Data-driven evaluations of HLbL scattering: a roundtable discussion

J. Bijnens, M. Hoferichter, E. Kaziukenas, E. Lymperiadou, A. Rebhan, H. Schäfer, E. Solomonidi, M. Zillinger

Aug 7, 2026

Ninth Plenary Workshop of the Muon $g-2$ Theory Initiative

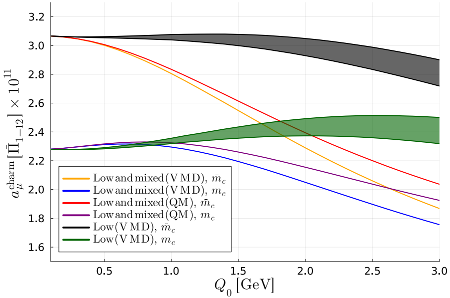
Storrs, Connecticut

Status of data-driven HLbL evaluations as of WP25: light quarks

Region		Dispersive	hQCD	Regge	DSE/BSE
$Q_i > Q_0$		$6.2^{+0.2}_{-0.3}$	6.3(7)	4.8(1)	2.3(1.5)
Mixed	A, S, T	3.8(1.5)			
	OPE	10.9(0.8)			
	Effective pole	1.2			
	Sum	15.9(1.7)	13.5(2.4)	12.8(5)	10.1(3.0)
$Q_i < Q_0$	$A = f_1, f_1', a_1$	12.2(4.3)	13.1(1.5)	10.9(1.0)	8.6(2.6)
	$S = f_0(1370), a_0(1450)$	$-0.7(4)$			$-0.8(3)$
	$T = f_2, a_2$	$-2.5(8)$	2.9(4)		
	Other	2.0	8.0(9)	3.2(6)	2.8(6)
	Sum	11.0(4.4)	24.0(2.8)	14.1(1.2)	10.6(2.7)
Sum		33.2(4.7)	43.8(5.9)	31.7(1.6)	23.0(7.4)

- Biggest differences concern **tensor-meson contributions** and **matching to short-distance constraints**, large uncertainties from **axial-vector resonances**

Status of data-driven HLbL evaluations as of WP25: charm quark



MH, Stoffer, Zillinger 2025

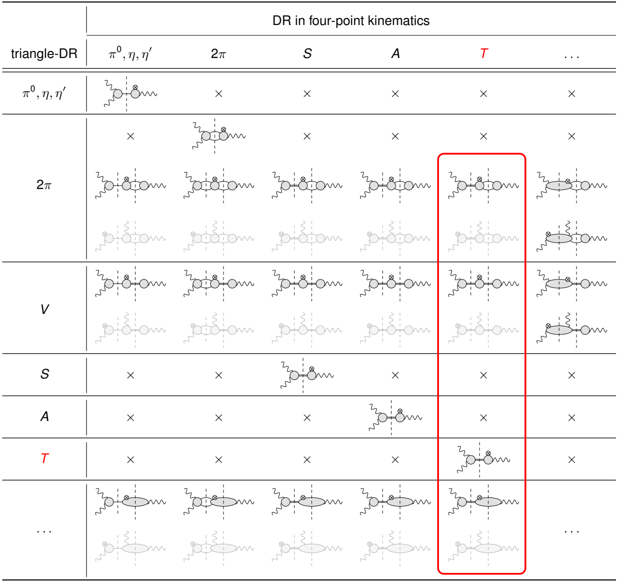
- WP25 used $a_\mu^{\text{HLbL}}[c] = 3(1) \times 10^{-11}$, with an error estimate from η_c pole
- Refined matching between pQCD and η_c contributions gives

$$a_\mu^{\text{HLbL}}[c] = 2.7(8) \times 10^{-11}$$

$\hookrightarrow$ main uncertainty from **pole mass m_c vs. $\overline{\text{MS}}$ mass $\bar{m}_c$**

- Beyond this level, need control over α_s corrections

Dispersive approaches: four-point and triangle kinematics



Lüdtke, Procura, Stoffer 2023

- **Four-point kinematics**:
  - Dispersion relation before taking $q_4 \to 0$
  - Tensor mesons require $\mathcal{F}_i^T(q_1^2, q_2^2)\mathcal{F}_j^T(q_3^2, 0)$
  - Only subset $\mathcal{F}_i^T \mathcal{F}_j^T$ free of kinematic singularities
- **Triangle kinematics**:
  - Dispersion relation after taking $q_4 \to 0$
  - Tensor mesons require $\mathcal{F}_i^T(q_1^2, q_2^2)\mathcal{F}_j^T(M_T^2, 0)$
  - Kinematic singularities absent for all $\mathcal{F}_i^T$
  - Need additional inputs: $\gamma^* T \to \pi\pi$

Outline for this session

- Few-slide updates:
  1. Johan Bijnens: α_s corrections to the charm loop
  2. Anton Rebhan: Tensor mesons in holographic QCD
  3. Maximilian Zillinger: Dispersive construction of the $f_2(1270)$ transition form factors and implications for HLbL
  4. Hannah Schäfer: a_1 and a_2 transition form factors from a $\rho\pi$ system
  5. Eirini Lymperiadou: Dispersive description of $\gamma^*\gamma^*\gamma \to \pi\pi$ subprocesses for tensor-meson contribution to muon $g-2$
  6. Emilis Kaziukenas: The $\gamma^* T \to \pi\pi$ subprocess and its role in the $f_2(1270)$ transition form factors
  7. Eleftheria Solomonidi: Reconstruction of $f_2(1270)$ transition form factors from 2π (and higher) cuts
- Open discussion

The Charm HLbL Contribution to the Muon $g - 2$

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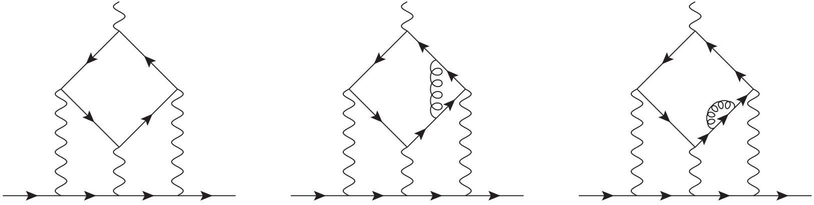
Ninth Plenary Workshop of the Muon g-2 Theory Initiative   Storrs, Connecticut, USA   3-7 August 2026

Charm contribution



(a) (b) (c)

- (a) LO (b) NLO (c) NLO mass renormalization
- IR problem regulated by m_c
- LO known to high order in m_μ/m_c Kuhn et al., arxiv:hep-ph/0301151
- Used to estimate $a_\mu^{HLbLc} = 3.0(1.0) \times 10^{-11}$ WP20,WP25,arxiv:1910.13432
- Problem: quite sensitive to m_c
- J. Bijnens, N. Hermansson-Truedsson, A. Rodríguez-Sánchez, Phys.Lett.B 873 (2026) 140178, [arxiv:2511.13097]

Charm contribution

Analytic result known in the case of on-shell mass in a different context

R. Boughezal, K. Melnikov, arXiv:1104.4510

$$a_\mu^{\text{HLbLc LO}} = \left(\frac{\alpha}{\pi}\right)^3 \left(\frac{3}{2}\zeta(3) - \frac{19}{16}\right) N_c q_c^4 \frac{m_\mu^2}{M_c^2}\,.$$

$$a_\mu^{\text{HLbLc NLO}} = a_\mu^{\text{HLbLc LO}} \frac{\alpha_s}{\pi} C_F \frac{\Delta_1}{\Delta_0}\,,$$

$$\Delta_0 = \frac{3}{2}\zeta(3) - \frac{19}{16}\,,$$

$$\Delta_1 = -\frac{473\pi^2}{1080}\ln^2 2 + \frac{52\pi^2}{405}\ln^3 2 - \frac{42853\pi^4}{259200} + \frac{5771\pi^4}{32400}\ln 2 + \frac{473}{1080}\ln^4 2 - \frac{52}{675}\ln^5 2 - \frac{8477}{2700} + \frac{473}{45}a_4 + \frac{416}{45}a_5 + \frac{34727\zeta_3}{2400} - \frac{23567\zeta_5}{1440}.$$

$C_F = 4/3$, $a_{4,5} = \mathrm{Li}_{4,5}(1/2)$, $C_F \Delta_1/\Delta_0 = 3.85174$

Charm contribution

- Pole mass scheme: renormalon ambiguity Beneke, Braun 1994
- $\overline{MS}$ much better
- Example: Charm two-point function leading term in Q^2/m_c^2
  A. Maier, P. Maierhofer, P. Marquard, A. V. Smirnov, 2010

$$\Pi_V(Q \ll m_c) \propto (1 + 0.295 + 0.036 - 0.010)\, z(m_c) = (1.295 \pm 0.089)\, z(m_c)$$

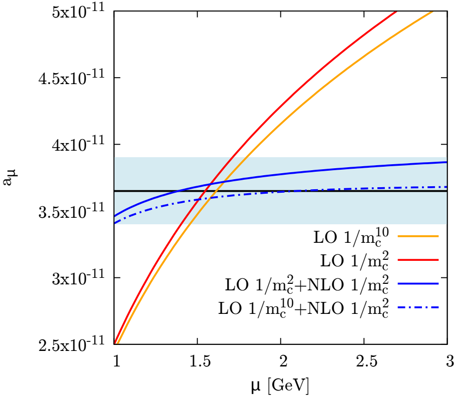
- First: the calculated orders; Second: only first two orders with error estimate
- So we need to rewrite to $\overline{MS}$

$$a_\mu^{\text{HLbLc LO}, \overline{\mathrm{MS}}} = \left(\frac{\alpha}{\pi}\right)^3 \left(\frac{3}{2}\zeta(3) - \frac{19}{16}\right) N_c q_c^4 \frac{m_\mu^2}{m_c(\mu)^2},$$

$$a_\mu^{\text{HLbLc NLO}, \overline{\mathrm{MS}}} = a_\mu^{\text{HLbLc LO}, \overline{\mathrm{MS}}} \frac{\alpha_s}{\pi}\left(C_F \frac{\Delta_1}{\Delta_0} - \frac{8}{3} + 4\log\left(\frac{m_c(\mu)}{\mu}\right)\right).$$

Charm contribution

- On shell $M_c = 1.49$ GeV
  $\alpha_s(M_c) = 0.3519$
  $a_\mu^{\text{HLbLc}} = (2.30 + 0.99 + \cdots) \times 10^{-11}$
- $m_c(m_c) = 1.2730 \pm 0.0046$ GeV,
  $\alpha_s(m_c) = 0.3876 \pm 0.0118\,.$
- 5 loop running of m_c and α_S
- Final result:
  $\boxed{a_\mu^{\text{HLbLc}} = 3.65(25) \times 10^{-11}}$
- Lattice (connected part only)
  BMWc: $3.92(26) \times 10^{-11}$
  Mainz/CLS: $3.1(4) \times 10^{-11}$
- Lattice with disconnected
  BMWc: $3.73(26) \times 10^{-11}$
  Mainz/CLS: $2.8(5) \times 10^{-11}$



Tensor mesons in hQCD

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w/ Luigi Cappiello (Napoli), Josef Leutgeb & Jonas Mager

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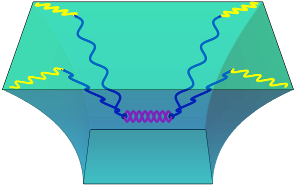
9th Plenary Muon g-2 TI Workshop, UCONN, August 7, 2026



Holographic tensor mesons – rationale & results

In hQCD,

- (axial) vector mesons are dual to nonabelian 5D flavor gauge fields, implementing automatically VMD; chiral anomalies through unique Chern-Simons action
  infinite tower of axials $\to$ correct MV limit of HLbL $\hat{\Pi}_1(Q,Q,Q_3)$ when $Q \ll Q_3$
  but symmetric limit $\lim_{Q\to\infty} \hat{\Pi}_1(Q,Q,Q)$ only 81% of quark loop
- 5D gravitons are dual to tensor mesons, but no flavor structure: tensor glueballs
- tensor quarkonia should be described by massive 5D strings;
  integrating them out assumed to lead to contributions to Einstein-Hilbert action, thus mixing with tensor glueballs, effectively describing whole multiplet
- infinite **tower of tensor mesons** contributes to symmetric limit of $\hat{\Pi}_1$;
  matching the missing 19% gives good description of a_2, f_2, f_2' photon couplings
- generates 2 out of 5 tensor TFFs $\mathcal{F}_i^T$: $\mathcal{F}_1^T$ similar to simple quark model, but inseparable $\mathcal{F}_3^T$, which changes sign of contribution to a_μ!

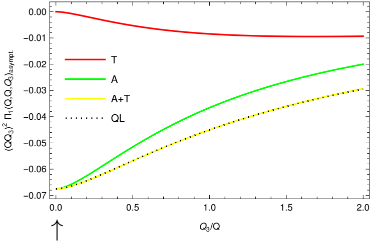
$$a_\mu^T \times 10^{11} = +11.1^{+1.3}_{-3.0}, \qquad [(Q_i < 1.5\text{GeV}): \underbrace{\mathbf{+8.5}}_{} {}^{+1.0}_{-2.3}] \qquad [\text{Mixed: } +1.9^{+0.2}_{-0.5}]$$

$\mathbf{+2.9}$ a_2, f_2, f_2' pole contributions
$\mathbf{+5.6}$ T^* tower

Short-distance behavior of hQCD results vs. Quark Loop

Asymptotic $\hat{\Pi}_1(Q,Q,Q_3) \times Q^2 Q_3^2$:

agrees with QL with max. deviation of 0.44%!



$\uparrow$
MV

[Bijnens et al. 2411.09578]

Miraculous SD identity for y-even corner integrands: only axial-sector contributions, tensors drop out

Summing the corner integrands $(F(Q_1^2,Q_2^2,Q_3^2)+F(Q_2^2,Q_3^2,Q_1^2)+F(Q_3^2,Q_1^2,Q_2^2)$ where F represents the integrand) and relabeling dummy variables one miraculously finds, when summing over the $36 = 3\times 12$ functions, keeping only the even pieces in the y variable (and thus also restoring full crossing of the integrand),

$$\sum_i^{36} T_i \overline{\Pi}_i = \frac{128}{3m_\mu^2\pi^2\overline{Q}^6}\Big(-Q^2(-1+\sigma)\left[3\omega_L(-1+y^2)+\omega_T(2+y^2)\right] + m_\mu^2\left[6\omega_L\sigma(-1+y^2)-2\omega_T(-3+2\sigma)(2+y^2)\right]\Big). \tag{4.30}$$

satisfied by SD limit of hQCD results, despite typical individual 25% corner contributions from tensors! [CLMR, 26xx.xxxxx]

Symmetric limits $\boxed{\lim_{Q\to\infty} Q^{a_i}\hat{\Pi}_i(Q,Q,Q)}$ [1] compared to the quark loop (QL) [2]:

$\hat{i}$	A	T	A/QL	T/QL	sum
1	-0.03657	-0.00846	81.22	18.78	100 %
4	-0.0061	-0.01144	37.50	70.40	108 %
7	-0.0061	-0.00256	75.00	31.55	107 %
17	$+0.0061$	$+0.00034$	59.04	3.33	62 %
39	$+0.0183$	$+0.00769$	68.80	28.94	98 %

$\rightarrow$ Tensors improve also all transverse SDCs significantly (except $\hat{\Pi}_{17}$)

[1] $a_i = 4$ for $i = 1, 4$ and $a_i = 6$ otherwise

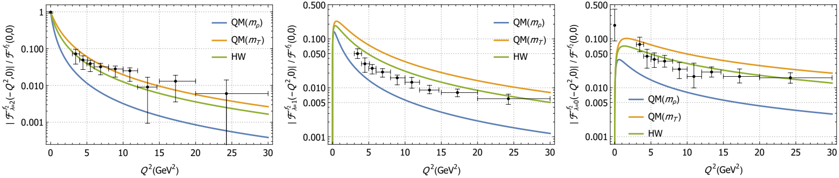
[2] comparison to gluonic corrections in preparation

Update: Comparison of singly virtual f2 TFFs with data

$\mathcal{F}_1^T$ from hQCD (HW) in between QM(M_ρ) (DV 2016) and QM(M_T) (HSZ24) (all w/o $\mathcal{F}_2^T$)

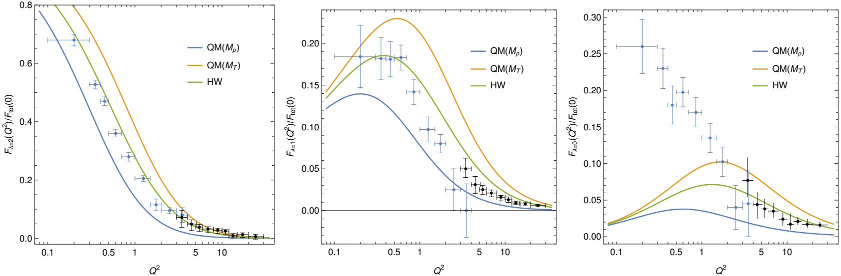
vs. BELLE ($3.45 \leq Q^2[\mathrm{GeV}^2] \leq 24.25$):

$\lambda = 2$ $\lambda = 1$ $\lambda = 0$



vs. BELLE and BESIII (Max Lellmann 2026):

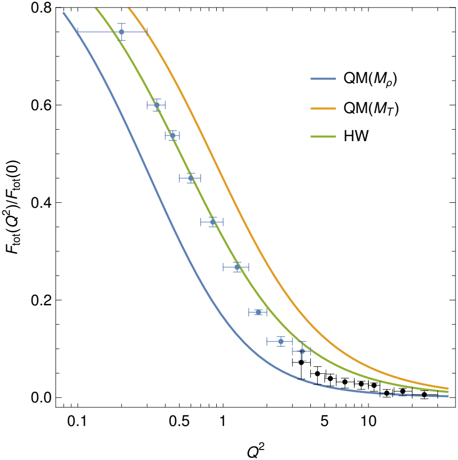
$\lambda = 2$ $\lambda = 1$ $\lambda = 0$



Comparison of normalized singly virtual $F_{\mathrm{tot}}^{f_2}$ with data

Experimental results for F_{tot} (combination of $\lambda = 2, 1, 0$) has slightly smaller errors
Blue: BESIII (Max Lellmann), black: BELLE

Agreement with hQCD (HW) result at low Q^2 even better
(but F_{tot} overestimated at large Q^2, in contrast to $\lambda = 2$ part):

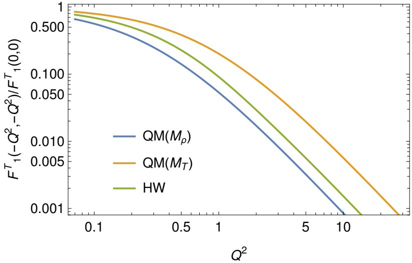


Doubly virtual TFFs in hQCD

Doubly virtual $\mathcal{F}_1^T$

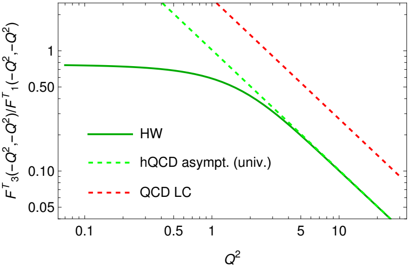
again in between QM(M_ρ) and QM(M_T):

$\mathcal{F}_1^T(-Q^2,-Q^2)/\mathcal{F}_1^T(0,0)$



Doubly virtual $\mathcal{F}_3^T$:

$\mathcal{F}_3^T(-Q^2,-Q^2)/\mathcal{F}_1^T(-Q^2,-Q^2)$



same sign as $\mathcal{F}_1^T$

asymptotically $\mathcal{F}_3^T \sim c\,\mathcal{F}_1^T/Q^2$

but $c|_{w=0}$ smaller than in LC expansion

Universal feature of hQCD with tensor mesons dual to 5d gravitons (not only for HW):

no zero and sign change possible for $\mathcal{F}_{1,3}^T$ of ground-state mesons

Caveats

Tensor mesons in hQCD treated as (effective) tensor glueballs

(i.e., all modes dual to fraction of full energy-momentum tensor)

- $\rightarrow \mathcal{F}^T_{1,3}$ fixed by *symmetries* (5D diffeomorphism and gauge invariance) and restriction to *lowest derivatives in 5D action* ($\Rightarrow$ only 1 free parameter)
- $\mathcal{F}^T_{2,4,5} \neq 0$ feasible as perturbations, but (too) many free parameters
- difficult to generalize to case where $\mathcal{F}^T_{2,4,5} \not\ll \mathcal{F}^T_{1,3}$, nontrivial flavor multiplets; would need new guiding principle

The $f_2(1270)$ Transition Form Factors and the D-Wave in $\gamma^*\gamma^* \to \pi\pi$

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07. August 2026

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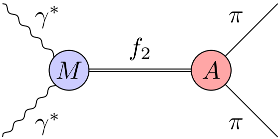
Dispersive Extraction of the f_2 TFFs

- **Assumption:** The f_2 strongly dominates the D-wave of $\gamma^*\gamma^* \to \pi\pi$, so we assume this partial wave is fully saturated by the f_2 resonance
- Matching procedure for helicity amplitudes in the limit $s \to M_{f_2}^2$:

$$iH^{J=2,\pi\pi}_{\lambda_1\lambda_2}(s,z) \quad = $$



$$iH^{J=2,\pi\pi}_{\lambda_1\lambda_2}(s,z) = \varepsilon^{\lambda_1}_{\mu}(q_1)\varepsilon^{\lambda_2}_{\nu}(q_2) iM^{\mu\nu\alpha\beta}(p,q_1,q_2)\frac{is^T_{\alpha\beta\gamma\delta}}{s - M_{f_2}^2 + iM_{f_2}\Gamma_{f_2}} iA^{\gamma\delta}(p,p_1,p_2)$$

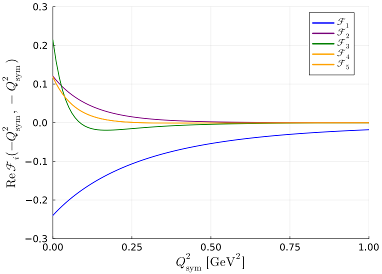
$$\lim_{s\to s_{f_2}}\left(\frac{s - s_{f_2}}{s - 4M_\pi^2} h_{J=2,\lambda_1\lambda_2}(s)\right) = \sum_{i=1}^{5} C_i(q_1^2, q_2^2, M_{f_2}^2)\mathcal{F}_i^T(q_1^2, q_2^2)$$

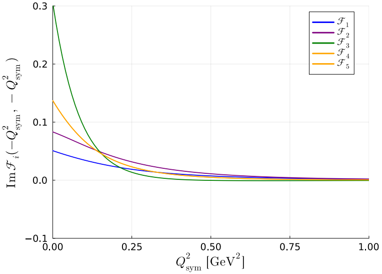
$$s_{f_2} = \left(M_{f_2} - i\frac{\Gamma_{f_2}}{2}\right)^2$$

- We use modified MO solution for $h_{2,\lambda_1\lambda_2}(s)$ including RHC and LHC
- Consistency check: z dependence in $H^{J=2,\pi\pi}_{\lambda_1\lambda_2}(s,z)$ drops out

Results for the Dispersive TFFs

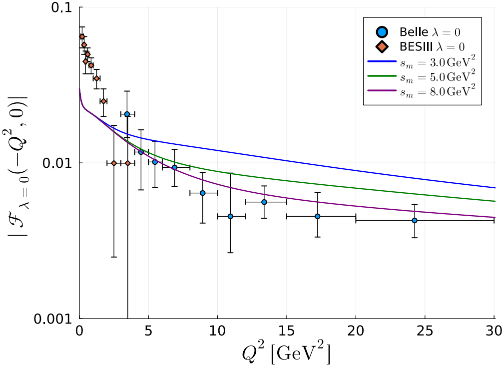
- **LHC vs RHC:** The real part of $\mathcal{F}_1^T$ is almost entirely driven by the Left-Hand Cut (ω exchange). $\mathcal{F}_3^T$ is driven by the Right-Hand Cut.
- **On-shell matching:** Two-photon rate matched to $\Gamma_{f_2\gamma\gamma} = 2.60$ keV.
- Dispersive helicity fraction: $r^{\text{disp}} = 0.015$.



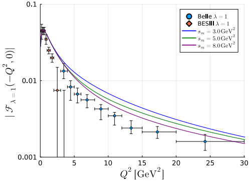


Comparison with Experimental Data

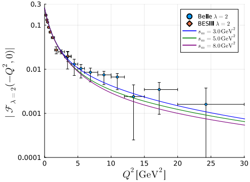
$\lambda = 0$: **tension at low Q^2**



$\lambda = 1$



$\lambda = 2$



Best global high-Q^2 agreement is obtained for $\boldsymbol{s_m = 8.0\,\mathrm{GeV}^2}$. The $\lambda = 0$ result shows significant tension with BESIII at low Q^2, with

$$r^{\mathrm{BESIII}} = 0.139^{+0.060}_{-0.032}$$

TFFs in Resonance ChPT

- An independent cross-check using Resonance ChPT.
- **Pion Loops (LO):** Generate imaginary parts and non-analytic momentum dependence.
- **Local Terms (NLO/NNLO):** Supplemented by VMD extensions (including excited ρ' states) to ensure proper UV falloff.

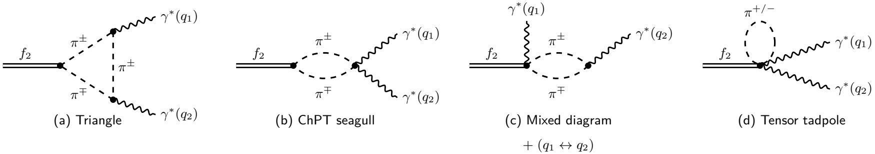
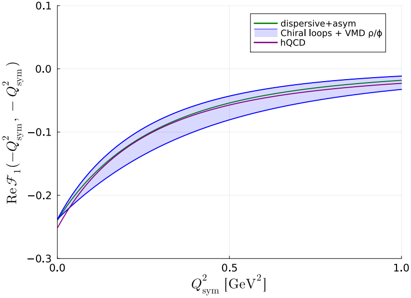
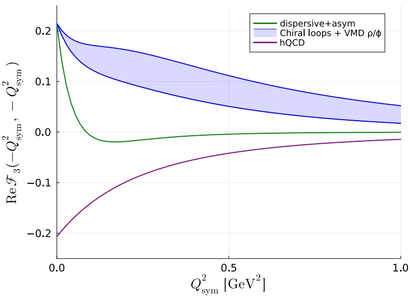


Abbildung: One-loop topologies contributing to $f_2 \to \gamma^* \gamma^*$.

Comparing Frameworks: Real Parts

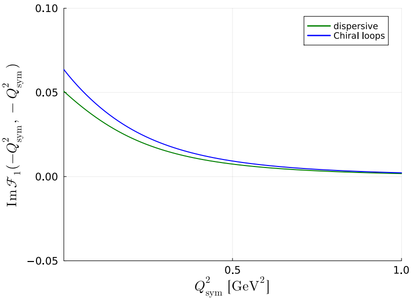
- **Agreement:** Excellent agreement across Dispersive, ChPT, and hQCD for the real part of $\mathcal{F}_1^T$.
- **Tension:** Major discrepancy in $\mathcal{F}_3^T$. hQCD predicts negative normalization; Dispersive solution predicts positive normalization and crossing to negative.

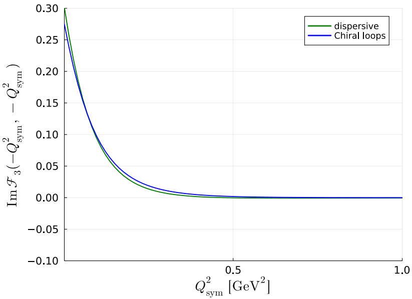




Comparing Frameworks: Imaginary Parts

- **Imaginary Parts:** Good agreement between dispersive analysis and loops from resonance ChPT loops. Especially good agreement for $\mathrm{Im}\ \mathcal{F}_3^T$, making the discrepancy in the real part particularly puzzling.





Final Results for Muon $g - 2$

- The $f_2(1270)$ contribution is calculated using the scalar functions $\bar{\Pi}_{1-12}$. Currently, kinematic singularities in the optimized basis allow us to include only $\mathcal{F}_1^T$ and $\mathcal{F}_3^T$.

- Contribution to low-energy region using central matching scale $Q_0 = 1.5$ GeV
$$a_\mu^{f_2}[\bar{\Pi}_{1-12}, \mathcal{F}_{1,3}^T] = -3.35(17) \times 10^{-11}$$

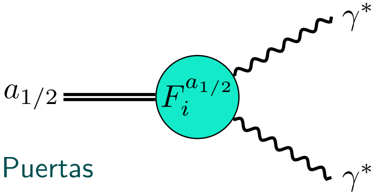
- **Component Breakdown:** The form factor $\mathcal{F}_3^T$ has a very mild impact. The total value is driven almost entirely by the real part of $\mathcal{F}_1^T$.

a_1 and a_2 transition form factors from a $\rho\pi$ system

Hannah Schäfer

in collaboration with

Martin Hoferichter, Bastian Kubis,
Eirini Lymperiadou, Pablo Sánchez-Puertas



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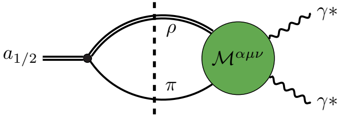
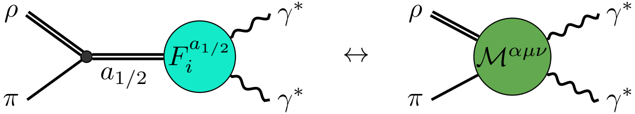
Ansatz: match to $\rho\pi \to \gamma^*\gamma^*$

- direct loop calculation not possible,

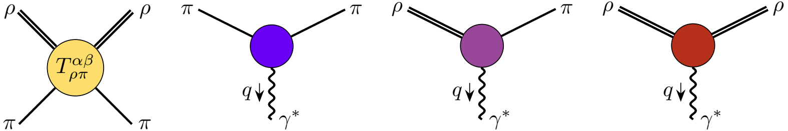


ambiguous

- match $a_{1/2}$ pole in $\rho\pi \to \gamma^*\gamma^*$ to $\mathcal{M}^{\mu\nu\alpha}$ (cf. MZ's part)

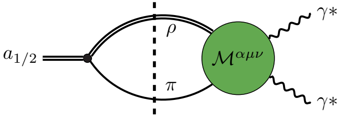


- extract a_1/a_2 TFFs, relate them to $\rho\pi$ rescattering and known form factors F_π^V, $F_{\rho\pi}$, G_1, G_2, G_3 via $\rho\pi \to \gamma^*\gamma^*$ system



Ansatz: match to $\rho\pi \to \gamma^*\gamma^*$

- direct loop calculation not possible,



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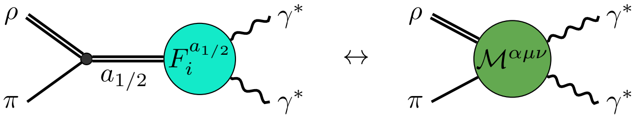

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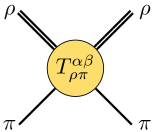
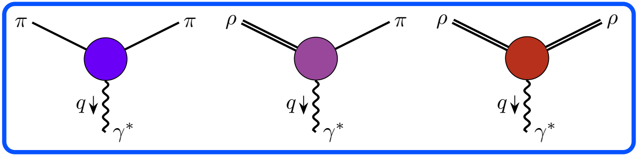
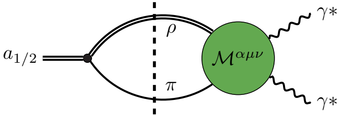




known

- direct loop calculation not possible,



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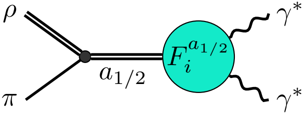
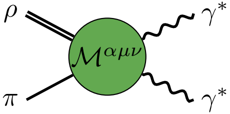


$\leftrightarrow$



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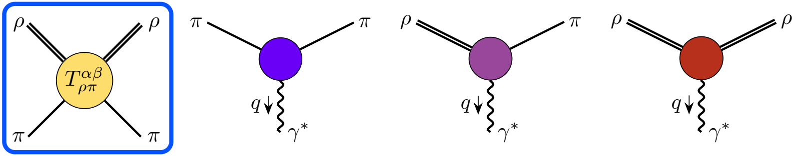
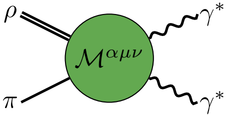


to be modelled

Full $\rho\pi \to \gamma^*\gamma^*$ amplitude

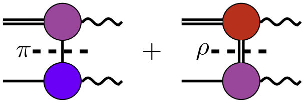
- gauge-invariant description of 

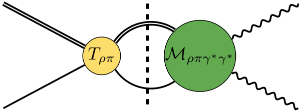
respecting

  all **symmetries**, BTT decomposition $\mathcal{M}^{\mu\nu\alpha} = \sum_{i=1}^{13} \mathcal{F}_i \, T_i^{\mu\nu\alpha}$,
  $\mathcal{F}_i$ **free of kinematic zeros and singularities**
- include **rescattering** and **crossed-channel** effects
  $\to$ **Muskhelishvili–Omnès** description

$$\mathcal{F}_i(s,t,u;q_1^2,q_2^2)$$

$$= \mathcal{F}_i^{\text{Born}}(s,t,u;q_1^2,q_2^2) + \frac{1}{2\pi \mathrm{i}} \int_{s_{\text{thr}}}^{\infty} \mathrm{d}s' \, \frac{\text{disc}_s^{\rho\pi} \mathcal{F}_i(s',t,u;q_1^2,q_2^2)}{(s'-s-\mathrm{i}\epsilon)}$$



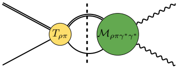


- Born terms: ρ/π exchange in t/u channel

$\rho\pi \to \gamma^*\gamma^*$ system: coupled equations

- discontinuity for partial-wave projected helicity amplitudes from unitarity relation (depend on only one variable)

$$\mathrm{disc}_s^{\rho\pi} m^J_{\lambda_1\lambda_2;\lambda_\rho}(s) \sim \sum_{\lambda_{\mathrm{RH}}} t^{J*}_{\lambda_{\mathrm{RH}}\lambda_\rho}(s)\, m^J_{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}(s)$$



$\to$ need DRs in terms of $m^J_{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}(s)$, $t^J_{\lambda_{\mathrm{RH}}\lambda_\rho}(s)$

- partial-wave projection & expansion of DR for $\mathcal{F}_i$

$\to$ system of coupled equations

$$m^J_j(s) = m^{\mathrm{Born},J}_j(s) + \sum_{j'} \frac{1}{2\pi \mathrm{i}} \int_{s_{\mathrm{thr}}}^{\infty} \mathrm{d}s'\, K^{jj'}(s,s')\, \mathrm{disc}_s^{\rho\pi} m^J_j(s'),$$

$K^{jj'}(s)$ kernel functions, to be determined—find optimised basis for $m^J_j(s)$

Solution and matching

- $t^J_{\lambda_{\mathrm{RH}}\lambda_\rho}(s) \sim \sin\delta^{\lambda_{\mathrm{RH}}\lambda_\rho}_J(s)e^{\mathrm{i}\delta^{\lambda_{\mathrm{RH}}\lambda_\rho}_J(s)}$, phases to be modelled
- Omnès functions $\Omega^{\lambda_{\mathrm{RH}}\lambda_\rho}_J(s) = \exp\left\{\frac{s}{\pi}\int_{s_{\mathrm{thr}}}^\infty \mathrm{d}s' \frac{\delta^{\lambda_{\mathrm{RH}}\lambda_\rho}_J(s')}{s'(s'-s)}\right\}$
- Muskhelishvili–Omnès solution

$$m^J_{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}(s) = \Delta^{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}_J(s) + \frac{\Omega^{\lambda_{\mathrm{RH}}\lambda_\rho}_J(s)}{\pi}\times \times\int_{s_{\mathrm{thr}}}^\infty \mathrm{d}s' \frac{\Delta^{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}_J(s')\sin\delta^{\lambda_{\mathrm{RH}}\lambda_\rho}_J(s')}{|\Omega^{\lambda_{\mathrm{RH}}\lambda_\rho}_J(s')|(s'-s)}$$

  with inhomogeneities $\Delta^{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}_J(s)$ including Born terms
- additional LHCs $\to$ modified MO solution
- match helicity amplitudes or scalar functions and solve for TFFs
- $K^*\bar{K}$ system $\to$ f_1' TFFs

Dispersive analysis of $\gamma^*\gamma^*\gamma \to \pi\pi$ subprocesses for tensor-meson contribution to muon $g-2$

Eirini Lymperiadou | 9th Plenary Workshop of the Muon g-2 Theory Initiative

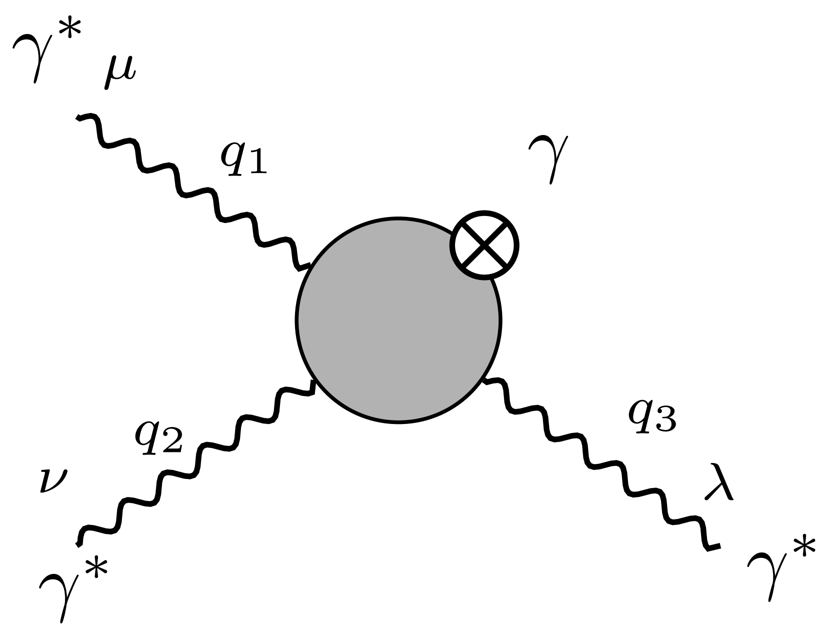
in collaboration with Peter Stoffer, Massimiliano Procura, Emilis Kaziukenas, Eleftheria Solomonidi

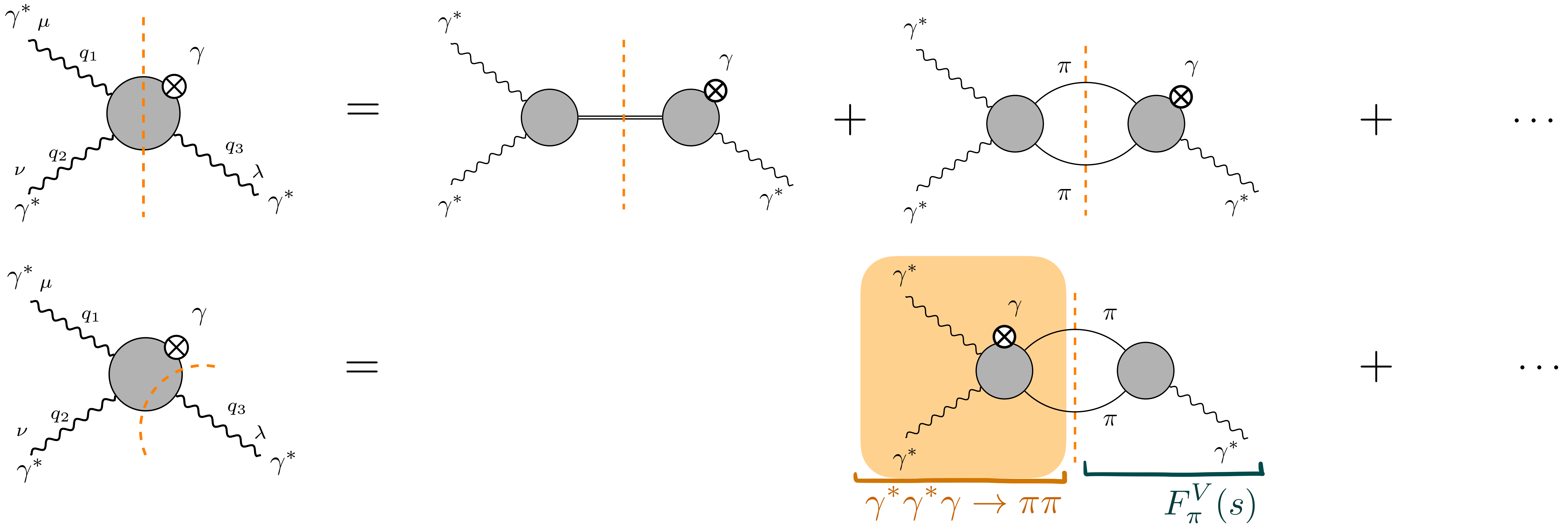
PSI

Universität Zürich[UZH]

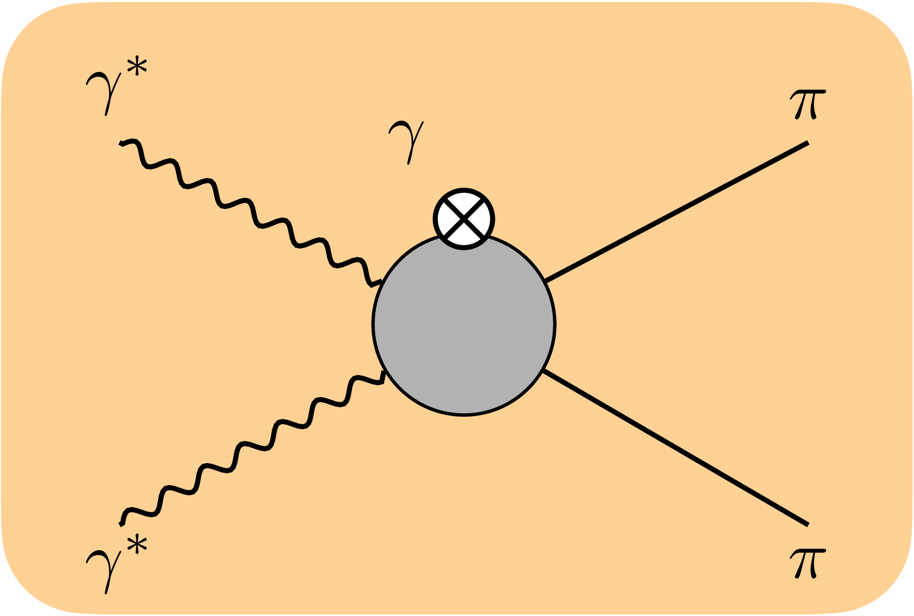
Unitary cuts in triangle kinematics framework

[arXiv:2302.12264]





[arXiv:2302.12264]



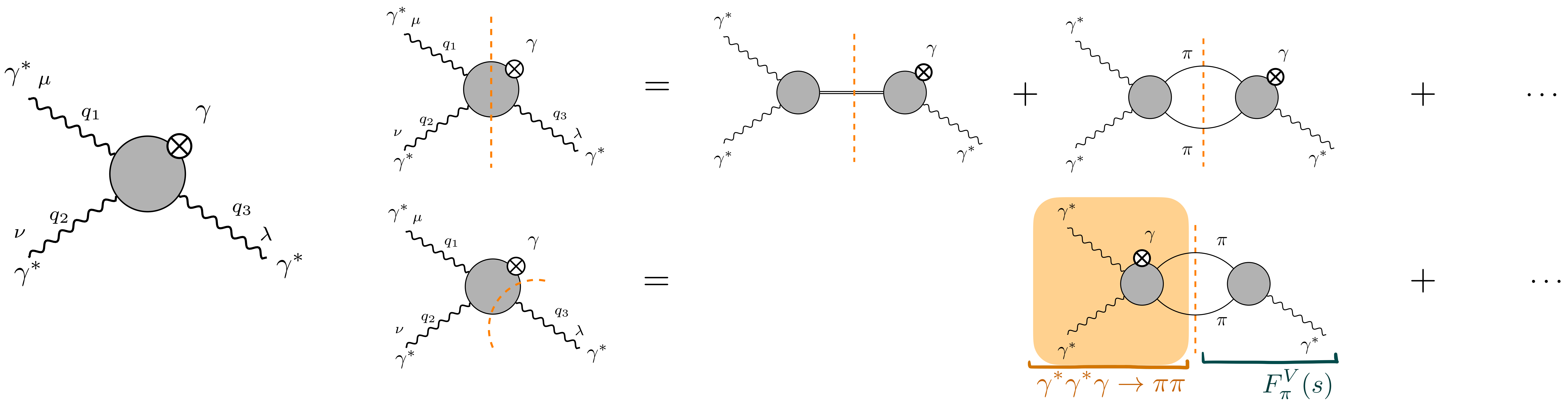
- Basis for $\gamma^*\gamma^*\gamma \to \pi\pi$ in the soft photon limit

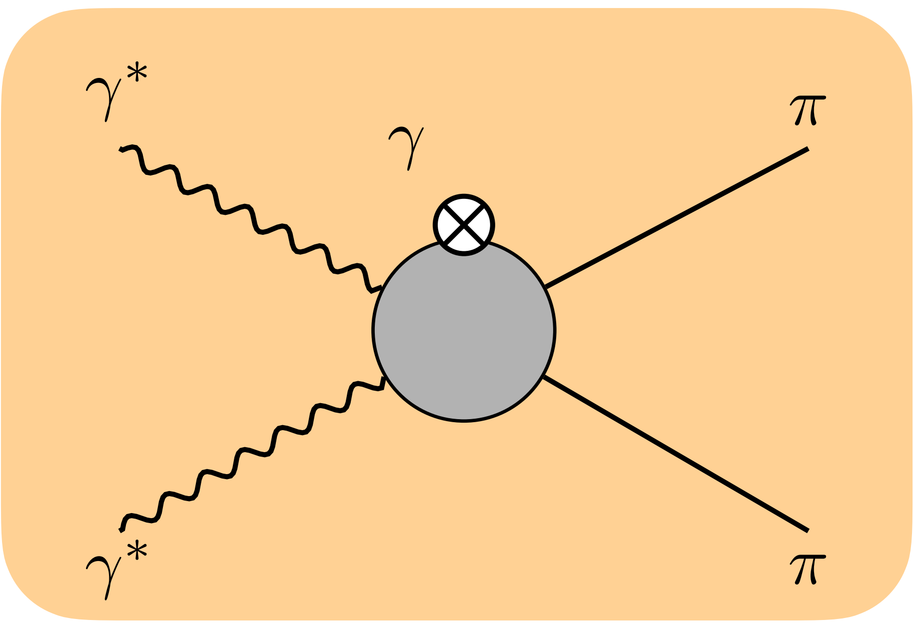
$$\frac{\partial}{\partial q_{3\sigma}} \mathcal{M}^{\mu\nu\lambda}_{\mathrm{reg}}(p_1, p_2, q_1, q_2)\Big|_{q_3=0} = \sum_{i=1}^{27} \mathcal{B}_i^{\mu\nu\lambda;\sigma}(q_1, q_2, q_5)\bar{A}_i(s, t-u, q_1^2, q_2^2)$$

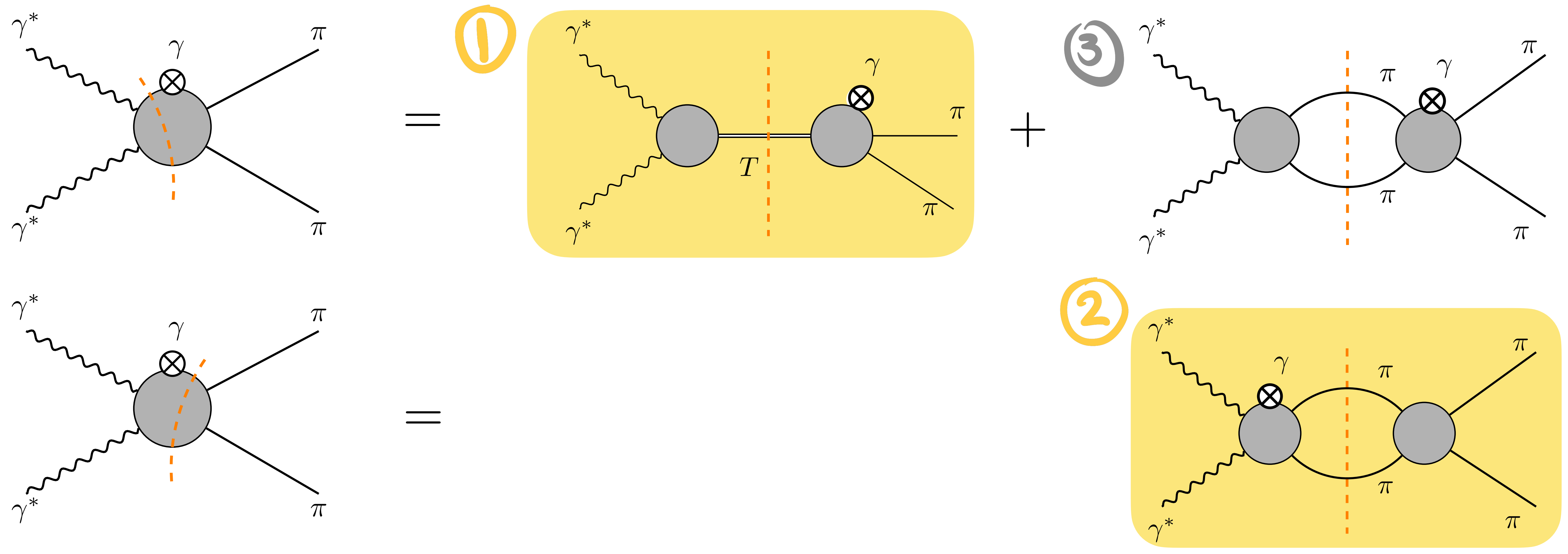
- Projectors for $\gamma^*\gamma^*\gamma \to \pi\pi$: constructed

Unitary cuts in triangle kinematics framework

[arXiv:2302.12264]



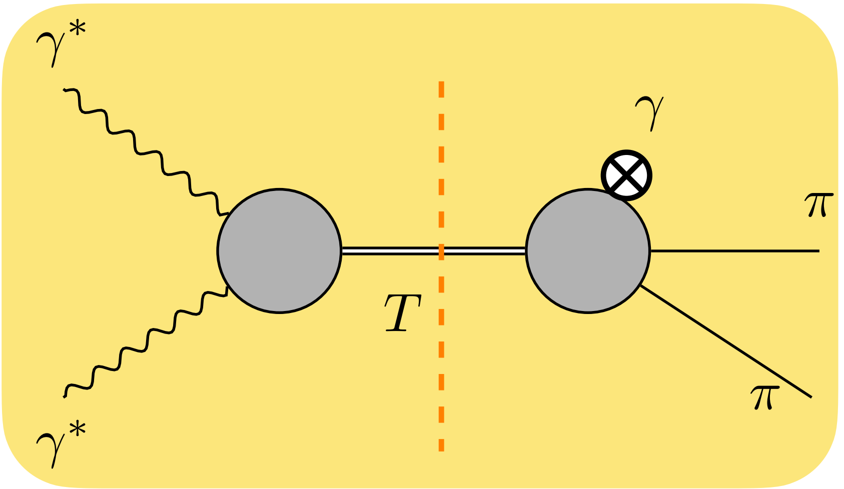




1 s-channel **:** Tensor-meson pole

2 $\tilde{s}$-channel **:** Two-pion cut

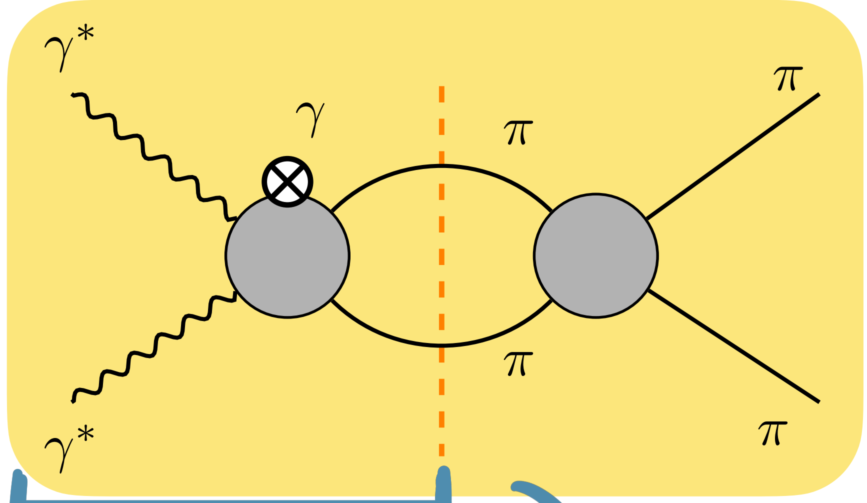
① Tensor-meson pole cut



$$\bar{A}_i^{\text{tensor},s} = \frac{1}{(m_T^2 - s)} \sum_{j=1}^{5} \frac{t_{j,1}^i(m_T^2, q_1^2, q_2^2)}{m_T^3} \mathcal{F}_j^T(q_1^2, q_2^2) \hat{\mathcal{F}}_5(m_T^2, 0)$$

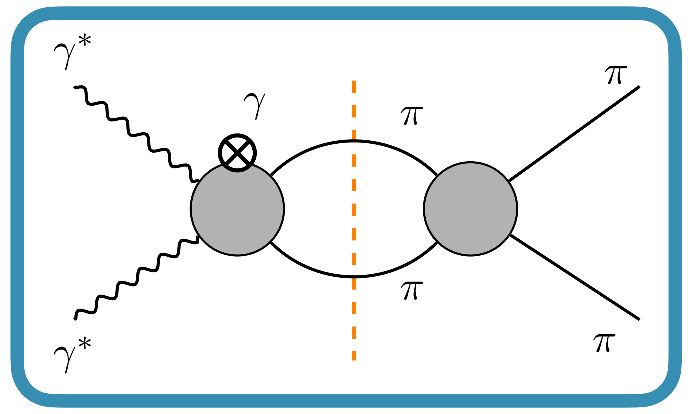
- $\mathrm{Im}_s^T \bar{A}_i = 0$, for $i \in \{11, 12, 21, 22, 24, 27\}$

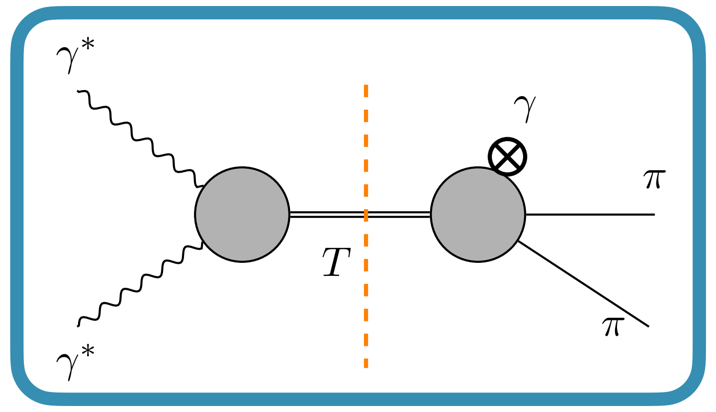
② Two-pion cut



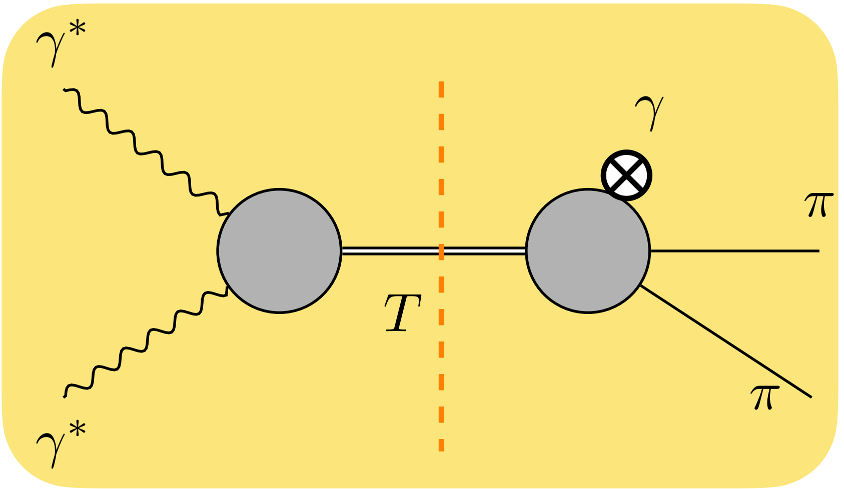
- Use general expansion $\bar{A}_i = \sum_j a_j^i(s) \cdot P_j(z)$
- Inhomogeneous contribution from tensor-meson pole

$$\mathrm{Im}_s^{\pi\pi} \bar{A}_i = \mathrm{Im}_s^{\pi\pi} a_0^{i\,\text{resc.}}(s) = \left[a_0^{i\,\text{resc.}}(s) + \hat{F}_i^{\text{inh.}}(s)\right] \cdot \sin\delta_1^1(s) e^{-i\delta_1^1(s)}$$





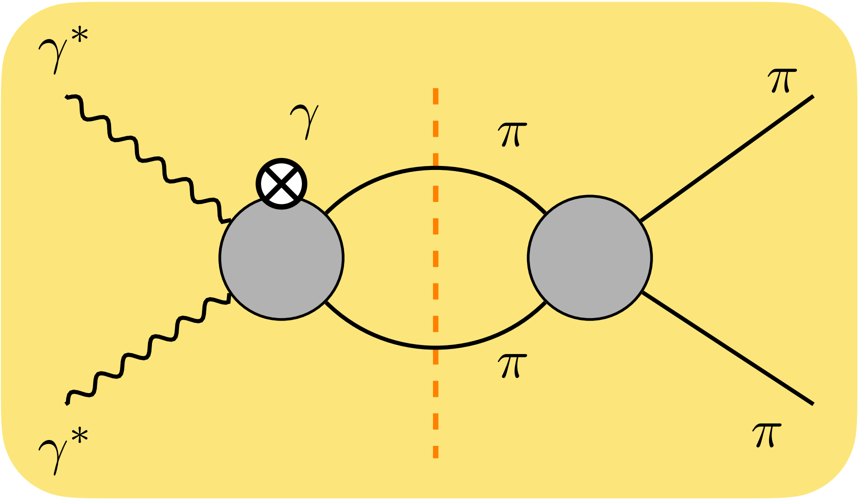
① Tensor-meson pole cut



$$\bar{A}_i^{\text{tensor},s} = \frac{1}{(m_T^2 - s)} \sum_{j=1}^{5} \frac{t^i_{j,1}(m_T^2, q_1^2, q_2^2)}{m_T^3} \mathcal{F}_j^T(q_1^2, q_2^2)\hat{\mathcal{F}}_5(m_T^2, 0)$$

- $\mathrm{Im}_s^T \bar{A}_i = 0$, for $i \in \{11, 12, 21, 22, 24, 27\}$

② Two-pion cut



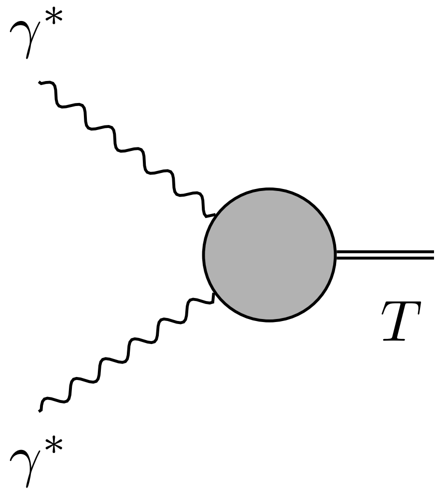
- Use general expansion $\bar{A}_i = \sum_j a^i_j(s) \cdot P_j(z)$

$$\bar{A}_i^{\pi\pi,\tilde{s}} = \left[\frac{\Omega^1_1(s)}{m_T^2} + \frac{\Omega^1_1(s)\frac{s}{m_T^2} - \Omega^1_1(m_T^2)}{(m_T^2 - s)} \right] \frac{1}{\Omega^1_1(m_T^2)} \sum_{j=1}^{5} \frac{t^i_{j,1}(m_T, q_1^2, q_2^2)}{m_T^3} \mathcal{F}_j^T(q_1^2, q_2^2)\hat{\mathcal{F}}_5(m_T^2, 0)$$

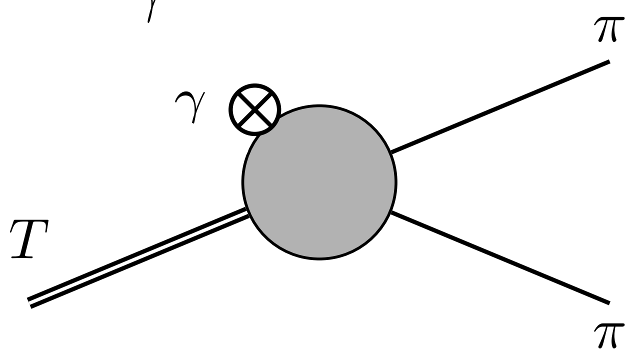
- 8 scalar functions with no imaginary part, 3 with only homogeneous contribution

NEXT STEPS

! Input for TFFs & scalar functions

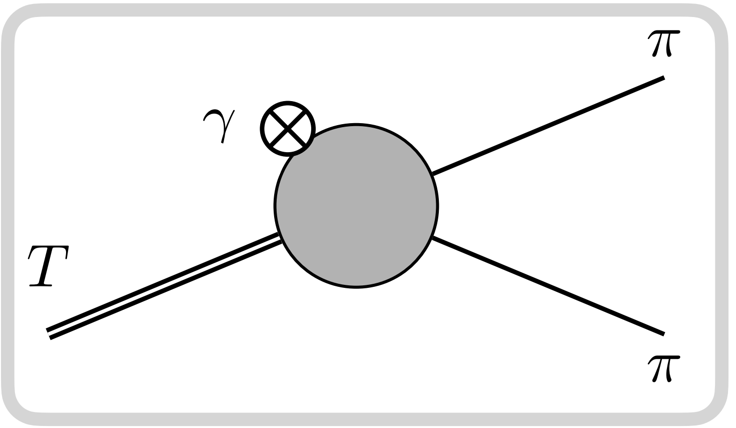


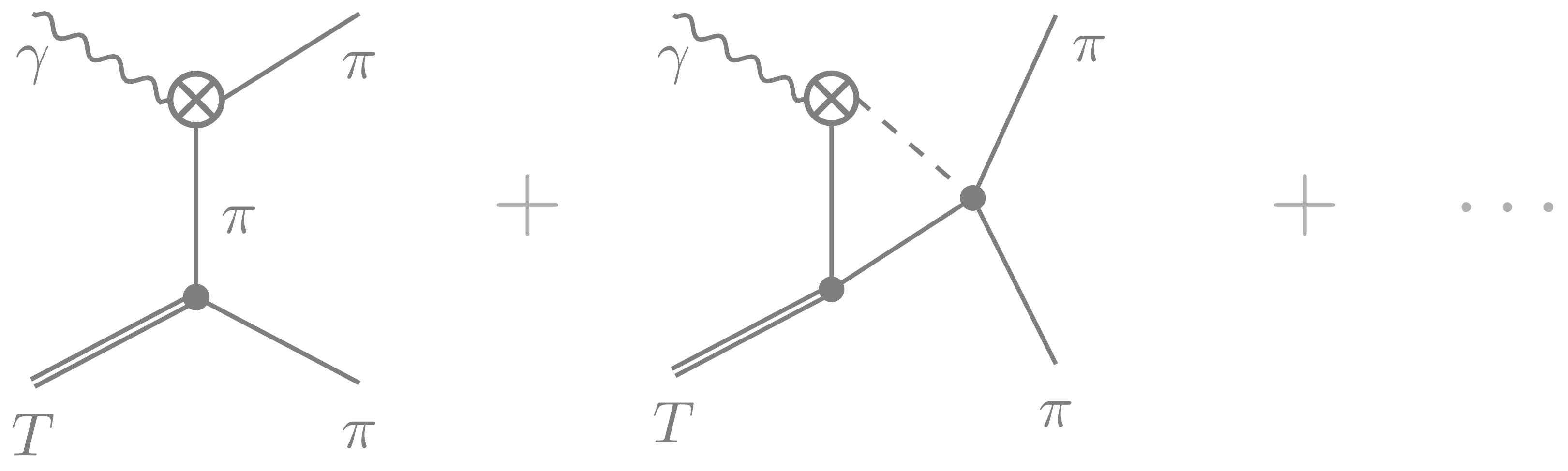
- previously known : Quark model / hQCD
- new constructions : Emilis & Elefteria (Vienna-Zurich) / Maximilian (Bern)



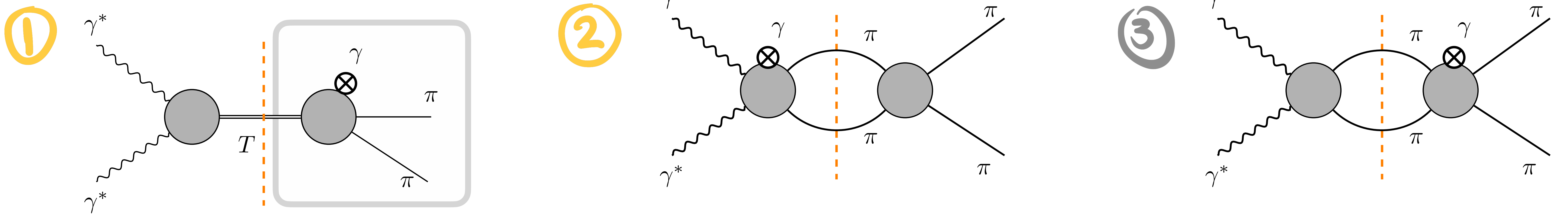
- new construction : Emilis & Elefteria (Vienna-Zurich)
- preliminary input : Omnès approximation $\sim \Omega(s) \cdot \text{const.}$

- preliminary input : include LHC and additional terms to the description of $T\gamma^* \to \pi\pi$





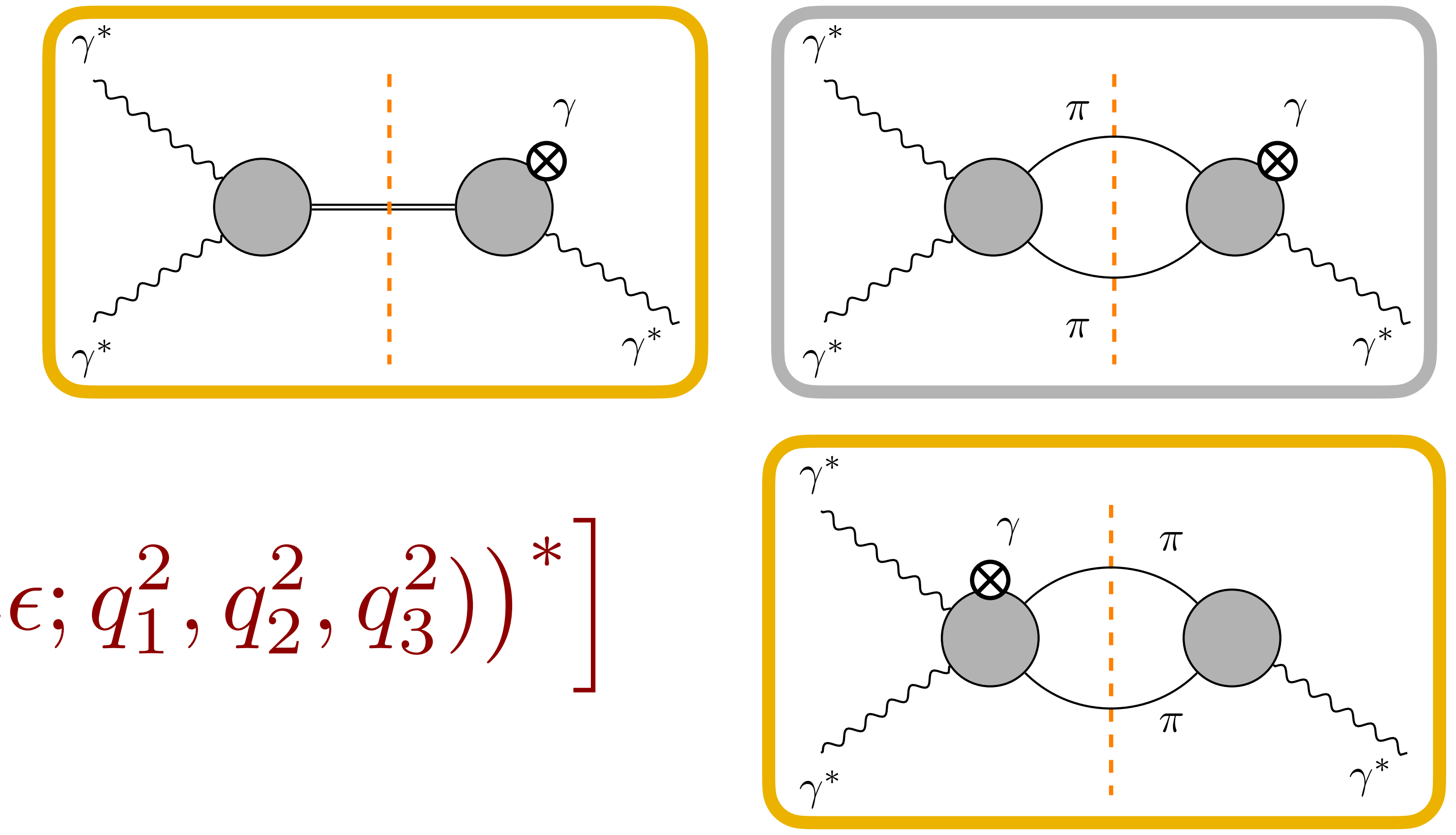
but then



Compute the contributions to a_μ^{HLbL}

- sum up both unitary cuts consistently

$$\Delta\check{\Pi}_i(q_1^2, q_2^2, s') = \lim_{q_3 \to s'} \left[\Delta_s\check{\Pi}_i(s'; q_1^2, q_2^2, q_3^2 + i\epsilon) + \left(\Delta_3\check{\Pi}_i(s' + i\epsilon; q_1^2, q_2^2, q_3^2)\right)^*\right]$$



✦ consistent inclusion of tensor-meson contributions within the dispersive framework

✦ constructed formalism for the subprocesses : necessary input for TFFs & scalar functions

The $\gamma^* T \to \pi^+\pi^-$ subprocess and its role in the $f_2(1270)$ transition form factors

Emilis Kaziukėnas [1,2]

with

E. Lymperiadou [3,4], M. Procura [1], E. Solomonidi [3,4], P. Stoffer [3,4], and J.-N. Toelstede[3,4]

[1]University of Vienna, Austria, [2]Vienna Doctoral School in Physics, Austria
[3]University of Zurich, Switzerland, [4]Paul Scherrer Institute, Villigen, Switzerland

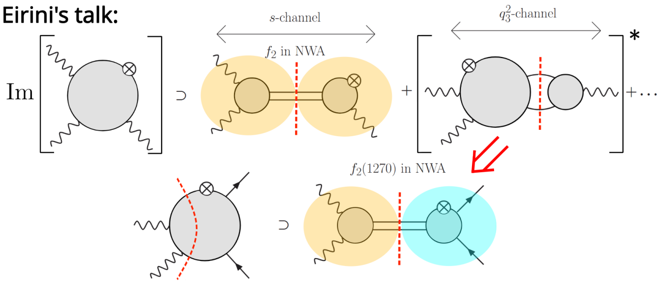
9th Plenary Workshop of the Muon g-2 Theory Initiative
August 7 2026, University of Connecticut

FWF Austrian Science Fund

universität wien
Vienna Doctoral School in Physics

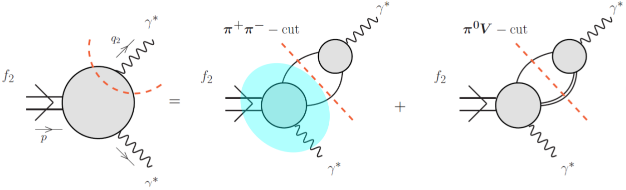
I) $\gamma^* T \to \pi^+\pi^-$ - ingredient of HLbL and tensor TFFs!

1 **Goal:** disp. evaluation of lightest spin-2 resonance $\boldsymbol{f_2(1270)}$.



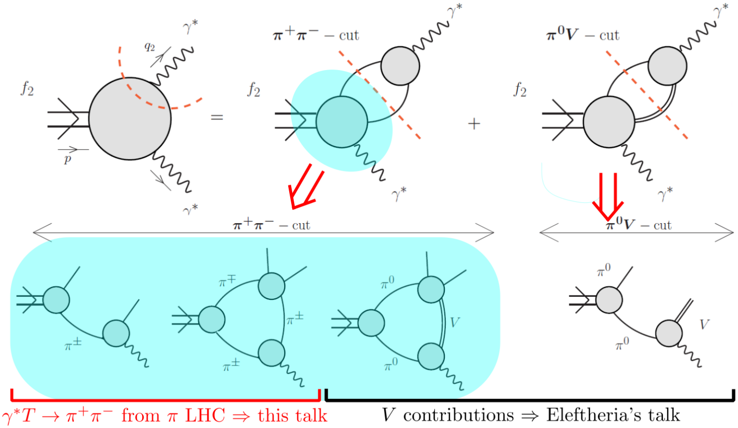
2 **s-Cut :** need TFFs $\mathcal{F}_i^T$, $q_i^2 < 0$ and $q_i^2 = 0, m_T^2$;
q_3^2-Cut : TFFs $\mathcal{F}_i^T$ and $\gamma^* T \to \pi^+\pi^-$ - new! $\Rightarrow$ **Use to construct TFFs too!**

3 **How? TFFs from q_2^2-cut:** take $\pi^+\pi^-$-, πV-cuts (match $\gamma^*\gamma^* \to \pi\pi$ D-wave pheno [1905.13198v2]) $\Rightarrow$ **Eleftheria's talk**



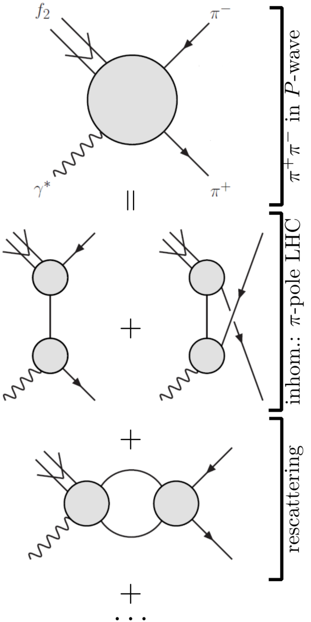
I) $\gamma^* T \to \pi^+\pi^-$ - ingredient of HLbL and tensor TFFs!

3 **How? TFFs from q_2^2-cut:** take $\pi^+\pi^-$-, πV-cuts (match $\gamma^*\gamma^* \to \pi\pi$ D-wave pheno [1905.13198v2]) $\Rightarrow$ **Eleftheria's talk**



4 **Assume only $T \to \pi\pi$ coupling:** BR $\approx 84\%$. Need $\gamma^* T \to \pi^+\pi^-$ and $\gamma^* T \to \pi V$ (both P-wave), $V = \rho, \omega$.

II) The $\gamma^* T \to \pi^+\pi^-$ subprocess: Inhom. Omnès problem



1 $\boldsymbol{\gamma^* T \to \pi^+\pi^-}$ **definition:** $\times 8$ scalar funcs from BTT (1-to-1 map to hel. amplitudes)

$$\mathcal{M}^{\mu\alpha\beta}(s,t,u) = \sum_i T^{\mu\alpha\beta}\hat{\mathcal{F}}_i(s,t,u).$$

2 Simplifying argument for Omnès problem:

Helicity amplitudes: $H_{\lambda_\gamma\,\lambda_T} = M_{ij}\hat{\mathcal{F}}_j(s,t,u)$ → **Pure P-wave:** $H_i = d^1_{m,0}(z)h^1_i(s)$ → **Pure s-channel DRs:** $M^{-1}_{ij}d^1_{m_j,0}(z)h^1_j(s) \overset{!}{=} \hat{\mathcal{F}}^P_i(s)$ → **MO problem directly for BTT funcs.:** $\mathrm{Disc}_s\hat{\mathcal{F}}^P_i(s) = \hat{\mathcal{F}}^P_i(s)e^{-\delta^1_1(s)}\sin\delta^1_1(s)\theta(s-4M^2_\pi)$

3 MO solution:

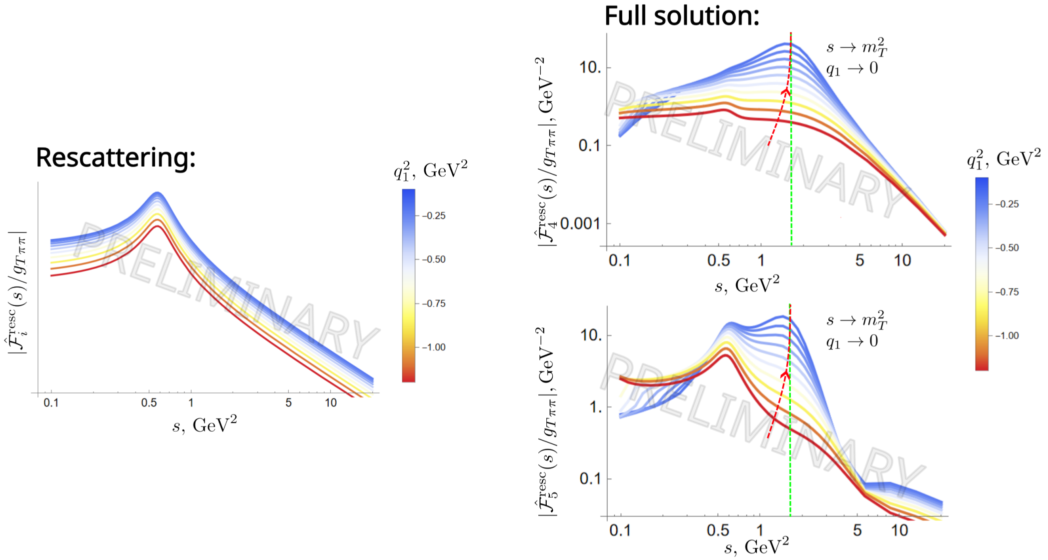
$$\hat{\mathcal{F}}^P_i(s,q_1^2) = \hat{\mathcal{F}}^P_{i,\mathrm{Born}}(s,q_1^2) + \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \, \frac{\hat{\mathcal{F}}^P_{i,\mathrm{Born}}(s',q_1^2)\sin\delta^1_1(s')}{(s'-s-i\epsilon)|\Omega_1(s')|},$$

$$\hat{\mathcal{F}}^P_{i,\mathrm{Born}}(s,q_1^2) = \frac{F^V_\pi(q_1^2)}{\lambda^{m_i}(s,q_1^2,m_T^2)}\left(P_i + Q_i\mathrm{Disc}_s\, C_0(s,q_1^2,m_T^2)\right),$$

with asymptotic ansatz (only $\times 5\,\hat{\mathcal{F}}_i$ out of 8 funcs. $\neq 0$ in P-wave):

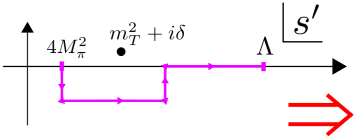
$$\hat{\mathcal{F}}_{1,2} \asymp \frac{1}{s^2}, \quad \hat{\mathcal{F}}_{3,4,5} \asymp \frac{1}{s}.$$

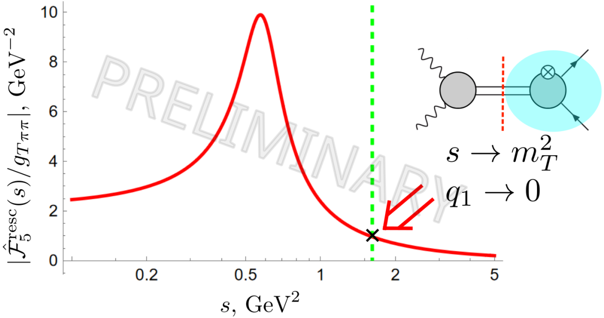
III) $\gamma^* T \to \pi^+\pi^-$ results: $q_1^2 < 0$ case



1. All $\hat{\mathcal{F}}_i(s, q_1^2)$ - similar resc. behavior, $s = M_\rho^2$ peak.
2. Two types of full (Born + resc.) behavior, $q_1^2 \to 0$ - Born part develops soft-γ singularity at $s = m_T^2$.

III) $\gamma^* T \to \pi^+\pi^-$ results: $q_1^2 = 0$, $q_1 \to 0$ cases





3 On-shell $\hat{\mathcal{F}}_i^{\mathrm{Born}} \propto 1/(s - m_T^2)^n$ - **deform resc. integral path** for stable integration.

4 **$\gamma T \to \pi^+\pi^-$ in $\gamma^*\gamma^*\gamma \to \pi\pi$ s−cut:** only $\hat{\mathcal{F}}_5^{resc}(m_T^2, 0) \neq 0$ in $q_1 \to 0$ (generic $\hat{\mathcal{F}}_i(s, q_1^2)$ for TFFs though):

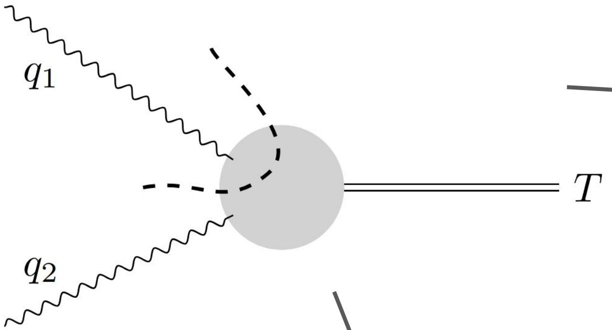
$$\hat{\mathcal{F}}_5^{resc}(m_T^2, 0) \approx -g_{T\pi\pi} \times (0.95 + 0.14i)\,\mathrm{GeV}^{-2}.$$

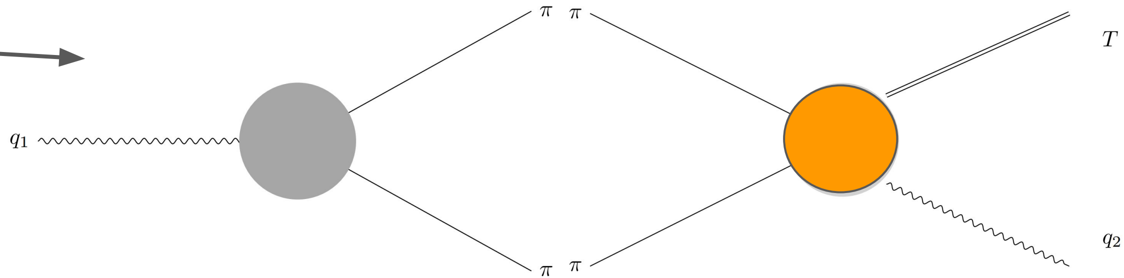
Further work:

- *V*-**res. contributions:** **⇒Eleftheria's talk.** Is multi-channel MO problem avoidable?
- Predict **branching fraction of $T \to 2(\pi^+\pi^-)$** for consistency: importance of $T \to VV$ couplings?
- Compare with $\gamma^* T \to \pi^+\pi^-$ from $\boldsymbol{T \to 2(\pi^+\pi^-)}$ with $T \to \pi\pi$ coupling - analogy to $\eta^{(\prime)} \to \pi\pi\gamma^*$ e.g. [2412.16281].

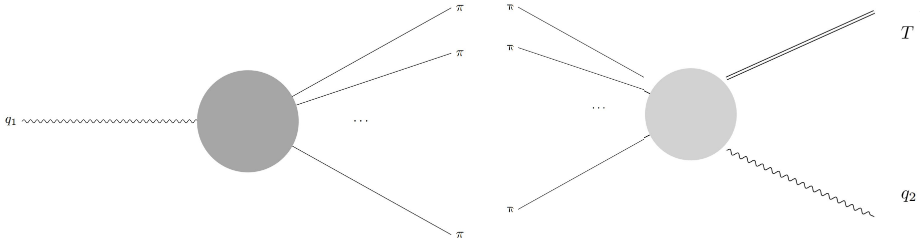
First results for tensor-meson Transition Form Factors with hadronic cuts

(continuing on the same method,
with Kaziukenas, Lymperiadou, Procura, Stoffer)

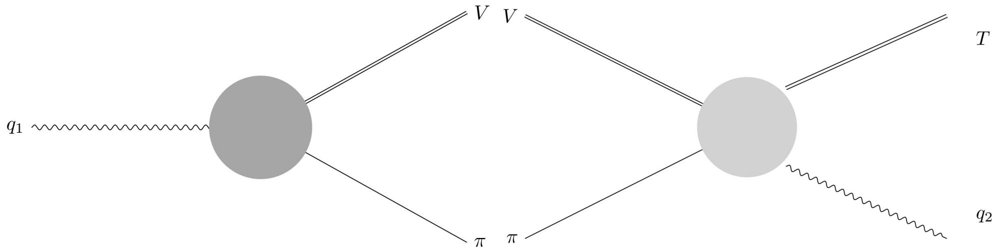




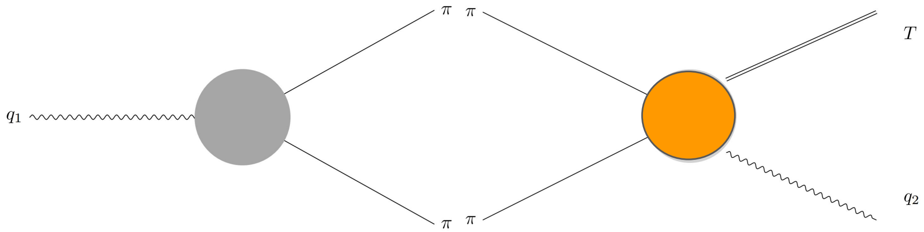
reconstructed as described before



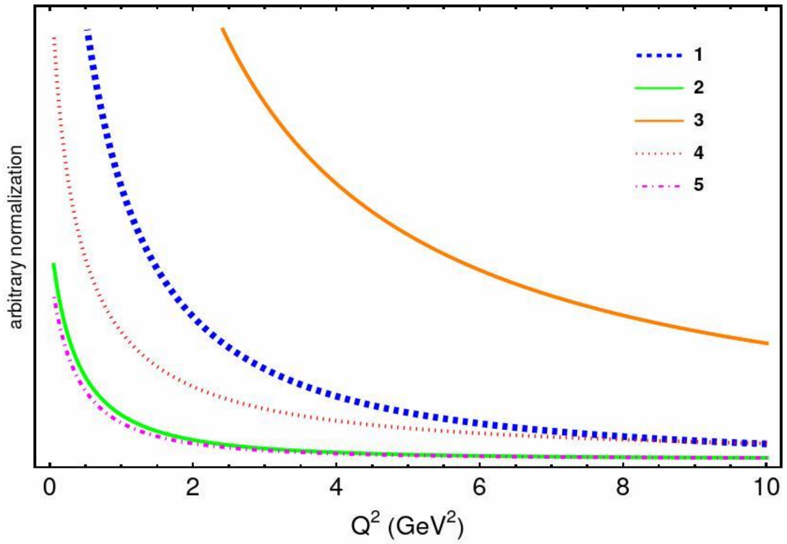
modelled as



TFFs from 2-pion cut

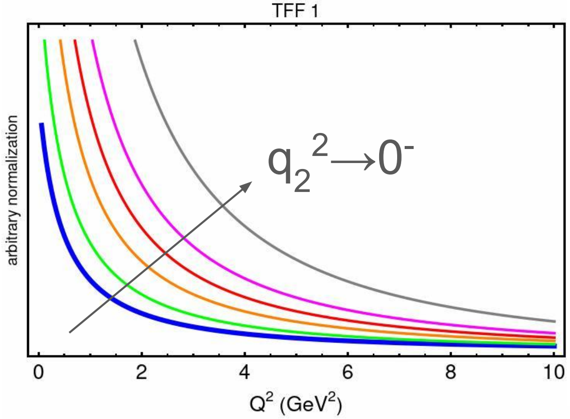


|TFFs| @ dispersed q_1^2=-Q^2, fixed q_2^2=-3 GeV^2

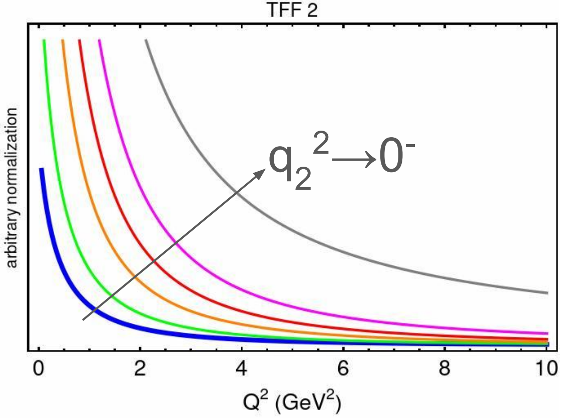


Doubly off-shell results

PRELIMINARY



TFF 3 consistently the largest, followed by TFF 1



Remarks & near-future plans

- 2-pion cut contribution to the TFFs somewhat **sizable** (significant fraction of the $f_2(1270)$ decay width to 2 photons)

  To be checked:

- Inclusion of higher cuts: $\gamma \to \pi\rho \to \gamma\, T$, $\gamma \to \pi\omega \to \gamma\, T$
- Relative size of each contribution & cancellations? (*πV cuts expected large*)
- *Consistency* of our assumptions *(e.g. contact coupling of $f_2(1270)$ only to two pions)*
- Impact of inelasticities in the $\pi\pi$ cut

Discussion points

- How to resolve the differences in $\mathcal{F}_3^T$?

  $\hookrightarrow$ hQCD, chiral loops, dispersion relations

- Consequences of new experimental results:
  - $\boldsymbol{f_1 \to e^+e^-}$ CMD-3, Logashenko: $\text{Br}[f_1 \to e^+e^-] = 8.3(2.3)(1.1)[2.6] \times 10^{-9}$

    $\hookrightarrow$ larger than expected, consequences for $\mathcal{F}_i^A$?
  - $\boldsymbol{\gamma^*\gamma \to \pi\pi}$ BESIII, Lellmann: helicity fraction $r \simeq 15(5)\%$

    $\hookrightarrow$ larger than expected, consequences for $\mathcal{F}_2^T$?
  - $\boldsymbol{\eta \to \gamma\gamma}$ PrimEx-η, Gan at Confinement 2026: $\Gamma[\eta \to \gamma\gamma] = 0.431(21)(25)[32]\,\text{keV}$

    $\hookrightarrow$ 2.3 σ below collider average 0.516(18) keV, consequences for η pole?

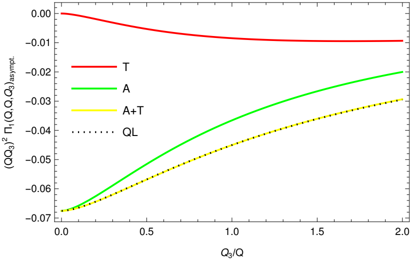
- How can we make connection to lattice QCD besides pseudoscalar poles?

- Which definition of pseudoscalar poles best describes the long-distance behavior in lattice calculations? Romiti

- ...

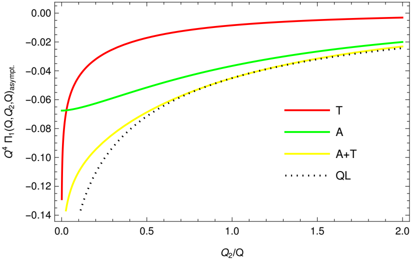
Longitudinal SDCs - MV and opposite corner

Asymptotic $\hat{\Pi}_1(Q, Q, Q_3) \times Q^2 Q_3^2$:



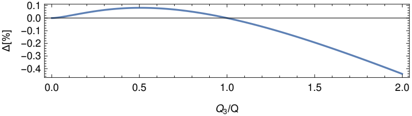
(MV limit $Q_3/Q = 0$: $-\frac{2}{3\pi^2} \approx -0.0675$)

Asymptotic $\hat{\Pi}_1(Q, Q_2, Q) \times Q^4$:

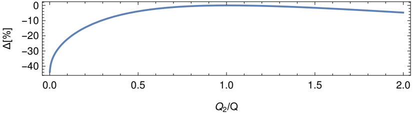


(log. div. in QL and T, but different coeff.)

error:



error:



anyway tiny contribution to a_μ from corner opposite to MV!

“Miraculous” SD identity for corner integrands $Q \ll \overline{Q}$

[J. Bijnens, N. Hermansson-Truedsson, A. Rodriguez-Sanchez, 2411.09578]]

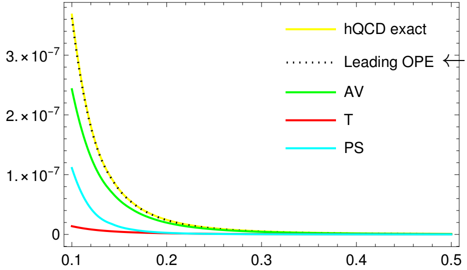
Summing 3×12 corner integrands $I(Q_1, Q_2, Q_3) + I(Q_2, Q_3, Q_1) + I(Q_3, Q_1, Q_2)$, only even pieces in y:

$$\sum_i^{36} T_i \bar{\Pi}_i \to \frac{128 N_c \sum e_q^4}{3 m_\mu^2 \pi^2 \overline{Q}^6} \Big(-Q^2(\sigma - 1)\left[3\underline{\omega_L}(y^2-1) + \underline{\omega_T}(2+y^2)\right] + m_\mu^2 \left[6\underline{\omega_L}\sigma(y^2-1) - 2\underline{\omega_T}(2\sigma-3)(2+y^2)\right]\Big) \tag{4.30}$$

$\Rightarrow$ in a_μ integrand, tensor contributions suppressed in SD corner regions!

obeyed by hQCD tensor results, even though individual contributions $\sim 25\%$ of AV ones

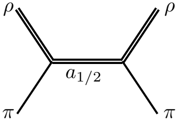
$\sum T_i \bar{\Pi}_i^{(a=3)}$ with fixed $\overline{Q} = 10$ GeV (as in 2411.09578, Fig.6)



$\leftarrow$ with holographic $\omega_{T,L}$

$Q/\overline{Q}$

Phases $\delta_J^{\lambda_{\text{RH}}\lambda_\rho}(s)$ for $\rho\pi$ rescattering

- in principle 5 different phases per J, but in practice not all independent
- first ansatz: extract from a_1/a_2 poles  ,

  take care of width, threshold and high-energy behaviour
- possible alternative: from 3π system, 3-body unitarity calculations Sadasivan et al. 2022