Discussion on HLbL, ETM collaboration

HLbL group in Bern

University of Bern, Switzerland

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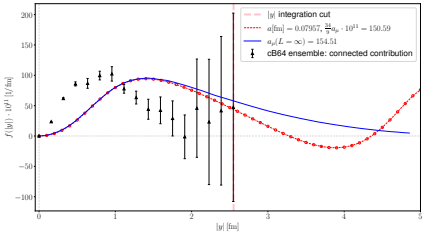
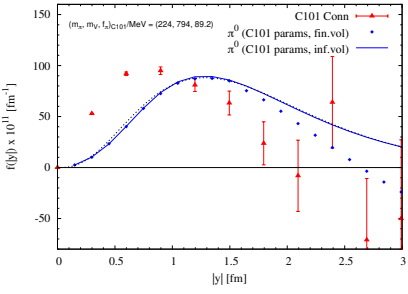
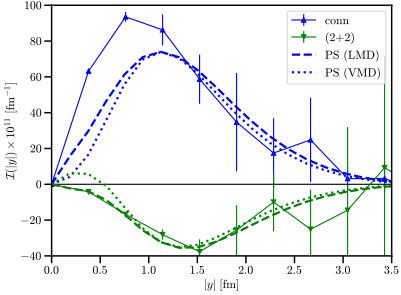
Pion pole contribution

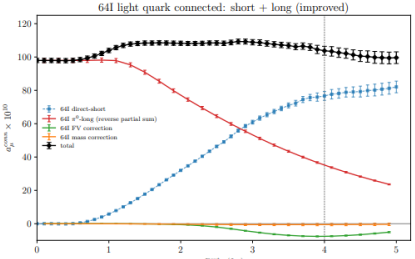
Light contribution and pion pole

Pole contribution









Pole contribution and Transition Form Factors

$$\Pi_{\mu\nu\sigma\lambda}(x,y,z) = \Pi^{(1)}_{\mu\nu\sigma\lambda}(x,y,z) + \Pi^{(2)}_{\mu\nu\sigma\lambda}(x,y,z) + \Pi^{(3)}_{\mu\nu\sigma\lambda}(x,y,z)\,,$$

where, e.g., (see Refs Blum et al., 2025; Fodor et al., 2025):

$$\Pi^{(1)}_{\mu\nu\sigma\lambda}(x,y,z) = -\int_{u,v} \widetilde{M}_{\mu\nu}(u,x,y)\left[\frac{e^{ipx}}{p^2+m_P^2}\right]\widetilde{M}_{\sigma\lambda}(v,z,0)\,,$$

$$\tilde{M}_{\mu\nu}(u,x,y) = -\epsilon_{\mu\nu\alpha\beta}\,\partial_{x_\alpha}\partial_{y_\beta}\int_{q_1,q_2}\mathcal{F}_{P\gamma^*\gamma^*}(-q_1^2,-q_2^2)\,e^{iq_1(x-u)}e^{iq_2(y-u)}, \qquad (1)$$

Lattice calculation of VMD approximation

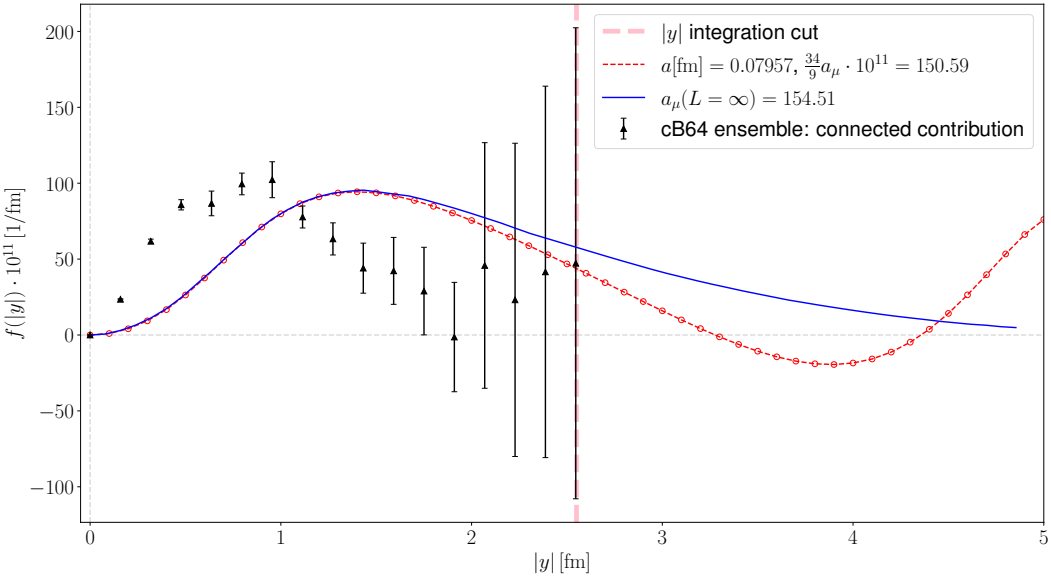
VMD parametrization

$$\mathcal{F}_{P\gamma^*\gamma^*}(-q_1^2,-q_2^2) = -\frac{N_c}{12\pi^2 F_\pi}\frac{M_V^4}{(q_1^2+M_V^2)(q_2^2+M_V^2)}$$

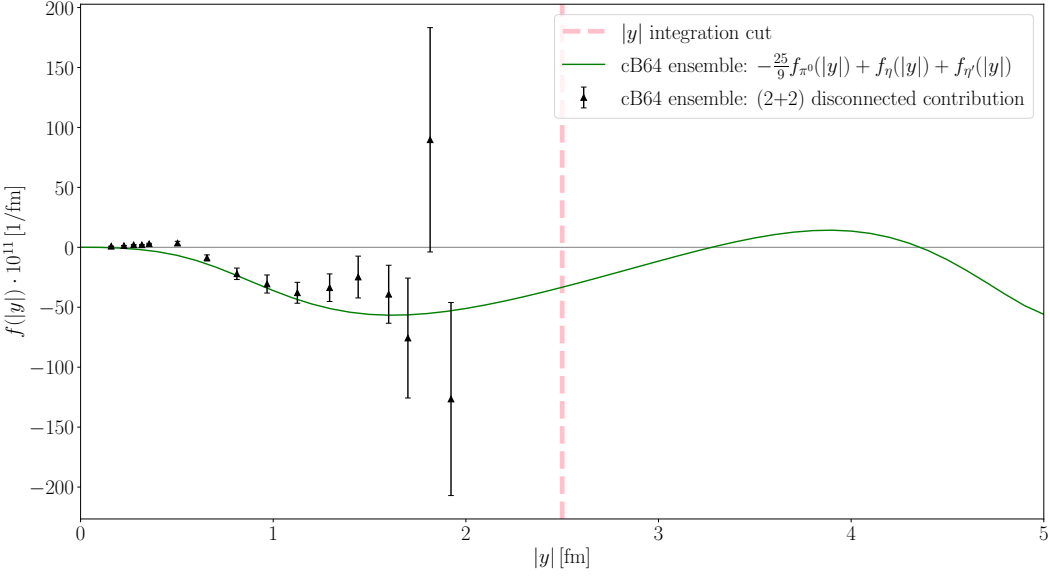
Lattice calculation: explicit sum over lattice points: $\int_w \to \sum_w$

$$\boxed{f(|y|) = \frac{m_\mu e^6}{3} 2\pi^2 |y|^3 \int_x \bar{\mathcal{L}}^{(\Lambda)}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\left[\int_z (-z_\rho) i\Pi^{\mathrm{VMD}}_{\mu\nu\sigma\lambda}(x,y,z)\right]}$$

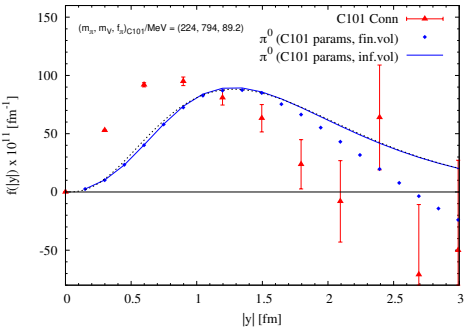
Pole contribution VS lattice data: connected contribution

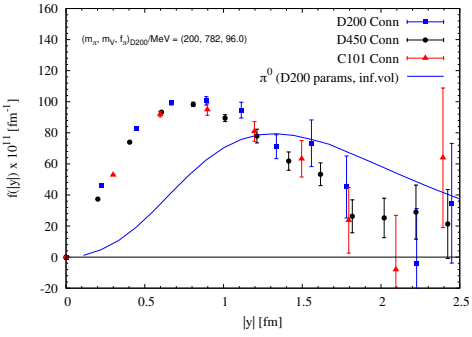


Pole contribution VS lattice data: 2+2 disconnected

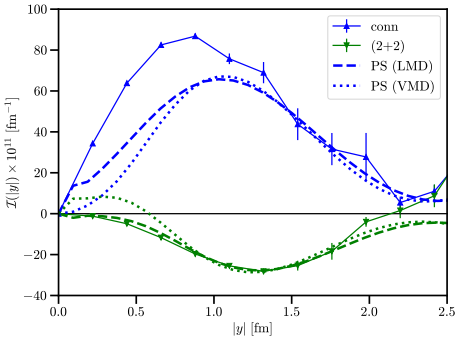


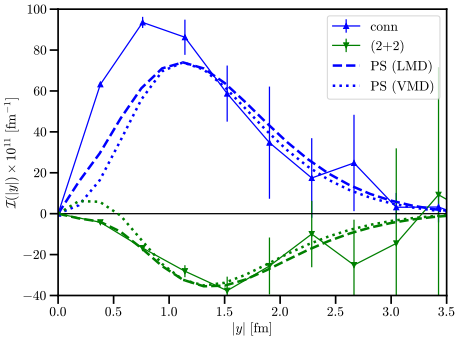
Mainz [Chao et al., 2021]





BMW: Pole contribution VS lattice data: connected contribution





NOTEs:

- They use a different kernel
- Even if I use their same kernel, I can't reproduce their data because the VMD parameters are not shared

References I

[1] Thomas Blum, Norman Christ, Masashi Hayakawa, Taku Izubuchi, Luchang Jin, Chulwoo Jung, Christoph Lehner, and Cheng Tu. "Hadronic light-by-light contribution to the muon anomaly from lattice QCD with infinite volume QED at physical pion mass". In: *Phys. Rev. D* 111.1 [2025], p. 014501. DOI: 10.1103/PhysRevD.111.014501. arXiv: 2304.04423 [hep-lat] [cit. on p. 4].

[2] Zoltan Fodor, Antoine Gerardin, Laurent Lellouch, Kalman K. Szabo, Balint C. Toth, and Christian Zimmermann. "Hadronic light-by-light scattering contribution to the anomalous magnetic moment of the muon at the physical pion mass". In: *Phys. Rev. D* 111.11 [2025], p. 114509. DOI: 10.1103/wdrk-7nrt. arXiv: 2411.11719 [hep-lat] [cit. on p. 4].

References II

[3] En-Hung Chao, Renwick J. Hudspith, Antoine Gérardin, Jeremy R. Green, Harvey B. Meyer, and Konstantin Ottnad. "Hadronic light-by-light contribution to $(g-2)_\mu$ from lattice QCD: a complete calculation". In: *Eur. Phys. J. C* 81.7 [2021], p. 651. DOI: 10.1140/epjc/s10052-021-09455-4. arXiv: 2104.02632 [hep-lat] [cit. on p. 8].