

Towards the hadronic light-by-light contribution to the muon $g-2$ with C^* boundary conditions

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1 Motivation

HLbL limits the SM precision.

WP25 [5]: $a_\mu^{\text{SM}} = 116\,592\,033(62) \times 10^{-11}$ vs. experiment $116\,592\,071.5(14.5) \times 10^{-11}$ (124 ppb) [6]: SM error $4\times$ the experimental one, entirely hadronic. Total budget $\approx 14 \times 10^{-11}$; HLbL alone $\pm 9 \times 10^{-11}$, even with a perfect HVP.

Both lattice HLbL programmes [7,8] integrate the four-current correlator; we pursue an independent estimate via the two-current subtraction proposed in [1,2], with different systematic and statistical properties, extending to fully dynamical QCD+QED. This poster: an investigation of that route in pure QED, with an exact benchmark for the physics and a measured noise scaling for the statistics.

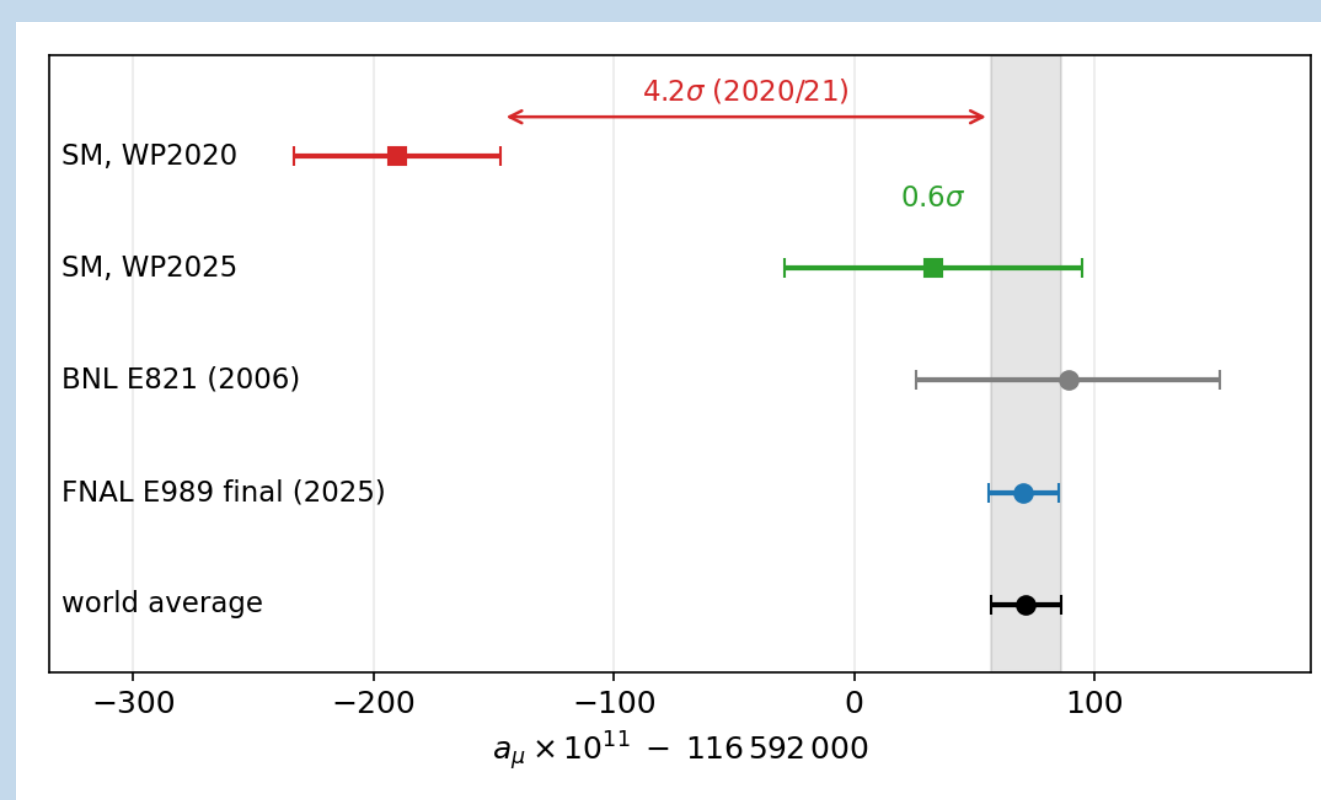


Figure 1: The muon $g-2$ landscape: the 4.2σ tension of WP20 collapsed to 0.6σ once WP25 adopted the lattice HVP; the burden is now on the SM error [4,5,6].

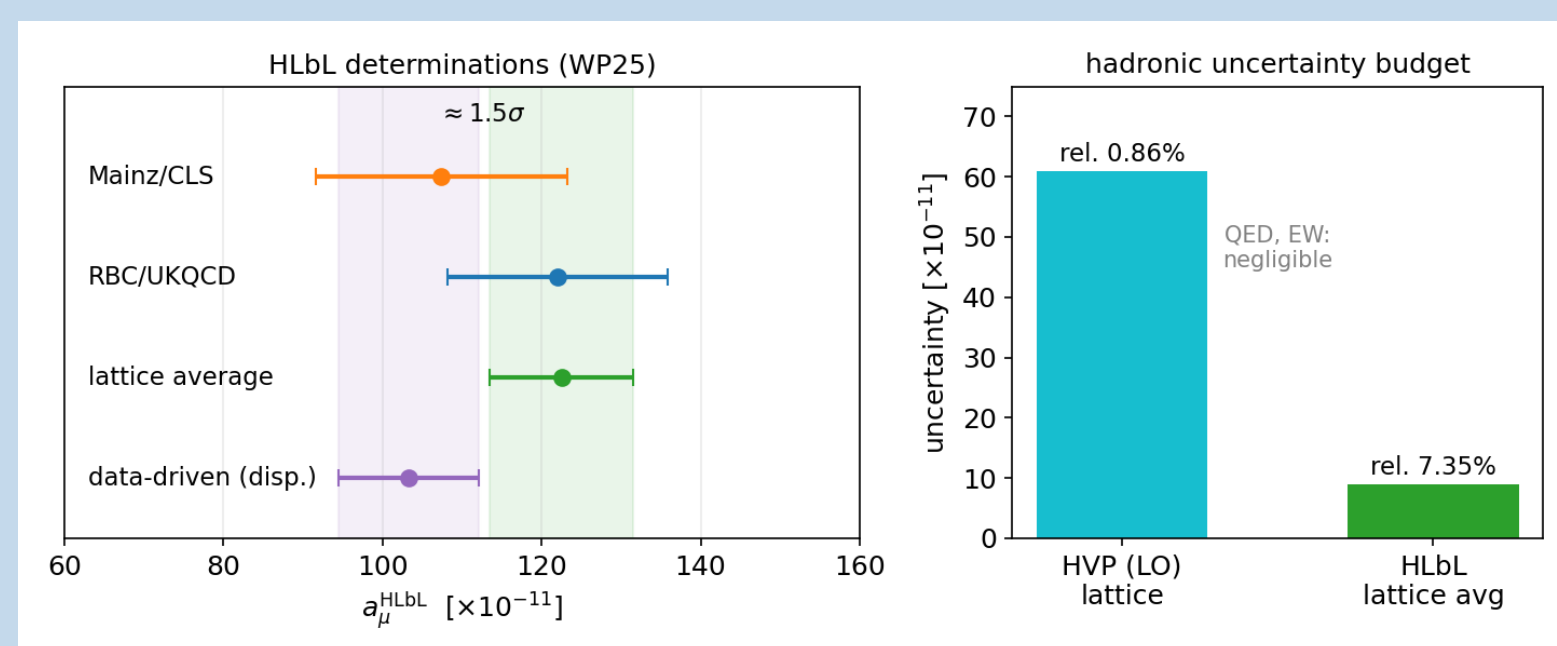
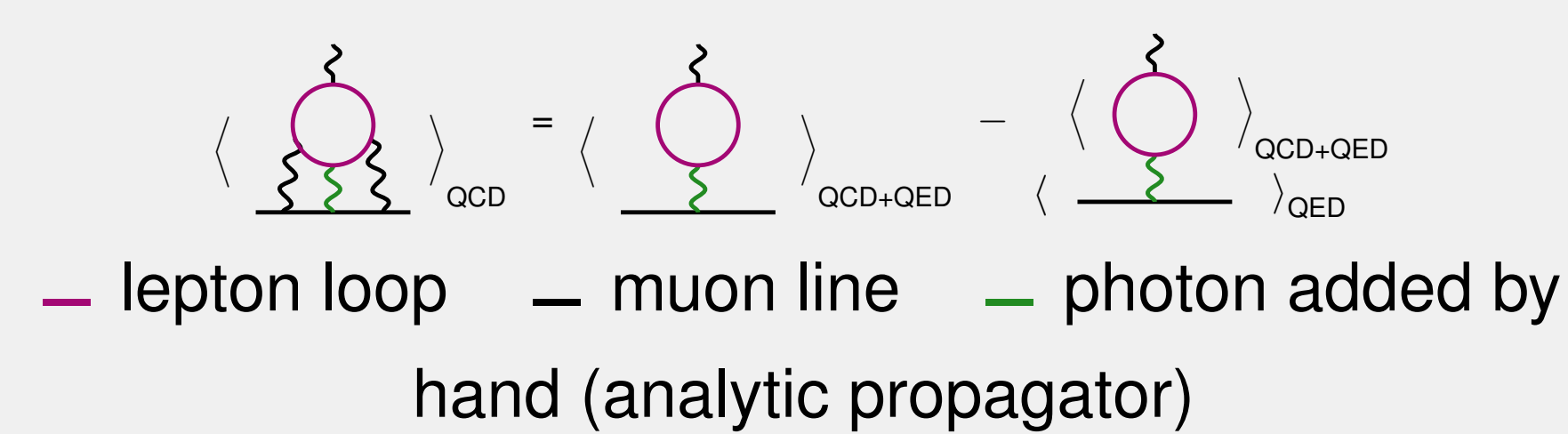


Figure 2: HLbL determinations and the hadronic error budget (WP25): at the 124 ppb goal, the HLbL uncertainty dominates.

2 Method: the two-current subtraction

Estimator via two-current correlators [1,2].

$$M = \langle AB \rangle_{\text{QCD+QED}} - \langle A \rangle_{\text{QCD+QED}} \langle B \rangle_{\text{QED}}$$



- A : loop with two currents (external + internal); B : muon line with one internal current; the green photon joins them by an explicit convolution; all other photon exchanges come from the sampled photon field [1,2].
- In the difference the lower-order contractions cancel in the path integral; the leading surviving term is the connected LbL at order α^3 .

3 Variance: predicted and measured

Scaling of the variance in α .

For $\sigma^2 \sim \alpha^n/N$ the signal-to-noise ratio is $\sqrt{N} \alpha^{3-n/2}$; with the measured $n = 6$, $\text{StN} \approx \sqrt{N}$. The leading terms in the leptonic case:

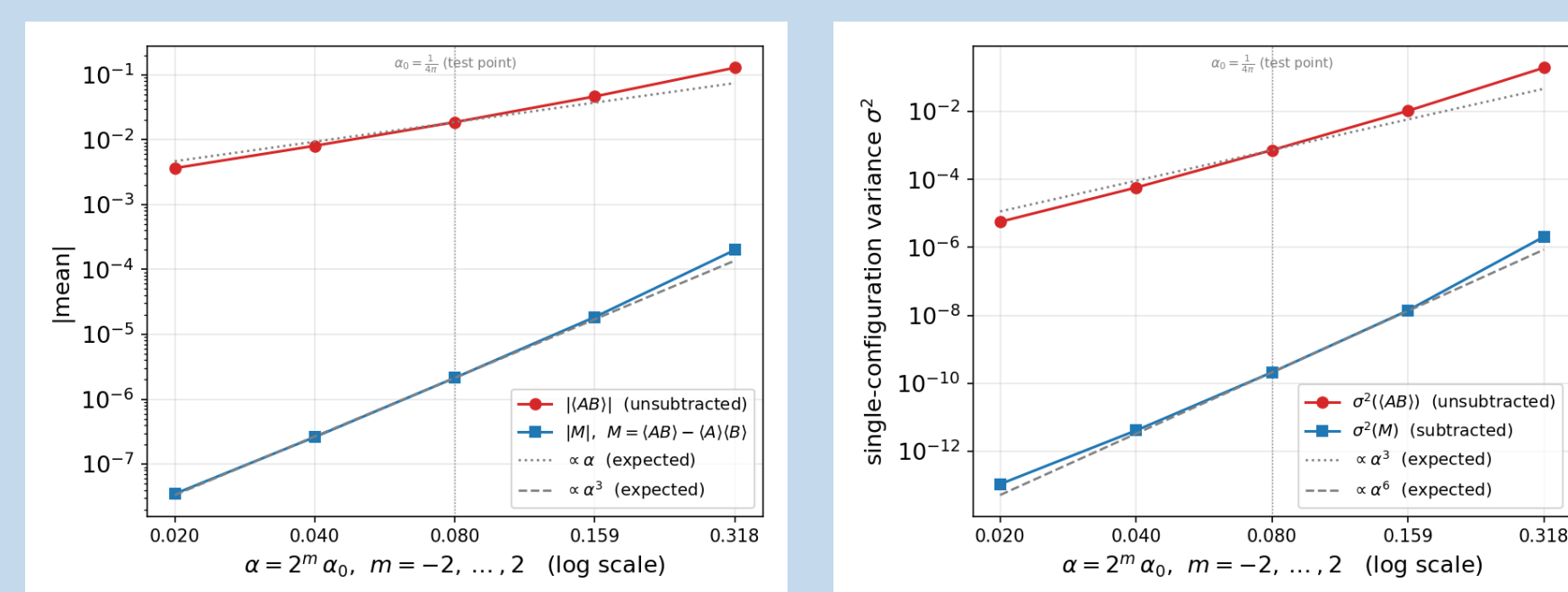
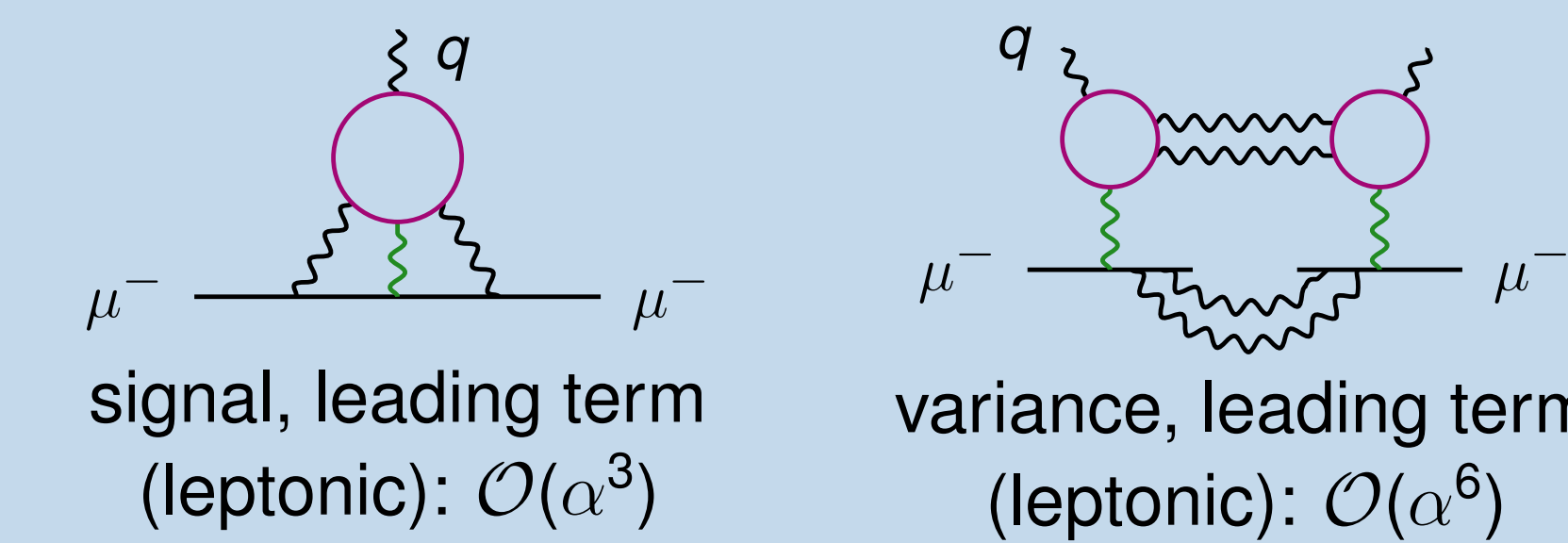


Figure 3: Coupling scan at fixed lepton mass, couplings $\alpha = 2^m \alpha_0$ with $m = -2, \dots, 2$, representative momentum (log-log; grey lines: expected scalings, to guide the eye). Top: means. The subtracted M follows α^3 ; the unsubtracted $\langle AB \rangle$ is dominated by the $\mathcal{O}(\alpha)$ lower-order piece, two orders above the signal. Bottom: variance of a single-configuration measurement (the error of the N -configuration average is $\sigma/\sqrt{N}$). The subtracted estimator follows α^6 ; the subtraction cancels noise by factors 600 to 8500.

4 Leptonic benchmark test

Testing the leptonic LbL against the known perturbative result.

- Exact benchmark [3]: $F_2(0)_{\text{LbL}} = 0.371 (\alpha/\pi)^3$; at larger-than-physical coupling $\alpha = 1/4\pi$ ($e = 1$) the target is $F_2(0) = +6.03 \times 10^{-6}$.
- Wilson muon, non-compact Feynman-gauge photons; all scales (m_μ , Z_μ , Z_V) measured on the same ensembles, no external inputs.
- Two geometries at the coarsest spacing: isotropic 24×8^3 ($N=200$, 4 shells, $Q_{\min}^2 = 0.398$) and anisotropic $24 \times 16 \times 8^2$ ($N=100$, 10 points, $Q_{\min}^2 = 0.121$).

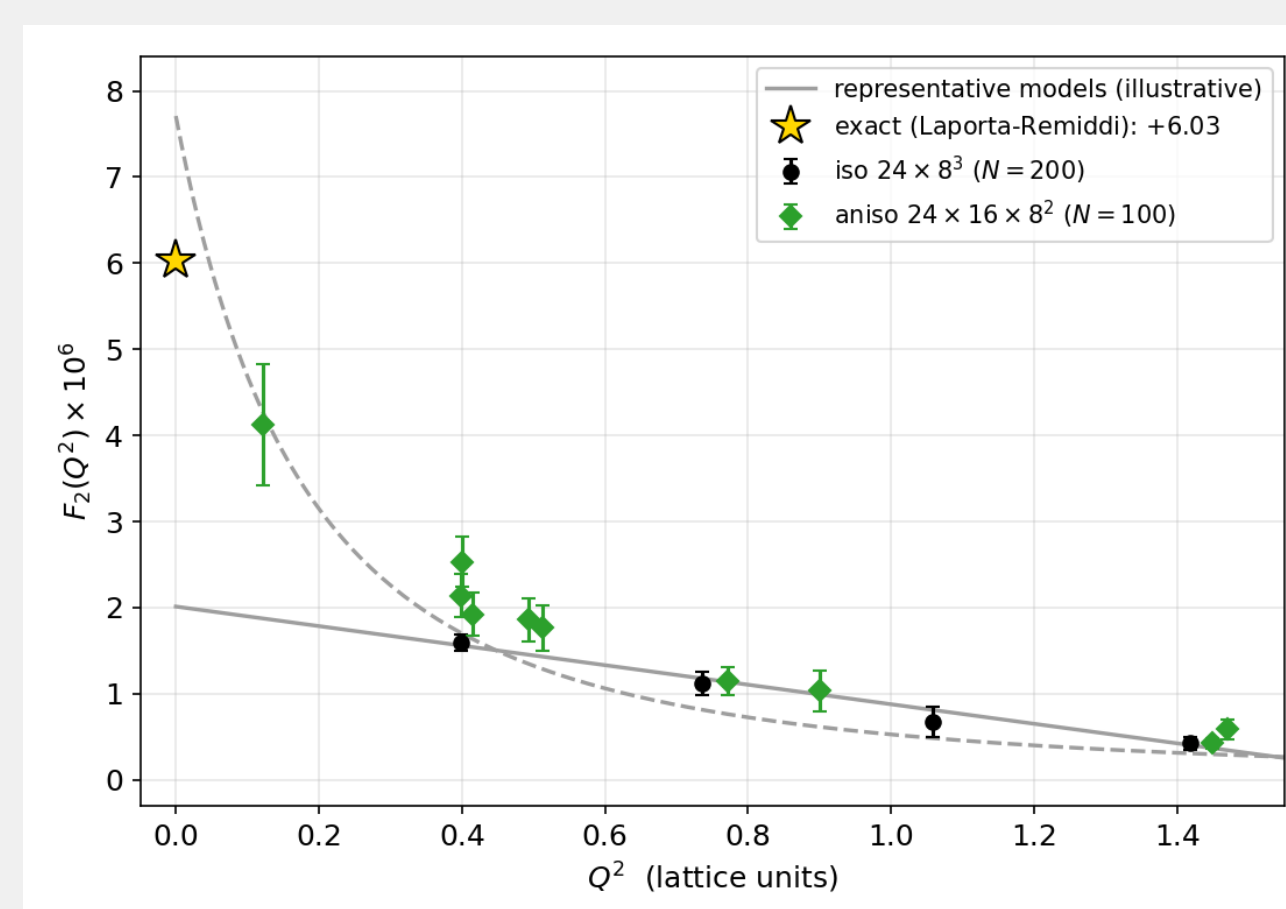


Figure 4: $F_2(Q^2)$ on both geometries vs. the exact target (star). The lowest point, $F_2(Q^2=0.121) = +4.12(70) \times 10^{-6}$, reaches two thirds of the target and rising, with statistical errors at the few-percent level of the target. Grey: representative extrapolation models, illustrative only; a controlled $Q^2 \rightarrow 0$ extrapolation needs lower Q^2 and is deliberately left to the next stage (larger volumes and C^* momenta, box 5).

5 From QED to QCD+QED

Improved estimator: predicted $\sigma^2 \sim \alpha^4/N$.

Dynamical QCD+QED will use C^* boundary conditions [9]: local finite-volume QED with no photon zero mode by construction, half-integer momenta lowering $Q_{\min}^2$; ensembles to be generated (RC* programme [10,11]).

Signal and variance across the settings:

setting	leading signal	leading variance
QED		α^6/N , measured
QCD + qQED		α^4/N , predicted
dyn. QCD+QED		α^4/N , predicted

Next: test the α^4/N prediction with the same coupling scan on available QCD ensembles:

a [fm]	volume	$M_\pi L$
0.098	32×16^3	3.3
0.075	48×24^3	3.8
0.049	64×32^3	3.3

Table 1: QCD ensembles for the quenched-QED step ($M_\pi = 420$ MeV).

6 Conclusions and outlook

Goal: connected + leading disconnected HLbL in dynamical QCD+QED with C^* .

- The subtraction method is investigated in pure QED against the exact benchmark: $F_2(Q^2)$ resolved, rising toward it.
- In QED the predicted scalings are confirmed numerically: signal α^3 , subtracted variance α^6 , so $\text{StN} = \sqrt{N} \alpha^{3-n/2} = \sqrt{N}$ at $n = 6$.
- For QCD + qQED and dynamical QCD+QED the prediction is $\sigma^2 \sim \alpha^4/N$, i.e. $\text{StN} \sim \sqrt{N} \alpha$: not yet confirmed numerically.
- The $Q^2 \rightarrow 0$ extrapolation is deliberately left open; it needs lower Q^2 .
- Next: the connected HLbL in QCD + quenched QED (box 5), then dynamical QCD+QED with C^* .

References

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