Towards a consensus evaluation of $\Delta\alpha_{\text{had}}(M_Z^2)$?

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The fine-structure constant at the Z pole

How to get α at M_Z in $\overline{\text{MS}}$?

$$\hat{\alpha}^{-1}(M_Z^2) = \alpha^{-1}\Big[1 - \underbrace{\Delta\alpha_{\text{lep}}(M_Z^2)}_{0.0314979(2)} - \Delta\alpha_{\text{had}}(M_Z^2) - \underbrace{\left(\Delta\hat{\alpha}(M_Z^2) - \Delta\alpha(M_Z^2)\right)}_{0.007122(1)(5)}\Big]$$

$$= \alpha^{-1}\left[1 - \Delta\alpha_{\text{had}}(M_Z^2)\right] - 5.2923(7)$$

- $\hat{\alpha}(M_Z^2)$ important input for electroweak precision physics (FCC-*ee* at Z pole)
- Usual strategy
  1. Start with $\alpha \equiv \alpha(0)$ in Thomson limit
  2. Evolve to M_Z in on-shell scheme ($\Delta\alpha_{\text{lep}}$ known at four-loop order)
  3. Convert to $\overline{\text{MS}}$ at M_Z

$\hookrightarrow$ need hadronic running $\Delta\alpha_{\text{had}}(M_Z^2)$

Current evaluations

Reference	Result	Comment
Krasnikov, Rodenberg (1997) [158]	0.02737 ± 0.00039	pQCD for $\sqrt{s} > 2.3$ GeV
Kühn & Steinhauser (1998) [159]	0.02778 ± 0.00016	full $\mathcal{O}(\alpha_s^2)$ for $\sqrt{s} > 1.8$ GeV
Groote *et al.* (1998) [157]	0.02787 ± 0.00032	use of QCD sum rules
Martin *et al.* (2000) [160]	0.02741 ± 0.00019	incl. new BES data
de Troconiz, Yndurain (2001) [161]	0.02754 ± 0.00010	pQCD for $s > 2$ GeV^2
Burkhardt, Pietrzyk (2011) [162]	0.02750 ± 0.00033	pQCD for $\sqrt{s} > 12$ GeV
Erler, Ferro-Hernández (2017) [163]	0.02761 ± 0.00010	conv. from $\overline{\text{MS}}$ scheme
Jegerlehner (2019) [164]	0.02755 ± 0.00013	Euclidean split technique
Davier *et al.* (2019) [165]	0.02761 ± 0.00010	pQCD for $\sqrt{s} = 1.8$–3.7 & > 5 GeV
Keshavarzi *et al.* (2019) [166]	0.02761 ± 0.00011	pQCD for $\sqrt{s} > 11.2$ GeV
Cè *et al.* (2022) [156]	0.02773 ± 0.00015	LQCD for Euclidian $Q^2 < 5$ GeV^2

PDG 2026

- PDG 2026: outdated compilation of $\Delta\alpha_{\text{had}}(M_Z^2)$, **no recommendation**
- PDG 2024: same compilation, recommended

$$\Delta\alpha_{\text{had}}(M_Z^2) = 0.02783(6) \quad \Rightarrow \quad \hat{\alpha}^{-1}(M_Z^2) = 127.930(8)$$

$\hookrightarrow$ unclear procedure, unrealistic error estimate (removed from PDG 2026)

- Differences among recent evaluations much bigger:

$$\Delta\alpha_{\text{had}}(M_Z^2) = \left\{ \begin{array}{c} 0.02761(10)[\text{DHMZ}, e^+e^-] \\ 0.02795(11)[\text{Erler, Ferro-Hernández, Mainz lattice}] \end{array} \right\} \quad \Rightarrow \quad \hat{\alpha}^{-1}(M_Z^2) = \left\{ \begin{array}{c} 127.962(14) \\ 127.914(15) \end{array} \right\}$$

Towards improved determinations of $\hat{\alpha}(M_Z^2)$

- Clear need for a consensus evaluation of $\Delta\alpha_{\rm had}(M_Z^2)$ and future improvements
- BSM people are starting to pay attention
  - New idea to extract $\hat{\alpha}(M_Z^2)$ from $e^+e^- \to \{e^+e^-, \mu^+\mu^-\}$ via interplay of t-channel photon and s-channel Z exchange Riembau 2501.05508

    $\hookrightarrow$ similar to MUonE, systematics to be studied
  - CERN report see Indico of Radiative Corrections at Future Lepton Colliders
- Should we, as Theory Initiative, take on recommendations for $\Delta\alpha_{\rm had}(M_Z^2)$?
  - Low-energy part falls obviously within our purview, just different energy weighting
  - Interplay between lattice QCD and data-driven methods again key aspect
  - Perturbative aspects new, and those are non-trivial talk by A. Conigli
  - Short-distance windows (HVP) and short-distance constraints (HLbL) precedents for interplay with pQCD
- If yes, what about a topical workshop in the first half of 2027 to get things started?