An Update on Light-Quark-Connected Hadronic Vacuum Polarization Contribution to the Muon $g-2$ with Staggered Fermions

9th Plenary Workshop of the Muon $g-2$ Theory Initiative

Vaishakhi Moningi

University of Connecticut

University of Connecticut, Storrs, Connecticut, USA
August 3 – 7, 2026

Collaborators

HVP with staggered fermions

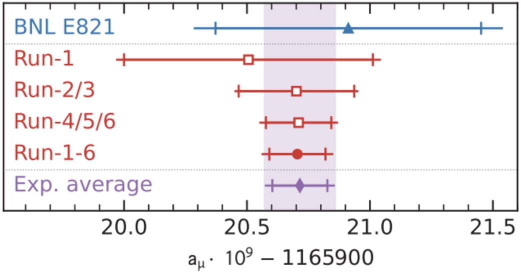
Thomas Blum (UConn), Luchang Jin (UConn), Christopher Aubin (Fordham), Maarten Golterman (SFSU), Santiago Peris (Barcelona)

Table of Contents

Muon $g-2$: Motivation and Experimental Status

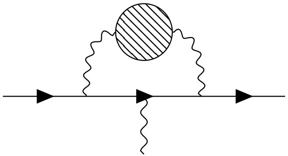
Why muon $g-2$?

- Anomalous magnetic moment $a_\mu = (g-2)/2$; $g = 2$ at tree level, $g > 2$ from quantum corrections
- A precision test of the Standard Model: compare high-precision experiment vs. theory to expose new particles / interactions
- Theory error dominated by the **hadronic vacuum polarization** (HVP) and hadronic light-by-light (HLbL)
- Lattice QCD gives a first-principles determination of the HVP



- Experimental world average now at 124 ppb [Donati et al. 2025]

Theoretical Framework: HVP in the Time-Momentum Representation



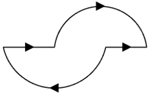
$$a_\mu^{\mathrm{HVP}} = 4\alpha^2 \int_0^\infty dq^2\, f(q^2)\, \hat{\Pi}(q^2)$$

$$\xrightarrow[\text{[Bernecker and Meyer 2011]}]{\text{time-mom. rep.}} 2 \sum_{t=0}^{T/2} w(t)\, C(t)$$

where,

- $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$ the subtracted HVP,
- $w(t)$ QED kernel and
- $C(t) = \frac{1}{3}\sum_{\vec{x},i} \langle J^i(\vec{x},t) J^i(0) \rangle$ is the two-point current-current function.

- On the lattice the single-site current is not conserved, so we use a single link point-split **conserved** current.



$$J^\mu(x) = \frac{1}{2}\eta_\mu(x)\big(\bar{\chi}(x+\hat{\mu})U_\mu^\dagger(x)\chi(x) + \bar{\chi}(x)U_\mu(x)\chi(x+\hat{\mu})\big)$$

Spectral decomposition of the propagator

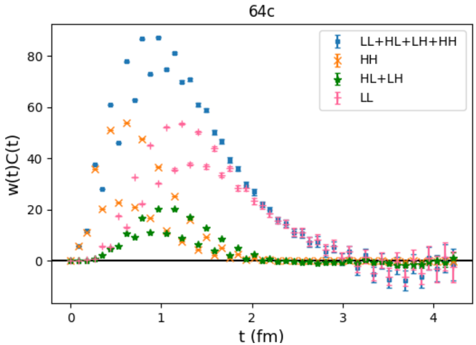
$$S(x,y) = \underbrace{S_L}_{\lambda \le \lambda_{\mathrm{cut}}} + \underbrace{S_H}_{\lambda > \lambda_{\mathrm{cut}}}$$

$$\boxed{C(t) = C_{LL} + C_{LH} + C_{HL} + C_{HH}}$$

gives four correlator contributions [Giusti et al. 2004; Moningi et al. 2025]

Table of Contents

Four-Part Decomposition of the Correlator



Low-Low (dominant, long distance)

- Constructed from contracting the low-mode **sparsenend meson fields** Λ_μ,

$$(\Lambda_\mu)_{n,m} = \sum_{\vec{x}} \langle n|x\rangle \; U_\mu(x) \; \langle x+\mu|m\rangle$$

[Moningi et al. 2026]

High-Low (sub-dominant)

- **Stochastic** linear combinations of low-mode sources per time slice is used as a source for the (high-mode part of the) propagator to effect the full-volume average. $\Rightarrow$ solves reduced from $N_T \times N_{\rm low}$ to $N_T \times N_{\rm hits}$ [Moningi et al. 2025]

High-High (dominant, short distance)

AMA with deflated, approximate CG:

$$\langle O\rangle = \langle O\rangle_{\rm ex} - \langle O\rangle_{\rm sub} + \frac{1}{N}\sum_i \langle O_i\rangle_{\rm app}$$

Table of Contents

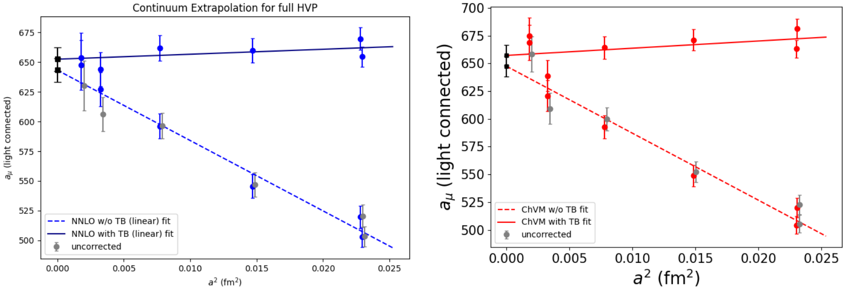
Lattice Ensemble Details

Label	$\approx a$ (fm)	$N_S^3 \times N_T$	L (fm)	m_π (MeV)	w_0/a	# configs
48II	0.15'	$48^3 \times 64$	4.85	134.73	1.13190(35)	40
32c	0.15	$32^3 \times 48$	4.85	134.44	1.13227(18)	48
48I	0.12	$48^3 \times 64$	5.81	134.53	1.41060(28)	32
64c	0.09	$64^3 \times 96$	5.61	128.12	1.95148(41)	78
96c	0.06	$64^3 \times 192$	5.45	134.61	3.01838(92)	77
144c	0.04	$144^3 \times 228$	6.12	134.17	4.03242(195)	20

- Configurations provided by the MILC collaboration (except 48II ensemble is a CalLat ensemble)
- The lattice spacings are set using using the $w_0 = 0.17187(68)$ fm which is is determined using the mass of Ω^- baryon [Bazavov et al. 2026] and w_0/a are obtained from v1 of Ref. [Bazavov et al. 2025a]

Full HVP: $a_\mu^{\mathrm{HVP,lqc}}$ (Preliminary)

FV, pion-mass mistuning and taste-breaking corrections, computed in NLO and NNLO SChPT, as well as in the ChVM model developed specifically for the case of staggered fermions in [Chakraborty et al. 2017] and applied further in [Borsanyi et al. 2021],[Aubin et al. 2022].



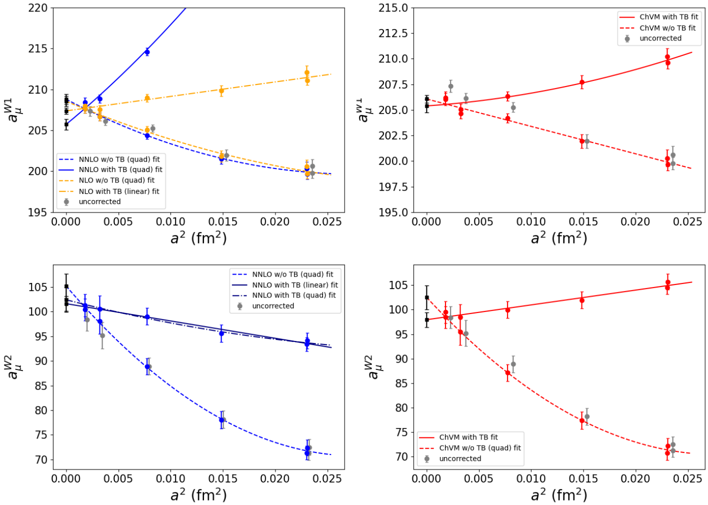
$$\text{NNLO} : a_\mu^{\mathrm{HVP,lqc}} = (660.5 \pm 9.2 \pm 4.0 \pm 1.9 \pm 0.3) \times 10^{-10} = 661(10) \times 10^{-10}$$

$$\text{ChVM} : a_\mu^{\mathrm{HVP,lqc}} = (652.4 \pm 9.2 \pm 4.9) \times 10^{-10} = 652(10) \times 10^{-10}$$

Errors: (stat, from fit)(half-distance between fits)(FV)(retuning to 144^3). Statistical errors uncorrelated across ensembles.

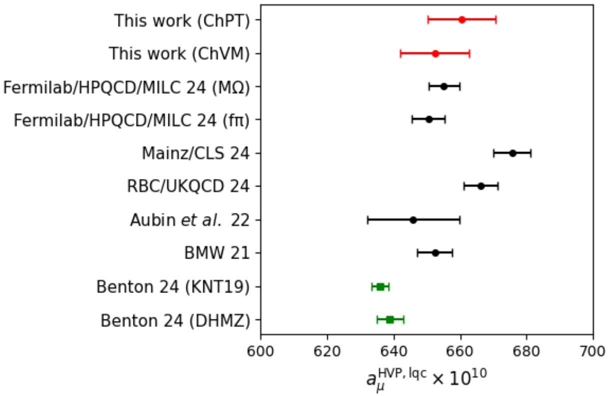
Window Quantities a_μ^{W} (Preliminary)

Intermediate windows[Blum et al. 2018]: standard $(0.4, 1.0, 0.15)$ fm and long-distance $(1.5, 1.9, 0.15)$ fm (more accessible to ChPT).



- ChPT less reliable for W1 (distances < 1 fm).

Comparison



Our $a_\mu^{\mathrm{HVP},lqc}$ agrees with existing lattice results and sits *above* the data-driven evaluation.

- This work
- FNAL/HPQCD/MILC '24[Bazavov et al. 2025b], Mainz '24[Djukanovic et al. 2025], RBC/UKQCD '24[Blum et al. 2025], Aubin '22[Aubin et al. 2022], BMW '21[Borsanyi et al. 2021]
- Data-driven: Benton '25[Benton et al. 2025]

a (fm)	$a_\mu^{\mathrm{HVP,lqc}}$ $[\times 10^{-10}]$	$a_\mu^{\mathrm{W1,lqc}}$ $[\times 10^{-10}]$	$a_\mu^{\mathrm{W2,lqc}}$ $[\times 10^{-10}]$
0.0426(2)	658(15)(5)[15]	207.3(5)(3)[6]	98.4(1.9)(1.0)[2.1]
0 (NNLO ChPT)	661(9)(4)[10]	—	103.4(2.5)(1.8)[3.1]
0 (ChVM)	652(9)(5)[10]	205.7(6)(3)[7]	100.2(2.5)(2.3)[3.3]

Table of Contents

Where the Error Comes From

On the finest ensemble $144^3 \times 288$, a=0.04 fm, we have

$$a_\mu^{\mathrm{HVP,\,lqc}} = (658.4 \pm 15.3) \times 10^{-10}$$

[Moningi et al. 2025; Moningi et al. 2026].

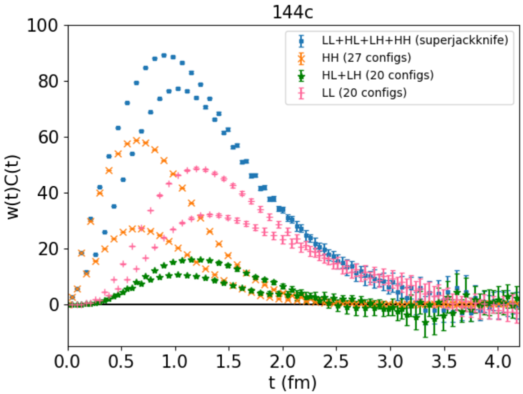
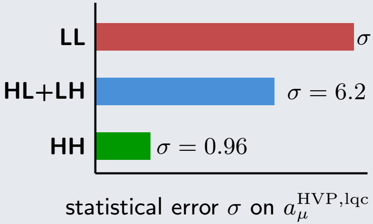


Figure 1: Summand $w(t)C(t)$ (units of 10^{-10}), statistical errors only.

Error budget breakdown



Challenge

Statistical precision of the HVP driven by C_{LL} is the current bottleneck

Idea: Skip Eigenvector Generation Entirely

Our strategy: eliminate the most expensive step

HISQ eigenvectors from MILC/Fermilab are **already available** $\rightarrow$ skip eigenvector generation, proceed directly to the meson-field contraction with sparsening. This removes the single most expensive step in the C_{LL} pipeline.

But the existing calculation uses a **fat-link** action, while the available eigenvectors are **HISQ**. The two actions differ, so we need a controlled way to use HISQ eigenvectors to compute the fat-link target – **AMA-style bias correction** (next).

The HISQ Dirac Operator

$$D_{\text{HISQ}} = c_1 D_{\text{Fat}} + c_2 D_{\text{Naik}}$$

Fat-Link Term D_{Fat}

- 1-link (nearest-neighbor) term with $c_1 = 1, c_2 = 0$
- ASQTAD fat7 smearing $\rightarrow$ reunitarization $\rightarrow$ another round of fat7 smearing
- Suppress taste-symmetry breaking
- Used in **our existing** calculation

Naik Term D_{Naik}

- 3-link (next-to-nearest-neighbor) term with $c_1 = 9/8, c_2 = -1/24$
- Removes free-field $\mathcal{O}(a^2)$ errors in the quark dispersion relation.
- Included in **MILC/Fermilab eigenvectors**

AMA-Style Bias Correction Strategy

The standard AMA estimator [Blum, Izubuchi, and Shintani 2013] applied to the action mismatch:

Our AMA mapping

$$\underbrace{C^F_{LL}}_{\text{exact}} = \underbrace{C^H_{LL}}_{\text{approx}} + \underbrace{(C^F_{LL} - C^H_{LL})}_{\Delta:\text{ bias correction}}$$

Exact
(Fat-Link Action)

- C_{LL} computed with fat-link operator D_F

→

Bias
(Both Actions)

- Small subset of exact measurements
- Estimate

$\Delta = (C^F - C^H)$

→

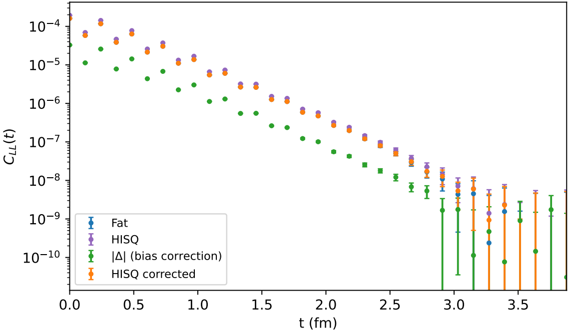
Approx
(HISQ Action)

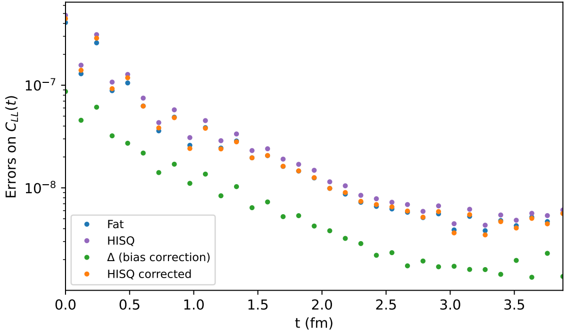
- C_{LL} from D_{HISQ}
- Cheap: evecs available

Key: the HISQ correlator (C^H) is the *approximation* (cheap, evecs already available); the fat-link correlator (C^F) is the *exact* target.

Validation on $48^3 \times 64$: Low-Low Correlator

- $48^3 \times 64,\ a = 0.1212$ fm
- $N_{\text{total}} = 51$
- $N_{\text{hisq}} = 51$
- $N_{\text{bias}} = 20$ configurations





Correlator comparison

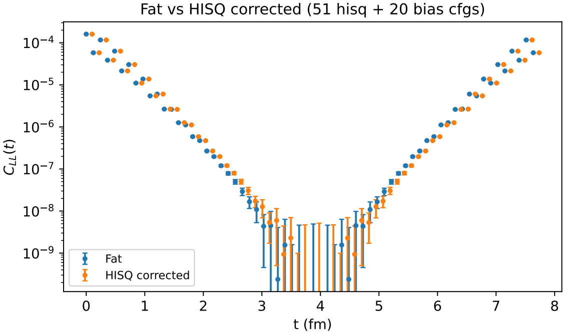
- $|\Delta|$ is 1–2 orders of magnitude below the fat-link correlator
- Fat and HISQ-corrected overlap
- Bias correction is a small perturbation at all timeslices

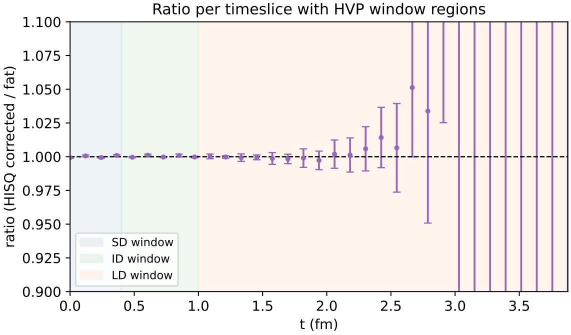
Error comparison

- $\sigma_\Delta < \sigma_{\text{HISQ}}$ across all timeslices
- Ratio $\sigma_\Delta / \sigma_{\text{HISQ}} \approx 0.27$
- Bias adds only $\sim 7\%$ to the variance

Validation on $48^3 \times 64$: Ratio Analysis

- $48^3 \times 64$, $a = 0.1212$ fm • $N_{\text{total}} = 51$ • $N_{\text{hisq}} = 51$ • $N_{\text{bias}} = 20$ configurations



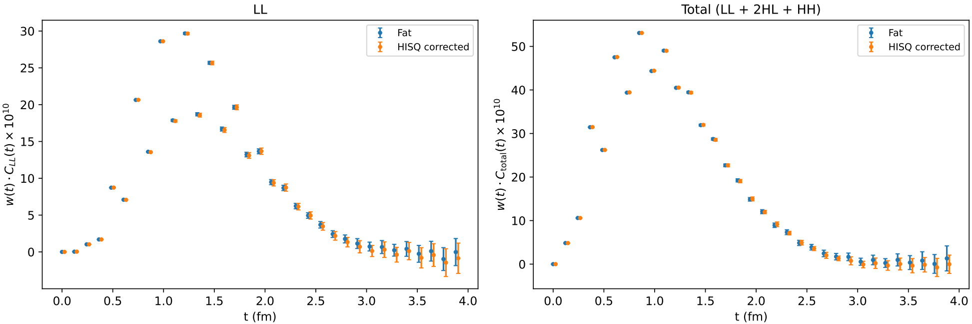


Region analysis

- $t = 0$–15: Ratio $= 1.0002 \pm 0.0008$, consistent with unity
- $t = 15$–22: Growing but controlled, still consistent with unity
- $t > 22$: Signal lost in noise

Validation on $48^3 \times 64$: $w(t)C(t)$

- $48^3 \times 64,\ a = 0.1212$ fm • $N_{\text{total}} = 51$ • $N_{\text{hisq}} = 41$ • $N_{\text{bias}} = 20$ configurations



- Fat-link and bias-corrected HISQ $w(t)C(t)$ overlap across all timeslices
- Errors grow at large t as the correlator signal decays exponentially

Validation on $48^3 \times 64$: All Windows

- $48^3 \times 64,\ a = 0.1212$ fm • $N_{\text{total}} = 51$ • $N_{\text{hisq}} = 51$ • $N_{\text{bias}} = 20$ configurations
- Ratio $= a_\mu^{\text{HVP,lqc}}$ (hisq corrected)$/a_\mu^{\text{HVP,lqc}}$ (fat)

	Intermediate $(0.4,\ 1.0,\ \Delta{=}0.15)$ fm	**Intermediate** $(1.5,\ 1.9,\ \Delta{=}0.15)$ fm	**Long distance** $(t_1{=}1,\ \Delta{=}0.15)$ fm
LL only			
ratio	1.0001 ± 0.0005	0.9988 ± 0.0041	1.0041 ± 0.0121
consistency	0.22σ ✓	0.31σ ✓	0.34σ ✓
Total			
ratio	1.00004 ± 0.00017	0.9991 ± 0.0030	1.0027 ± 0.0081
consistency	0.22σ ✓	0.31σ ✓	0.34σ ✓

Ratio consistent with unity at $< 0.35\sigma$ in **all** windows – no t-dependent systematic shift; the correction is unbiased.

Application to $144^3 \times 288$, $a = 0.042$ fm

Current status

- LL+HL on 20 configurations and HH on 27 configurations
- Fat-link eigenvectors only
- $a_\mu^{\mathrm{HVP,lqc}} = 658(15) \times 10^{-10}$ on this ensemble.

Proposed strategy

- Leverage 38 configurations with HISQ eigenvectors ($N_{\mathrm{low}} = 4000$) already available from MILC/Fermilab
- Apply AMA strategy bias correction to account for the fat-link $\leftrightarrow$ HISQ action difference

Projected error reduction: $\sqrt{(38+20)/20} \approx 1.7\times$

Bringing the statistical uncertainty on $a_\mu^{\mathrm{HVP,lqc}}$ from **2.3%** to **1.4%** on this ensemble.

- Continue generating eigenvectors and correlators, targeting **120 configurations total** to achieve sub-precent precision.

Table of Contents

Summary

What we showed:

- ✓ Preliminary full HVP and window results at continuum using the AMA/LMA (LL+HL+LH+HH) decomposition and **sparsened meson fields**: $a_\mu^{\mathrm{HVP,lqc}} \approx 661(10)$ (NNLO), $652(10)$ (ChVM)$\times 10^{-10}$.
- ✓ Validated a strategy to boost C_{LL} statistics by **reusing HISQ eigenvectors**.
- ✓ AMA bias correction removes the action difference: $< 0.35\sigma$ agreement on $48^3 \times 64$ across all windows, $< 2\%$ error overhead at $N_{\mathrm{bias}} = 20$.

Goals and Roadmap

Milestones

Near-term

- HISQ-eigenvector meson fields + AMA on the 38 available configs; validate LL agreement on 144^3 (as demonstrated on 48^3).
- Extend the AMA bias correction to the HL+LH part

Mid-term

- Reach **120 configs** on Leadership Computing Facility.

Longer-term

- Improve the continuum extrapolation and reduce the statistical error on the 96c ensemble as well.

Projected error reduction:

$$\sqrt{(100+20)/20} \approx 2.4\times$$

Bringing the statistical uncertainty on $a_\mu^{\mathrm{HVP,lqc}}$ from **2.3%** to **0.9%** on 144c ensemble.

Projected error budget (units 10^{-10}) on 144c

	now σ	target σ $(20 \to 120)$
LL	13.7	≈ 5.6
HL+LH	6.2	≈ 2.5
HH	0.96	≈ 0.4
Total	**15.1**	$\approx \mathbf{6.2}$

- Projected by $1/\sqrt{N}$ from the current budget

Thank you for your attention!

Any questions?

Table of Contents

References I

- Aubin, Christopher et al. (2022). "Muon anomalous magnetic moment with staggered fermions: Is the lattice spacing small enough?" In: *Phys. Rev. D* 106.5, p. 054503. DOI: 10.1103/PhysRevD.106.054503. arXiv: 2204.12256 [hep-lat].
- Bazavov, Alexei et al. (2025a). "Hadronic vacuum polarization for the muon $g-2$ from lattice QCD: Complete short and intermediate windows". In: DOI: https://doi.org/10.1103/PhysRevD.111.094508. arXiv: 2411.09656 [hep-lat]. URL: https://arxiv.org/abs/2411.09656.
- Bazavov, Alexei et al. (2025b). "Hadronic Vacuum Polarization for the Muon $g-2$ from Lattice QCD: Long-Distance and Full Light-Quark Connected Contribution". In: *Phys. Rev. Lett.* 135 (1), p. 011901. DOI: 10.1103/d583-yhfs. URL: https://link.aps.org/doi/10.1103/d583-yhfs.
- Bazavov, Alexei et al. (2026). "High-Precision Scale Setting with the Omega-Baryon Mass and Gradient Flow". In: *Phys. Rev. D*, pp. –. DOI: 10.1103/t18c-bpqy. URL: https://link.aps.org/doi/10.1103/t18c-bpqy.
- Benton, Genessa et al. (2025). "Data-driven results for light-quark connected and strange-plus-disconnected hadronic g- 2 short-and long-distance windows". In: *Phys. Rev. D* 111 (3), p. 034018. DOI: 10.1103/PhysRevD.111.034018. URL: https://link.aps.org/doi/10.1103/PhysRevD.111.034018.

- Bernecker, David and Harvey B. Meyer (2011). "Vector Correlators in Lattice QCD: Methods and applications". In: *Eur.Phys.J.* A47, p. 148. DOI: 10.1140/epja/i2011-11148-6. arXiv: 1107.4388.
- Blum, T. et al. (2018). "Calculation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment". In: *Phys. Rev. Lett.* 121.2, p. 022003. DOI: 10.1103/PhysRevLett.121.022003. arXiv: 1801.07224.
- Blum, T et al. (2025). "Long-Distance Window of the Hadronic Vacuum Polarization for the Muon g-2". In: *Physical Review Letters* 134.20, p. 201901. DOI: 10.1103/PhysRevLett.134.201901. URL: https://link.aps.org/doi/10.1103/PhysRevLett.134.201901.
- Blum, Thomas, Taku Izubuchi, and Eigo Shintani (2013). "New class of variance-reduction techniques using lattice symmetries". In: *Phys.Rev.* D88.9, p. 094503. DOI: 10.1103/PhysRevD.88.094503. arXiv: 1208.4349.
- Borsanyi, Sz et al. (2021). "Leading hadronic contribution to the muon magnetic moment from lattice QCD". In: *Nature* 593.7857, pp. 51–55. DOI: 10.1038/s41586-021-03418-1. URL: https://doi.org/10.1038/s41586-021-03418-1.
- Chakraborty, Bipasha et al. (2017). "Hadronic vacuum polarization contribution to a μ from full lattice QCD". In: *Physical Review D* 96.3, p. 034516. DOI: https://doi.org/10.1103/PhysRevD.96.034516.

Djukanovic, Dalibor et al. (2025). "The hadronic vacuum polarization contribution to the muon g-2 at long distances". In: *Journal of High Energy Physics* 2025.4, pp. 1–57. DOI: 10.1007/JHEP04(2025)098. URL: https://doi.org/10.1007/JHEP04(2025)098.

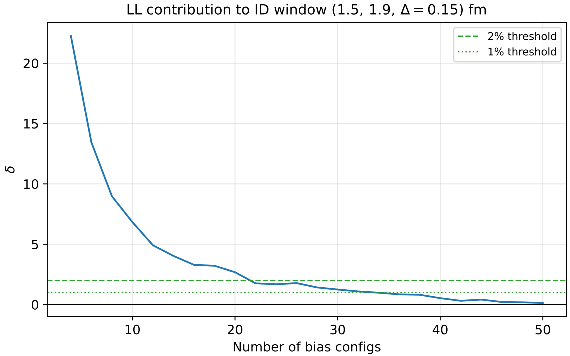
Donati, S et al. (2025). "Measurement of the positive muon anomalous magnetic moment to 127 ppb". In: *PHYSICAL REVIEW LETTERS*.

Giusti, Leonardo et al. (2004). "Low-energy couplings of QCD from current correlators near the chiral limit". In: *JHEP* 04, p. 013. DOI: 10.1088/1126-6708/2004/04/013. arXiv: /0402002.

Moningi, Vaishakhi et al. (2025). "Progress on computing the hadronic vacuum polarization contribution to the muon anomalous magnetic moment with staggered fermions". In: *PoS* LATTICE2024, p. 247. DOI: 10.22323/1.466.0247.

— (Feb. 2026). "The Light Quark Connected Hadronic Vacuum Polarization Contribution to the muon anomaly via Sparsened Meson Fields". In: *42th International Symposium on Lattice Field Theory*. arXiv: 2602.24190 [hep-lat].

Bias Correction Overhead vs. Number of Bias Configs



What is plotted

We define the **relative error overhead**:

$$\delta = \frac{\sigma_{\text{hisq corrected}} - \sigma_{\text{fat}}}{\sigma_{\text{fat}}} \times 100\%$$

$\delta = 0$ means the correction adds **no extra noise**.

Key thresholds

- $N_{\text{bias}} \sim 5$: $\sim 22\%$ increase
- $N_{\text{bias}} \sim 10$: $\sim 7\%$ increase
- $N_{\text{bias}} \sim 20$: $\sim 2\%$ increase
- $N_{\text{bias}} \sim 30$: $\sim 1\%$ increase