Improved estimators for the HVP

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Challenges to matching FNAL E989 prec. from lattice QCD

$$\lim_{a \to 0}$$

Critical slowdown of
Markov Update

$$m_u \neq m_d$$

$$\alpha \neq 0$$

QED / Isospin-breaking

$$C_{ij} = \langle O_i^\dagger O_j \rangle$$

Variational approach

Quick overview of improvements to all three

Towards the continuum limit

- Markov update has a diffusion process that scales naively as a^{-2} and a tunneling process that scales exponentially in the lattice spacing

Addressing the diffusion aspect:

- Applying a field transformation, U = f(V), and integrating in V instead of U can speed up the diffusive process by slowing down the movement of high-frequency modes and rescaling the trajectory length (FTHMC)

see arXiv:2502.05452

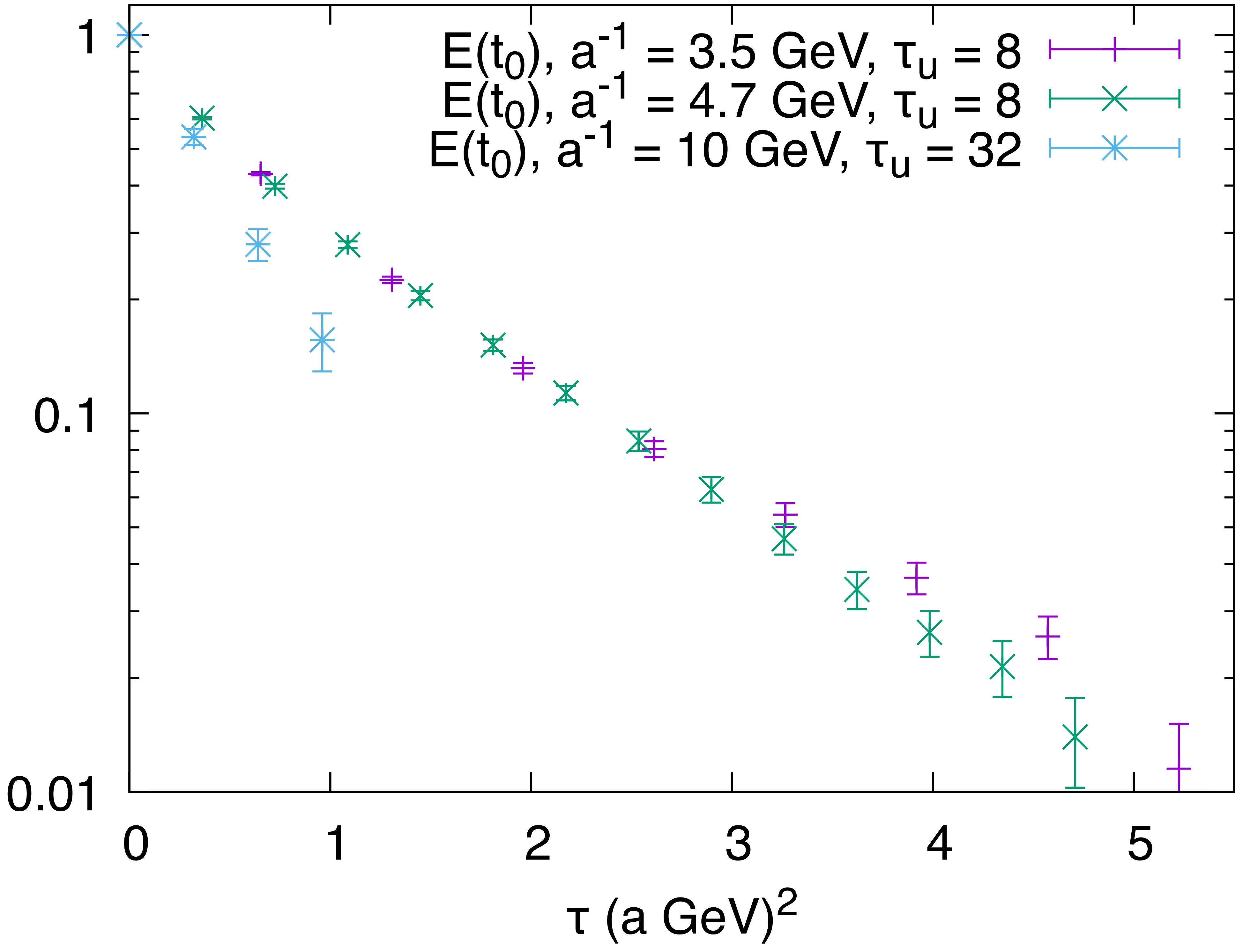
Addressing the tunneling aspect:

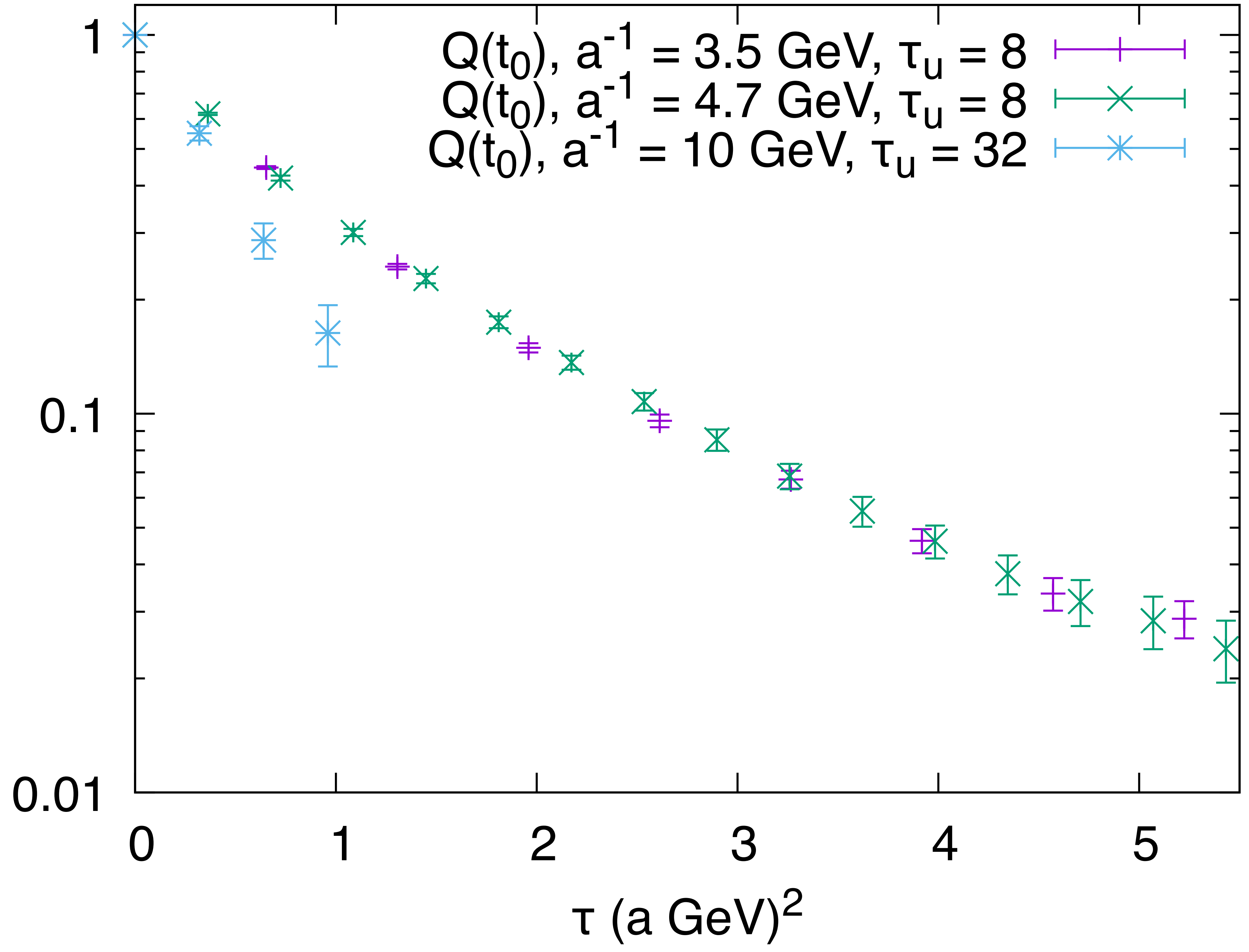
- Add bias potential V(Q) to the action with appropriately smeared Q to re-gain control over global topological sectors.

for bias potential, see arXiv:2307.04742

Diffusive scaling

- Let's look at FTHMC with stout smearing ($\rho = 0.12$), Iwasaki gauge, 2+1+1f Mobius DWF, physical m_s and m_c and $m_u = m_d = m_s$; dimensionless trajectory length is τ_u
- FTHMC and large τ_u approximately 3x faster than regular HMC, effects from FTHMC and large τ_u compound
- Autocorrelation (with stochastically local measure):



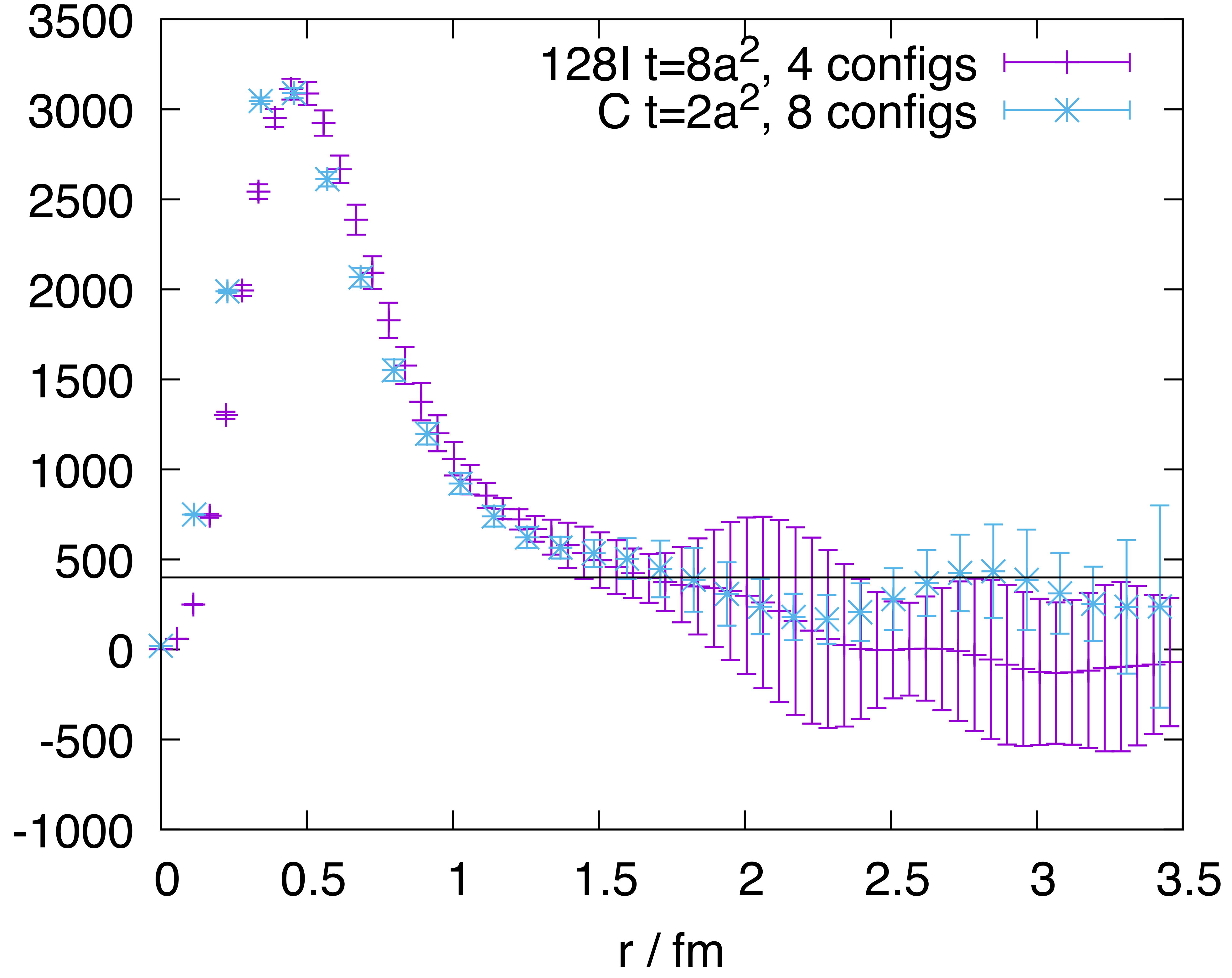


- With very large τ_u simulation at 10 GeV scales better than a^{-2}

Tunneling

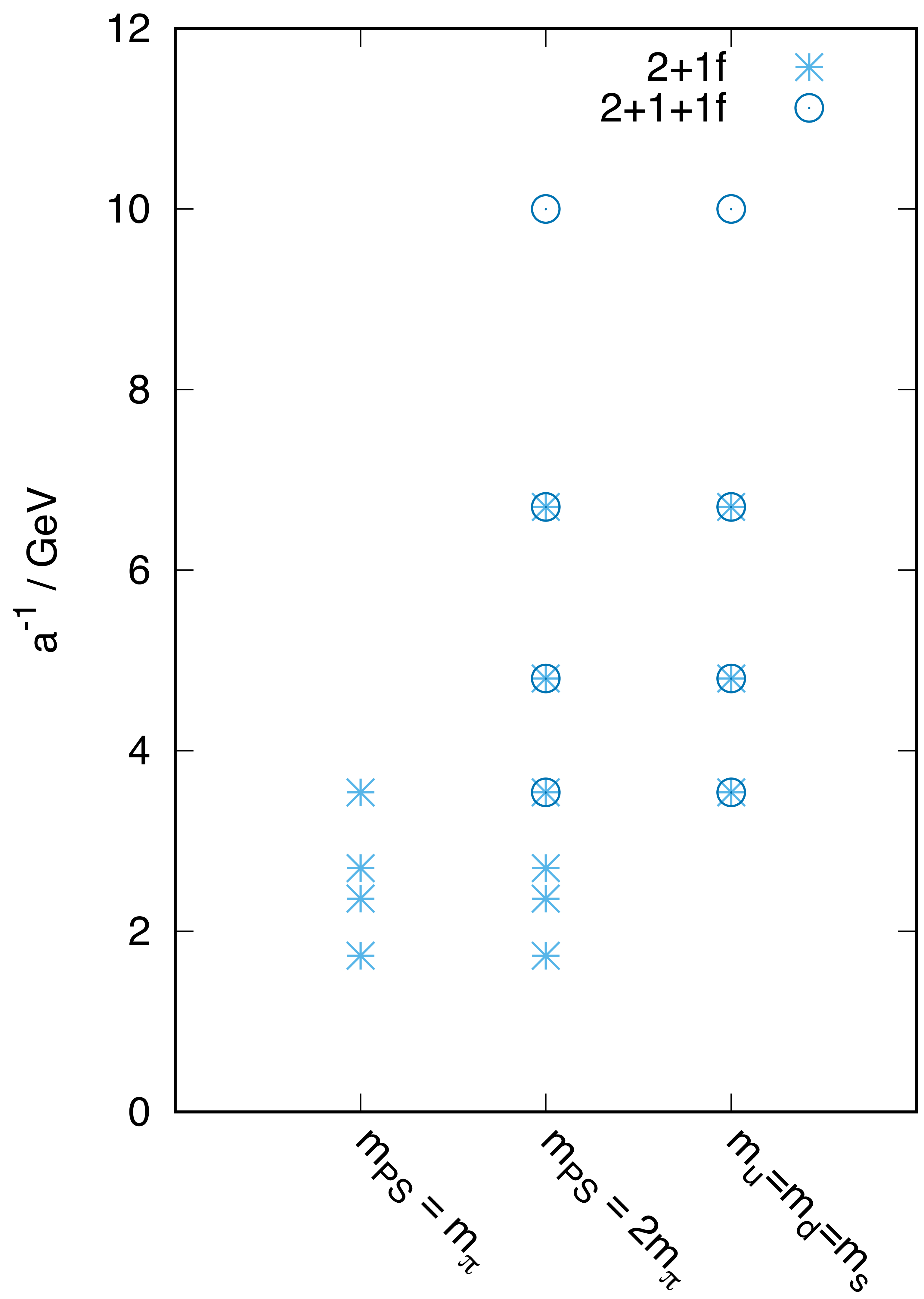
- Topological tunneling needs separate attention, problem is mild at large physical volumes
- We take control again of topological sectors by adding a bias potential V(Q) with GPT's automatic differentiation features; then perform separate streams in fixed sectors and combine them with locally determined susceptibility:

$$\langle Q^2 \rangle = \lim_{r\to\infty} \langle Q^2 \rangle_r \quad \text{with} \quad \langle Q^2 \rangle_r = \sum_{x,|\delta|<r} \langle Q(x)Q(x+\delta) \rangle$$



- Plot shows $\langle Q^2 \rangle_r$ for two ensembles after adjusting for their
  volume and lattice spacing
- C is a $64^3 \times 128$ physical pion mass ensemble at $a^{-1} = 1.73~\mathrm{GeV}$
- 128I is a $128^3 \times 288$ physical pion mass ensemble at $a^{-1} = 3.54~\mathrm{GeV}$

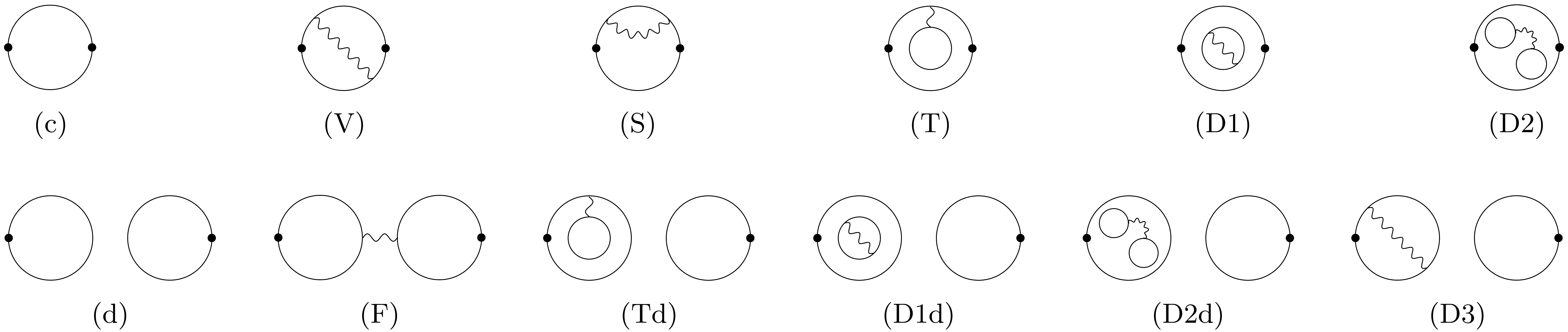
New 2+1 and 2+1+1 flavor ensembles



- Unitary charm for 2+1+1 flavor (M_5=1.8)
- Crucial for precision in short-distance HVP window and for τ decays

data produced at Aurora (< 10 GeV) and JUPITER = 10 GeV using GPT (production code at https://github.com/lehner/gpt)

Noisy QED diagrams



- Needed for muon g-2 HVP at desired precision, long-distance component not yet well studied
- Burger diagrams (D1) and (D1d) are a particular challenge (reweighting) that universally applies to all quantities at sufficiently high precision
- See Julian Parrino's talk for an update on all apart from (D1) and (D1d) and (D3)

1

Long-distance reconstruction of QED corrections to the hadronic vacuum polarization for the muon g-2

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(Dated: September 1, 2025)

The long-distance contribution of QED corrections to the hadronic vacuum polarization is particularly challenging to compute in lattice QCD+QED. Currently, it is one of the limiting factors towards matching the precision of the recent result by the Fermilab E989 experiment for the muon g-2. In this work, we present a method for obtaining high-precision results for this contribution by reconstructing exclusive finite-volume state contributions. We find relations between the pion-photon contributions of individual diagrams and demonstrate the reconstruction method with lattice QCD+QED data at a single lattice spacing of $a^{-1} \approx 1.73$ GeV and $m_\pi \approx 275$ MeV.

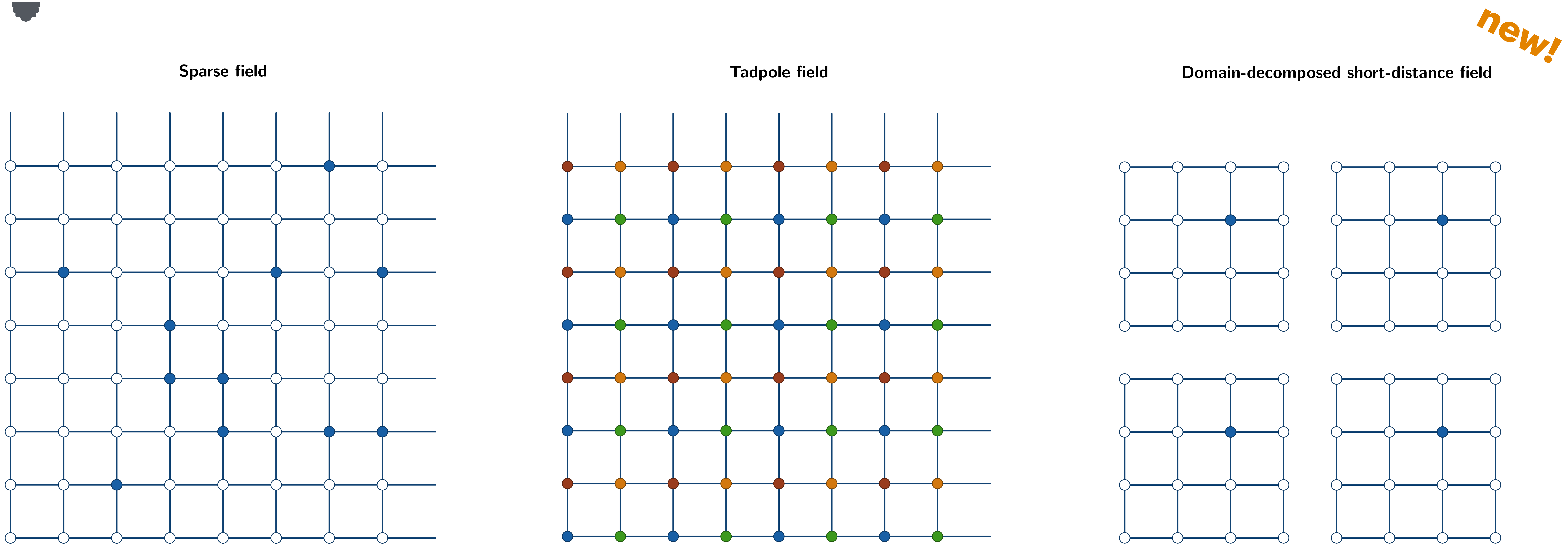
PACS numbers: 12.38.Gc

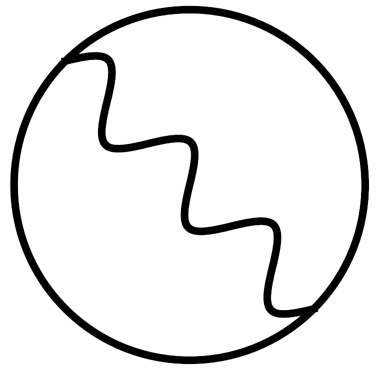
Phys. Rev. D 113 (2026) 5, 054503, arXiv:2508.21685

2

See C. Lehner talk at CERN on July 8, 2026 on how to simulate dynamical DWF QCD+QED with local vector currents (https://indico.cern.ch/event/1616325/)

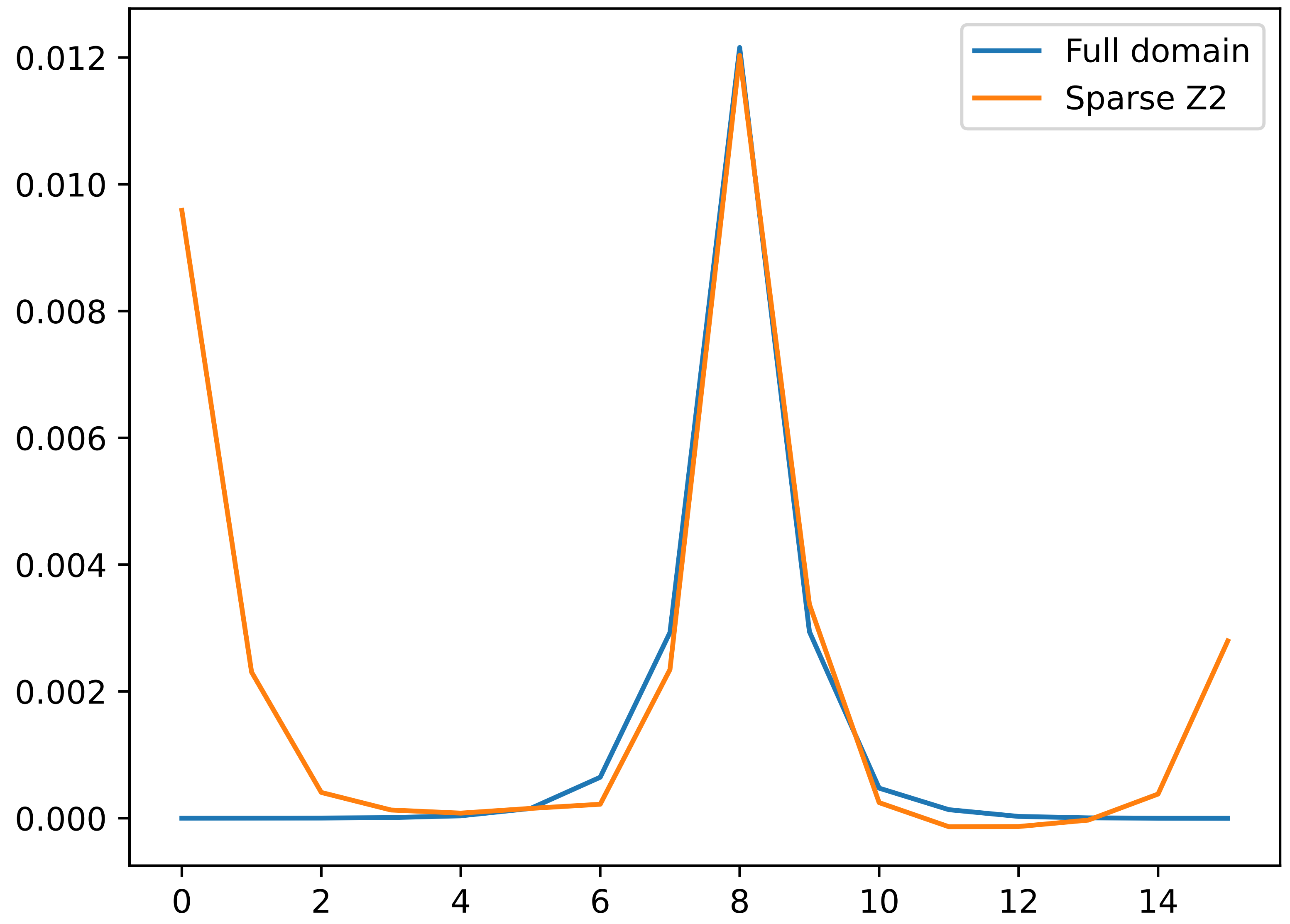
Domain decomposed short-distance fields

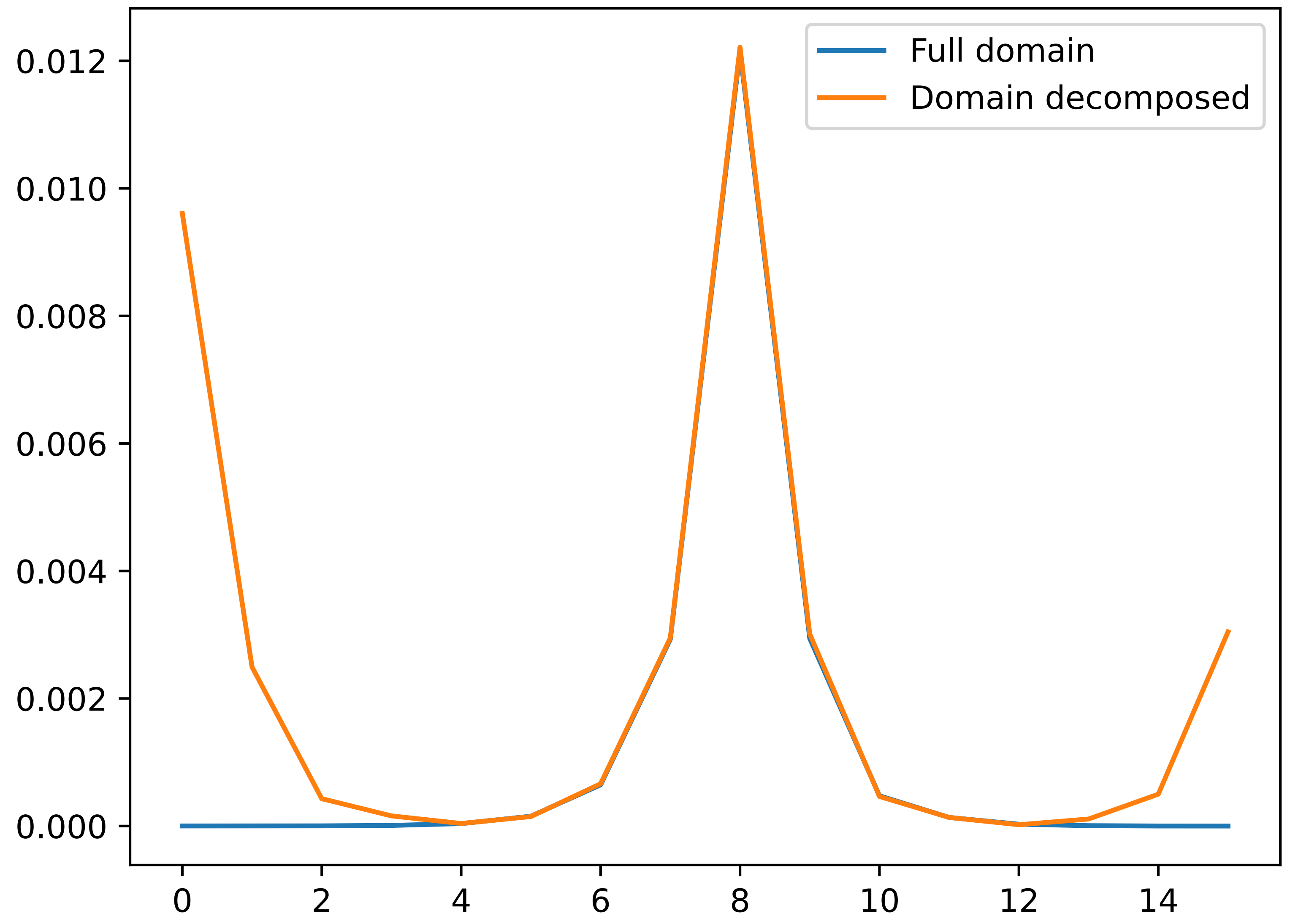




Idea: two vertices never far from each other, so quark propagators should not depend on gauge fields far away. Setup small (8^4) Dirichlet subvolumes that can all be solved at same time. For a $64^3 \times 128$ ensemble get 8192 source positions for cost of one propagator.

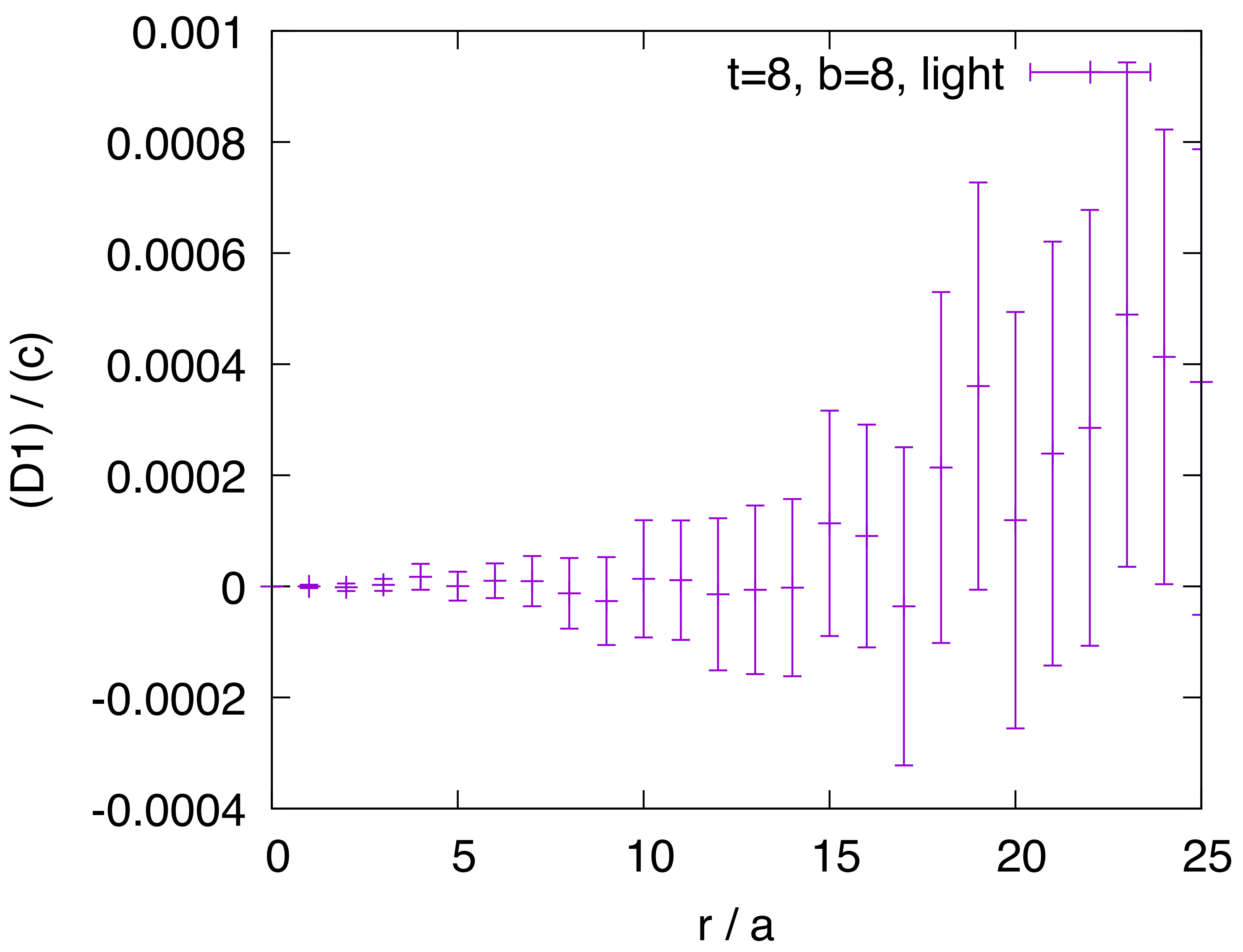
Why are domain decomposed Dirichlet boxes better than a sparse Z2 grid?

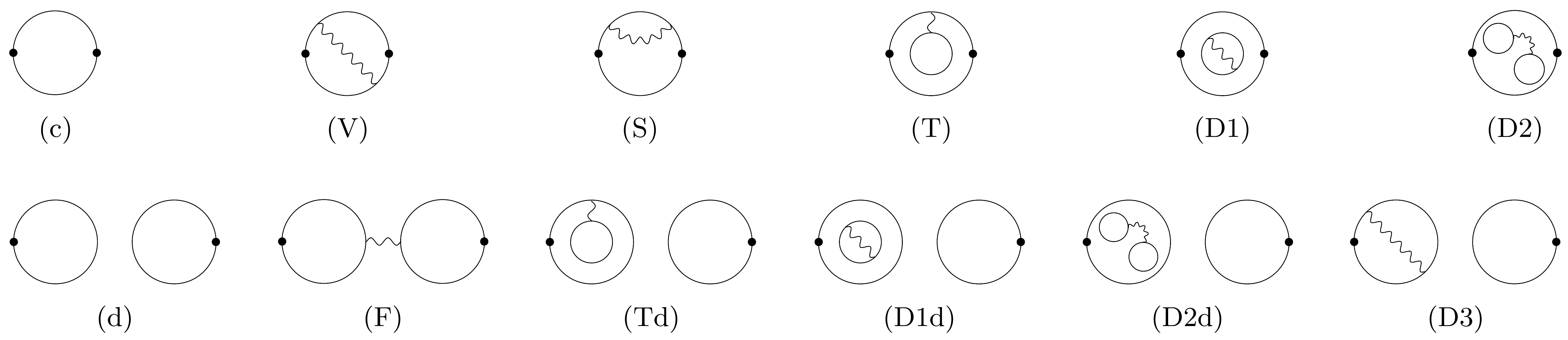




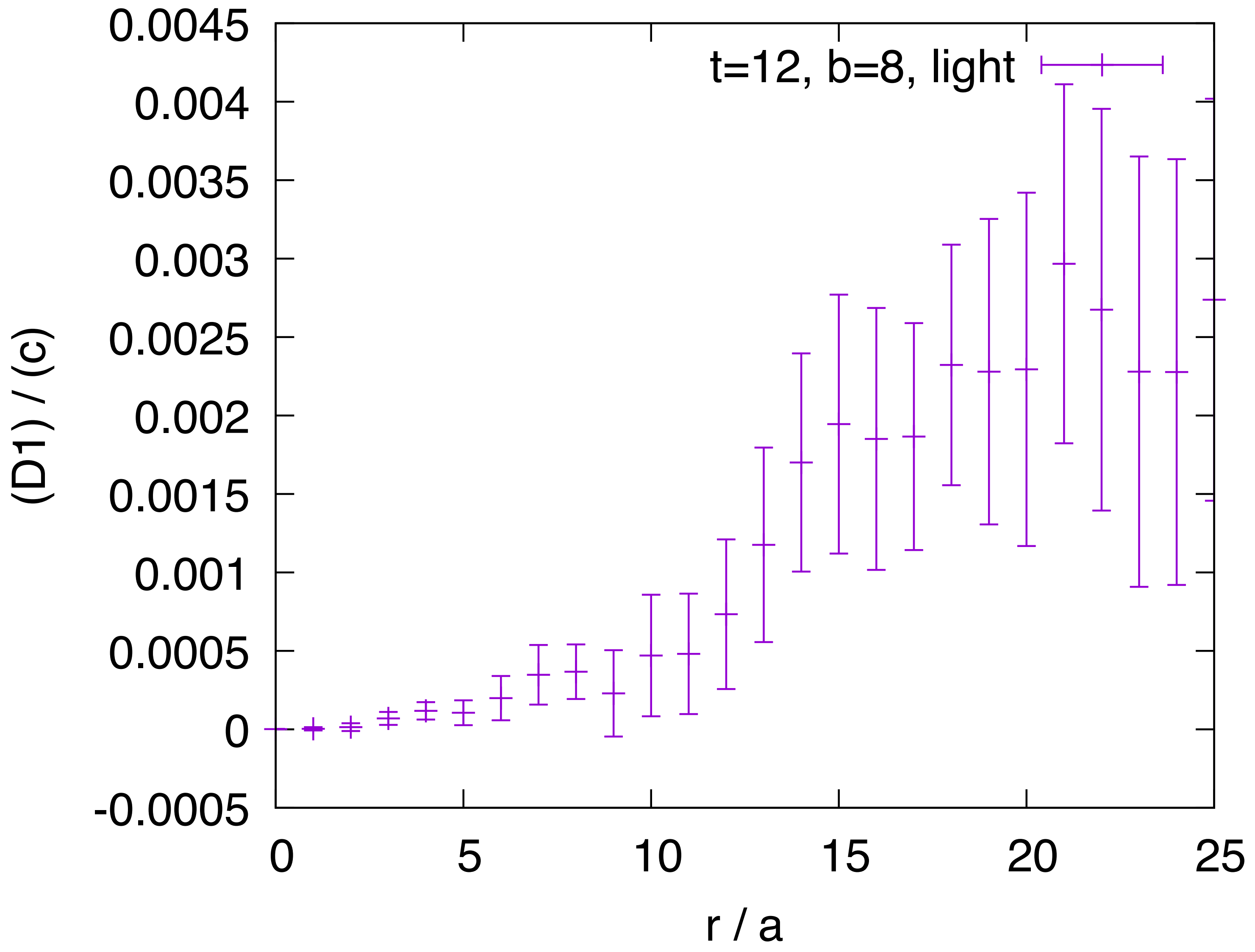
Spurious Z2 sources spoil crucial short-distance quark propagators! Domain decomposed short-distance fields do not have this issue.

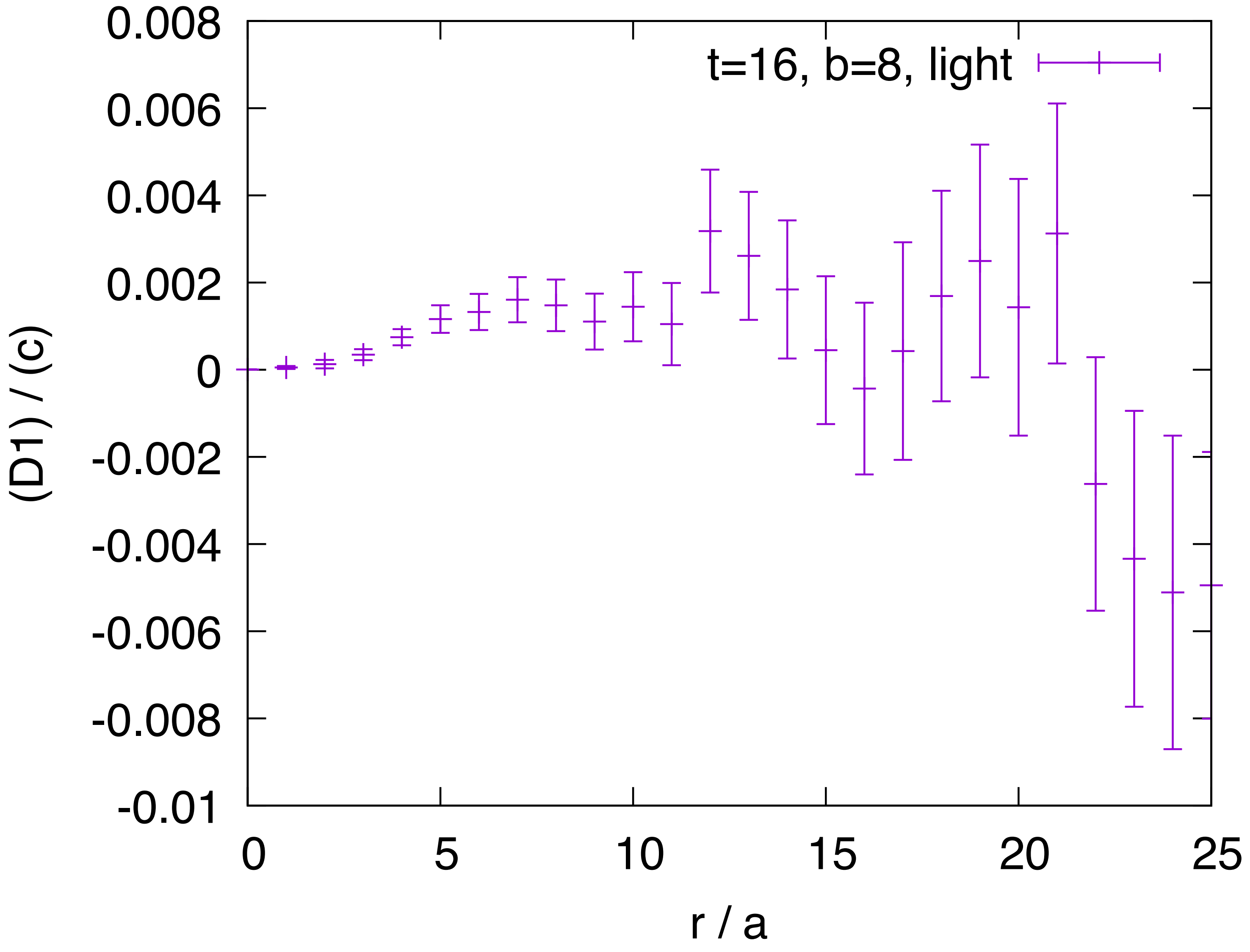
First results of (D1) with new method on large-volume physical pion mass ensemble (C)





Time separation between external vector currents (t), Dirichlet box size (b^4), distance between one external current and one burger current (r)





Improved variational approach

So far, we fared well with exact distillation based on a particularly smeared-gauge-field Laplace operator’s eigenmodes:

PHYSICAL REVIEW LETTERS **134,** 201901 (2025)

Editors' Suggestion

Long-Distance Window of the Hadronic Vacuum Polarization for the Muon $g-2$

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We provide the first *ab initio* calculation of the Euclidean long-distance window of the isospin-symmetric light-quark connected contribution to the hadronic vacuum polarization for the muon $g-2$ and find $a_\mu^{\mathrm{LD,iso,conn,ud}} = 411.4(4.3)(2.4) \times 10^{-10}$. We also provide the currently most precise calculation of the total isospin-symmetric light-quark connected contribution, $a_\mu^{\mathrm{iso,conn,ud}} = 666.2(4.3)(2.5) \times 10^{-10}$ in the BMW20 world, which is more than 4σ larger compared to the data-driven estimates of Boito *et al.* [Phys. Rev. D **107**, 074001 (2023)] and 1.7σ larger compared to the lattice QCD result of BMW20.

DOI: 10.1103/PhysRevLett.134.201901

PHYSICAL REVIEW D **113,** 114502 (2026)

Spectral analysis for nucleon-pion and nucleon-pion-pion states in both parity sectors using distillation with domain-wall fermions

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(Received 29 October 2025; accepted 17 April 2026; published 2 June 2026)

We present a study using the distillation method to analyze the spectra of nucleon, nucleon-pion, and nucleon-pion-pion states in the positive-parity sector, as well as nucleon and nucleon-pion states in the negative-parity sector. The study uses seven domain-wall fermion ensembles with varying pion masses ($m_\pi = 139$–279 MeV), lattice spacings ($a^{-1} = 1.730$ GeV and $a^{-1} = 2.359$ GeV) and volumes ($m_\pi L = 3.8$–7.5). To address the large number of contractions in this project, we implemented an algorithm to automate the contraction of nucleon-pion correlation functions that contain an arbitrary number of pions. In the positive-parity sector, we extrapolate the nucleon mass to the physical point. This study demonstrates the effectiveness of the distillation method for baryonic quantities with a focus on multihadronic states and establishes a foundation for future work.

DOI: 10.1103/lmtl-c92f

But points to optimize:

- more efficient creation of operators with a smaller smearing radius
- Laplace eigenmodes not translationally covariant, so distillation matrix space not optimized for well-defined momentum operators, this becomes important for baryons where tensor contractions are no longer of matrix-matrix and trace kind

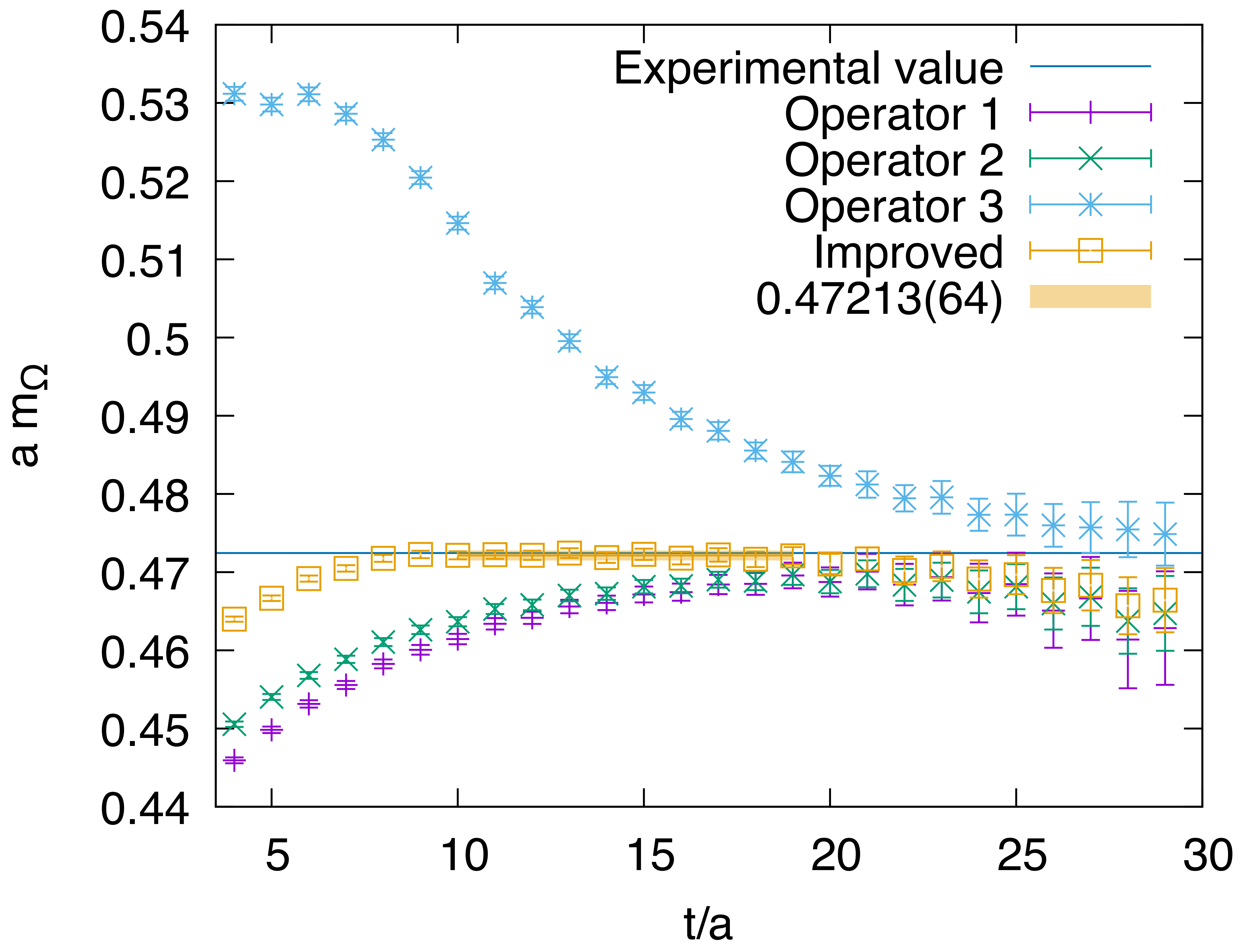
Improved (transl. cov.) variational approach

New idea:

- Translationally covariant Coulomb-Gauge-Fixed momentum sources with fixed spatial momentum
- Employ similar smearing before gauge fixing that we had used for exact distillation
- Straightforward to localize operators in particular dimension by only picking subset of all momentum modes
- Drastically cuts the relevant subspace which reduces contraction costs

First applications:

- Omega baryon mass on new 128I physical pion mass ensemble at 3.54 GeV cutoff

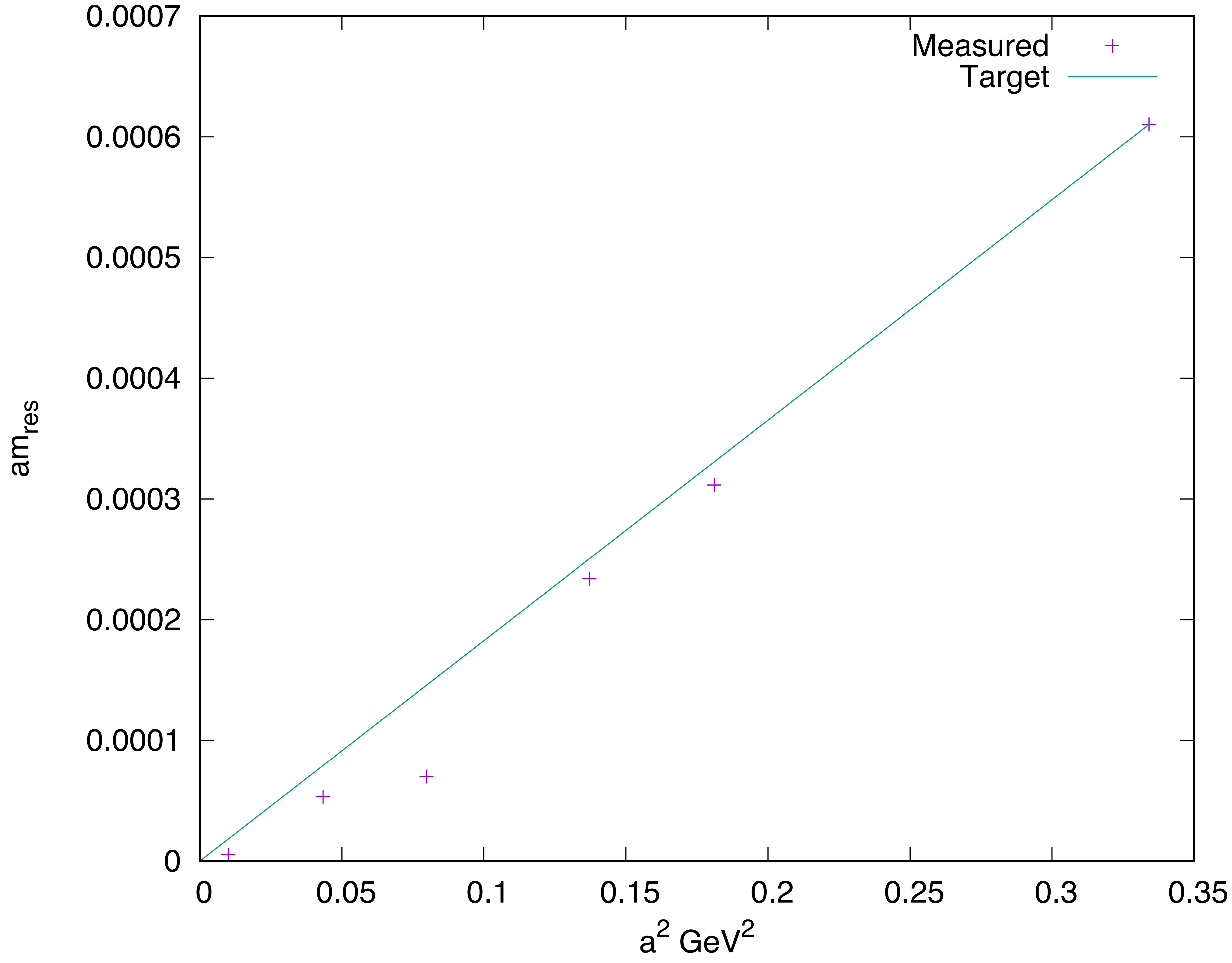


Summary

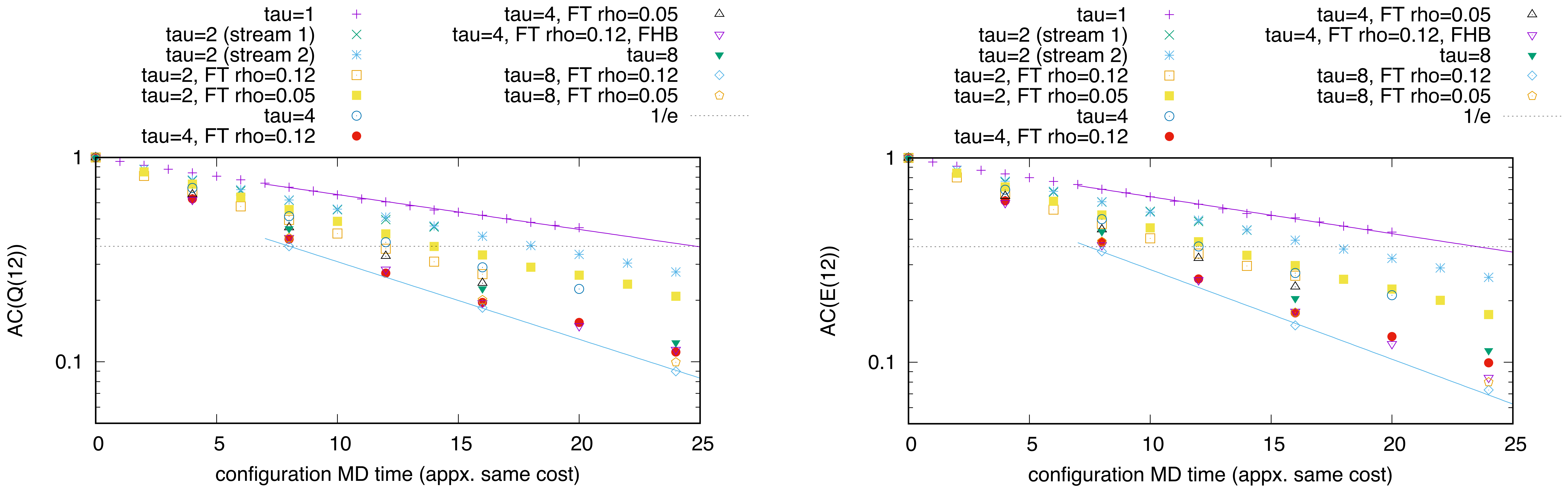
- Preparing data needed to match E989 experimental precision
- Large roadblocks addressed with new ultrafine ensembles up to 10 GeV cutoff, new sampling techniques, and a new optimized variational approach

Backup slides

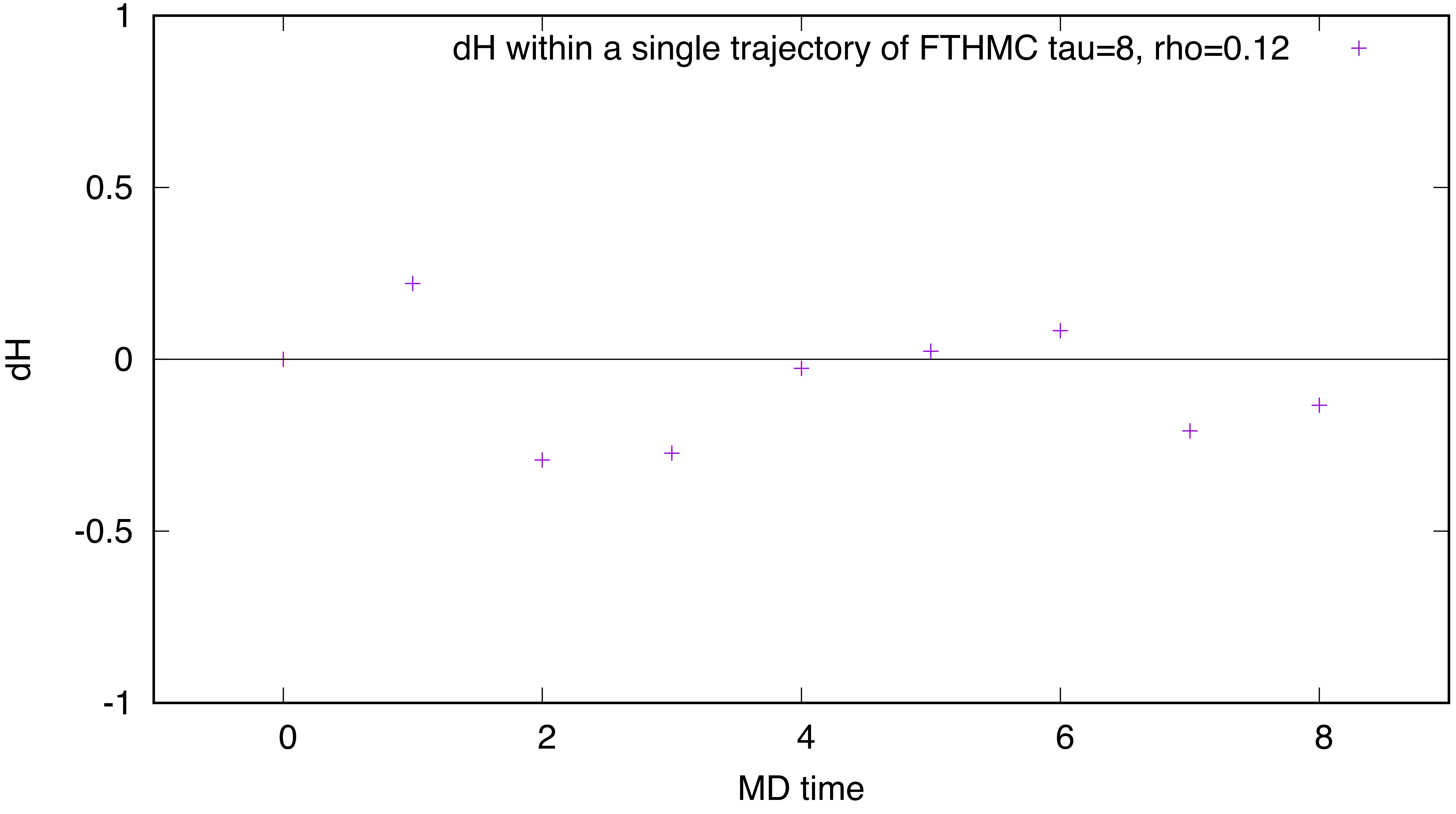
Residual chiral symmetry breaking



- Tuned such that linear discretization effects are always negligible compared to quadratic ones
- Continuum effective theory of corresponding Overlap applies



tau=1
tau=2 (stream 1)
tau=2 (stream 2)
tau=2, FT rho=0.12
tau=2, FT rho=0.05
tau=4
tau=4, FT rho=0.12
tau=4, FT rho=0.05
tau=4, FT rho=0.12, FHB
tau=8
tau=8, FT rho=0.12
tau=8, FT rho=0.05
1/e
AC(Q(12))
configuration MD time (appx. same cost)
AC(E(12))
configuration MD time (appx. same cost)


dH within a single trajectory of FTHMC tau=8, rho=0.12
dH
MD time
1
0.5
0
-0.5
-1
0
2
4
6
8