary conditions: preserve locality, gauge-invariance and translation-invariance at finite volume

$$A_\mu(x + L_k \hat{k}) = -A_\mu(x),$$

$$\psi(x + L_k \hat{k}) = C^{-1} \bar{\psi}(x)^\top,$$

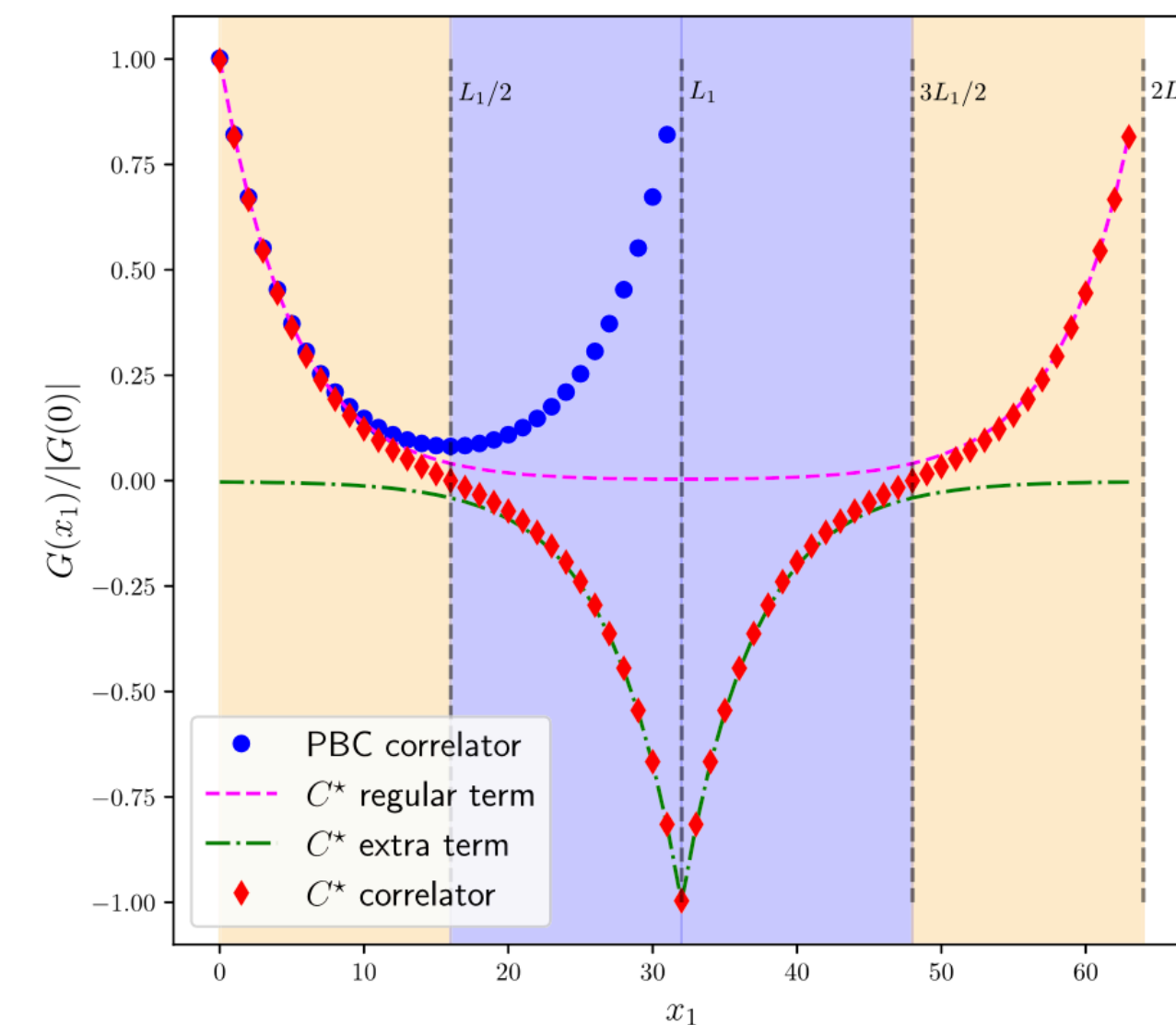
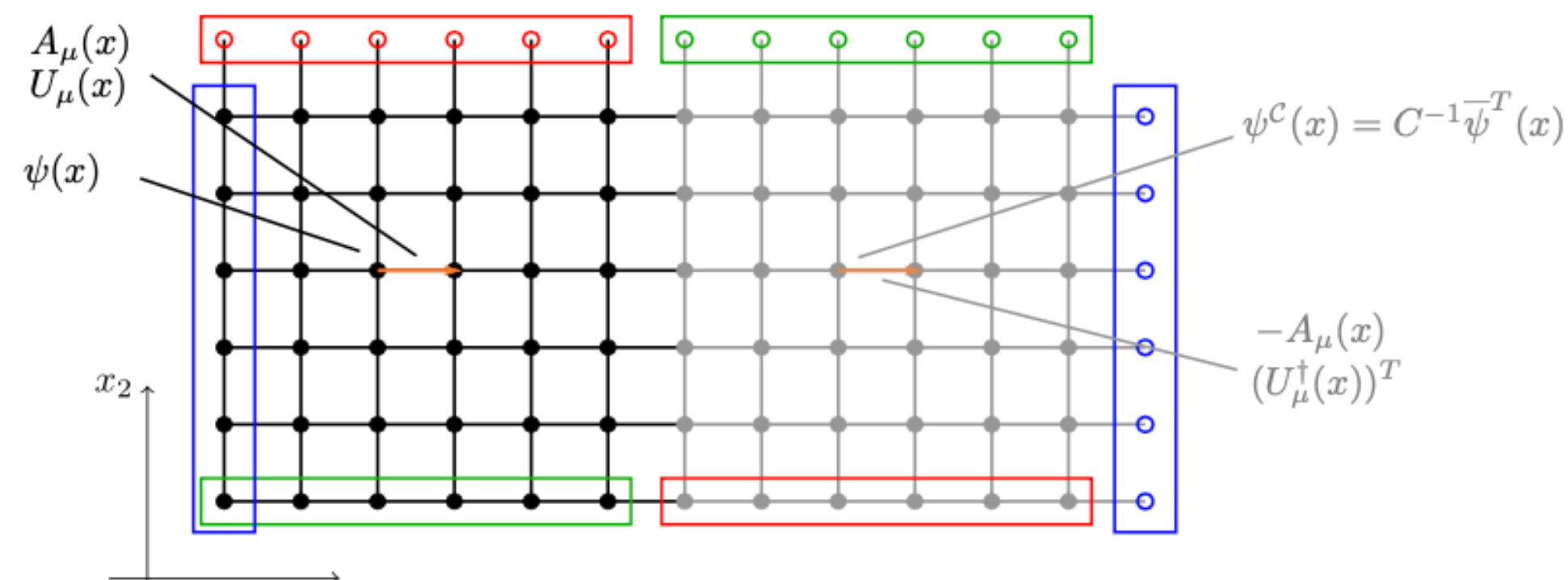
$$U_\mu(x + L_k \hat{k}) = U_\mu^*(x),$$

$$\bar{\psi}(x + L_k \hat{k}) = -\psi^\top(x) C.$$

- zero mode of A_μ absent by construction

Implementation: [openQ^{*}D: <https://gitlab.com/rcstar/openQxD>]

- Lattice is doubled in direction $\hat{1}$: $L_1 = 2L$, while $L_k = L$ for $k = 2, 3$.
- Orbifold construction: $\psi(x + L_k \hat{k}) = \psi(x + \frac{L_1}{2} \hat{1})$ for $k = 2, 3$



Comparing the two approaches to HVP in QCD+QED

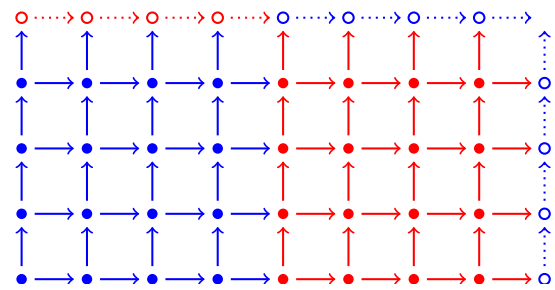
- Which approach gives better statistical precision (for HVP)?
- RM123 or $\text{QCD} + \text{QED}_q$

$$\Delta_{\text{stoch}} < \Delta_{\text{pert,summed-}\mu} < \Delta_{\text{pert,single-}\mu}$$

[RBC/UKQCD: Boyle et al, 1706.05293 JHEP 09 (2017) 153]

$a_\mu^{\text{U,w}} \times 10^{11}$	$a_\mu^{\text{U,w}} \text{ (LL)}$	$a_\mu^{\text{U,w}} \text{ (CL)}$
isoQCD+RM123 _{eq}	1084(5)	1087(5)
isoQCD+RM123	1093(20)	1094(21)
non-perturbative QCD+QED	1082(8)	1085(7)

[RC*: Alther et al. 2506.19770 JHEP10(2025)158]



- RM123 or dynamical QCD + QED

	IB type	from action			from V_μ^c at sink
		mass	QED		
valence connected	vv				
	vs				
	ss				
valence disc.ed	vv				
	vs				
	ss				

[RC*: Paola Tavella @ 8th plenary workshop of the Muon g-2 TI Initiative; PoS LATTICE2025 (2026) 198, arXiv:2605.01085]



Paola Tavella



Tim Harris



Letizia Parato



Javad Komijani



Roman Gruber



Alessandro Cotellucci



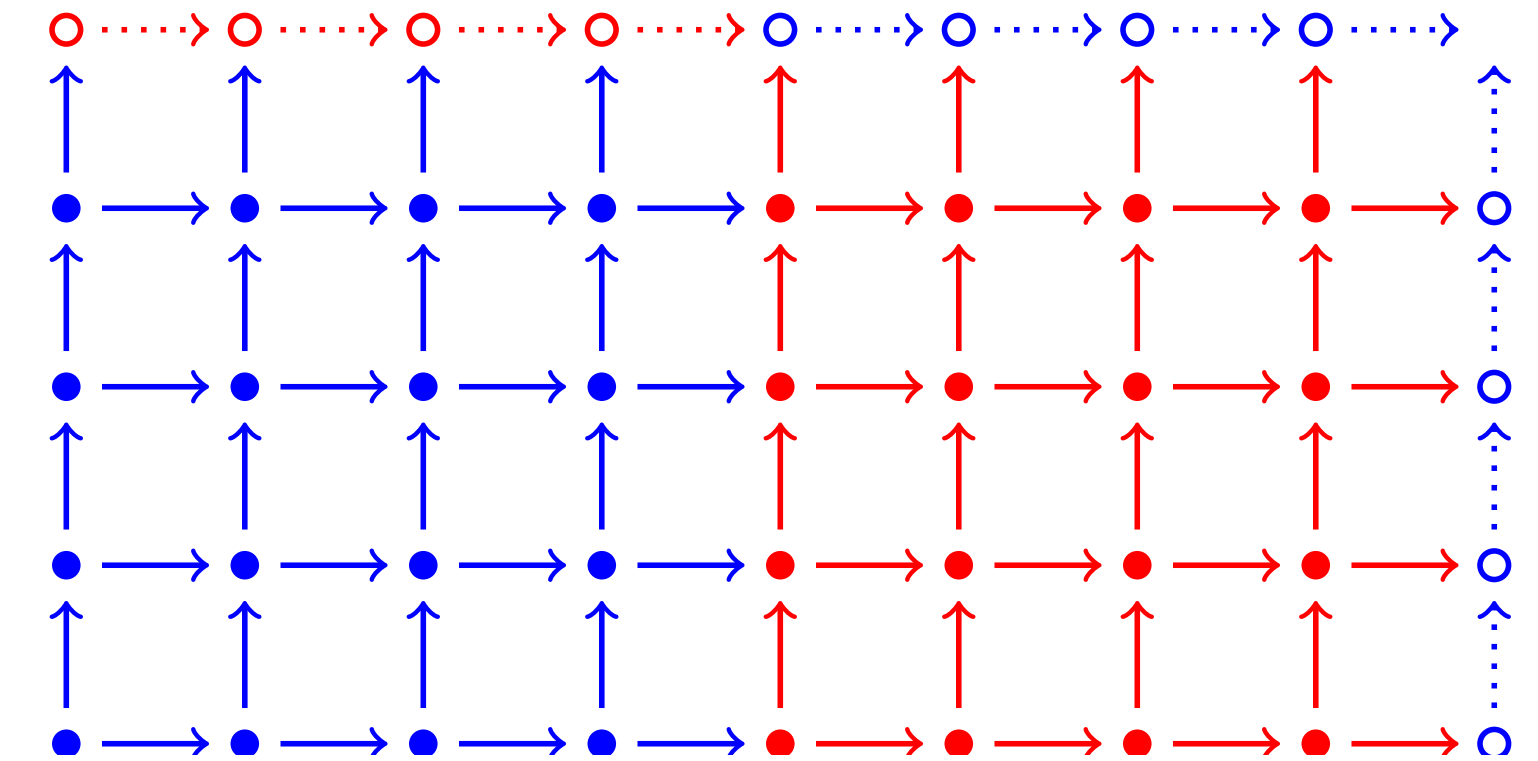
Anian Altherr



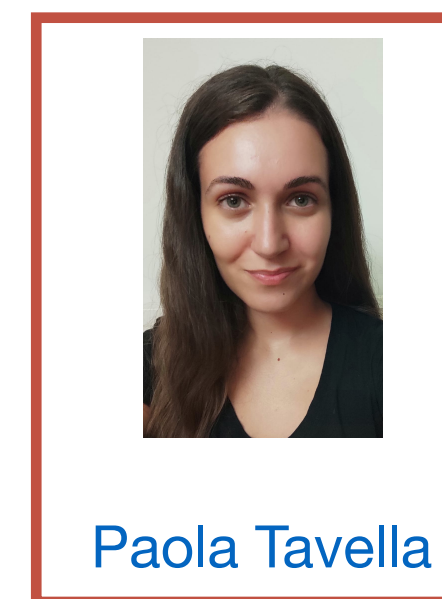
Klemen Kersic

Comparing the two approaches to HVP in QCD+QED

- Which approach gives better statistical precision (for HVP)?
- RM123 or full QCD + QED simulation



	IB type	from action			from V_μ^c at sink
		mass	QED		QED
valence connected	vv				
	vs				
	ss				
valence disc.ed	vv				
	vs				
	ss				



Tim Harris



Letizia Parato



Javad Komijani



Roman Gruber



Anian Altherr

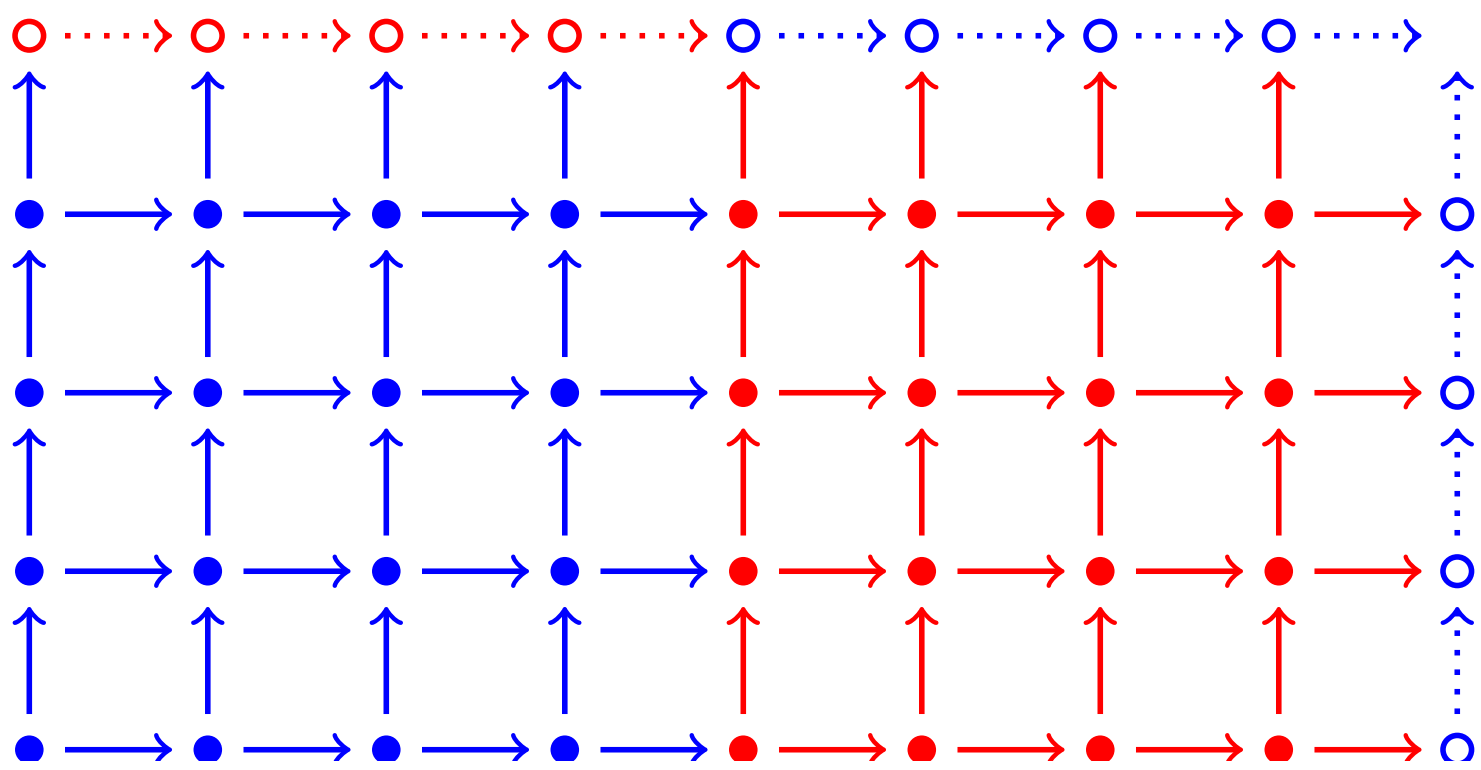


Klemen Kersic

[RC*: Paola Tavella @ 8th plenary workshop of the Muon g-2 TI Initiative;
PoS LATTICE2025 (2026) 198, arXiv:2605.01085]

Comparing the two approaches to HVP in QCD+QED

- Which approach gives better statistical precision (for HVP)?
- RM123 or full QCD + QED simulation



		from action			from V_μ^c at sink
		IB type	mass	QED	QED
valence connected	vv				
	vs				
	ss				
valence disc.ed	vv				
	vs				
	ss				



Paola Tavella



Tim Harris



Letizia Parato



Javad Komijani



Roman Gruber



Alessandro Cotellucci



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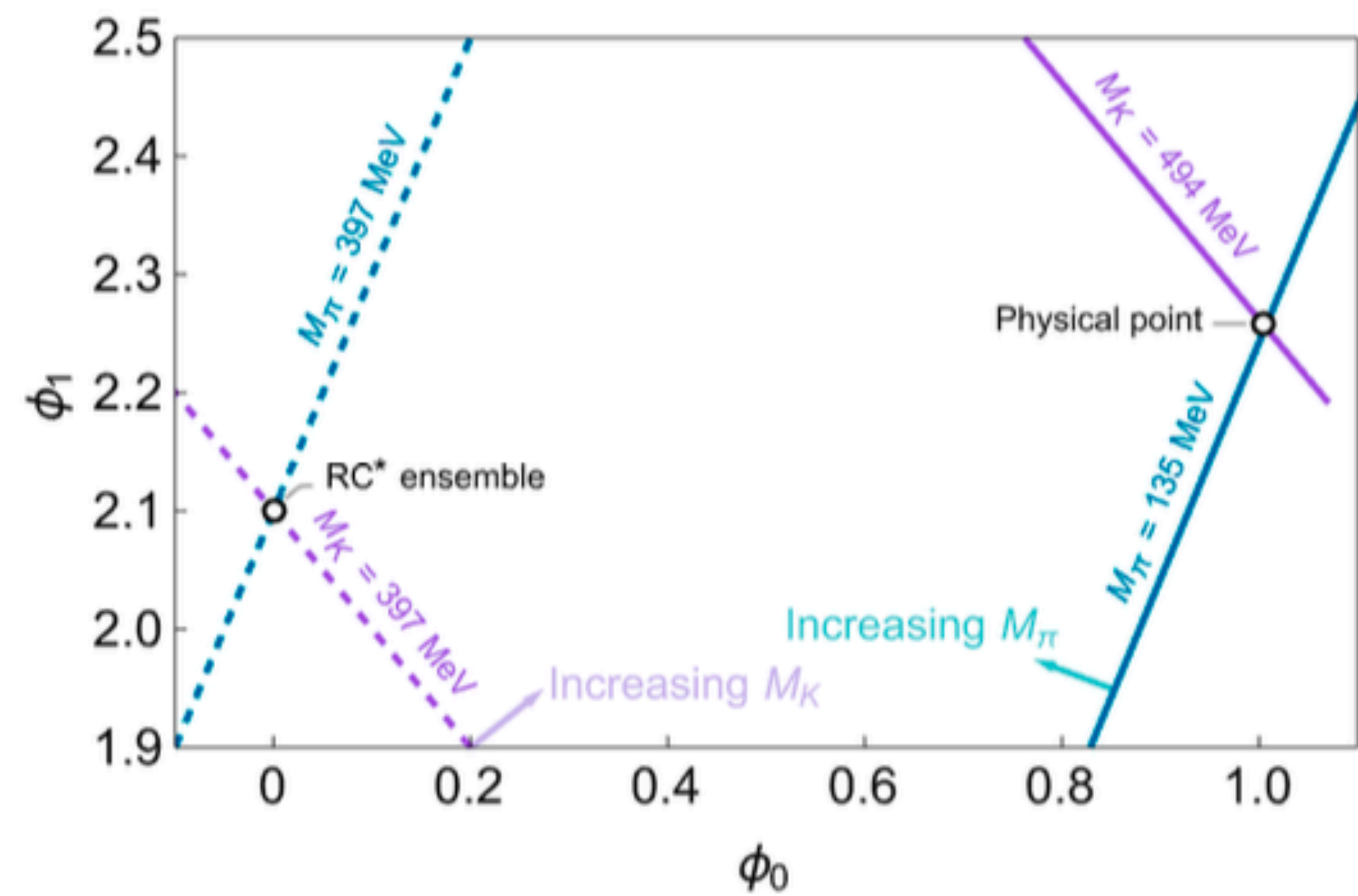
Klemen Kersic

[RC*: Paola Tavella @ 8th plenary workshop of the Muon g-2 TI Initiative;
PoS LATTICE2025 (2026) 198, arXiv:2605.01085]

Scheme choice for QCD+QED and isoQCD

Observable	“Fix”	Physical value	Target RC* value
$\sqrt{8t_0}$	a	0.415*	0.415 fm
$\phi_0 = 8t_0(M_{K^\pm}^2 - M_{\pi^\pm}^2)$	$m_s - m_d$	0.992	0
$\phi_1 = 8t_0(M_{K^\pm}^2 + M_{\pi^\pm}^2 + M_{K^0}^2)$	$m_u + m_d + m_s$	2.26	2.11
$\phi_2 = 8t_0(M_{K^0}^2 - M_{K^\pm}^2)/\alpha_R$	$m_u - m_d$	2.36	2.36, 0
$\phi_3 = \sqrt{8t_0}(M_{D_s^\pm} + M_{D^0} + M_{D^\pm})$	m_c	12.0	12.1
α_R	e^2	0.007297	α^{phys} , 0

* isoQCD computation by [CLS: Bruno, Korzec, Schaefer 1608.08900 PRD 95 (2017) 074504]



- ◆ unphysical point $M_\pi \approx 400$ MeV
- ◆ SU(3) symmetry in isoQCD
- ◆ U-spin symmetry in QCD+QED: $m_d = m_s$
- ◆ $N_f = 1 + 2 + 1$ $O(a)$ improved Wilson fermions

[Credit: P. Tavella]

U-spin symmetric hadronic vacuum polarization: $a_\mu^{U,w}$

$$a_\mu^{U,w} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt G^U(t) \tilde{K}(t; m_\mu) w_I(t).$$

$$a_\mu^{U,w} = \left(\frac{\alpha}{\pi}\right)^2 a \sum_{t=0}^{T/2} G^{U,\ell}(t) \tilde{K}(t; m_\mu) w_I(t), \quad \ell = 1, c$$

Local-local and local-conserved corr.

$$G^{U,1}(t) = (Z_V^m)^2 G_{\text{bare}}^{U,1}(t), \quad G^{U,c}(t) = Z_V^m G_{\text{bare}}^{U,c}(t).$$

- $G^U(t)$ correlator of the non-singlet current $V_\mu = \frac{1}{2}(\bar{\psi}_s \gamma_\mu \psi_s - \bar{\psi}_d \gamma_\mu \psi_d)$
- $w_I(t)$ gives the intermediate window [\[Blum et al. PRL121, 022003 \(2018\)\]](#)
- disconnected-free due to SU(3)/U-spin symmetry in our setup

$$\begin{aligned} & \frac{1}{4} \left(\text{diagram with two s quarks in a loop} + \text{diagram with two s quarks in a loop} - \text{diagram with one s and one d quark in a loop} \right) + \\ & + \frac{1}{4} \left(\text{diagram with two d quarks in a loop} + \text{diagram with two d quarks in a loop} - \text{diagram with one d and one s quark in a loop} \right) = \frac{1}{2} C \end{aligned}$$

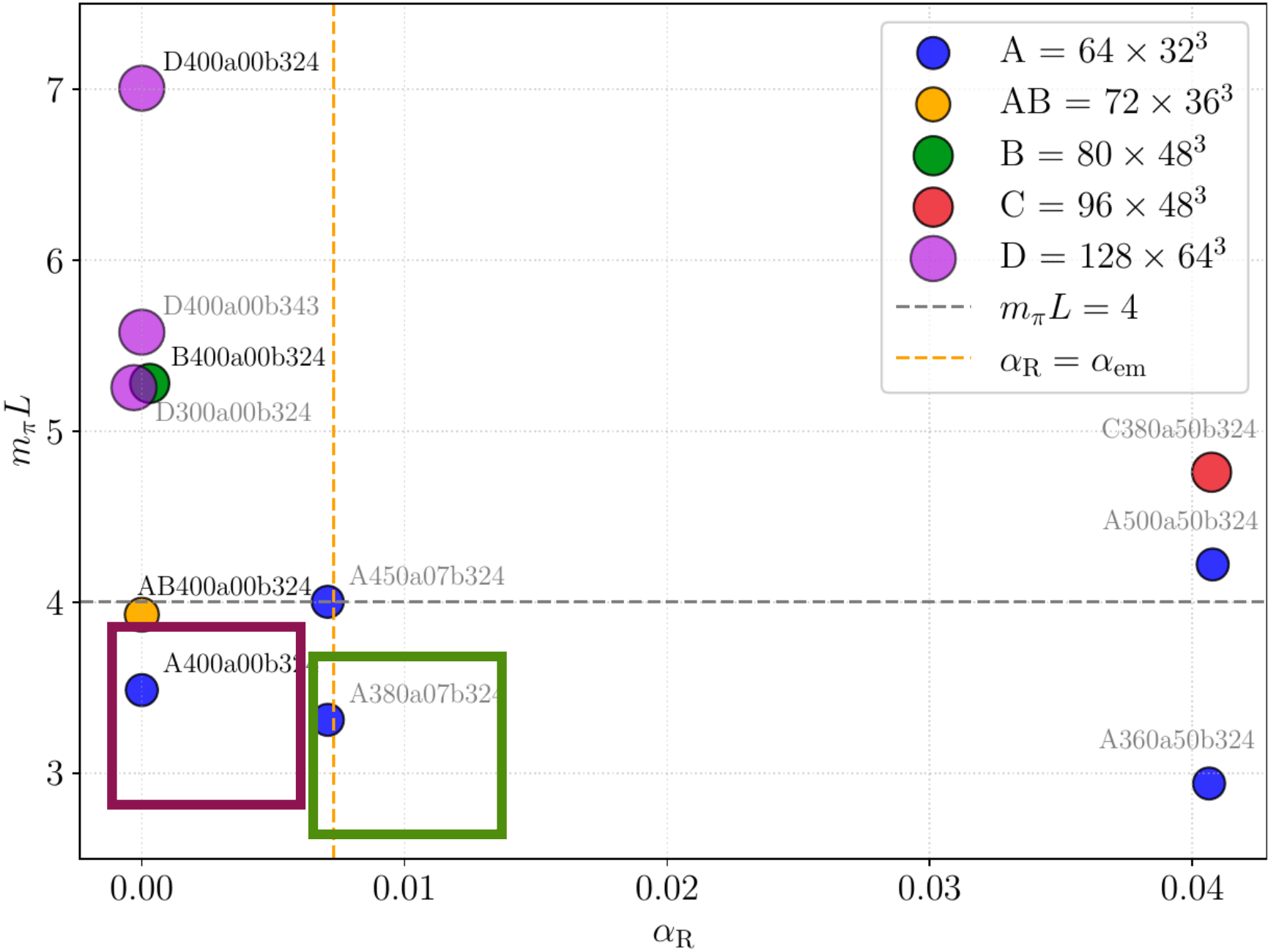
Note: in isoQCD with SU(3) symmetry $\rightarrow a_\mu^U = \frac{3}{4} a_\mu^{33}$

[\[Gerardin, Harris, Meyer, PRD99, 014519 \(2019\)\]](#)

Two ensembles with C* bc's: isoQCD and QCD+QED

[RC*: Alther et al. 2506.19770 JHEP10(2025)158]

- Comparison between two ensembles (QCD: $\alpha_R = 0$; full QCD+QED: $\alpha \approx \alpha_{\text{em}}$)
- Similar volume and similar pion mass, the same $\beta \longrightarrow$ different lattice spacing a
- $O(a)$ improved Wilson fermions; Improvement coeff. $c_{\text{SW}}^{\text{SU}(3)} = 2.18859$ [Fritzsch et al. JHEP06 (2018)025] and $c_{\text{SW}}^{\text{U}(1)} = 1$
- Similar hopping parameters; same statistics: 2000 gauge configurations

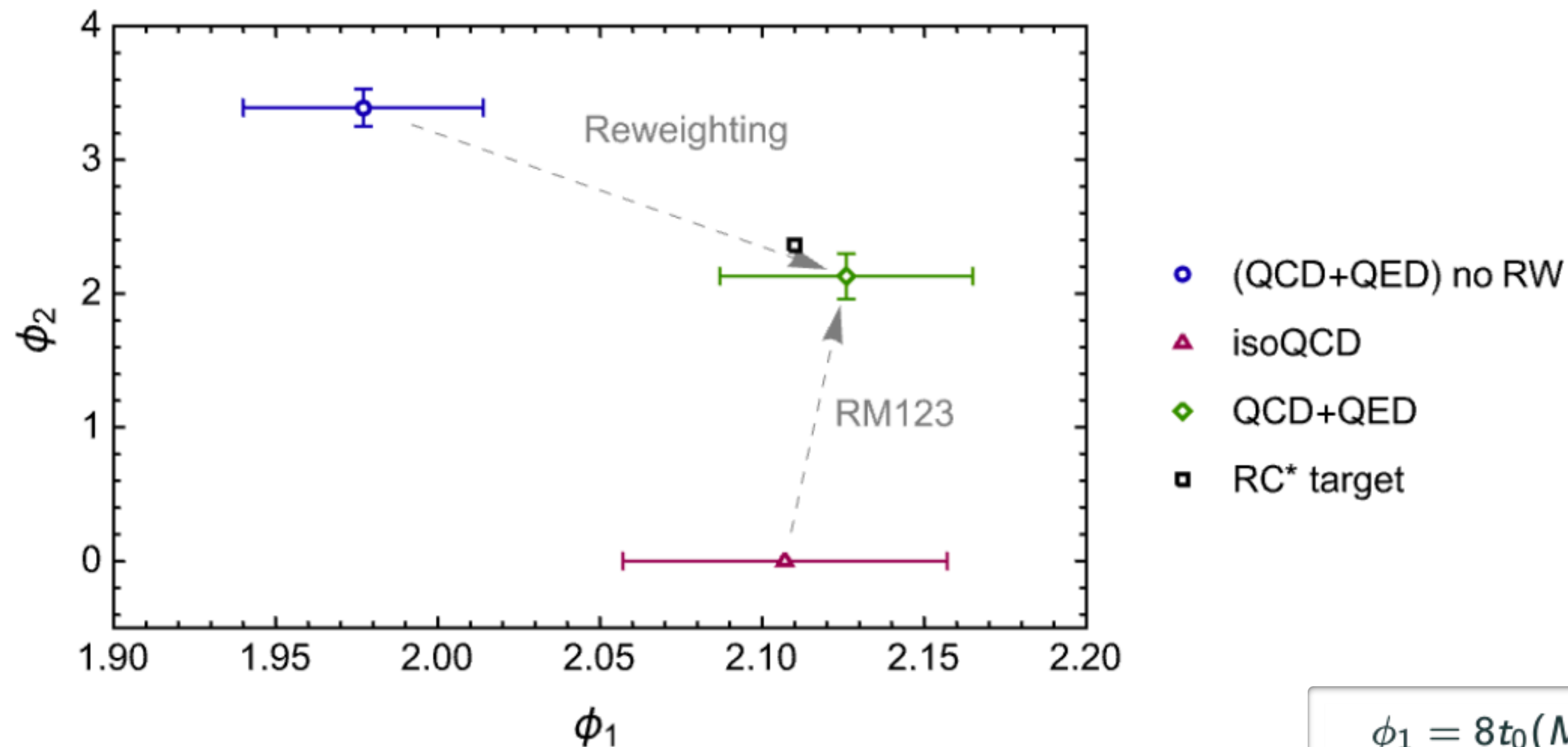


[<https://rcstar.gitlab.io/ensembles/>]

ensemble	lattice	β	α	κ_u	$\kappa_d = \kappa_s$	κ_c
A400a00	64×32^3	3.24	0	0.13440733	0.13440733	0.12784
A380a07	64×32^3	3.24	0.007299	0.13457969	0.13443525	0.12806355

Matching isoQCD and QCD+QED [RC*: Alther et al. 2506.19770 JHEP10(2025)158] [RC*: Alther et al. 2605.01085 PoS(LATTICE2025)198]

- ♦ On the QCD+QED ensemble mass reweighting (RW) is applied to compare at the same point in the (ϕ_1, ϕ_2)
- ♦ RW coefficients taken from [\[Bushnaq et al. JHEP03, 012 \(2023\)\]](#); small RW $\rightarrow$ no significant effect on the uncertainties



$$\phi_1 = 8t_0(M_{K^\pm}^2 + M_{\pi^\pm}^2 + M_{K^0}^2)$$

$$\phi_2 = 8t_0(M_{K^0}^2 - M_{K^\pm}^2)/\alpha_R$$

Two strategies for comparison

[RC*: Alther et al. 2506.19770 JHEP10(2025)158]

[RC*: Alther et al. 2605.01085 PoS(LATTICE2025)198]

1. Fixed bare parameters:

- **isoQCD+RM123**: we use the exact quark mass shifts from $\kappa_{QCD+QED}^{-1} - \kappa_{isoQCD}^{-1}$

$$a\Delta m_u = -0.00476435, \quad a\Delta m_{d,s} = -0.00077259, \quad a\Delta m_c = -0.00682735$$

- we compare the QCD+QED results for all obs: $t_0, \phi_i, a_\mu^{U,w}$

$$X^{n.p.} \text{ vs } X(0) + e^2 \partial_{e^2} X(0) + \sum_{f=u,d,s,c} \Delta m_f \partial_{m_f} X(0)$$

2. Fixed line of constant physics (LCP):

- **isoQCD+RM123**: we tune the quark mass shifts through 4 eqs.

$$\phi_i^{(0)} + e^2 \partial_{e^2} \phi_i + \sum_f \Delta m_f \partial_{m_f} \phi_i - \Delta_L \phi_i = \phi_i^*.$$

The uncertainties on Δm_f propagates to $a_\mu^{U,w}$

- **n.p. QCD+QED** we propagate the tuning error to the final observable through

$$(da_\mu^{U,w})^2 = \sum_f \left(\partial_{m_f} a_\mu^{U,w} \right)^2 (dm_f)^2, \quad dm_f(\vec{\phi}) = \sum_i (J^{-1})_{fi} d\phi_i,$$

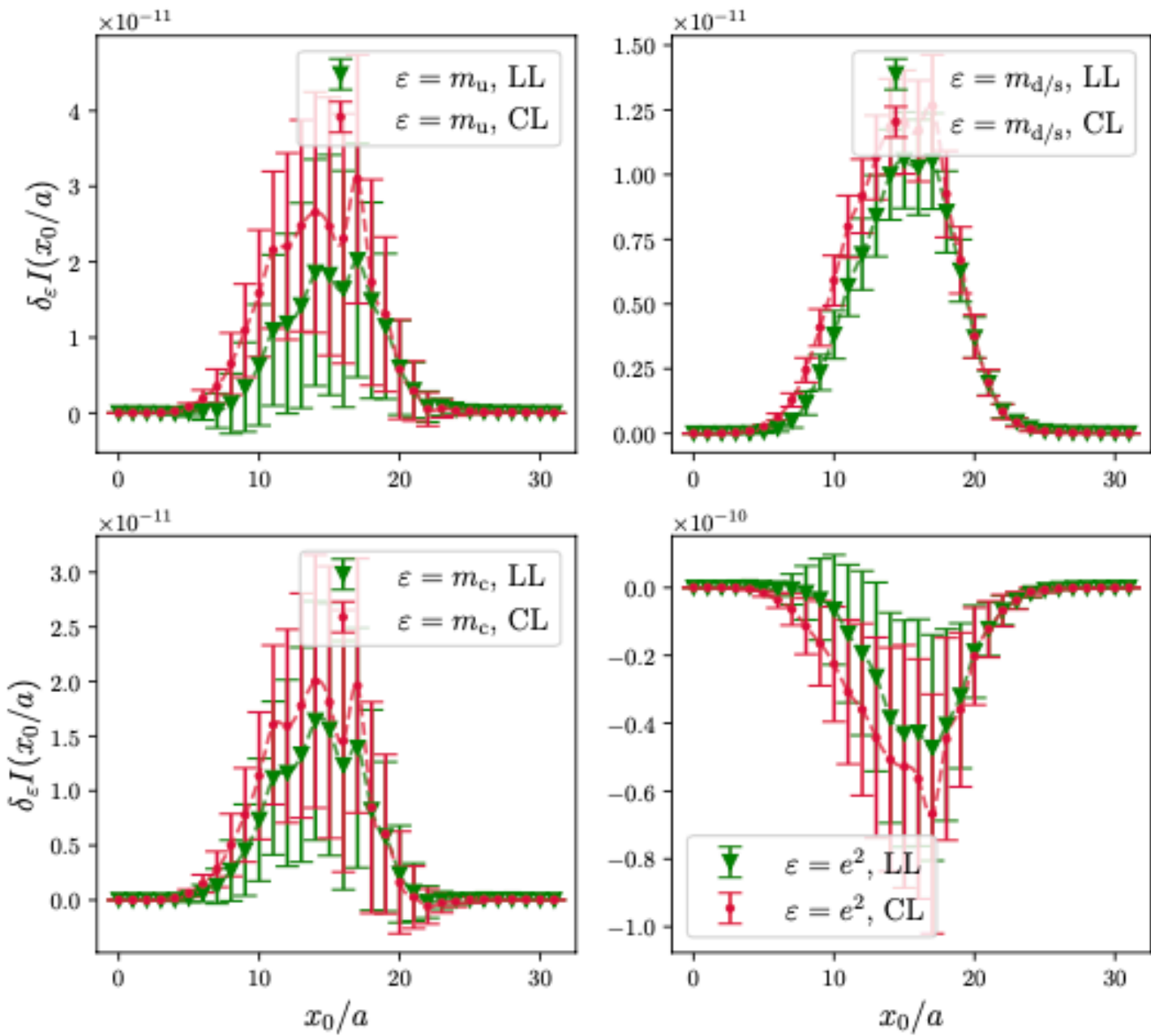
[Credit: P. Tavella]

RM123: correction to $a_\mu^{U,w}$

[RC*: Alther et al. 2506.19770 JHEP10(2025)158]
[RC*: Alther et al. 2605.01085 PoS(LATTICE2025)198]

$$a_\mu^{U,w} = \left(\frac{\alpha}{\pi}\right)^2 \sum_{\hat{t}=0}^{\hat{T}/2} \underbrace{\hat{K}(\hat{t}; am_\mu)}_{\delta_a a_\mu^{U,w}} \underbrace{w_I(a\hat{t})}_{\delta_{Z_V} a_\mu^{U,w}} \underbrace{\hat{G}_{\text{bare}}^{U,c}(\hat{t})}_{\delta_G a_\mu^{U,w}}$$

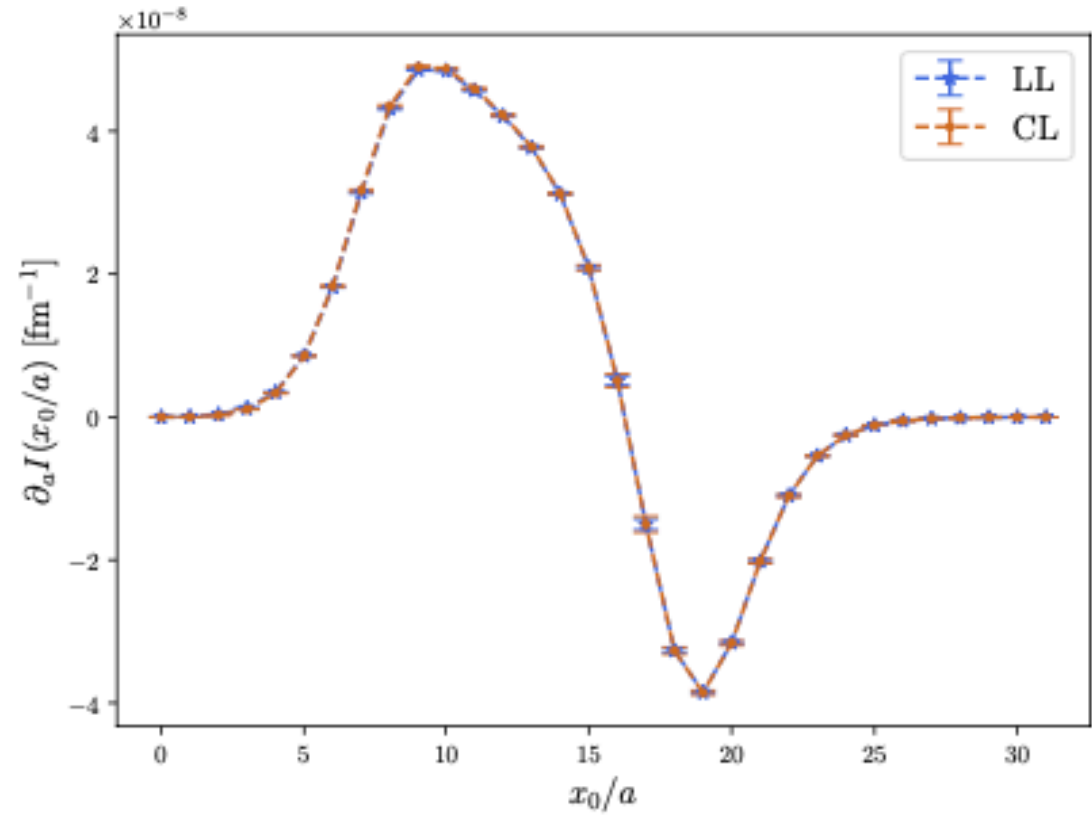
■ Integrand for $\delta_G a_\mu^{U,w}$:



■ Derivatives of Z_V^m :

$\partial_{am_u} Z_V^m$	$\partial_{am_{d,s}} Z_V^m$	$\partial_{am_c} Z_V^m$	$\partial_{e^2} Z_V^m$
-1.30(26)	-1.75(27)	-0.39(10)	-0.11(2)

■ Integrand for $\partial_a a_\mu^{U,w}$:



Corrections to $a_\mu^{U,w}$ at fixed bare parameters (input exact Δm_f):

units of 10^{-11}	$\ell = 1$	$\ell = c$
$\delta_G a_\mu^{U,w}$	11(16)	12(17)
$\delta_{Z_V} a_\mu^{U,w}$	3.8(3.4)	1.9(1.7)
$\delta_a a_\mu^{U,w}$	-8.4(2.3)	-8.4(2.4)
$\delta a_\mu^{U,w}$	7(18)	6(18)

[Credit: P. Tavella]

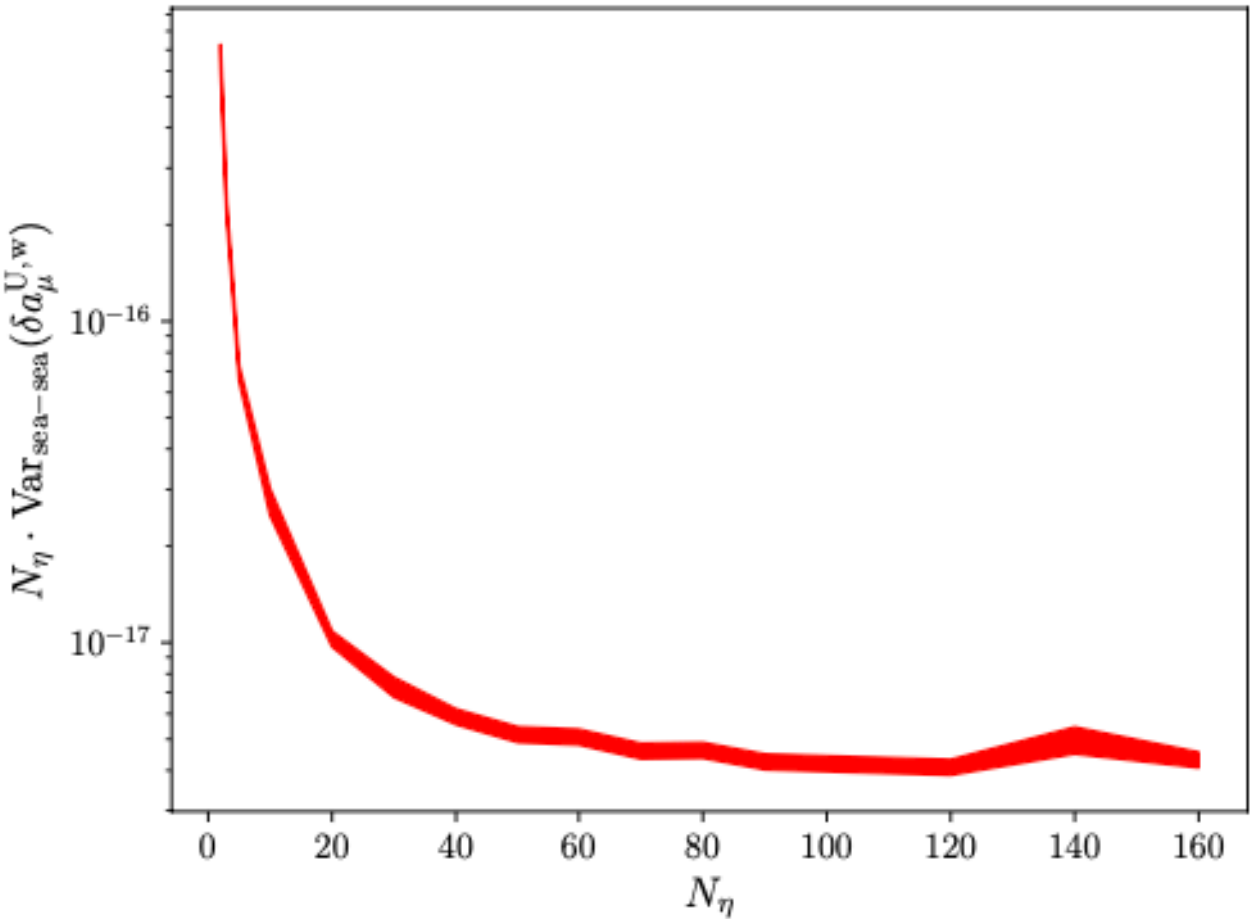
Ingredients at fixed bare parameters (valence vs. sea effects)

[RC*: Alther et al. 2506.19770 JHEP10(2025)158]
[RC*: Alther et al. 2605.01085 PoS(LATTICE2025)198]

- Decomposition of the IB correction to $a_\mu^{U,w}$

Class	$\ell = 1$	c
vv	-6.6(4)	-6.6(4)
sv	-0.24(1)	-0.24(1)
ss	14(18)	13(18)
Tot. $\delta a_\mu^{U,w}$	7(18)	6(18)

Uncertainty dominated by sea-sea effects!

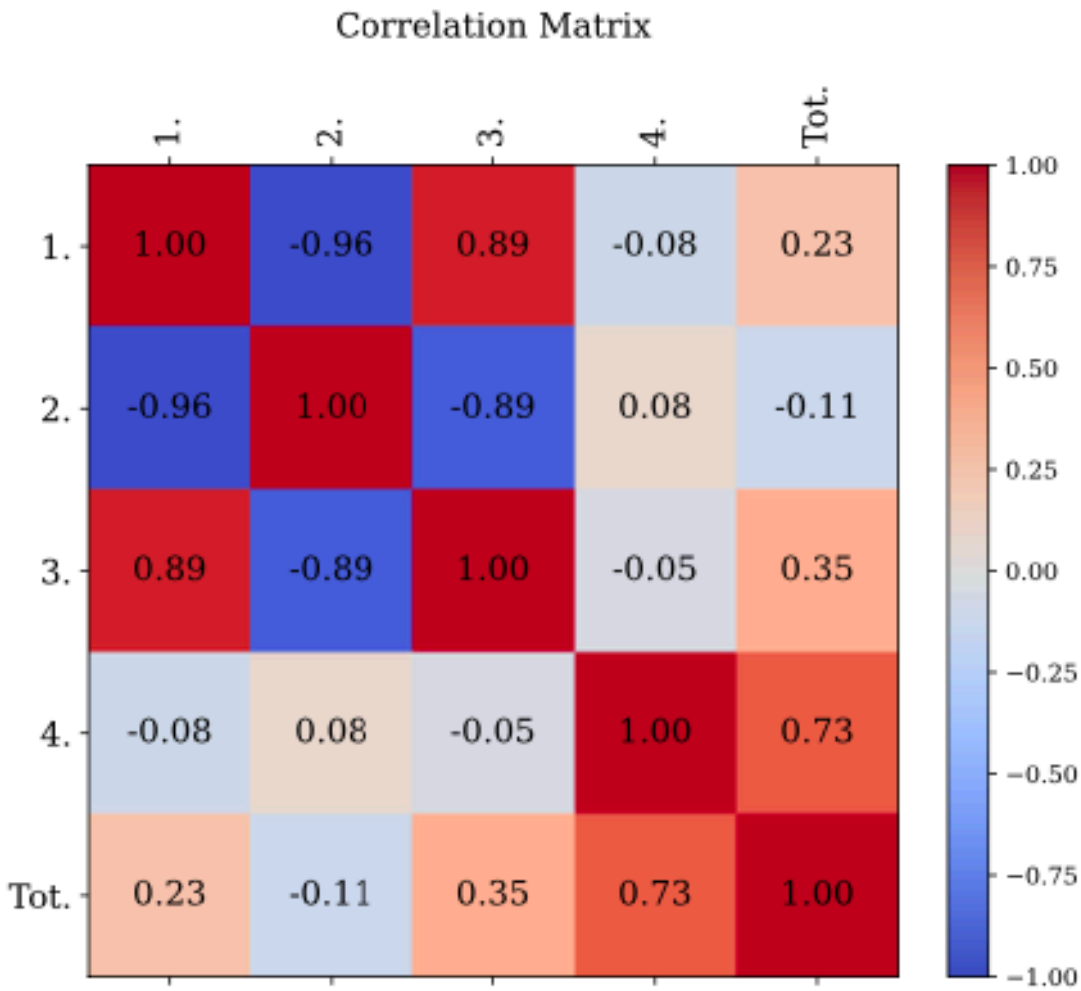


Alessandro Cotellucci

[A. Cotellucci et al. 2502.03145]

- Detailed sea-sea effects to $\delta a_\mu^{U,w} \times 10^{11}$

	Sea-diagram	$\ell = 1$	c
1.	$\sum_f \Delta m_f$	14(30)	13(30)
2.	$e^2 \sum_f q_f^2$	-19(43)	-18(43)
3.	$e^2 \sum_f q_f^2$	7(19)	6(19)
4.	$e^2 \sum_{fg} q_f q_g$	13(14)	12(14)
Tot.	$\delta a_\mu^{U,w}(ss)$	14(18)	13(18)



Paola Tavella

[Credit: P. Tavella]

Mass shifts and final comparison for $a_\mu^{U,w}$ (incl. sea-sea effects)

[RC*: Alther et al. 2506.19770 JHEP10(2025)158]
[RC*: Alther et al. 2605.01085 PoS(LATTICE2025)198]

	From fixed bare parameters	From fixing LCP
$a\Delta m_u$	-0.00476435	-0.00477(17)
$a\Delta m_{d,s}$	-0.00077259	-0.00082(17)
$a\Delta m_c$	-0.00682735	-0.0083(28)

- from fixed bare parameters: $\kappa_{QCD+QED}^{-1} - \kappa_{isoQCD}^{-1}$
- from fixing LCP: solution of $\phi_i^{isoQCD+RM123} = \phi_i^\star$

$a_\mu^{U,w} \times 10^{11}$	From fixing bare parameters		From fixed LCP	
	$\ell=l$	$\ell=c$	$\ell=l$	$\ell=c$
isoQCD	1083(6)	1086(5)	1083(6)	1086(5)
isoQCD+RM123 eq	1078(5)	1080(5)	1084(5)	1087(5)
isoQCD+RM123	1090(18)	1092(18)	1093(20)	1094(21)
n.p. QCD+QED	1082(7)	1085(7)	1082(8)	1085(7)

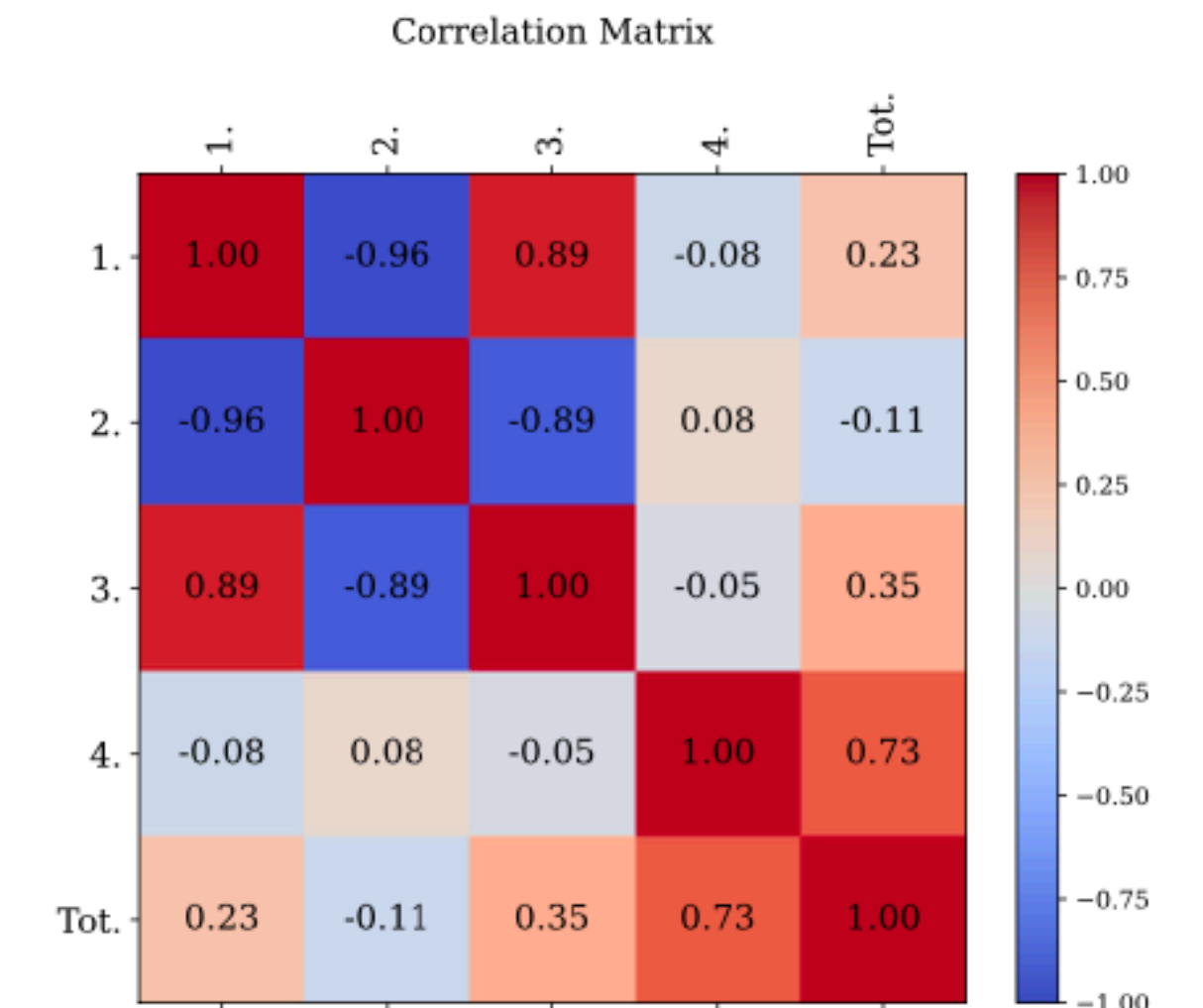
- no significant changes observed between the two strategies

Summary: comparing the two approaches to QCD+QED

- ♦ Sampling of full QCD+QED path integral and perturbative QED (RM123) [[Altherr et al. \(RC*\), 2506.19770 JHEP10\(2025\)158](#)]
- ♦ U-spin symmetric intermediate window contribution $a_\mu^{\text{U,w}}$, no disconnected contributions due to isospin symmetry
- ♦ RM123 includes sea-quark contributions at $O(a)$ [[A. Cotellucci et al. 2502.03145](#)] $m_\pi \approx 400 \text{ MeV}$, $L \approx 1.7 \text{ fm}$, fixed $a \approx 0.05 \text{ fm}$
[Sea-quark effects also fully included by BMW, see talk by [Bálint Tóth](#), Wed. 10.30am]

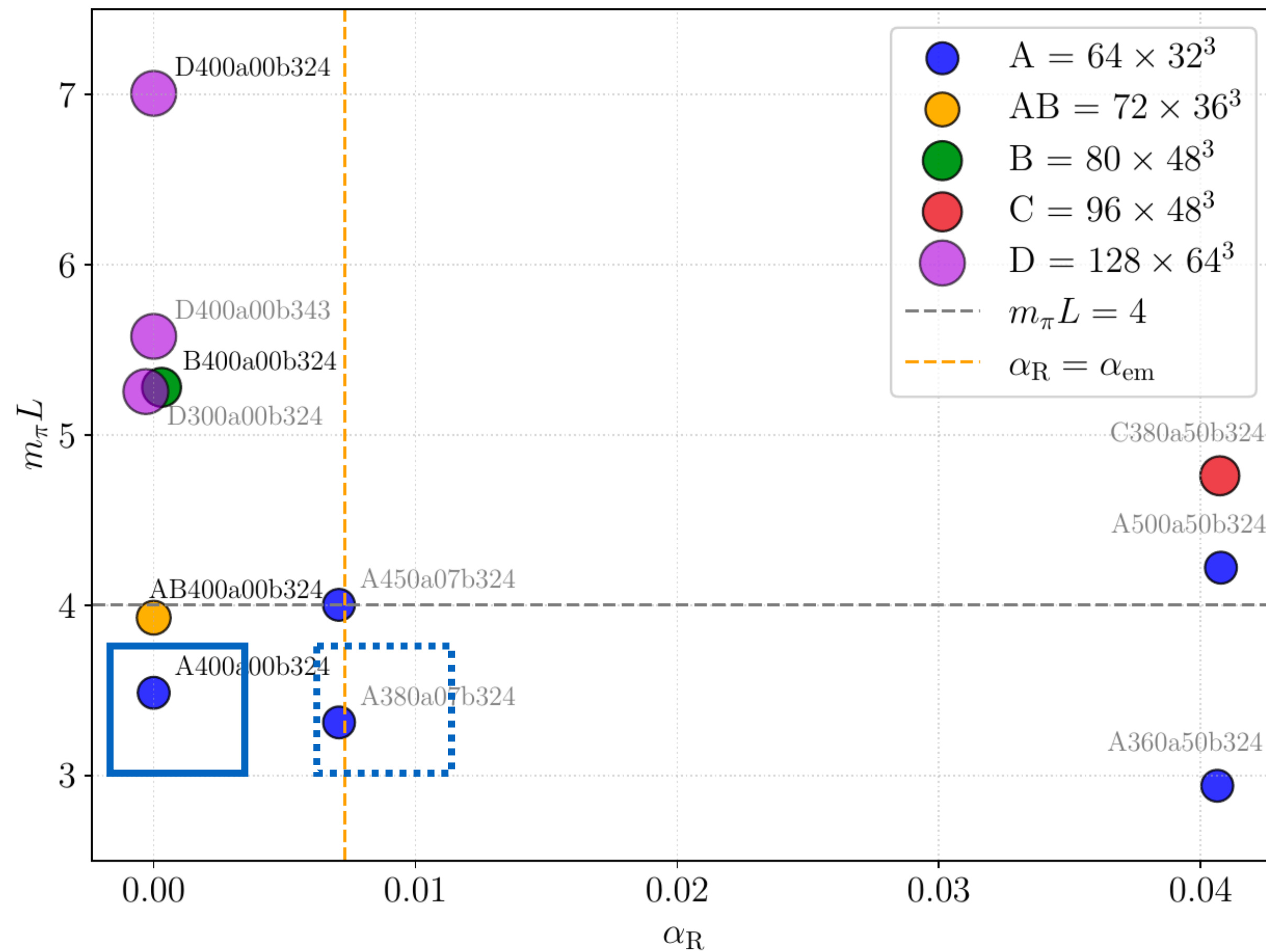
$$a_\mu^{\text{U,w}} \times 10^{11} = \begin{cases} 1094(21) & \text{RM123} \\ 1085(7) & \text{dynamical QCD + QED} \end{cases}$$

	Sea-diagram
1.	$\sum_f \Delta m_f$
2.	$e^2 \sum_f q_f^2$
3.	$e^2 \sum_f q_f^2$
4.	$e^2 \sum_{fg} q_f q_g$
Tot.	$\delta a_\mu^{\text{U,w}}(ss)$



Finite volume effects to the HVP

- ◆ Sampling of full QCD+QED path integral and QCD+RM123 approach [Altherr et al. (RC*), 2506.19770 JHEP10(2025)158]



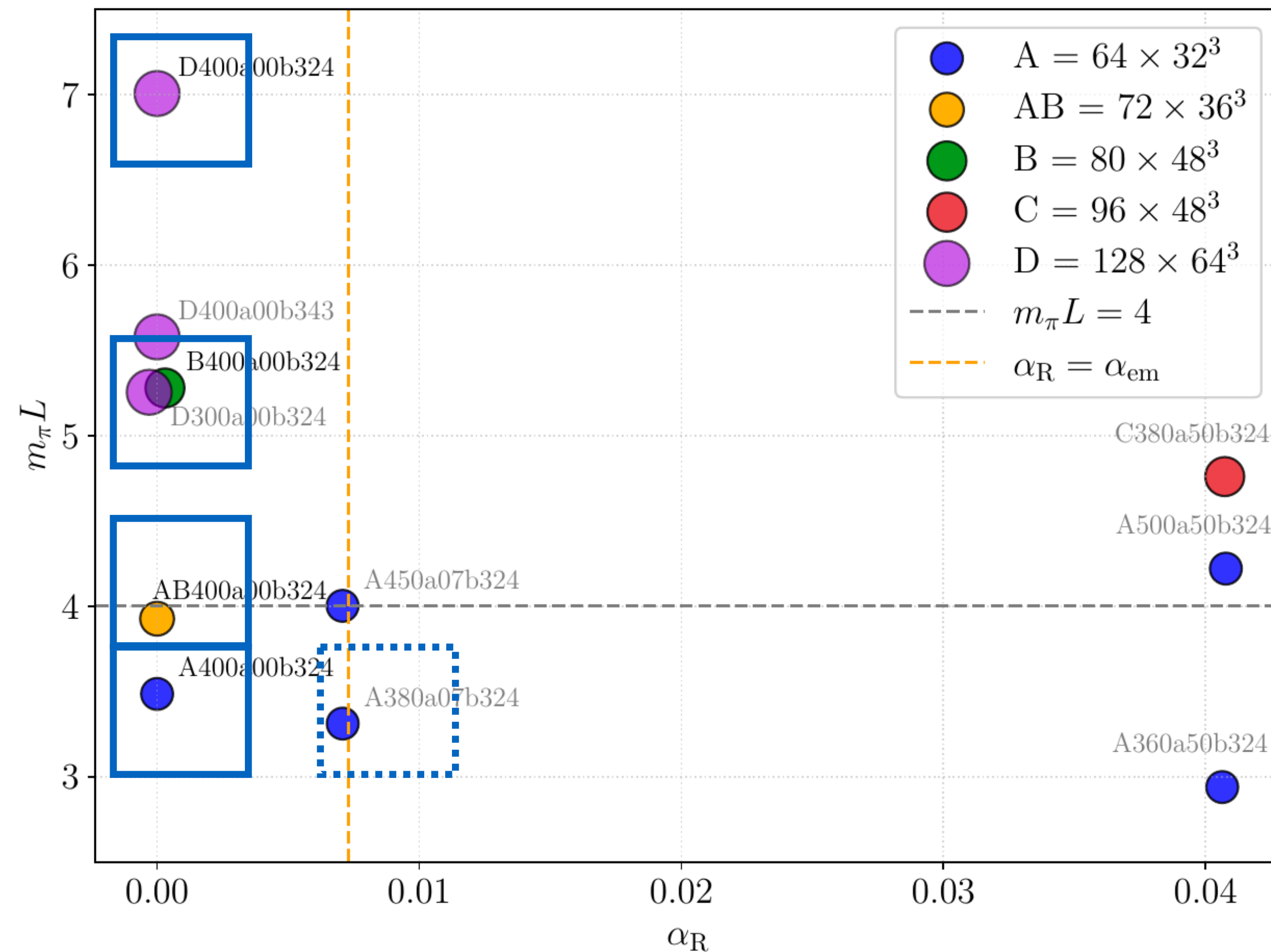
[<https://rcstar.gitlab.io/ensembles/>]

Finite volume effects to $a_\mu^{33,w}$ in isoQCD

- ◆ Three additional gauge ensembles with larger volume (isoQCD) in the usual RC* setup

$$a_\mu^{33,w} = \left(\frac{\alpha}{\pi} \right)^2 \int dt G^{33}(t) \tilde{K}(t; m_\mu) w_I(t)$$

[Bernecker, Meyer EPJA 47 (2011) 148]



[<https://rcstar.gitlab.io/ensembles/>]

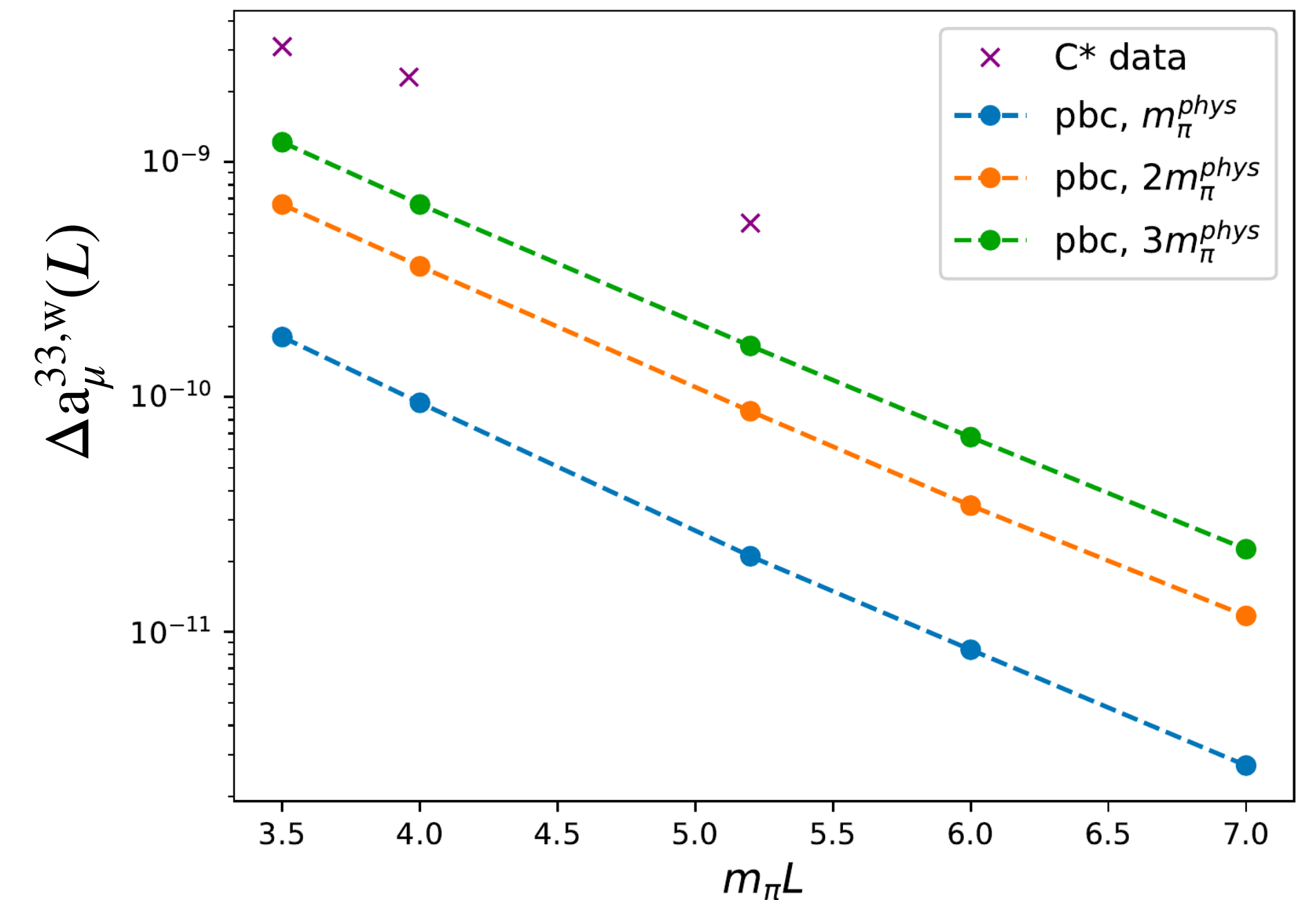
Finite volume effects to $a_\mu^{33,w}$ in isoQCD

- ◆ Numerical estimate of the finite volume (FV) effects on four ensembles with same lattice spacing and similar pion mass $m_\pi \approx 400$ MeV

Ensemble	L [fm]	m_π [MeV]	$m_\pi L$
A400	1.73	398.5(4.7)	~ 3.5
AB400	1.93	407.5(4.2)	~ 4.0
B400	2.59	401.9(1.4)	~ 5.2
D400	3.44	402.4(1.5)	~ 7

C* bc's: $\Delta a_\mu^{33,w} = a_\mu^{33,w}(\infty) - a_\mu^{33,w}(L)$

↓
D400



- ◆ Expected pion mass dependence of the FV effects vs. **measurements on RC* ensembles** [Hansen, Patella, Phys.Rev.Lett. 123 (2019) 172001]

Finite volume effects to $a_\mu^{33,w}$ in isoQCD

- ♦ Analytic estimate of the leading FV effects, Hamiltonian formalism: quantization along a spatial direction [Hansen, Patella, Phys.Rev.Lett. 123 (2019) 172001]

- ♦ Periodic bc's [Hansen, Patella, Phys.Rev.Lett. 123 (2019) 172001] see also [Aubin et al. PRD93, 054508 (2016), Bijnens, Relefors JHEP 12, 114 (2017)]

$$\Delta G^{1+3}(x_0|L) = -\frac{1}{3} \sum_{q=0,\pm 1} \int \frac{dp_3}{2\pi} \frac{e^{-L\sqrt{m_\pi^2 + p_3^2}}}{4\pi L} \times \text{Re} \int \frac{dk_3}{2\pi} e^{ik_3 x_0} T_q(-k_3^2, -k_3 p_3)$$

- T_q forward Compton scattering amplitude
- Dominant contribution: pion pole $T_{\pm 1} \approx F_\pi^2(-k_3^2)$ [space-like form factor]

- ♦ C* bc's [P. Tavella, LATT2026, 28 July, 4.30p] see also [Martins, Patella, PoS LATTICE2022 (2023) 323, 2212.09565]

$$\rightarrow \Delta G^{1,C^*}(x_0|L) = -\frac{1}{3} \int \frac{dp_3}{2\pi} \frac{e^{-L\sqrt{m_\pi^2 + p_3^2}}}{4\pi L} \times \text{Re} \int \frac{dk_3}{2\pi} e^{ik_3 x_0} T_0(-k_3^2, -k_3 p_3)$$

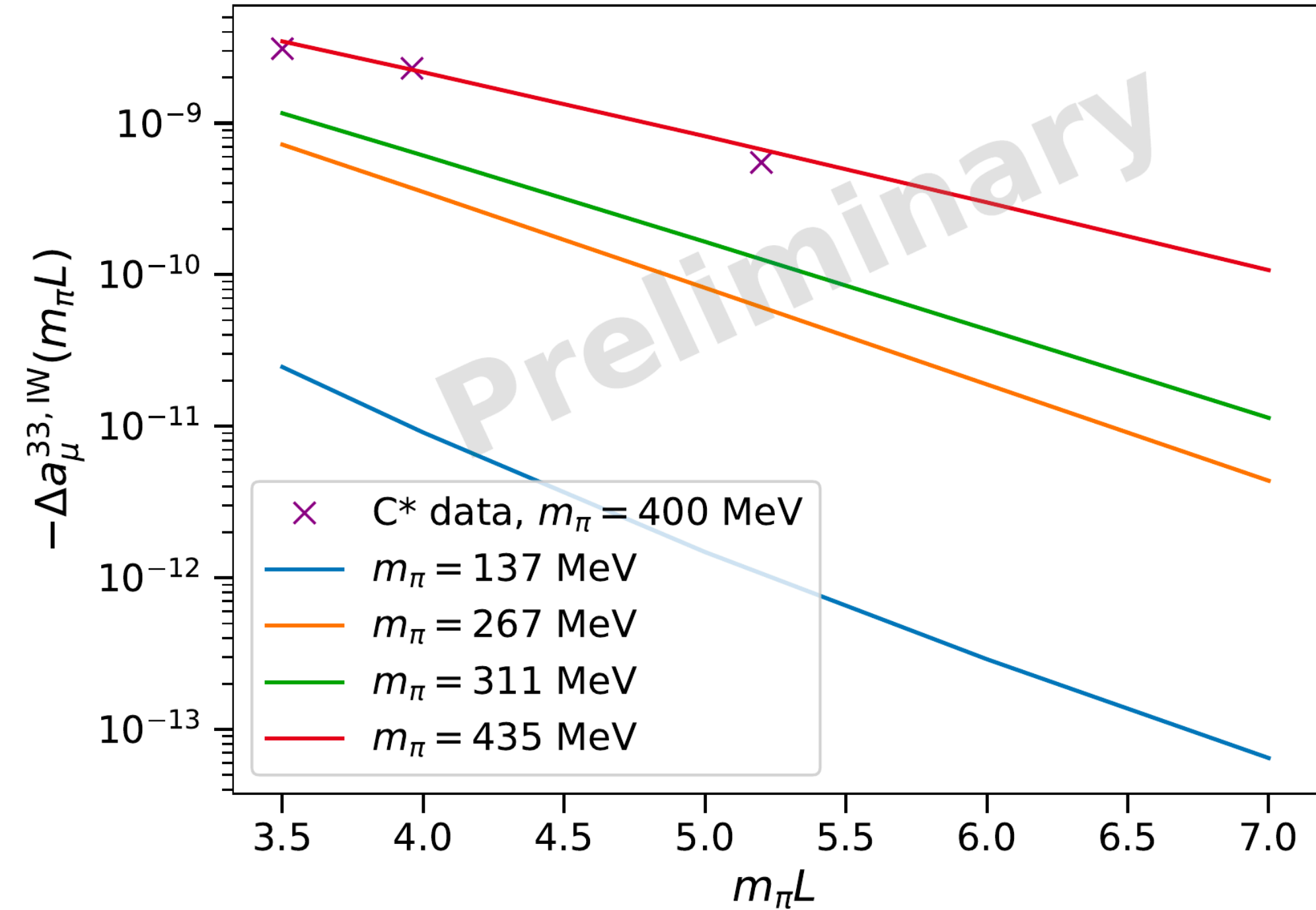
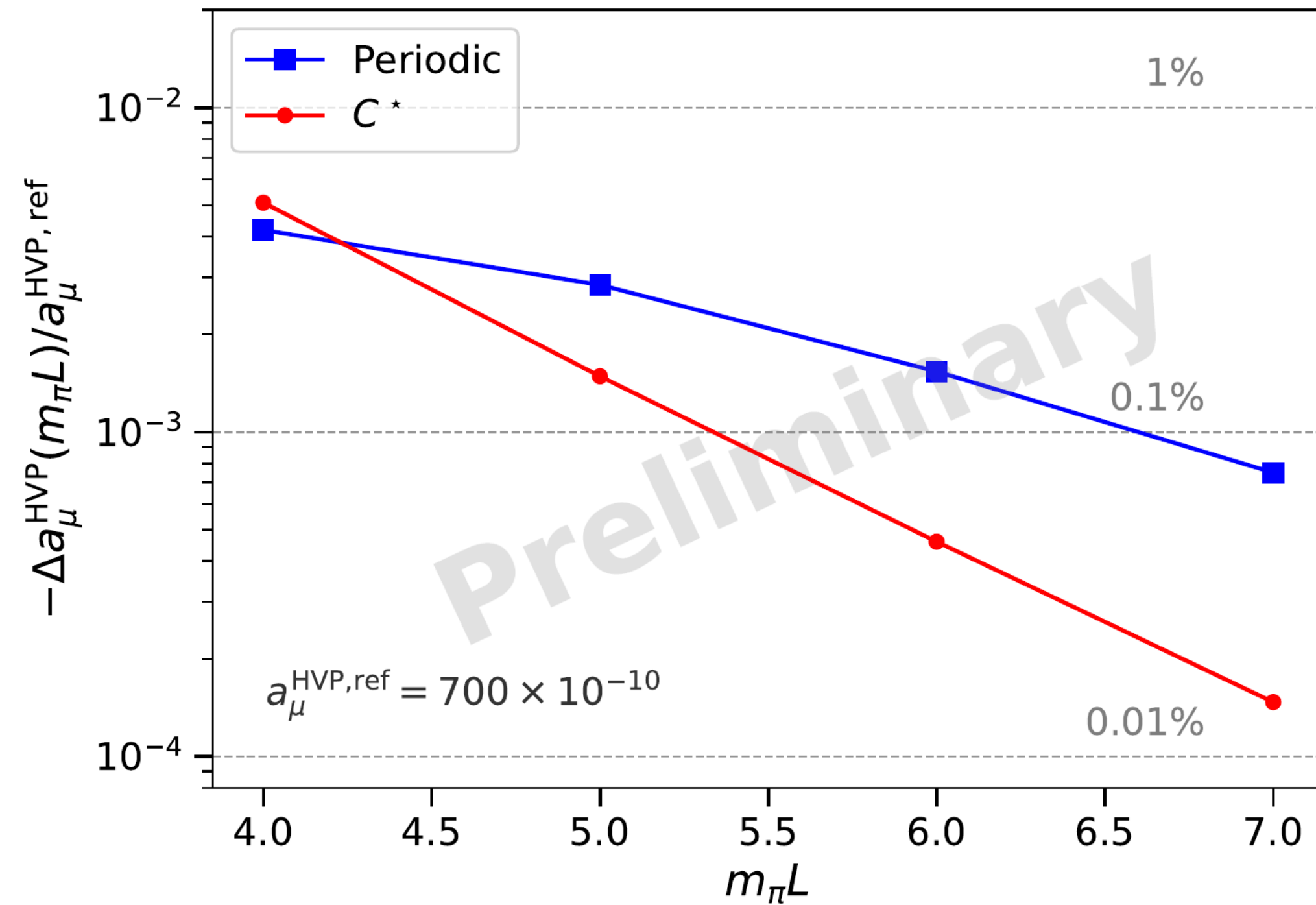
$$\rightarrow \text{Additional contribution: } \Delta G^{2a+2b}(x_0|L) = -\frac{1}{2\pi} \int dP_3 e^{iP_3 x_0} \int_{4m_\pi^2}^{\infty} ds \frac{e^{-\frac{L}{2}\sqrt{s + P_3^2}}}{s + P_3^2} s \rho_{\pi\pi}(s) + O(e^{-\frac{3}{2}Lm_\pi}) \quad [\text{Dominant: time-like form factor } |F_\pi(s)|^2]$$



Paola Tavella

Finite volume effects to $a_\mu^{33,w}$ in isoQCD

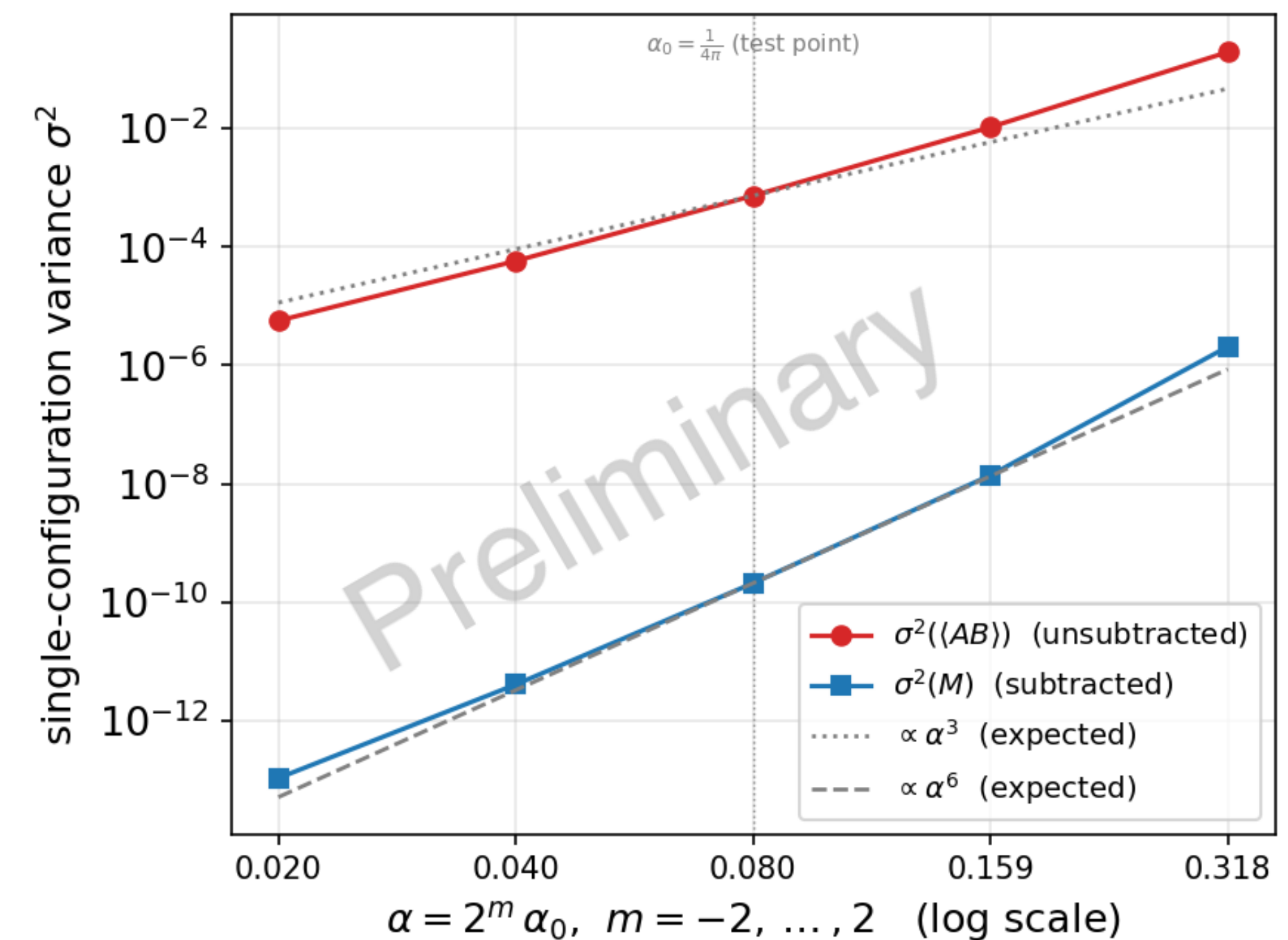
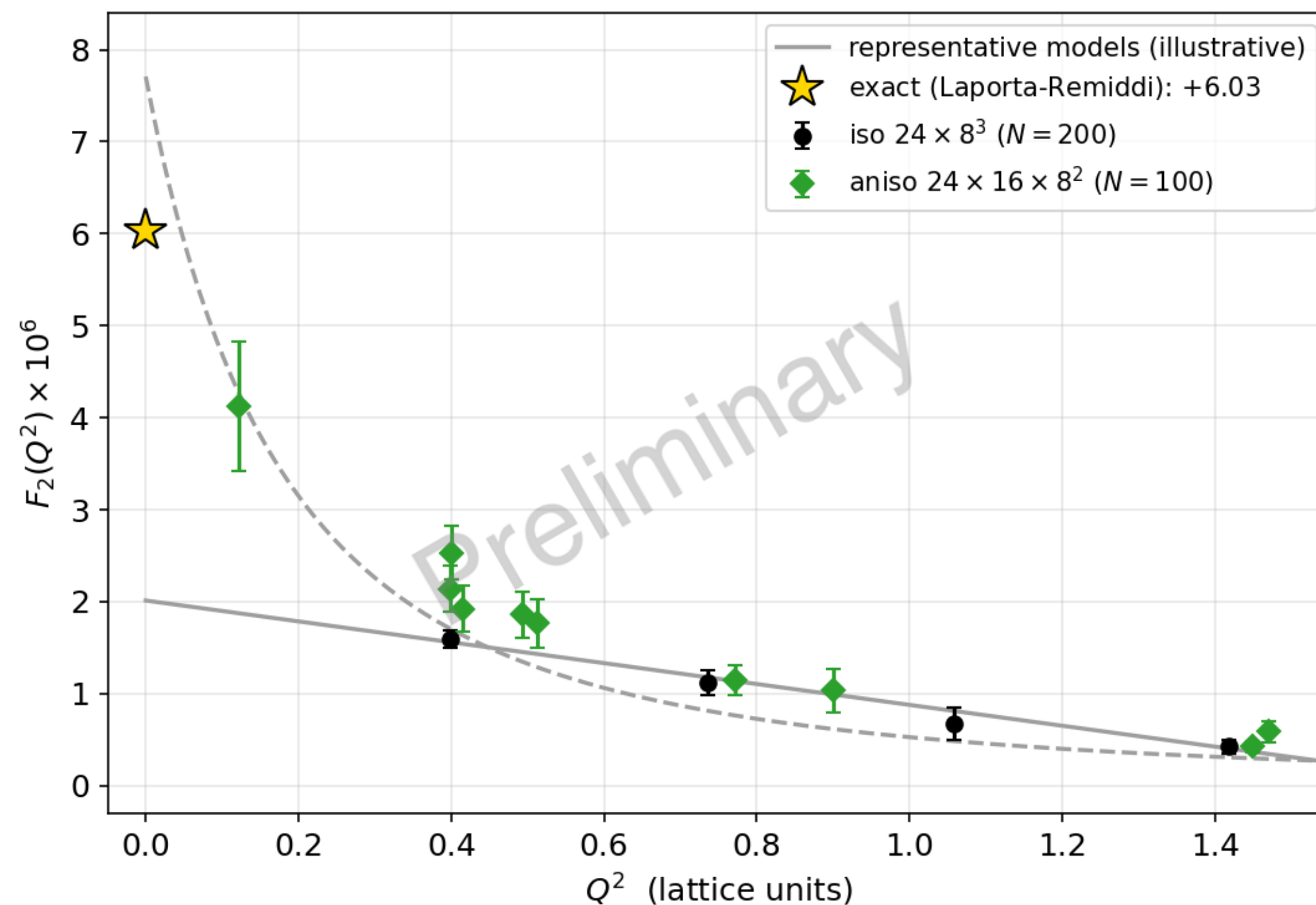
- ♦ Prediction for the leading* FV effects at physical pion mass [left]
- ♦ Plugging in time-like pion form factor [Erben, Green, Mohler, Wittig, PRD 101 (2020) 5, 054504] describes measured RC* data [right]



- ♦ NB: corrections beyond leading still need to be included

Hadronic Light by Light with C* bc's

- ◆ Alternate route to a_μ^{HLbL} : the two-current subtraction [Blum et al., PRL 114 (2015) 012001], no four-point function; extends to fully dynamical QCD+QED with C* bc's (leading disconnected automatically included)
- ◆ Leptonic benchmark against the exact perturbative target [Laporta, Remiddi, PLB 301 (1993) 440]; $F_2(Q^2)$ resolved on two geometries [left]
- ◆ Estimator statistics as a function of electromagnetic coupling α : signal $\propto \alpha^3$, subtracted variance $\propto \alpha^6$ [right]



[See poster and talk by Khai Phan, Thu. 12.15p]

Summary & Outlook

- ◆ Compared two approaches to lattice QCD+QED: sampling of full QCD+QED path integral and perturbative QED (RM123)
- ◆ $a_\mu^{\text{U,w}} \times 10^{11} = \begin{cases} 1094(21) & \text{RM123} \\ 1085(7) & \text{full QCD + QED} \end{cases}$ [Altherr et al., 2506.19770 JHEP10(2025)158] $m_\pi \approx 400 \text{ MeV}, L \approx 1.7 \text{ fm}, a \approx 0.05 \text{ fm}$
- ◆ Extend to other observables: baryons [Rosso et al., PoS(LATTICE2025)080, 2602.23910], $\Gamma(\tau \rightarrow X \nu_\tau)$ including IB; [E. Bäske @ LATT26]; renormalization of H_W [L. Holan @ LATT26];
- ◆ Better understanding of volume dependence in QCD [P.Tavella @ LATT26]; see also [Martins,Patella, PoS LATTICE2022 (2023) 323, 2212.09565]
- ◆ Ongoing: QED FV effects to meson masses [K. Kersic @ LATT26], disconnected HVP [A.Altherr, K. Kersic, T. Harris], HLbL contribution [see poster and talk by Kai Phan, Thu. 12.15p]
- ◆ Next steps HVP-related: QED FV effects to a_μ^{HVP} , m_π dependence of a_μ^{HVP} , continuum limit

RC* Collaboration



Anian Altherr, Erik Bäske, Isabel Campos
Plasencia, Tim Harris, Lukas Holan, Klemen
Kersic, Marina Marinkovic, Francesca Margari,
Agostino Patella, Khai Phan, Sara Rosso,
Nazario Tantalo, Paola Tavella

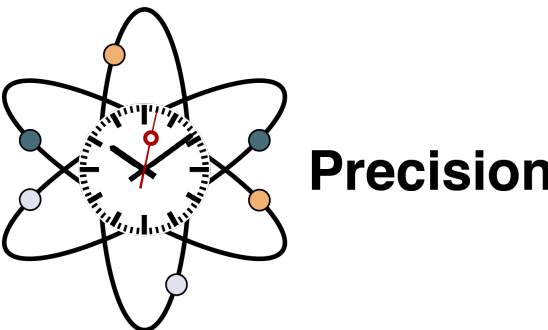
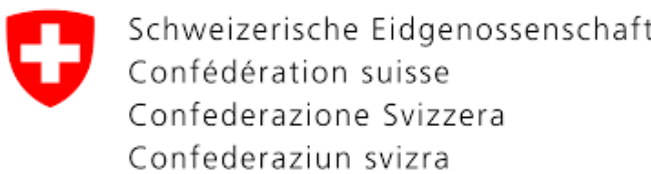
Alumni: Lucius Bushnaq, Alessandro Cotellucci, Madeline
Dale, Roman Gruber, Leo Filipovic, Martin Hansen, Javad
Komijani, Jens Lücke, Sofie Martins, Letizia Parato, ...

Founding alumni: Patrick Fritzs, Alberto Ramos

9th Plenary Workshop of the Muon g-2 Theory Initiative

Aug 3 – 7, 2026
Department of Physics, University of Connecticut
America/New_York timezone

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Additional slides

Mass corrections to $\pi^\pm$ meson masses

- ◆ Expand about isosymmetric theory: isoQCD (C* bc's) + RM123 [RM123: Divitiis et al. 1303.4896 PRD 87 (2013) 11]

- Perturbatively add IBE onto isospin-symmetric QCD (isoQCD):

$$\begin{array}{c} \text{isoQCD} \\ (e^2 = 0, m_u = m_d) \end{array} + \begin{array}{c} \text{strong} \\ (m_u \neq m_d) \end{array} + \begin{array}{c} \text{electromagnetic} \\ (e^2 \neq 0) \end{array}$$

A diagram showing the decomposition of perturbative additions. A bracket from the word "IBE" in the bullet point above points to two arrows. One arrow points to the "strong" box, and the other points to the "electromagnetic" box.

- Observable expansion:

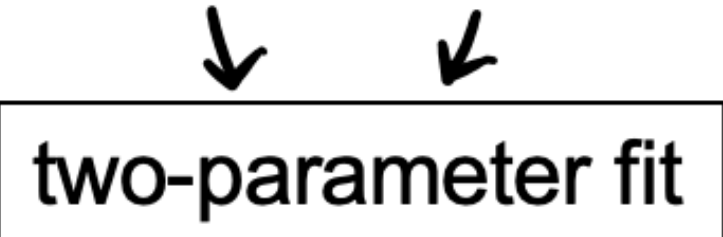
$$X = \underbrace{X^{\text{isoQCD}}}_{\mathcal{O}(e^{-mL})} + \underbrace{\sum_f \Delta m_f \partial_{m_f} X + (\Delta g_s)^2 \partial_{g_s^2} X + e^2 \partial_{e^2} X}_{\mathcal{O}\left(\frac{1}{L}\right)} + \mathcal{O}(e^4, (\Delta g_s)^2)$$

[Credit: K. Kersic]

Mass corrections to $\pi^\pm$ meson masses

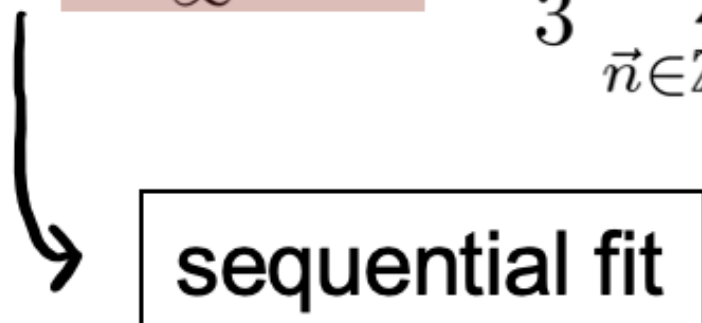
IBE component:

$$\begin{aligned}\Delta M_{\text{IBE}}(L) &= \Delta M_{\infty}^{\text{IBE}} + c_1 L^{-1} + \frac{c_2}{M_{\infty}} L^{-2} + \mathcal{O}(L^{-4}) + \mathcal{O}(e^{-m_{\infty} L}) \\ &= \Delta M_{\infty}^{\text{IBE}} + c_1 L^{-1} + \frac{c_2}{M_{\infty}^{\text{isoQCD}} + \Delta M_{\infty}^{\text{IBE}}} L^{-2} + \mathcal{O}(L^{-4}) + \mathcal{O}(e^{-M_{\infty} L}).\end{aligned}$$



isoQCD component:

$$M_{\text{isoQCD}}(L) = M_{\infty}^{\text{isoQCD}} - \frac{\xi}{3} \sum_{\vec{n} \in \mathbb{Z}^3 \setminus \{0\}} \frac{1 - 3(-1)^{\sum_k n_k}}{|n|L} K_1(|n| M_{\infty}^{\text{isoQCD}} L)$$

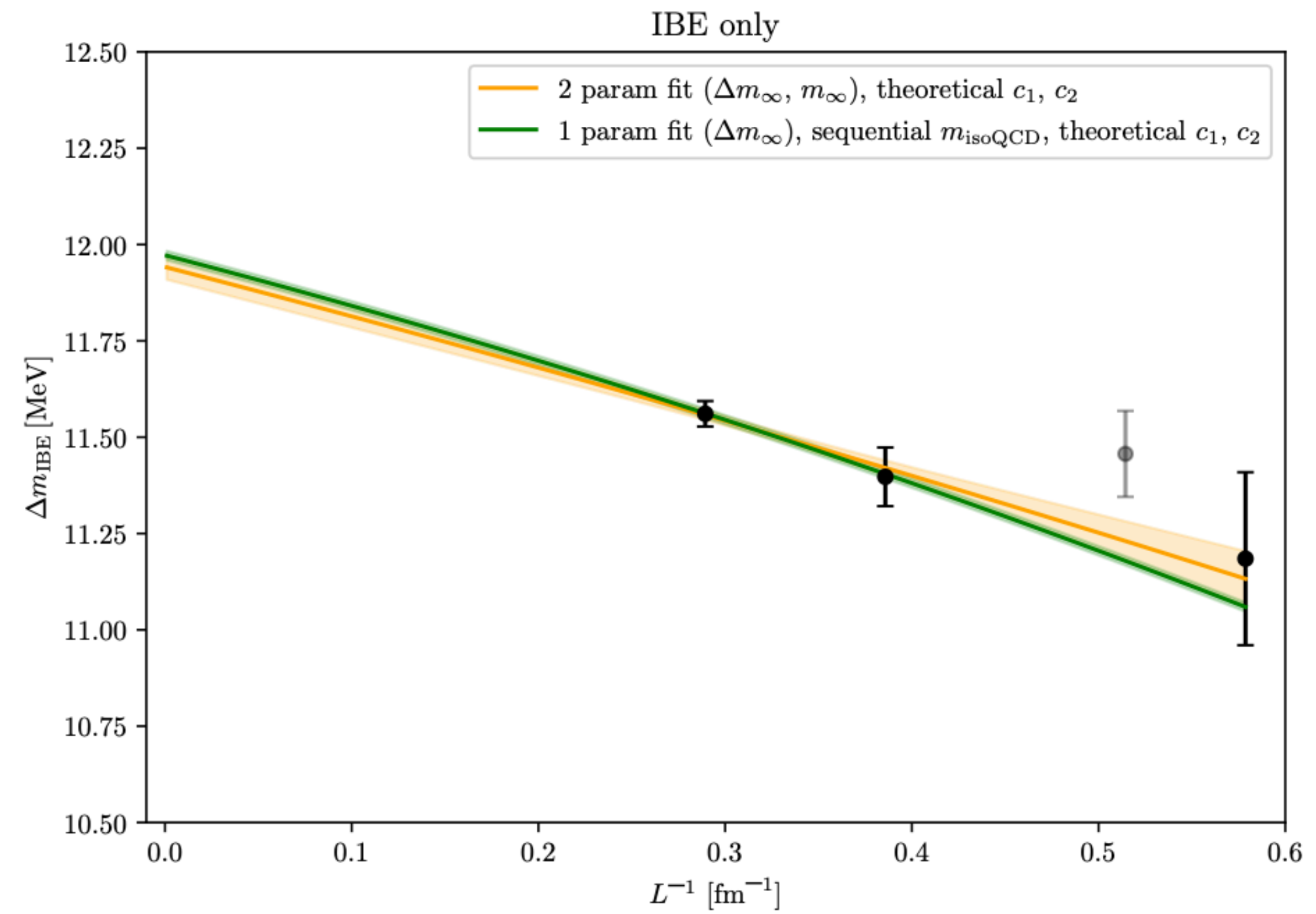
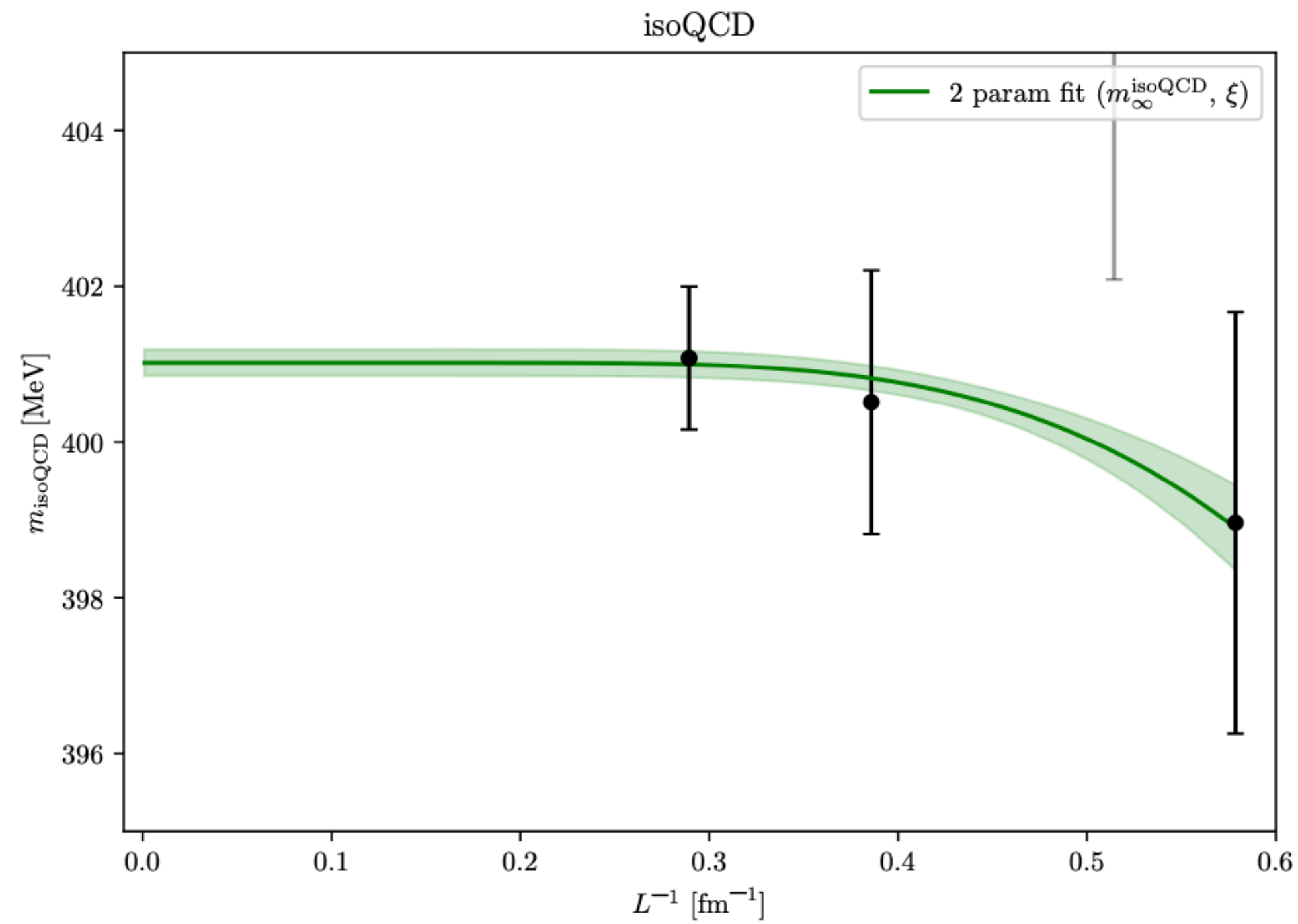


[Bushnaq et al. JHEP03, 012 (2023)]

[Credit: K. Kersic]

Mass corrections to $\pi^\pm$ meson masses

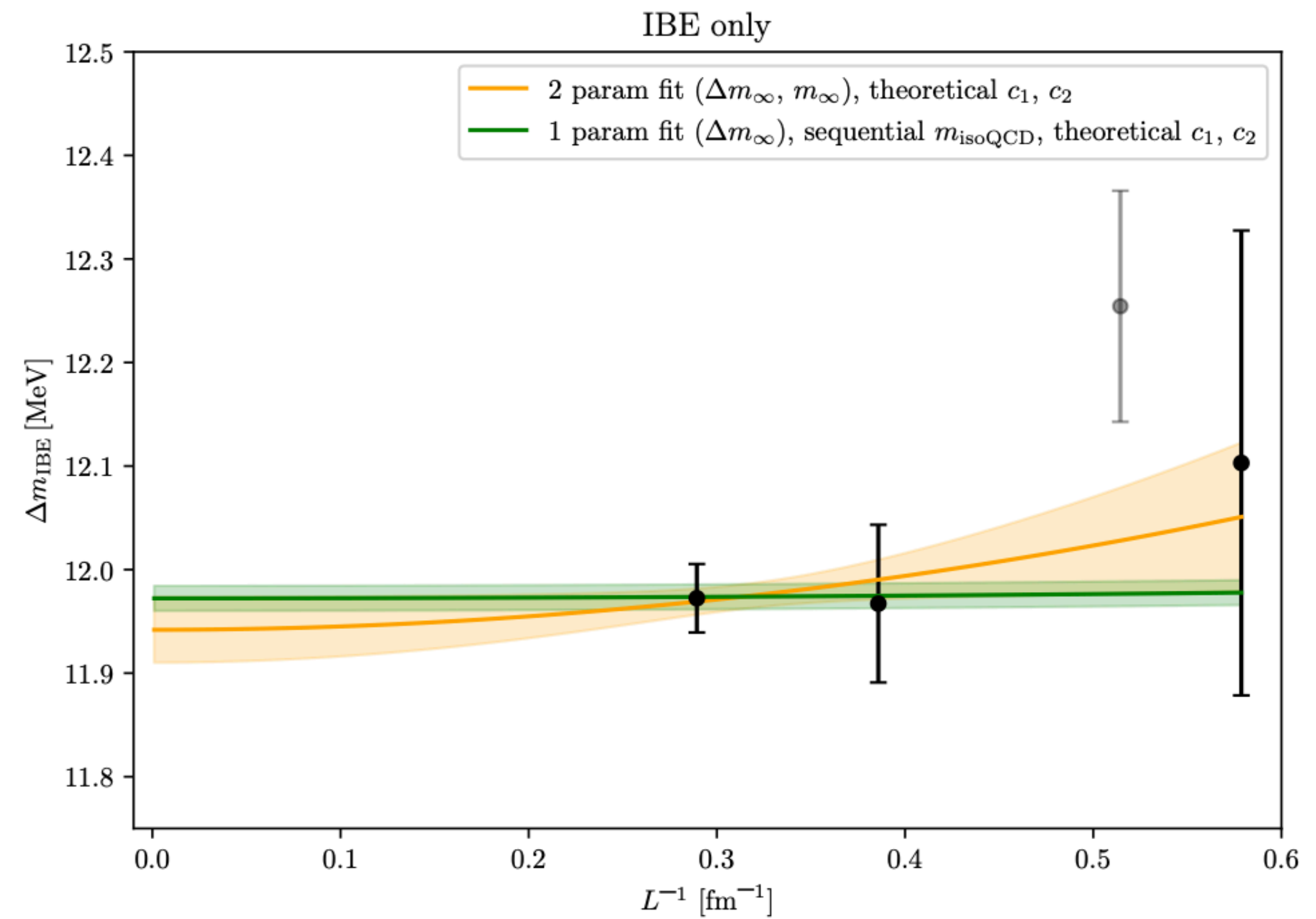
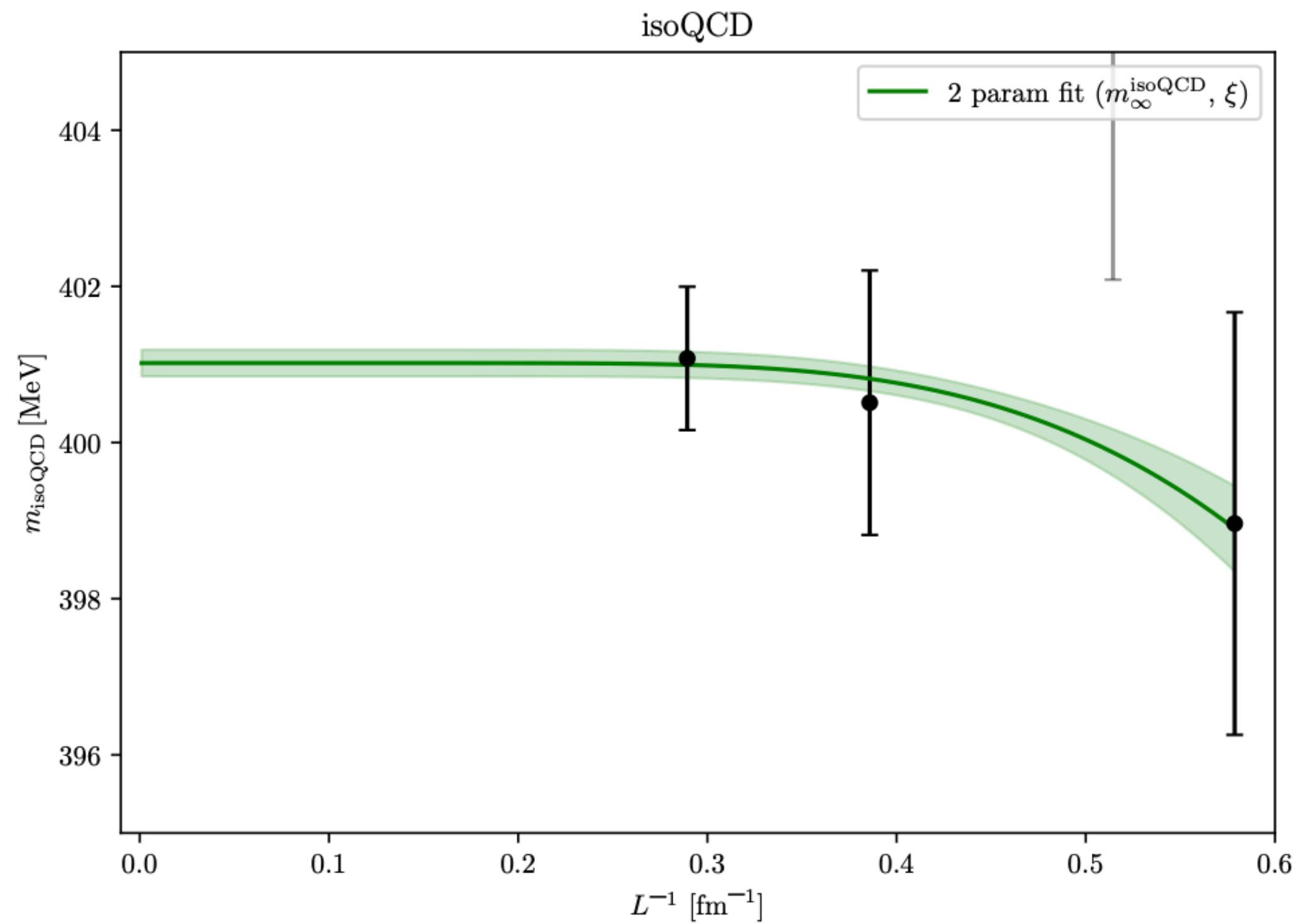
- ◆ Including universal terms; one-parameter fit: using fitted isoQCD values, two-parameter fit: directly fitting IBE data



[Credit: K. Kersic]

Mass corrections to $\pi^\pm$ meson masses

- ◆ Universal term subtracted using theoretical c_1, c_2 ; $1/L^3$ not present with C* bc's [Lucini et al. JHEP02(2016)076]

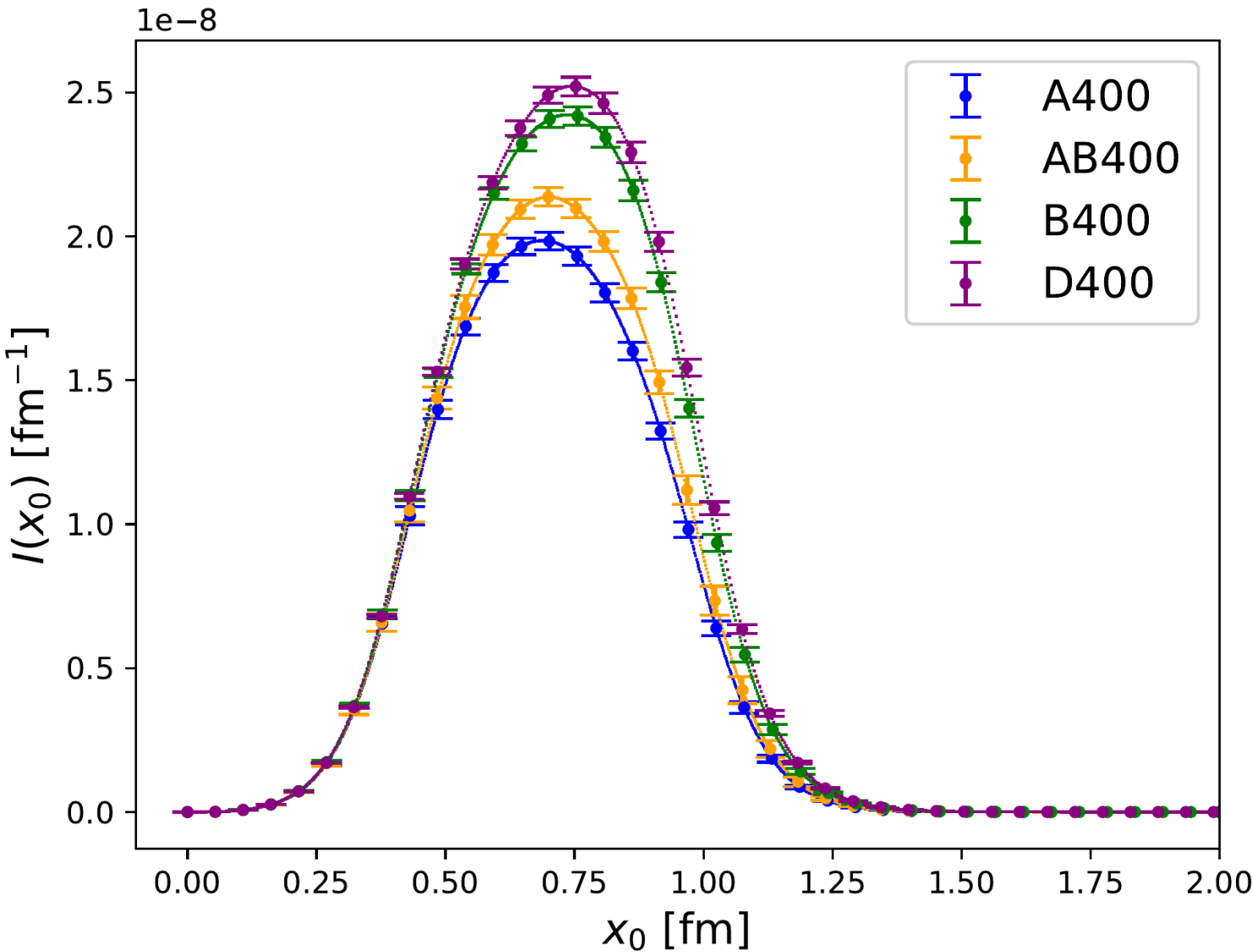


[Credit: K. Kersic]

Finite volume effects to $a_\mu^{33,w}$ in QCD

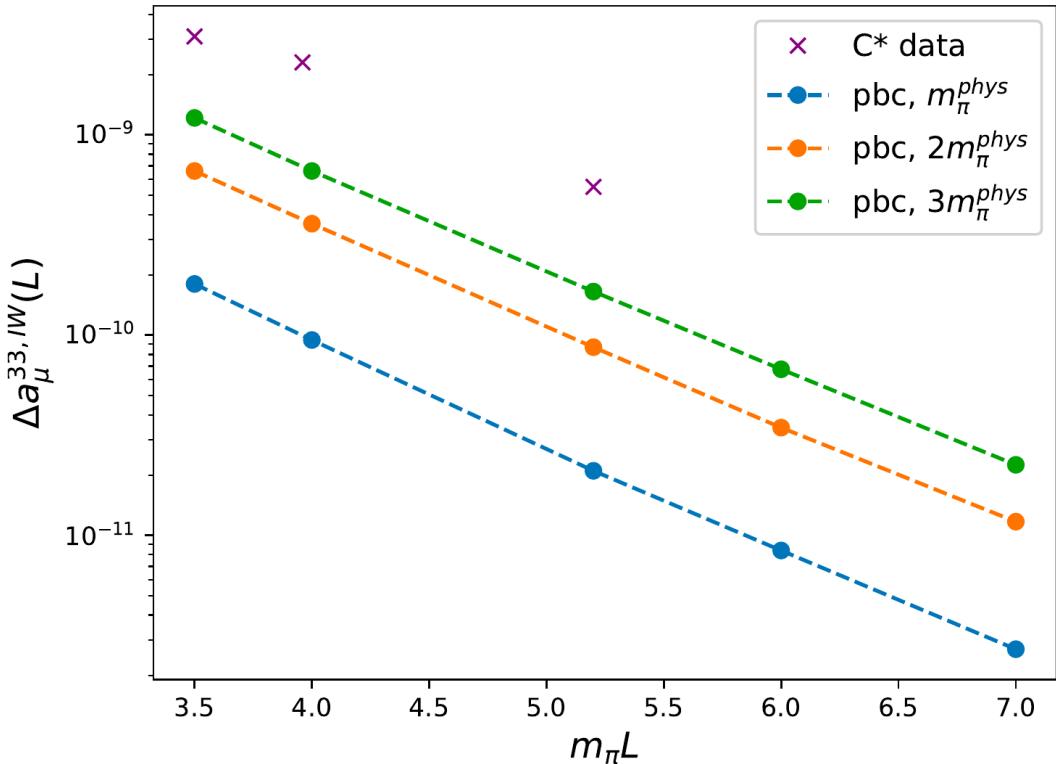
- Numerical estimate of the finite volume (FV) effects on four ensembles with same lattice spacing and similar pion mass $m_\pi \approx 400$ MeV

Ensemble	L [fm]	m_π [MeV]	$m_\pi L$
A400	1.73	398.5(4.7)	~ 3.5
AB400	1.93	407.5(4.2)	~ 4.0
B400	2.59	401.9(1.4)	~ 5.2
D400	3.44	402.4(1.5)	~ 7



- Expected pion mass dependence of the FV effects vs. measurements on RC* ensembles [\[Hansen, Patella, Phys.Rev.Lett. 123 \(2019\) 172001\]](#)

$m_\pi L$	m_π^{phys}	$2m_\pi^{\text{phys}}$	$3m_\pi^{\text{phys}}$	From C* data
3.5	$1.8 \cdot 10^{-10}$	$6.6 \cdot 10^{-10}$	$1.3 \cdot 10^{-9}$	$3.1(1) \cdot 10^{-9}$
4.0	$9.5 \cdot 10^{-11}$	$3.6 \cdot 10^{-10}$	$6.9 \cdot 10^{-10}$	$2.3(1) \cdot 10^{-9}$
5.2	$2.10 \cdot 10^{-11}$	$8.7 \cdot 10^{-11}$	$1.7 \cdot 10^{-10}$	$5(1) \cdot 10^{-10}$
6.0	$8.40 \cdot 10^{-12}$	$3.5 \cdot 10^{-11}$	$7.0 \cdot 10^{-11}$	—
7.0	$2.70 \cdot 10^{-12}$	$1.2 \cdot 10^{-11}$	$2.4 \cdot 10^{-11}$	—



Leading FV effects to $a_\mu^{33,w}$ in QCD: steps

$$G_{L\rho}(x_0) = -\frac{1}{3} \int_0^{L_\perp} d^2\vec{x}_\perp \int_0^L dx_3 \frac{\text{tr}[\mathcal{B} e^{-LH} T_3 \{j_\mu(x_0, \vec{x}_\perp, x_3) j_\mu(0, \vec{0}, L/2)\}]}{\text{tr}[\mathcal{B} e^{-LH}]}$$

1. Insert complete basis of eigenstates of H, p, U_C : $|n\rangle$

2. Integration over x_3

$$G_{L\rho}(x_0) = -\frac{1}{3} \int_0^{L_\perp} d^2\vec{x}_\perp \sum_{n,n'} \frac{M_{nn'}(x_0, \vec{x}_\perp)}{\mathcal{Z}} \chi_n \begin{cases} \textcircled{1} & \textcircled{2} \\ 2 \frac{e^{-LE_n} - e^{-L/2(E_n+E'_n)}}{E_{n'} - E_n}, & E_n \neq E_{n'} \\ \textcircled{3} & Le^{-LE_n}, & E_n = E_{n'} \end{cases}$$

- $M_{nn'}(x_0, \vec{x}_\perp) = \langle n | j_\mu(x_0, \vec{x}_\perp) | n' \rangle \langle n' | j_\mu(0) | n \rangle$
- $\chi_n = 1$, $\mathcal{Z} = \sum_n e^{-LE_n}$, $\chi_n = \eta_n^c$, $\mathcal{Z} = \sum_n \eta_n^c e^{-LE_n} \rightarrow$ selection rule for $|n\rangle, |n'\rangle$

3. Expand for large L and isolate $\Delta G(x_0|L_\perp, L) = G(x_0|L_\perp, L) - G(x_0|L_\perp, \infty)$

$$\text{pbc} : \textcircled{1} + \textcircled{3}, \quad \text{C}^* : \textcircled{1} + \textcircled{2}$$

4. Take the limit for $L_\perp \rightarrow \infty \Rightarrow$ **FV effects parametrized in terms of infinite volume quantities**

Dynamical QCD+QED

1. Sampling from the probability distribution

$$p(U, z) \propto \prod_f \text{Pf}(CKD_f[U, z]) e^{-S_{g, \text{SU}(3)}[U] - S_{g, \text{U}(1)}[z]}$$

- U(1) link: $z_\mu(x) = e^{ieaA_\mu(x)}$
- $\text{Pf}(CKD_f[U, z]) = (\det(CKD_f[U, z]))^{1/2}$

2. Compute two-point correlation functions on QCD+QED configurations:

$$\mathcal{O}_{2pt}(x, y) = \text{Tr}\{D_f^{-1}(y|x)\Gamma_A D_g^{-1}(x|y)\Gamma_B\} = x \begin{array}{c} \text{---} g \text{---} \\ \text{---} f \text{---} \end{array} y$$

Perturbative treatment of IB corrections: RM123

1. Expansion of the probability density $\text{Pf}(CKD_f[U, z])e^{-S_{\text{g}, \text{SU}(3)}[U]}e^{-S_{\text{g}, \text{U}(1)}[z]}$

$$p(U, A) \propto \prod_f \underbrace{\text{Pf}(CKD_f^{(0)}[U])e^{-S_{\text{g}, \text{SU}(3)}[U]}}_{\text{sampled } p(U)} \underbrace{e^{-S_\gamma[A]} \left[1 + O(\Delta D_f[U, A]) \right]}_{\text{obs. comp.}}$$

2. Expansion of the observable

$$D_f^{-1} : \text{fermion line with self-energy} = \text{fermion line} + \Delta m_f \text{fermion line with mass insertion} + eq_f \text{fermion line with photon insertion} + \frac{1}{2}e^2 q_f^2 \text{fermion line with photon loop}$$

$$\Gamma_{\tilde{V}} : \text{fermion-fermion vertex} = \text{fermion-fermion vertex} + eq_f \text{fermion-fermion vertex with photon insertion} + \frac{1}{2}e^2 q_f^2 \text{fermion-fermion vertex with photon loop}$$

Coulomb gauge:

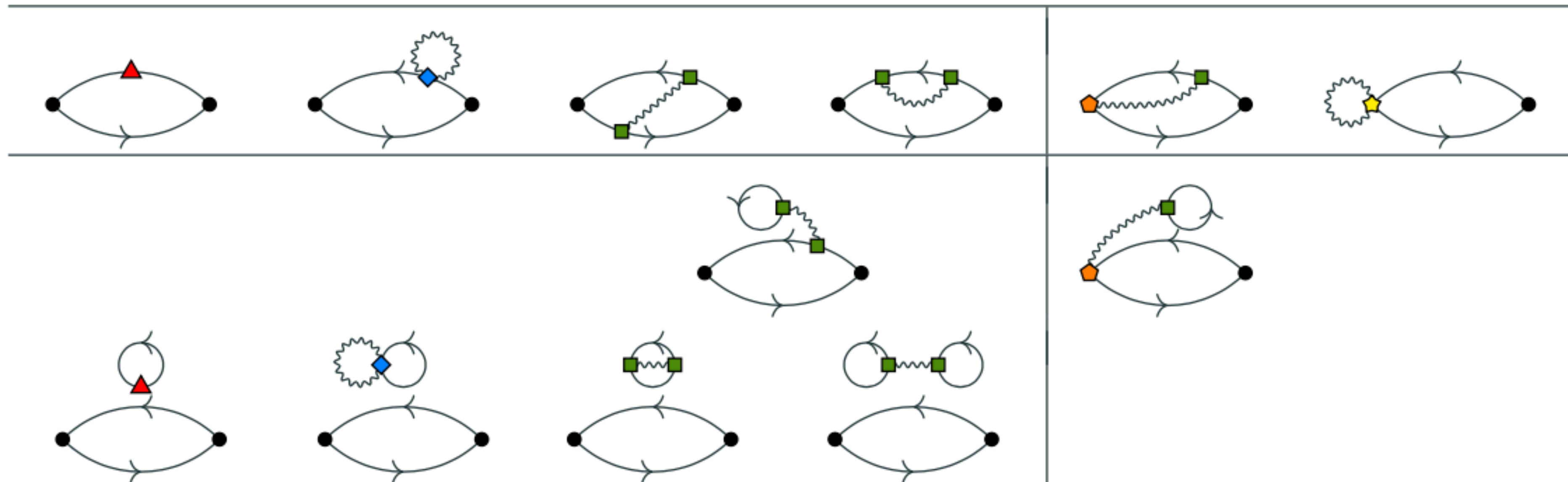
$$\sum_{k=1}^3 \partial^* A_k(x) = 0$$

Perturbative treatment of IB corrections: RM123

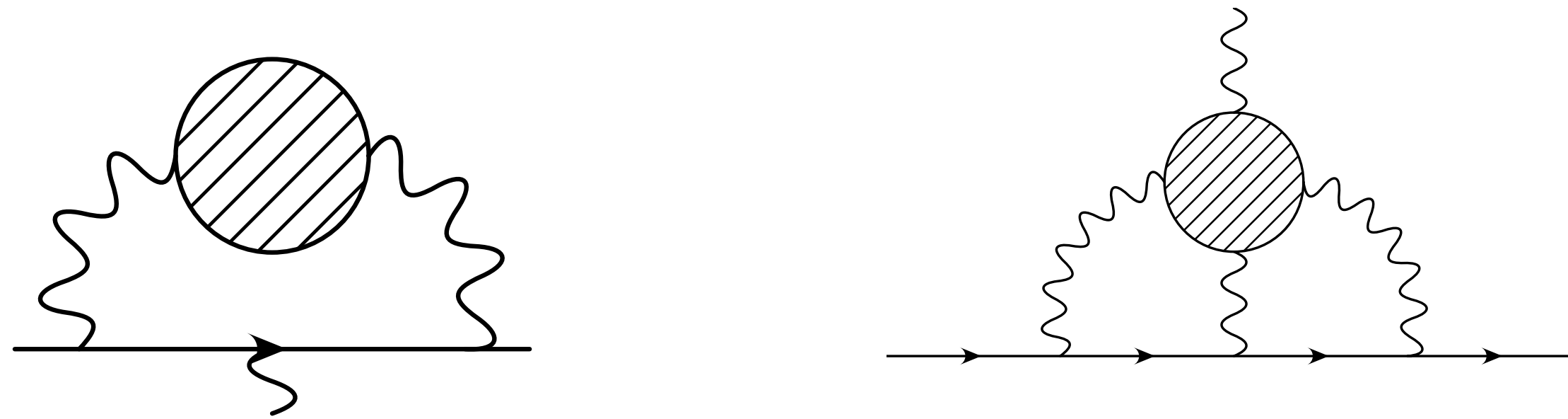
3. Computation of the LO two-point correlators

$$\mathcal{O}_{2pt}^{(0)}(x, y) = \text{Tr}\{D_f^{(0)-1}(y|x)\Gamma_A D_g^{(0)-1}(x|y)\Gamma_B\} = x \cdot \begin{array}{c} \text{---} \text{ } g \text{---} \\ \text{---} \text{ } f \text{---} \end{array} \cdot y$$

4. Computation of the IB corrections: valence-valence, sea-valence, sea-sea



Non-perturbative calculations of hadronic contributions to the muon g-2



[Muon g-2 theory Initiative, White Paper Update
[WP25], Phys.Rept.1143
(2025), 2505.21476 [hep-ph]]

- ♦ Hadronic contributions are obtained by integrating over all possible virtual photon momenta

$$\text{wavy line} \text{---} \text{red circle} \text{---} \text{wavy line} \quad \hat{\Pi}(q^2) \quad a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2) \hat{\Pi}(q^2)$$

- ♦ P.T. cannot reliably compute full hadronic contribution
- ♦ Two-point & four-point correlation functions:

$$\text{HVP: } \langle 0 | T\{j_{\mu} j_{\nu}\} | 0 \rangle \quad \text{HLbL: } \langle 0 | T\{j_{\mu} j_{\nu} j_{\rho} j_{\sigma}\} | 0 \rangle$$

