Discussion session on isospin-breaking for τ

Mattia Bruno

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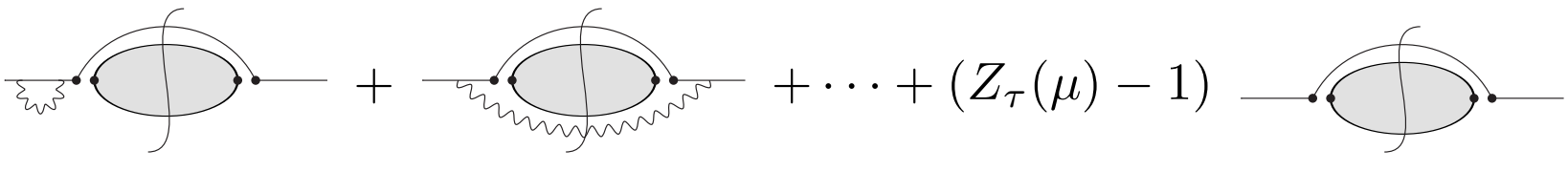
Provide IB correction to relate τ with ee

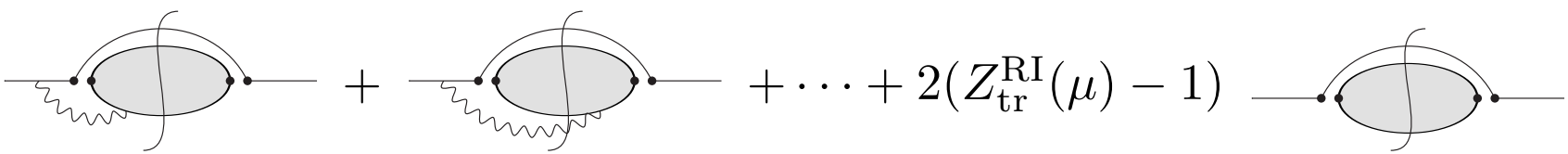
Lattice methodology established

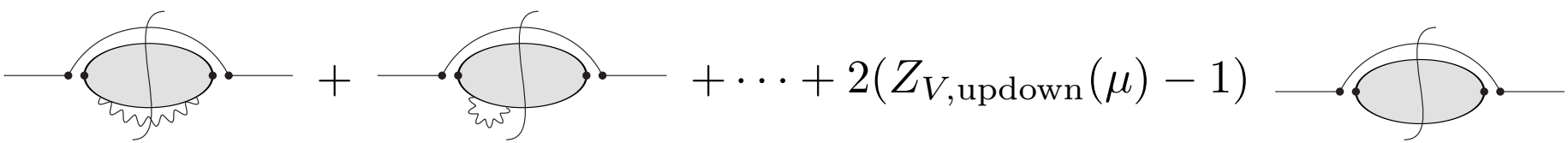
Connection w/ pheno analysis is the difficult part
but needed to make progress in short run

Radiative corrections

A reminder from the talk



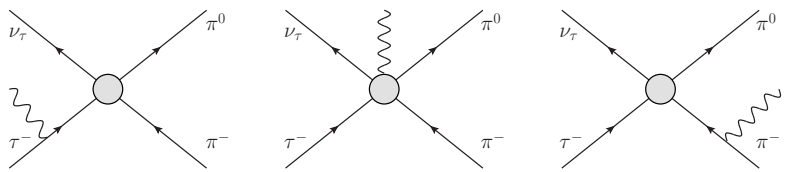


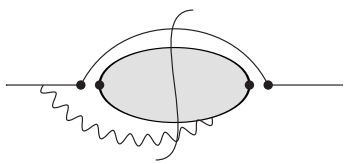


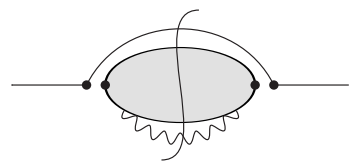
Radiative corrections

Can this "topological" (cit. Cirigliano) separation be achieved on EFT/dispersive side? In my opinion

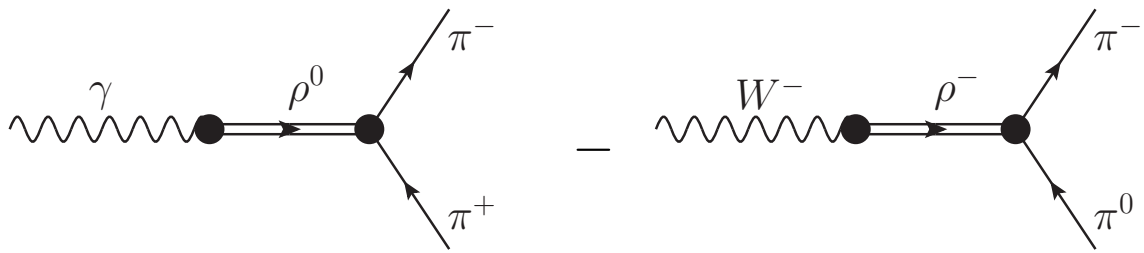
i. yes but experts must answer
ii. mapping of virtual corrections is easy
iii. special care with real emission
iv. do not think in terms of amplitude but contributions to rate

$A, B, C =$ 

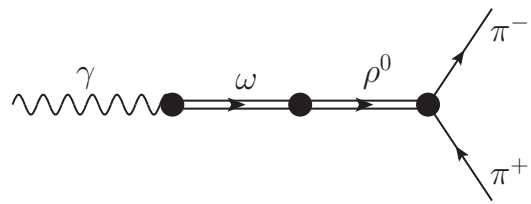
$A^\dagger(B+C) \rightarrow$ 

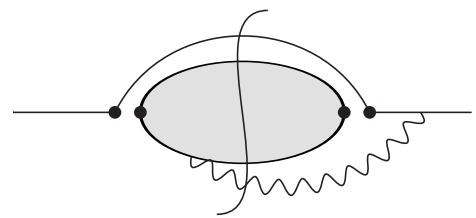
$|B+C|^2 \rightarrow$ 

A reminder: we focus on δG_{11}

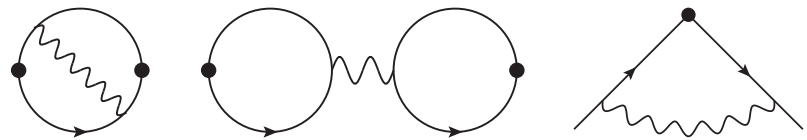
$\delta G_{11} = O(\alpha) \quad \leftrightarrow$ 

$m_{\rho^0} \neq m_{\rho^\pm}$, $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$, $m_{\pi^0} \neq m_{\pi^\pm}$

$G_{01} \leftrightarrow \quad \delta_{\rho\omega} \simeq O(m_{\text{up}} - m_{\text{down}}) + O(e^2)$ 

Severe inverse problem makes it difficult 

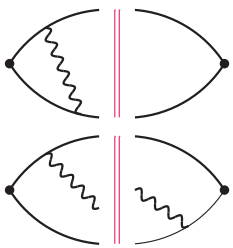
Neutral vs charged correlators

$\delta G_{11}(t,\mu) = G_{11}^{\gamma}(t) - G^{W}(t,\mu) \rightarrow$ 

Pheno analysis of $\delta G_{11}(t,\mu)$: what's inside it?

fully inclusive therefore 2π and 4π, and more ...

intermediate/long distance windows preferable quantities



1. difference of 2π form factors w/o $\rho\omega$ mixing
   the charged FF is prescription dependent!
2. difference of phase space due to $m_{\pi^-} - m_{\pi^0}$
3. difference in final-state radiation

all of them inside δG_{11}

additional syst. errs $G^{W} - G^{W,exp}$

Challenge

Identify a scheme where both pheno and lattice can meet

The word "scheme" might have conflicting meanings

- how we renormalize the charged current defines a scheme
  - eg. RIMOM
- how we separate short and long-distance effects
  - eg. phase-space vs FSR/G_{EM} vs ratio FF

A possible strategy to be coordinated

do not separate real vs virtual but instead look at topologies $= \delta G_{11}$

a.) we will calculate δG_{11} in RI scheme. useful for comparison?
I am afraid not

b.) calculate δG_{11} in $\overline{\rm MS}$? (question by Martin). doable?
yes isolating quark topologies is trivial
but 2-loop at $O(\alpha\alpha_s)$ hence difficult

c.) matching lattice vs disp is possible via physical scheme
Example: renormalization of bilinear current j in pure QCD
if $j \to \pi\pi$ then $Z_j \langle \pi | j(0) | \pi \rangle = \text{const}$
if $j \to \pi$ then $Z_j \langle 0 | j(0) | \pi \rangle = \text{const}$
ongoing work on extension in LQCD+QED